NOZZLES

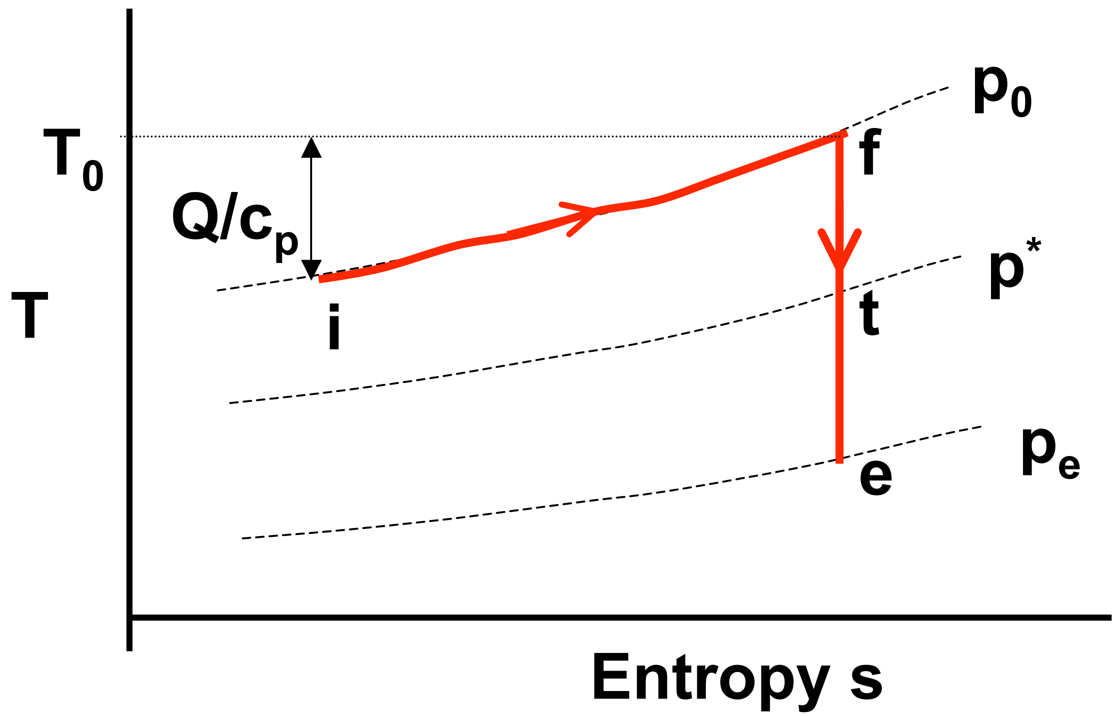
The function of the rocket nozzle is to convert the thermal energy in the propellant into kinetic energy as efficiently as possible, in order to obtain high exhaust velocity along the desired direction.

The mass of a rocket nozzle is a large part of the engine mass. Many of the failures encountered in rocket engines are also traceable to failures of the nozzle – historical data suggest that 50% of solid rocket failures stemmed from nozzle problems.

The design of the nozzle must trade off:

1. Nozzle size (needed to get better performance) against nozzle weight penalty.
2. Complexity of the shape for shock-free performance vs. cost of fabrication

T-s Diagram (or h-s Diagram) for a nozzle: Ideal



i: reactants at chamber pressure p_0

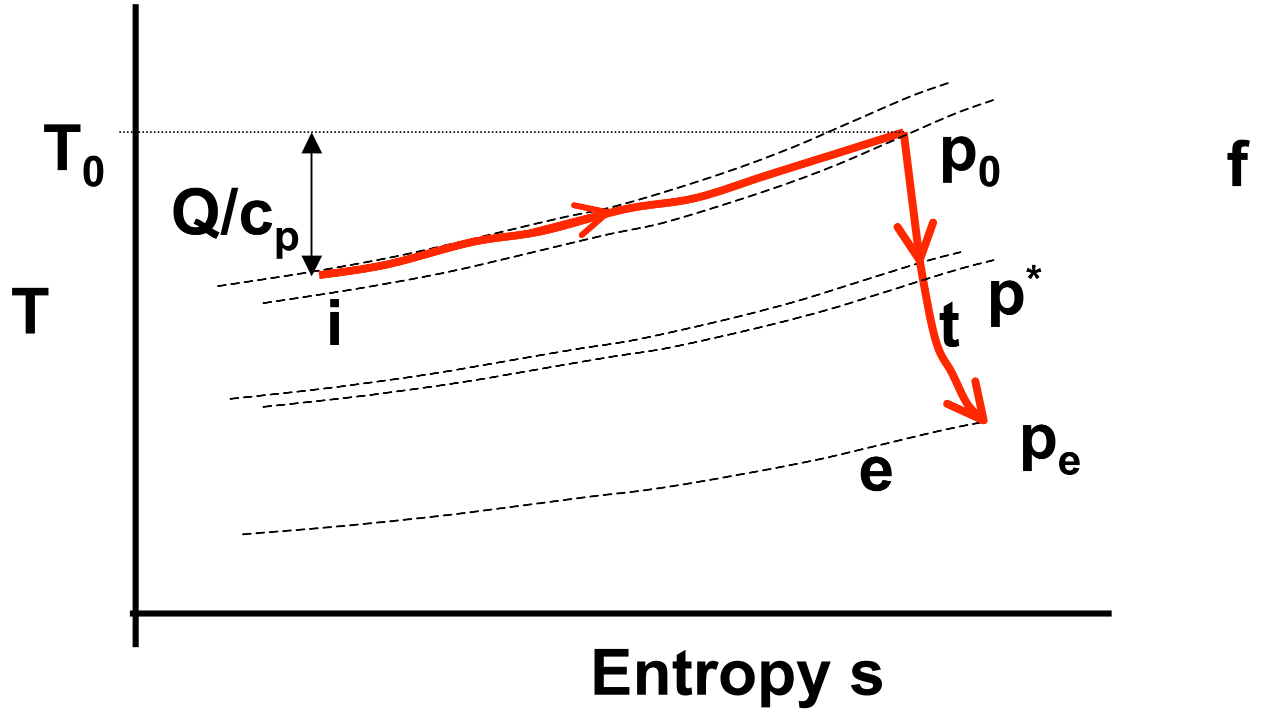
f: final mixture at combustion chamber stagnation conditions p_0 ,T_0

t: throat conditions. Mach 1, p^*, T^*

e: exit conditions p_e ,T_e (assuming fully expanded)

T-s Diagram (or h-s Diagram) for a nozzle: with losses

Losses show up as increases in entropy for each step – usually accompanied by a loss of pressure. At the exhaust, note that exhaust temperature is higher than ideal when the final pressure is reached.



i: reactants at chamber pressure p_{0i}

f: final mixture at combustion chamber stagnation conditions p_0 ,T_0

t: throat conditions. Mach 1, p^*, T^*

e: exit conditions p_e ,T_e (assuming fully expanded).

Recall – Expressions for Thrust Coefficient - 1

$$C_F = \frac{F}{A_t / p_0}$$

where A_t is nozzle throat area and p_0 is chamber pressure (N/m^2)

Thus, $u_t = \sqrt{\gamma R T_t}$

For sonic conditions at the throat, $\rho_t = \rho_0 \left(\frac{2}{\gamma + 1} \right)^{1/(\gamma - 1)}$

and

$$F = A_t p_0 \gamma R \left\{ \frac{2 T_t T_0}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right)^{1/(\gamma - 1)} \left[1 - \left(\frac{p_e}{p_0} \right)^{(\gamma - 1)/\gamma} \right] \right\}^{1/2} + (p_e - p_0) A_e$$

Thrust Coefficient - 2

Using isentropic flow relations,

$$T_t T_0 = \left(\frac{p_0}{\rho_t R}\right)\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}$$

$$\frac{1}{\rho_t^{\,2}}\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma+1}} = \frac{1}{\rho_0^{\,2}}$$

$$F = A_t p_0 \gamma \left\{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_0}\right)\right]^{\frac{\gamma-1}{\gamma}}\right\}^{1/2} + (p_e - p_a)A_e$$

and Thrust Coefficient

$$C_F = \left\{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[1-\left(\frac{p_e}{p_0}\right)\right]^{\frac{\gamma-1}{\gamma}}\right\}^{1/2} + \frac{(p_e - p_a)A_e}{p_0 A_t}$$

Depends entirely on nozzle characteristics. The thrust coefficient is used to evaluate nozzle performance.

Characteristic Velocity c*

Used to characterize the performance of propellants and combustion chambers independent of the nozzle characteristics.

$$\dot{m} = \frac{p_0 A_t}{\sqrt{\gamma R T_0}} \left\{ \gamma \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right\} = \frac{p_0 A_t}{\sqrt{\gamma R T_0}} \Gamma$$

where Γ **is the quantity in brackets. Note:** $a_0 = \sqrt{\gamma R T_0}$

So $\dot{m} = \frac{p_0 A_t}{a_0} \Gamma$

Characteristic velocity $c^* = \frac{p_0 A_t}{\dot{m}}$

Assuming steady, quasi-1-dimensional, perfect gas. The condition for maximum thrust is ideal expansion: nozzle exit static pressure being equal to the outside pressure. In other words,

$$p_e = p_a$$

Nozzle Types

The subsonic portion of the nozzle is quite insensitive to shape – the subsonic portion of the acceleration remains isentropic. The divergent nozzle is where the decisions come into play.

Conical Nozzle

- **Easier to manufacture – for small thrusters**
- **Divergence losses: Exit velocity is not all in the desired direction.**

Bell Nozzle

- **Complex shape**
- **Highest efficiency (near axial flow at exhaust)**
- **Large base drag during atmospheric flight after burnout**

Plug Nozzle or Aerospike Nozzle (linear or annular) (X-33, VentureStar, 1960s concept)

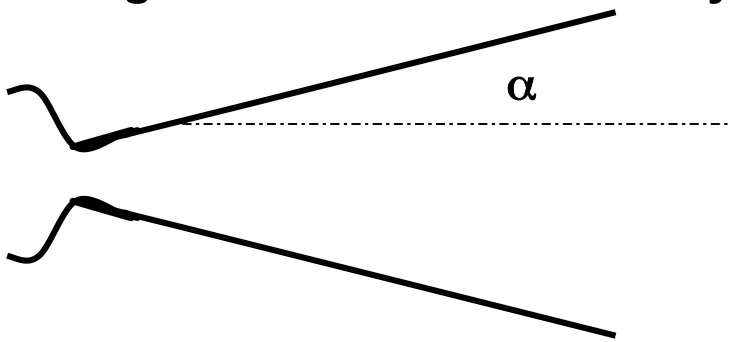
- **Altitude compensating (see Chang 3-30c)**

Expansion- Deflection Nozzle (E-D)

- **Shortest nozzle of the “enclosed” types.**

Conical Nozzle

- Easier to manufacture – for small thrusters
- Divergence losses: Exit velocity is not all in the desired direction.



$$\lambda = \frac{1}{2}(1 + Cos\alpha)$$

$$98.3\% @ \alpha \approx 15^0$$

Note: α can be as large as 12 to 18 degrees.

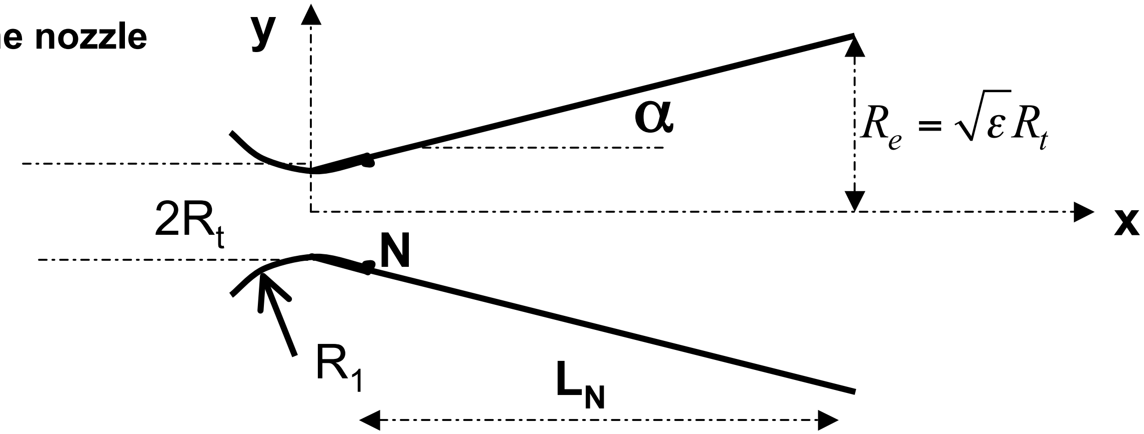
$$C_F = \lambda \left\{ \frac{2\gamma^2}{\gamma - 1} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{p_e}{p_0} \right) \right]^{\frac{\gamma-1}{\gamma}} \right\}^{1/2} + \varepsilon \frac{(p_e - p_a)}{p_0}$$

Here λ is the "thrust efficiency", defined as ratio of actual to ideal thrust , accounting for flow divergence.

ε is the nozzle Area Ratio (ratio of exit area to throat area).

Geometric Representations of Nozzles

Cone nozzle



R_t **Throat internal radius**

R_1 **Radius of curvature of the nozzle contraction**

N **Transition point from circular contour to conical contour. Located along at angle a downstream of throat.**

$$X_N = R_1 Sin\alpha$$

$$YN = R_t + R_1(1 - Cos\alpha)$$

L_N **Nozzle Length:**

$$L_N = \frac{R_t\left(\sqrt{\varepsilon} - 1\right) + R_1\left[\frac{1}{Cos\alpha} - 1\right]}{Tan\alpha}$$

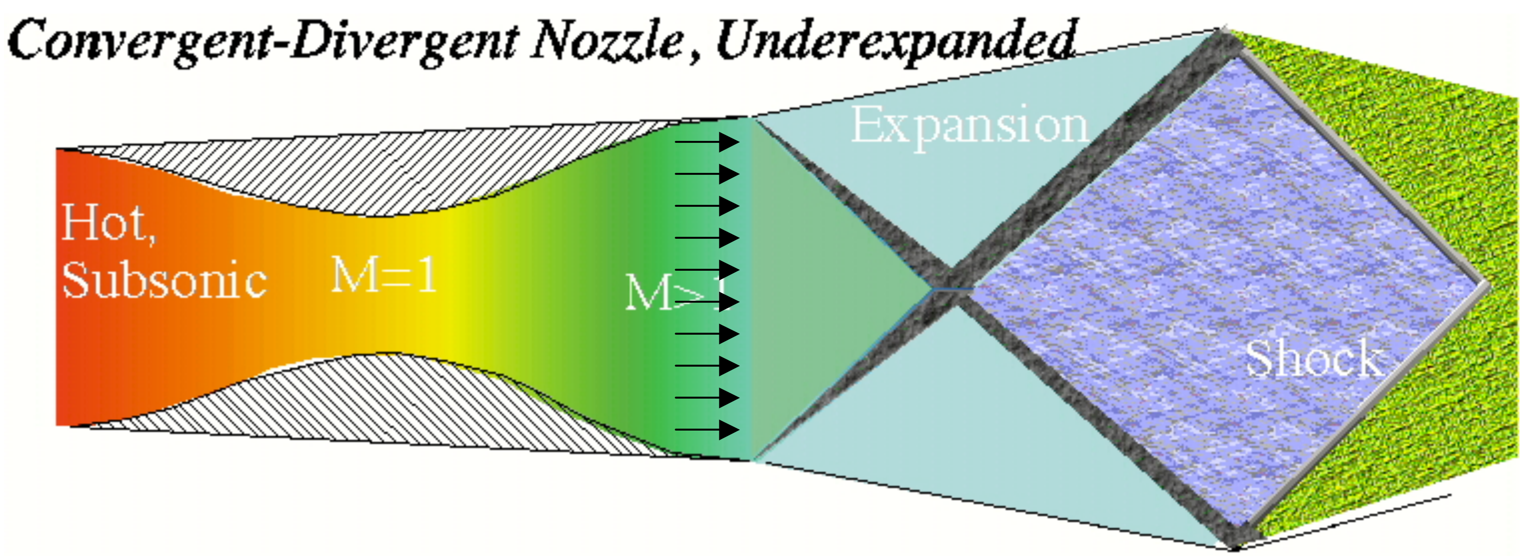
R_e **Exit radius**

For a conical nozzle, $R_1 \approx 1.5 R_t$ **is a typical choice.**

Bell Nozzle

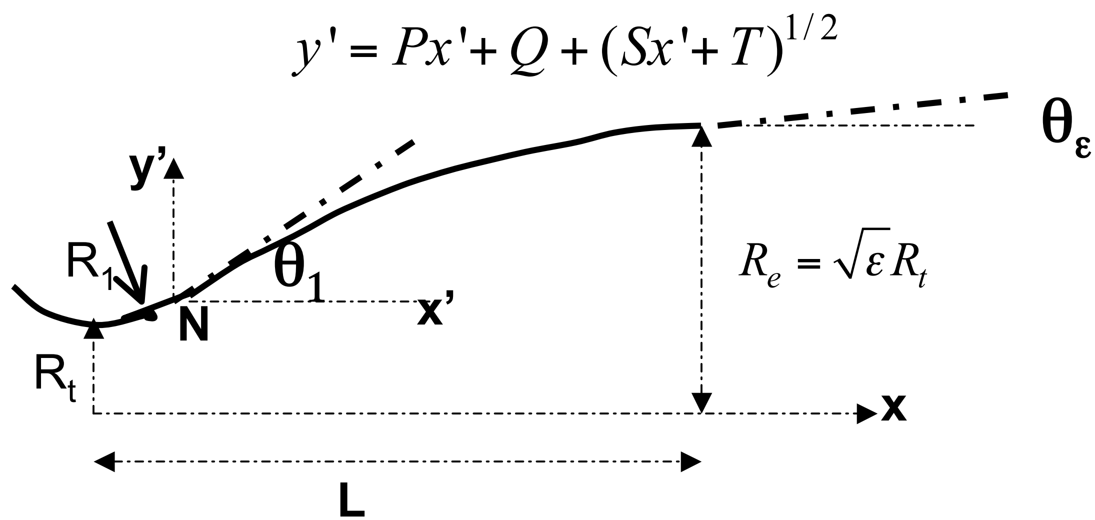
- **Complex shape**
- **Highest efficiency (near axial flow at exhaust)**
- **Large Base Drag during atmospheric flight after burnout**

A true bell nozzle is contoured to minimize the turning (compression) shock losses at the wall as the flow expands, but still turning the flow towards an axial exhaust. Note that the flow may still be under-, over- or fully-expanded at the exit, and hence shocks / expansions may exist downstream.



Bell Nozzle

An approximate shape can be formed from a parabola (after G.V.R. Rao)



$R_1 = 1.5 R_t$ **Upstream of the throat**

$R_1 = 0.382 R_t$ **Downstream of the throat**

Parabolic Bell Nozzle Contour, cont'd (Rao, 1958)

There are 4 unknowns in the rotated parabolic segment equation (P,Q,S,T) and 4 boundary conditions

1. At N: $X'_N = 0; Y'_N = 0$

2. At e: $X'_e = X_e - X_N; Y'_e = Y_e - Y_N$

Or, $X'_e = X_e - X_N = L - X_N;$

$Y'_e = Y_e - Y_N = \sqrt{\varepsilon} R_t - Y_N$

3. At N: θ_N **is given (Rao, 1958, plots)**

2. At e: θ_e **is given (Rao, 1958, plots)**

So,

1. $Y'_N = PX'_N + Q + (SX'_N + T)^{1/2}$

and hence $T = Q^2$ ……………………(A)

2. $Y'_e = PX'_e + Q + (SX'_e + T)^{1/2}$

$(SX'_e + T)^{1/2} = Y'_e - PX'_e - Q$

and

3. $Tan\theta = \left(\frac{dy'}{dx'}\right)_N = P + \frac{S}{2\left(SX'_N + T\right)^{1/2}}$ ………..(B)

(from (A))

$$Tan\theta_N - P = \frac{S}{2Q}$$

$$Q = \frac{S}{2\left(Tan\theta_N - P\right)}$$ ……………………………. (C)

4. $Tan\theta_e = P + \frac{S}{2(SX'_e + T)^{1/2}}$

$$(SX'_e + T)^{1/2} = \frac{S}{2(Tan\theta_e - P)}$$ …………… (D)

Equating (B) = (D) leads after some manipulation, to

$$S = \frac{2(Tan\theta_e - P)(Tan\theta_N - P)(Y'_e + PX'_e)}{Tan\theta_N - Tan\theta_e} \qquad \text{............... (E)}$$

Also, squaring (B)

$$SX'_e + T = \left[Y'_e - PX'_e - Q\right]^2$$

Substituting for Q from (C),

$$S = \frac{(Y'_e - PX'_e)^2 (Tan\theta_N - P)}{X'_e Tan\theta_N - Y'_e} \qquad \text{............... (F)}$$

Eventually gives

$$P = \frac{Y'_e Tan\theta_N + Y'_e Tan\theta_e - 2X'_e Tan\theta_e Tan\theta_N}{2Y'_e - X'_e Tan\theta_N - X'_e Tan\theta_e} \qquad \text{...... (G)}$$

Down to one unknown.

Use eqn. (G) to find $P = P(X'_e, Y'_e, \theta_N, \theta_e)$

Then either (F) or (E) to get S

Then use (C) to get Q

Then use (A) to get T in terms of the original X,Y axes.

$$Y = Y_N + P(X - X_N) + Q + [S(X - X_N) + T]^{\frac{1}{2}}$$

Recall: $Y'_e = Y_e - Y_N = \sqrt{\varepsilon} R_t - [R_t + R_1(1 - Cos\theta_N)]$

and $X'_e = X_e - X_N = L - R_1 Sin\theta_N$

where L is generally a fraction of that for a 15-degree half-angle cone with the same R_1 (i.e., $0.382R_t$)

$$L = f\left[\frac{R_t(\sqrt{\varepsilon} - 1) + R_1\left[\frac{1}{\cos 15^0} - 1\right]}{\tan 15^0}\right]$$

f = 100%; 90% or 80% of a 15-degree cone.

Comparison of bell and cone nozzles

For the same ε, we would expect $\lambda_{bell} > \lambda_{cone}$

A bell nozzle, while more complex to build, will generally yield a more efficient exhaust than a cone in a shorter nozzle length.

Same nozzle efficiency factor can be reached with about 70% of the length of a cone nozzle.

Alternatively, efficiency factor can be increased from about 98% for a cone to about 99.2% for a bell of the same length

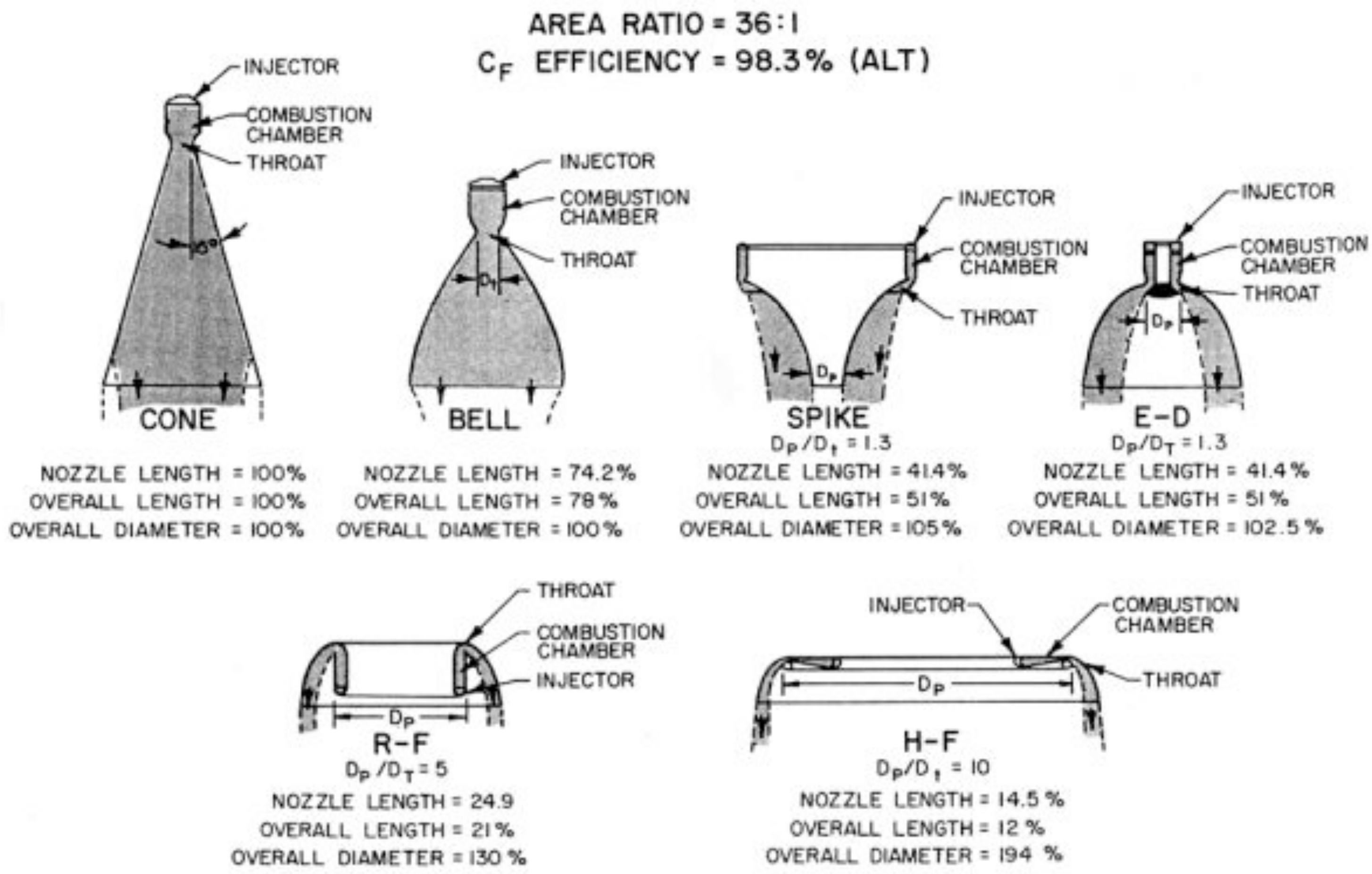
Types of Nozzles

In this lecture we will continue the discussion on nozzles:

- Types of nozzles
- Expansion Ratio criteria
- Nozzle heat transfer and materials issues

Comparison of Optimal Nozzles (source: Huzel & Huang, 1967)

E-D: Expansion-Deflection **Spike: "Plug"** **R-F: Reverse Flow** **H-F: Horizontal Flow**



http://www.aerospaceweb.org/design/aerospike/shapes.shtml

Annular Nozzles

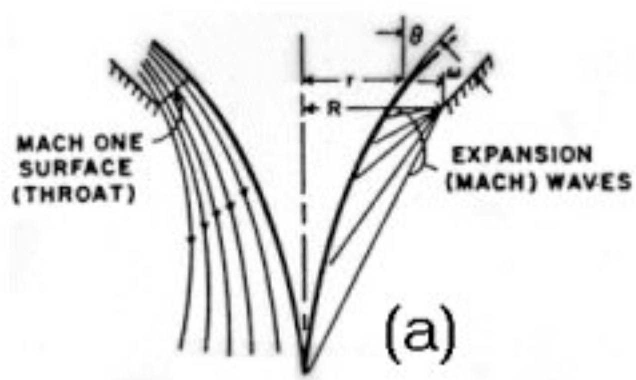
Expansion-Deflection Nozzle and Plug Nozzle

•Substantially shorter than conical or bell nozzles for the same thrust and area ratio.
•Design-point performance is nearly as good.
•Off-design performance is better under conditions where conventional nozzles would be over-expanded.

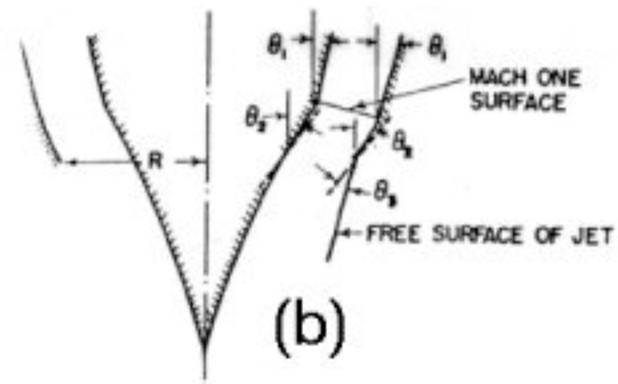
•Common feature: Free shear layer bounding nozzle flow.

Spike Nozzle Flow Geometries

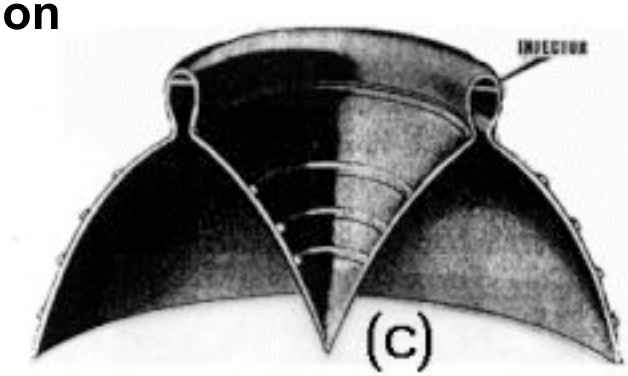
[from Berman and Crimp, 1961]



External expansion



Internal-external expansion

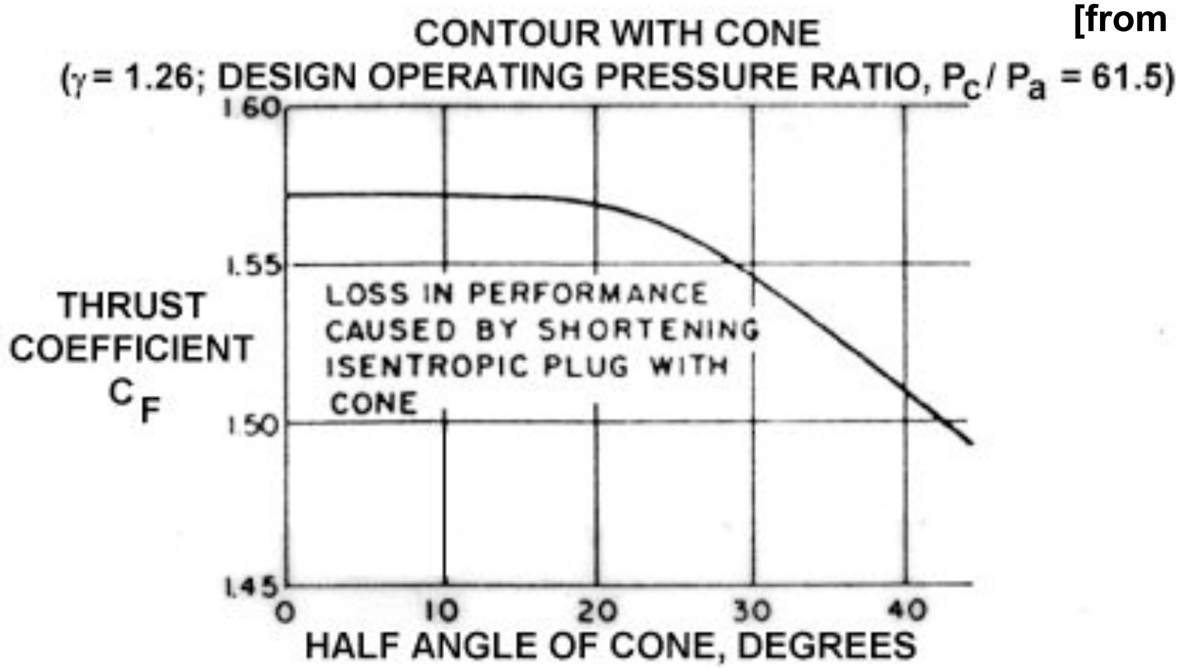


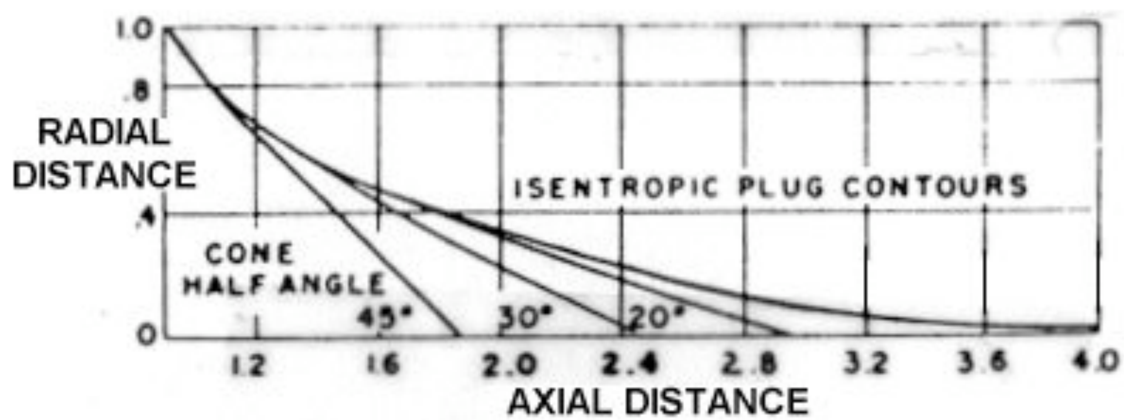
Internal expansion

http://www.aerospaceweb.org/design/aerospike/inflow.shtml

Effect of Replacing Spike with Cone

[from Berman and Crimp, 1961]



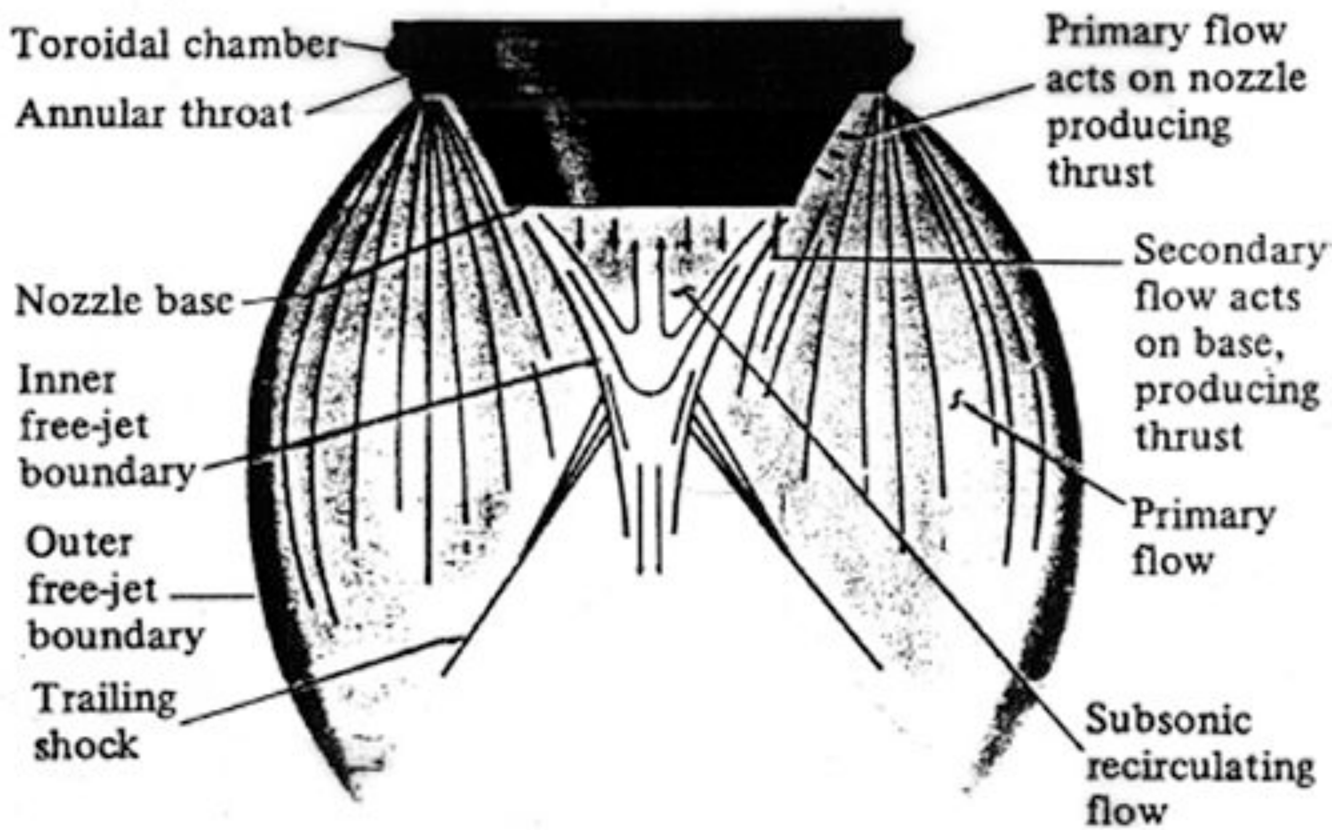


http://www.aerospaceweb.org/design/aerospike/inflow.shtml

Aerospike

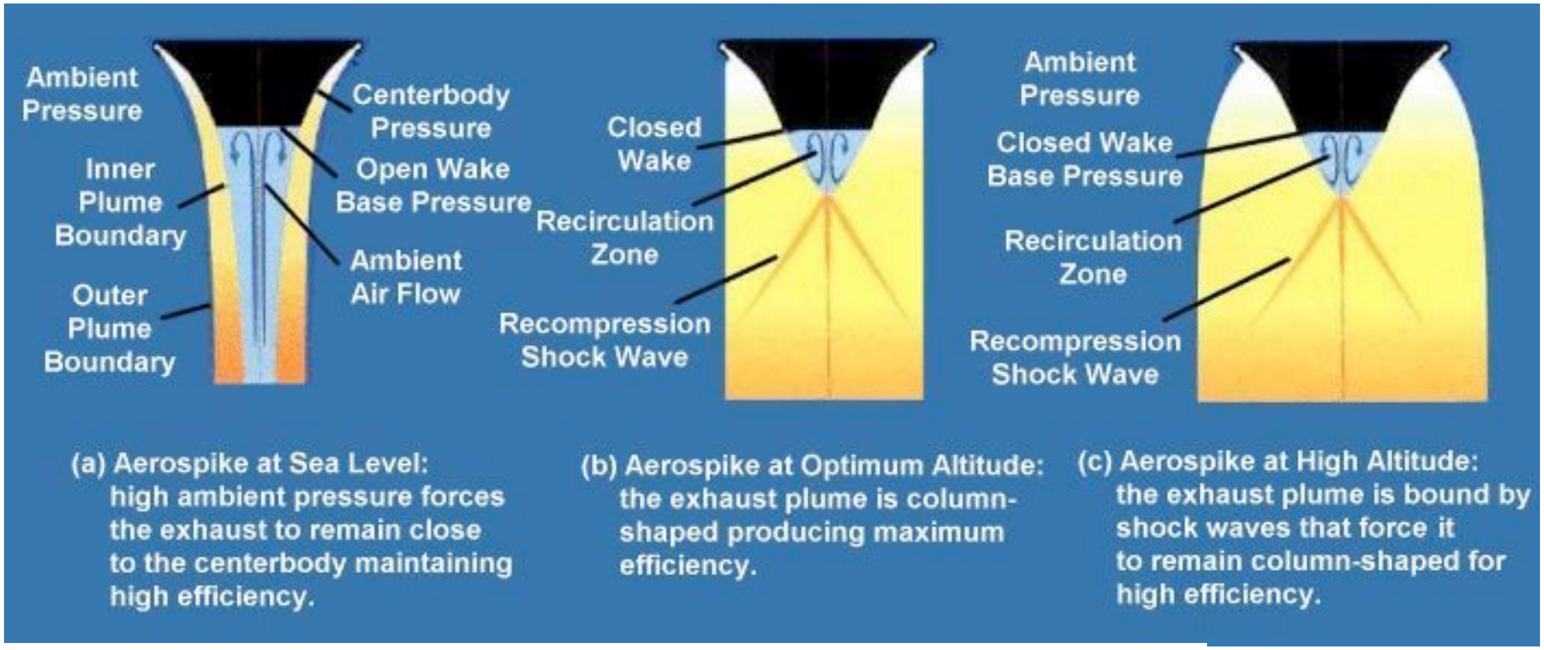
Source: Hill & Peterson, p. 540

Truncates full spike and adds secondary base flow to help contour the inner flow.

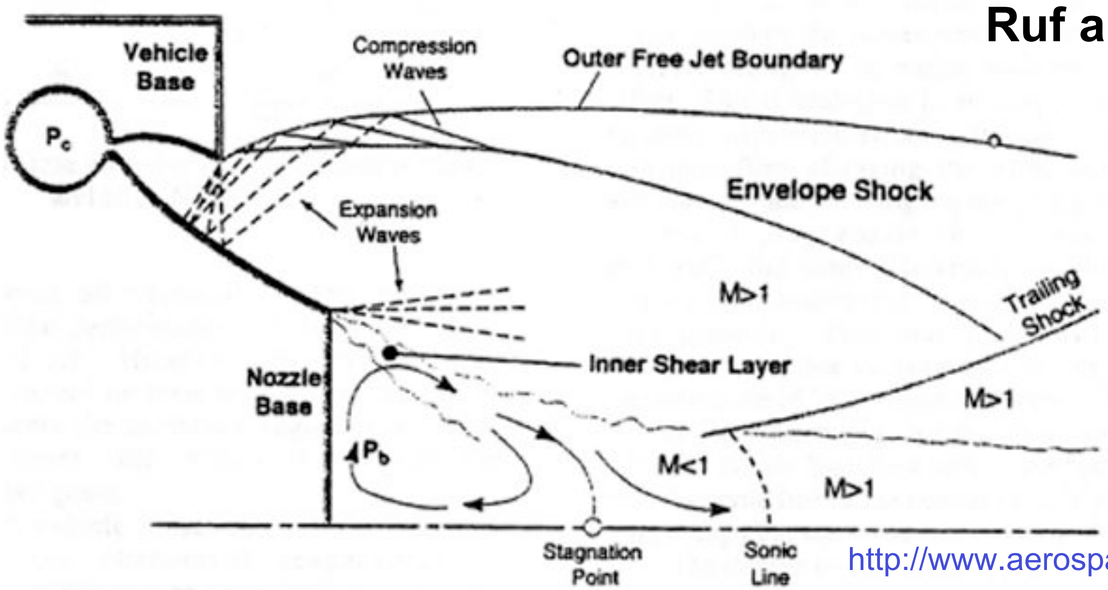


http://www.aerospaceweb.org/design/aerospike/aerospike.shtml

Aerospike Flow Features



Rocketdyne RS2200. Flynn, 1996

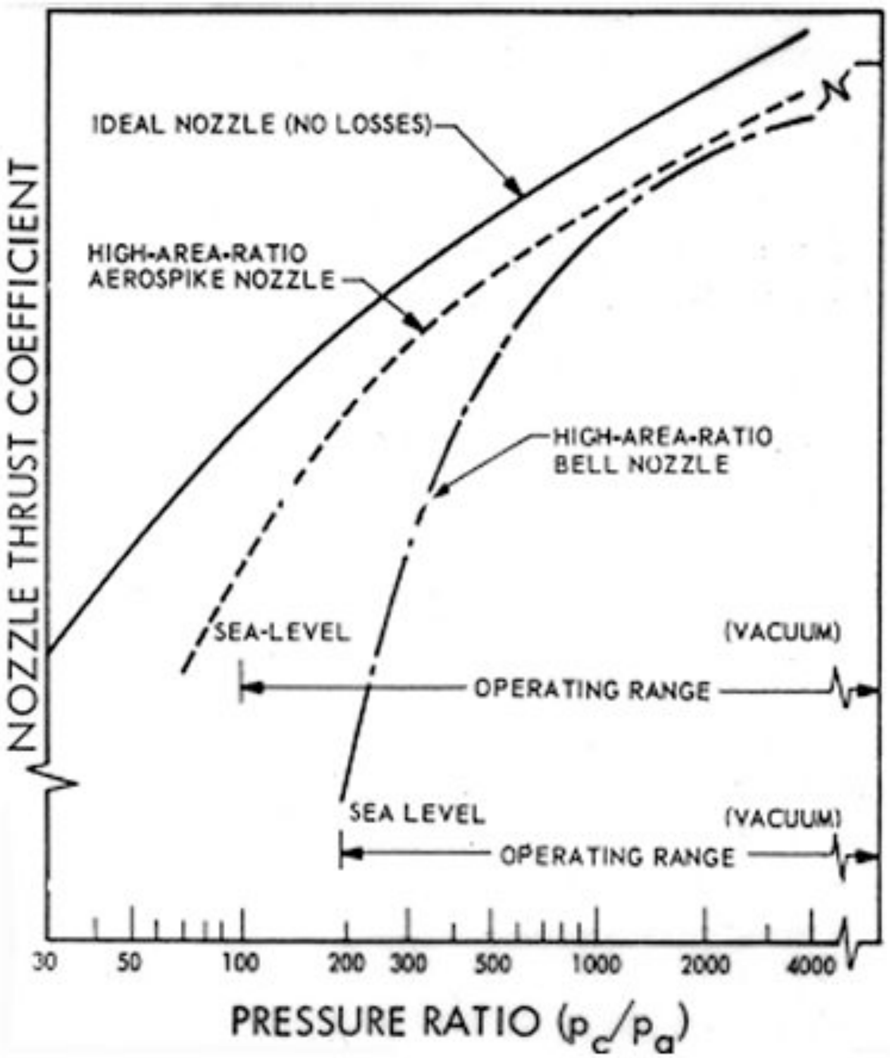


Ruf and McConnaughey, 1997

Thrust = $F_{thruster} + F_{centerbody} + F_{base}$

http://www.aerospaceweb.org/design/aerospike/aerodynamics.shtml

Theoretical Performance Comparison



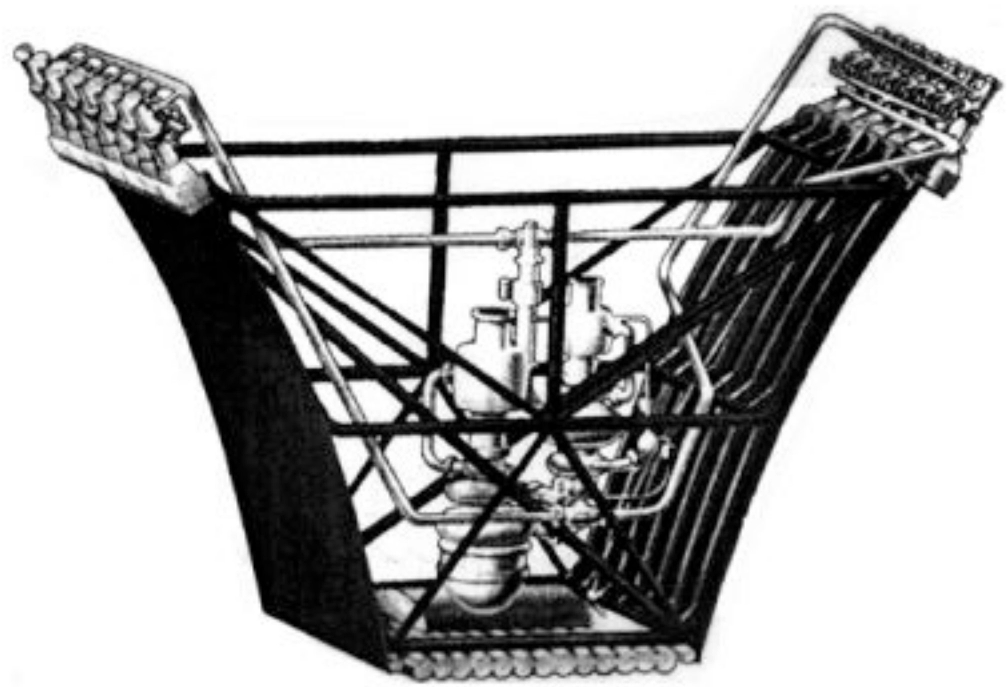
[from Huzel and Huang, 1967]

Note: The aerospike is susbtantially better at low altitudes where the bell and cone are likely to be overexpanded.

http://www.aerospaceweb.org/design/aerospike/compensation.shtml

Linear Aerospike

Combustion chamber modules placed along the inside.



Advantages and Disadvantages

Advantages

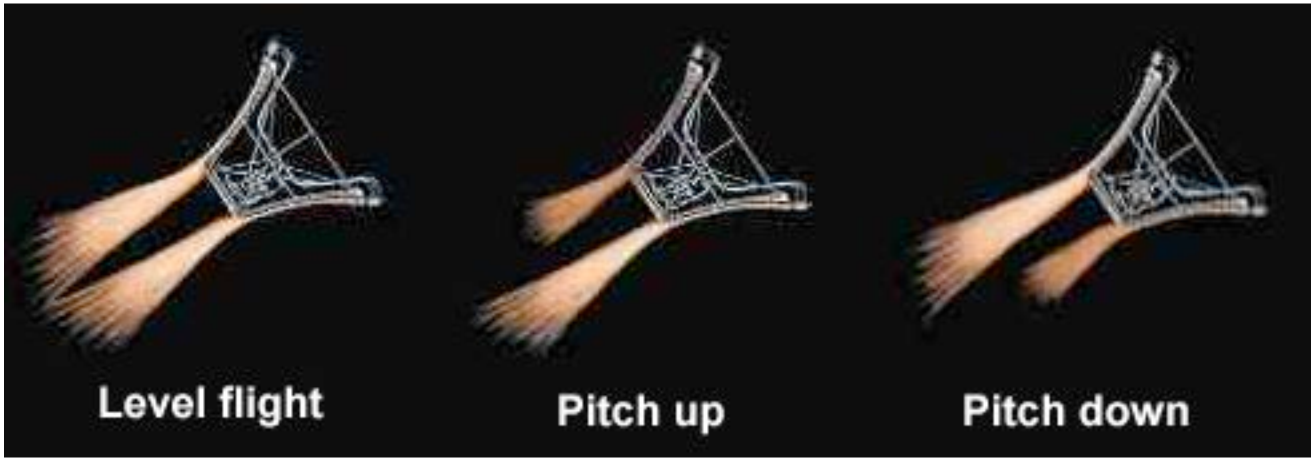
- **Much smaller size**
- **Altitude compensation**
- **Lower chamber pressure / higher expansion ratio: safer**
- **Lower vehicle drag**
- **Lower vehicle weight**
- **Modular: lower manufacture and repair costs; lower downtime**
- **Thrust Vectoring: mass flow through individual elements can be controlled.**

Disadvantages

- **Heating.**
- **More complex geometry**
- **??? (not much experience to-date)**

http://www.aerospaceweb.org/design/aerospike/x33.shtml

Rocketdyne, '99



METHOD OF CHARACTERISTICS

The name comes from a method used to solve hyperbolic partial differential equations: Find "characteristic lines" (combinations of the independent variables) along which the partial differential equation reduces to a set of ordinary differential equations, or even, in some cases, to algebraic equations which are easier to solve.

In aerodynamics, the method is easily related to physical features of the flow.

1. Introduction using physical arguments.

In supersonic flow, small disturbances cannot propagate upstream. (Why SMALL? Because large disturbances such as detonations travel at the speed of sound appropriate to their internal conditions, which may be a lot faster than the ambient speed of sound. If a missile explodes behind a fighter flying at Mach 2, chances are that it is all over for the fighter.)

"Mach cones" or "Mach Waves", are inclined at the "Mach angle" μ to the flow direction. So, Disturbance at a point along a streamline in a supersonic flow is first felt along a Mach wave originating from that point on the streamline.

The flow upstream of the Mach wave is "undisturbed" - properties unchanged. Thus, changes in properties occur across Mach waves. Each Mach wave can be considered to be the dividing line between regions of slightly different properties.

In a 2-dimensional flow, we can think of Mach waves going out on both sides of the streamline. We might call these the "left-running Mach wave" and the "right-running Mach wave".

Mach waves are "characteristic directions".

ON the Mach wave, properties are “indeterminate”. A "determinant goes to zero" to keep the solution from blowing up: suddenly a new, simpler equation is available...

MOC calculation:

We step short, finite distances along streamlines, and assume that properties remain constant unless we cross a Mach wave emanating from somewhere.

Each time there is such a change, new Mach waves are generated.
We determine properties at a new point along each streamline by seeing what Mach waves intersect there (where did those Mach waves come from, and hence, what were the properties there?)

We march downstream.

Detailed derivations:

1. Show that the Mach waves are indeed characteristic directions of the partial differential equation describing the flow (in this case, the linearized potential equation for 2-D supersonic flow).
2. Obtain relations between pressure and velocity along those lines, and use them to solve simultaneous algebraic equations to get conditions at each point.

Results:

$$d\theta \mp \sqrt{M^2 - 1}\,\frac{dV}{V} = 0$$

Integrating, we get:

$$\theta \mp \int \sqrt{M^2 - 1}\,\frac{dV}{V} = const$$

The integral may be familiar. You encountered it (many moons ago) in AE 3004 under the section on Prandtl-Meyer expansion fans In other words, the integral is simply the Prandtl-Meyer function n(M).

$$\theta \mp \nu = const$$

The above equation is called the compatibility relationship, and must be satisfied by the flow angle and the Mach number at every point on the two families of characteristic lines derived earlier. Specifically,

Along the line, called the C- characteristic

$$\frac{dy}{dx} = \tan(\theta - \mu)$$

$$\theta + \nu = const$$

Along the line, called the C+ characteristic

$$\frac{dy}{dx} = \tan(\theta + \mu)$$

$$\theta - \nu = const$$

Some questions

1. **Which is a wing? Which is an airfoil?**
2. **What is the difference between a Supercritical Airfoil and a Supersonic Airfoil**
3. **What is the difference between a Supersonic leading edge and a Supersonic Airfoil leading edge?**

4. What method would you use to analyze:

a. **(incomp. flow, airfoil)**
b. **(incomp. flow, high AR wing)**
c. **(incomp. flow, low AR wing)**
d. **(subsonic flow, high-AR wing)**
e. **(subsonic flow, low-AR wing)**
f. **(supercritical flow, airfoil)**
g. **(supersonic flow, airfoil)**
h. **(supersonic flow, wing with subsonic le)**
i. **(supersonic flow, wing with supersonic le)**
j. **(incomp. flow, airfoil, lift and pitching moment)**
k. **(incomp. flow, airfoil, lift, pitching moment, drag)**
l. **(subsonic flow, airfoil, lift, pitching moment)**
m. **(incomp flow, wing, lift, pitching moment, induced drag)**
n. **(incomp flow, wing, lift, pitching moment, total drag)**