Soft X-Rays and Extreme Ultraviolet Radiation: Principles and Applications

Chapter 9 Homework Problems

9.1 (a) Compute diffraction efficiencies to orders $m = 1$ to 5 for an opaque transmission grating with line to space ratios of 1:2, that is, where the opaque line widths are $d/3$ and the spaces are $2d/3$. (b) Compare this with the diffraction efficiencies of a symmetric (1:1) grating. (c) What does this suggest about the appearance of even order diffraction from gratings and zone plates?

9.2 Following the introductory text to Section 9.2, and the nomenclature in Figure 9.2, draw a right triangle containing the on-axis focal length f as one leg, the radial distance to the outer zone r_n as the second leg, and hypotenuse $f + \frac{n\lambda}{2}$. (a) Confirm, using the Pythagorean theorem, that zone plate radii are given by

$$r_n^2 = n\lambda f + \frac{n^2\lambda^2}{4}$$

(b) Show that for a lens of small numerical aperture, $NA \ll 1$, the $n^2\lambda^2/4$ term can be neglected, yielding the simpler expression for zone plate radii

$$r_n \simeq \sqrt{n\lambda f}$$

(c) Show that in this limit sequential zones (and thus all zones) have equal area (hint: form the difference $\pi r_n^2 - \pi r_{n-1}^2$). (d) Why is this important to the focusing properties of a Fresnel zone plate lens?

9.3 Give the specifications for a zone plate lens designed to achieve a spatial resolution of 25 nm with uniform spherical wave illumination ($R \to \infty$) at a wavelength of 3.1 nm. Assume the radiation source has a relative spectral bandwidth $\Delta\lambda/\lambda = 10^{-3}$. Design the lens to have the largest possible focal length. Give the outer zone width Δr, the number of zones, N, the diameter, focal length, numerical aperture, $F^{\#}$, and the depth of focus.

9.4 You are designing a soft x-ray microscope for imaging thick hydrated biological samples at high spatial resolution. Your goal is to achieve a spatial resolution ten times better than that of a scanning confocal visible light microscope operating at 486 nm wavelength with a 1.4 NA ($n \sin\theta$) oil immersion objective lens. You choose a soft x-ray wavelength that will provide the greatest contrast by absorption between the organic material and water. (a) What wavelength will you use? (b) What outer zone width will you require for the lens in a scanning microscope? (c) What will be the zone plate lens's numerical aperture (NA) and F number ($F^{\#}$)? (d) How might relative spectral bandwidth of the source affect performance? (f) Describe the proper illumination for the scanning microscope, and also for a full field microscope. How might the illumination characteristics affect achievable spatial resolution? (g) Are equal outer zone widths required for both microscopes to achieve the same spatial resolution?

9.5 (a) Discuss potential aberrations in a zone plate lens. (b) How would each affect imaging properties? (c) What measurements would reveal these aberrations? (d) How might each be avoided in fabrication of the zone plate or corrected in use?

9.6 For a scanning zone plate microscope what experimental features are important for achieving near-theoretical ("diffraction limited") performance?

9.7 You are designing a scanning photoemission microscope for studying surfaces containing iron, cobalt, and nickel. The studies will require observations of the respective L-edge structure with a relative spectral resolution of 3×10^{-4}, and a spatial resolution of 80 nm. You wish to have a long focal length zone plate lens so as to provide the greatest working distance (sample to lens separation) and thus least complexity for photoelectron collection. (a) What specifications will you set for the zone plate lens? Give all parameters. (b) What type of experiments could be performed with this microscope?

9.8 You are designing the pinhole illumination for a scanning zone plate microscope. To obtain near diffraction limited resolution you choose a small pinhole so as to overfill the zone plate. If your choice of wavelength and pinhole diameter lead to an Airy pattern with first null diameter equal to twice the zone plate diameter, (a) what will be the intensity variation across the zone plate, and (b) what fraction of the radiation through the pinhole would be captured by the lens? (c) How does this intensity variation affect microscope performance? Express your answer as the ratio of focal spot FWHM to that of the ideal Airy pattern obtained with uniform illumination of the same lens. (d) Extend your analyses to situations where the illuminating Airy pattern null diameter ranges from one to four times the zone plate diameter. Graph intensity variation (min/max), ratio of focal spot diameters (FWHM), and the fractional radiation collection by the lens, as a function of the illumination parameter (Airy null to zone plate diameter).

9.9 (a) Determine the efficiency to first order of 100 nm thick gold, nickel, and germanium phase zone plates, at a photon energy of 500 eV, using the Kirz formula [J. Kirz, *JOSA 64*, 301 (1974)]

$$\eta_m = \frac{1}{m^2\pi^2}\left(1 + e^{-2\phi\beta/\delta} - 2e^{-\phi\beta/\delta}\cos\phi\right)$$

where η_m is the efficiency to odd order m, $m = \pm 1, \pm 3, \ldots$ for the above formula, δ and β are the real and imaginary components of refractive index at wavelength λ, and $\phi = 2\pi t\delta/\lambda$ is the phase shift for material zones of thickness t. (b) What are the efficiencies for these three materials at a photon energy of 800 eV? For δ and β values of nickel and gold consult Appendix C. For germanium and other materials of interest to you, consult the website [http://www-cxro.lbl.gov].

9.10 What are the potential advantages and disadvantages of soft x-ray microscopes with respect to other techniques such as SEM (scanning electron microscope), TEM (transmission electron microscope), visible light microscopes (conventional high NA, phase contrast, scanning confocal, near-field), and hard x-ray fluorescent microprobes?