Soft X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

Chapter 2 Homework Problems

2.1

(a) Confirm Eq. (2.15) by showing that

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\mathbf{E}(\mathbf{r},t) = \left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\iint_{k\omega} \mathbf{E}_{k\omega}\, e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}\,\frac{d\omega d\,\mathbf{k}}{(2\pi)^4}$$

$$= \iint_{k\omega} \left[(-i\omega)^2 - c^2(i\,\mathbf{k})^2\right]\mathbf{E}_{k\omega}\, e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}\,\frac{d\omega d\,\mathbf{k}}{(2\pi)^4}$$

(b) Explain in a few sentences how Eq. (2.15) leads to the free-space (vacuum) dispersion relation $\omega = kc$, or equivalently $f\,\lambda = c$, in the absence of current sources.

2.2

(a) Confirm the expression for time averaged power per unit area, as given in Eq. (2.37), in terms of the electric and magnetic fields.

(b) Making assumptions for a plane wave in vacuum, as led to Eq. (2.31), express this as the wave intensity I (power per unit area, or magnitude of the cycle-averaged Poynting vector) in terms of the electric field. For assistance with the complex quantities consult Ref. 3 of this chapter.

2.3

(a) Using the expression for intensity I obtained in the previous example calculate the magnitude of the electric field at the focus of an 805 nm wavelength, 25 fs duration Ti:sapphire laser (see Chapter 6, Section 6.8) brought to a focal intensity of 1.6×10^{14} W/cm^2.

(b) Compare the result of (a) with the magnitude of the electric field an electron 'feels' inside an atom. For simplicity, take the hydrogen atom as example and assume the electron is at a distance of a_0 (Bohr radius) from the nucleus.

(c) What energy could a free electron gain if located in the focal region of the laser at the intensity calculated from part (a)? Assume the electron is in simple harmonic oscillation.

(d) What intensity would be required to bring a free electron to an equal energy as the ionization energy of a neutral Neon atom (from Ne to Ne^{+1})? Consult Table 6.2, p. 247.

2.4

(a) Give a physical interpretation of scattering cross-sections.

(b) While this might equal the geometric area for uncharged particles ("billiard balls"), why is this not necessarily the case for charged objects?

(c) Why would one expect this to vary widely for a bound electron in the vicinity of a resonant frequency?

2.5

Calculate the classical electron radius and Thomson scattering cross-section (free electron) as given in Eqs. (2.44) and (2.45), using physical constants given in Appendix A, p. 419. Calculate the scattering cross-section for a semi-classical bound electron for $\omega = 2\omega_s$, $\omega = 0.9\omega_s$, and $\omega = 0.5\omega_s$, in the approximation $\gamma/\omega_s \ll 1$. Express your answers in terms of the free electron cross-section σ_e.

2.6

(a) Calculate the scattering cross-section for a multi-electron atom in the approximation $f \rightarrow f_0$, using Eq. (2.76) and the data from Appendix C, pp. 428−436. Calculate the scattering cross-section of an iron atom ($Z = 26$) for photon energies of 100 eV, 300 eV, 700 eV, 1 keV, and 10 keV. Express your results in terms of the free electron (Thomson) cross-section. You can use the tabulated data on p. 432, or for a more complete data set consult the website http://www.cxro.lbl.gov/optical_constants.

(b) In what circumstances is the approximation $f \rightarrow f_0$ appropriate, i.e., for which of these photon energies and/or for what angular range?

(c) Repeat the calculations for a silicon atom ($Z = 14$) and a molybdenum atom ($Z = 42$) at the same photon energies.