# 1 Fuzzy Logic

## 1.1 Introduction

First proposed at 1965 and based on the concept of fuzzy sets, fuzzy set theory provides means for representing uncertainty. Probability theory is the primary tool for analyzing uncertainty and assumes that the uncertainty is a random process. However, uncertainty is not always random though and fuzzy set theory is used to model the kind of uncertainty associated with imprecision, and lack of information.

Conventional set theory distinguishes between those elements that are members of a set and those are not, there being very, clear or crisp boundaries. Figure 1 shows the crisp set "medium temperature". Temperatures between 20 and 30 C lie within crisp set, and have a membership value of one.

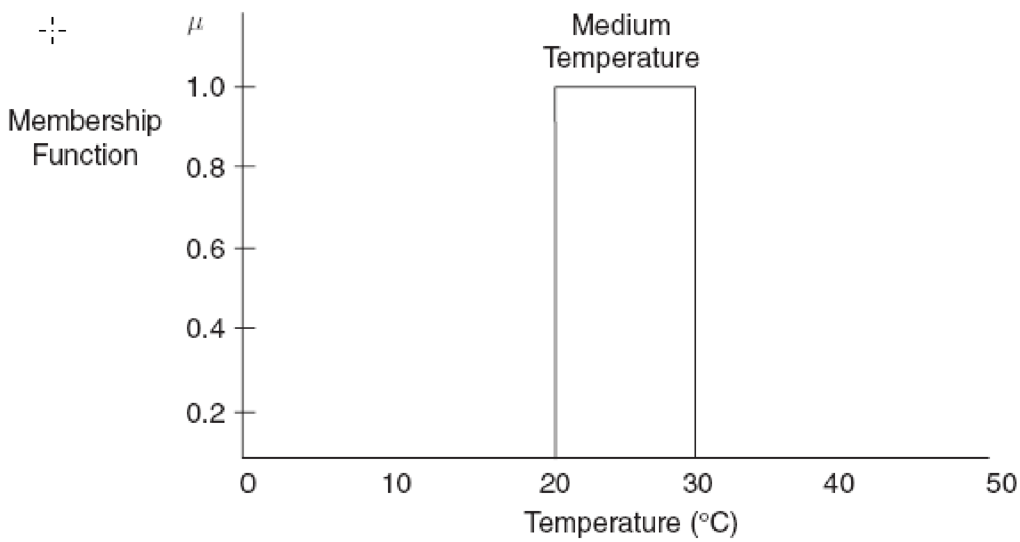


**Figure 1. Crisp Set**

The central concept of fuzzy set theory is that the membership function μ , like probability theory, can have value of between 0 and 1. In Figure 2, the membership function μ has linear relationship with the x-axis, called the universe of discourse U. This produces a triangular shaped fuzzy set.

Fuzzy set represented by symmetrical triangles are commonly used because they give good results and computation is simple. Other arrangement includes non-symmetrical triangles, trapezoids, and Gaussian.

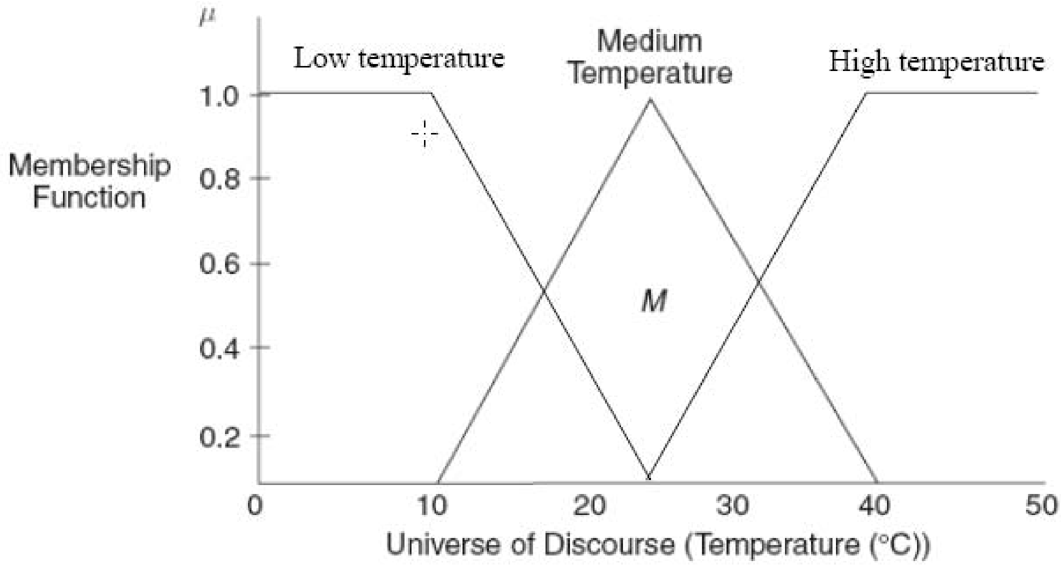


**Figure 2. Fuzzy set**

Let the fuzzy set "medium temperature" be called fuzzy set M. If an element u of the universe of discourse U lies within fuzzy set M, it will have a value of between 0 and 1. This is expressed as

$$\mu_M \in [0,1]$$

When the universe of discourse is discrete and finite, fuzzy set M may be expressed as

$$M = \sum_{i=1}^{N} \frac{\mu_M(u_i)}{u_i}$$

## 1.2 Operations on Fuzzy Sets

Let A and B be two fuzzy sets within a universe of discourse *U* with membership function $\mu_A \in [0,1]$ and $\mu_B \in [0,1]$ respectively. The following fuzzy set operations can be defined

*Equality*: Two fuzzy sets A and B are equal if they have the same membership function within a universe of discourse *U*

$$\mu_A(u) = \mu_B(u), \forall\, u \in U$$

*Union*: The union of two fuzzy sets A and B corresponds to Boolean OR function and is given by

$$\mu_{A \cup B}(u) = \max[\mu_A(u), \mu_B(u)], \forall\, u \in U$$

*Intersection*: The intersection of two fuzzy sets A and B corresponds to the BooleanAND function and is given by

$$\mu_{A \cap B}(u) = \min[\mu_A(u), \mu_B(u)], \forall\, u \in U$$

*Complement*: The complement of fuzzy set A corresponds to the Boolean NOT function and is given by

$$\mu_{-A}(u) = 1 - \mu_A(u), \forall\, u \in U$$

## 1.3 Fuzzy relations

An important aspect of fuzzy logic is the ability to relate sets with different universes of discourse. Consider the relationship

IF L THEN M

L is known as the *antecedent* and M as the *consequent*. The relationship is denoted by

$$A = L \times M$$

And is defined as

$$L \times M = \begin{bmatrix} \min\{\mu_L(u_1), \mu_M(v_1)\} & \dots & \min\{\mu_L(u_1), \mu_M(v_k)\} \\ \vdots & \ddots & \vdots \\ \min\{\mu_L(u_j), \mu_M(v_1)\} & \dots & \min\{\mu_L(u_j), \mu_M(v_k)\} \end{bmatrix}$$

Where $u_i, i = 1: j$ and $v_i, i = 1: k$ are the discrete universe of discourse.

## 1.4 Control using Fuzzy Logic

The structure of a Fuzzy Logic Controller (FLC) system is shown in Figure 3.

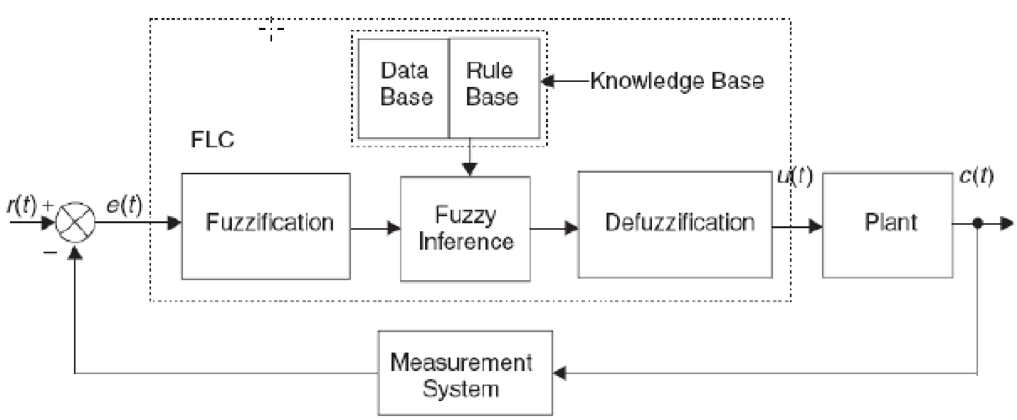


**Figure 3. Fuzzy Logic Controller**

### *1.4.1* Fuzzification

Fuzzification is the process of mapping input to the FLC into fuzzy membership values in the various input universes discourse. Decisions need to be made regarding

1. Number of inputs
2. Size of universes of discourse
3. Number and shape of fuzzy sets

### *1.4.2* **Fuzzy *rule base***

The fuzzy rule base consists of a set of antecedent-consequent linguistic rules of the form

IF *e* is PS AND *ce* is NS THEN *u* is PS

This style of fuzzy conditional statement if often called a 'Mamdani' type rule.

### *1.4.3* **Fuzzy *inference***

Consist of translating all expressions to fuzzy language notation.

### *1.4.4* **Defuzzification**

Defuzzification is the procedure for mapping from a set of inferred fuzzy control signals contained within a fuzzy output window to a non-fuzzy control signal. The center of area method is the most well known defuzzification technique, for discrete system can be expressed as

$$u(kT) = \frac{\sum_{i=1}^{N} u_i \mu_M (u_i)}{\sum_{i=1}^{N} \mu_M (u_i)}$$