Section 2-1—Rational Numbers on a Number Line

Background:

1. Natural numbers are {1, 2, 3, . . . }

2. Whole numbers are {0, 1, 2, 3, . . .}
   - The whole numbers are the same as natural numbers, but they include zero.

3. Integers are {. . ., -3, -2, -1, 0, 1, 2, 3, . . .}
   - The integers are the whole numbers and their opposites.

4. Negative numbers are separated from the positive numbers by 0.

5. Infinite—(∞), the set of numbers never ends

6. Positive Numbers—greater than zero

7. Negative Numbers—less than zero

8. A rational number is any number that can be written as
   - a fraction (the denominator cannot be 0 and both the numerator and denominator must be integers) OR
   - a decimal that terminates (ends) or repeats itself in a pattern

9. The absolute value of a number is the quantity of steps that the number is from 0.

10. The absolute value of a number is always positive.

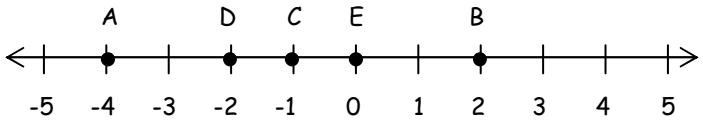
11. The coordinate of the point is the number that shows the location of the point.

12. On a number line the graph of the smallest number is to the left of the graph of a larger number.

Procedure and Examples:

A. Naming coordinates and graphing points--
   1. To name a coordinate, write the number below the letter
   2. To graph a point, find the number on the number line, put a colored circle on the number line, and write the letter above the number line.

A. Examples:



1. Name the coordinates of A, B, and C.
2. On a separate sheet of paper, graph the points x and y if the coordinate of x is 4 and y is -5.

B. Graphing a set of numbers on a number line—

1. Draw a number line
2. Label the number line with equal intervals
3. To graph a point find the number on the number line

- To graph tenths, divide an interval that is equal to 1 into ten smaller intervals.
- Label the intervals by starting with the point that is closest to 0. This will be the .1, the next interval will be .2, followed by .3 and so on.

4. Put a colored circle on the number line
5. Write the letter above the number line

B. Examples: On a separate sheet of paper, graph the following sets of numbers—

1. {-3, -1, 1, 3, . . . }
2. {. . ., -3.5, -3, -2.5, -2, -1.5, . . .}

C. Evaluating expressions using absolute values--

1. If necessary, substitute the value for the variable inside parentheses.
2. Use order of operations, to simplify the expression within the absolute value signs.
3. Count the number of steps it would take to go from zero to the number inside the absolute value signs | |
4. Write this number without the absolute value signs (it will always be positive)
5. If necessary, use the order of operations to simplify the expression

C. Examples: Evaluate the following expressions—

1. | -5 |
2. | 7 |
3. | -8| - | 2|
4. | 4 | + | -7 |

On a separate sheet of paper, evaluate each expression if y = 12:

1. | y – 8 | + 5
2. | 2 – y | - 17
3. | y | - 8
4. | 3y – 12 | + 2y

Section 2-2 —Adding and Subtracting Rational Numbers

Background:

1. A positive 1 and a negative 1 cancel each other out to make zero:

2. The opposite of a positive is a negative

3. The opposite of a negative is a positive

4. The additive inverse of a number is the same as the opposite of a number. For example:
   - The additive inverse of a +3 is -3.
   - The additive inverse of a -5 is +5.

Procedure and Examples:

A. Adding or Subtracting Integers—
   1. Circle each number and the operation before the number, but after the pervious number. For example:
   - (-3)(-(-4))(+ 7)(- 9)
   2. Decide whether each number is positive or negative
   - If a circle has exactly one operation or sign in it, then use that sign to decide if the number is positive or negative
   - If there is a "+" (plus sign) and another sign or operation in the circle, then ignore the "+" and use the other sign to decide if the number is negative or positive
   - If there is a "-" (minus sign) then replace it with the words "the opposite of" to decide if the number is negative or positive
   3. Determine the number of players on the positive team by adding all the positive numbers together
   4. Determine the number of players on the negative team by adding all the negative numbers together
   5. If one negative cancels with one positive, then the team number of players left after canceling is the answer
   6. If one negative cancels with one positive, then the name of the team with players left after canceling is the sign of the answer

A. Examples: On a separate sheet of paper, add or subtract the following integers—
   1. -5 - 8
   2. -8 + 7
   3. 5 - 9
   4. 11 -2
   5. -6 – 5 + 7
   6. (-2) + (-1)

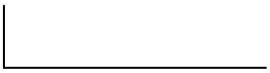
7. 8 – (-4)
8. -3 – (-7)
9. 4 + (-5)
10. 2 – 9 – (-4) + (-6)

B. Adding and Subtracting Decimals—
   1. Find the largest absolute value of each number
   - Find the absolute value of each number (what are the numbers without any negative signs
   - Decide which absolute value is the largest
   2. Write the largest number found in step 1 including the decimal point
   3. Write another decimal point below the first decimal point.
   4. Use the decimal point in step 3 to write the smallest number found in step 1 below the number written in step 2
   5. Use the steps 1 and 2 in the directions for adding and subtracting integers above to determine if each number in the original problem is positive or negative.
   6. If both numbers are the positive or both numbers are negative, then write add beside the decimal problem
   7. If one of the numbers is positive and the other number is negative, then write subtract beside the decimal problem
   8. Add or subtract the numbers (don't forget to bring down the decimal point)
   9. If the first number in the problem was originally a positive number, then the answer found in step 8 is positive
   10. If the first number in the problem was originally a negative number, then the answer found in step 8 is negative

B. Examples: On a separate sheet of paper, find each sum or difference—
   1. -8.4 - 5
   2. -2.37 + (-5.1)
   3. 5.9 – 11.98
   4. -17.84 – (-8.952)

C. Adding and Subtracting Fractions and Mixed Numbers—
   1. Use the steps 1 and 2 in the directions for adding and subtracting integers above to determine if each number in the original problem is positive or negative. Write a "P" for positive or an "N" for negative above each number.
   2. For the following steps, pretend as though both fractions are positive
   3. If necessary, change all mixed numbers to an improper fraction

4. Find the Least Common Denominator (LCD)
- Draw an upside down division box



- Write both denominators in the box putting a small amount of space in between them
- Find a number that will divide into both of the denominators in the upside down division box
- Write this number to the left of the box
- Divide the number on the left of the box into the numbers on the inside of the box.
- Write the answer to each division problem below the number in the box
- Repeat the division steps above if possible
- The LCD is found by multiplying all of the numbers on the outside of the box together

5. Find equivalent fractions. For each fraction in the problem, cover
- Write "the original fraction" $\bullet \frac{2nd}{1st} = \frac{3rd}{LCD}$
- Cover up the numerators of the entire problem
- Find the number that would go in the denominator labeled 1st to make the multiplication problem true
- Write the same number that is in the 1st location in the 2nd location also
- Multiply the number in the numerator of the first fraction by the numerator in the second fraction to find the number in the 3rd location.

6. Draw a large fraction bar with the LCD in the denominator.
7. In the numerator write the largest numerator from the 3rd location found in step 5, a small space, and then the smallest numerator from the 3rd location found in step 5.
8. In step 1, if both numbers are positive or both numbers are negative, then write a plus sign in the small space between the numbers
9. In step 1, if one of the numbers is positive and the other number is negative, then write a minus sign in the small space between the numbers
10. Add or subtract the numerators to find the numerator of the answer
11. The denominator of the answer is the LCD
12. To find the sign of the answer
- Find the largest numerator in step 7
- The original sign of this numerator is also the sign of the answer

C. Examples: On a separate sheet of paper, find each sum or difference—

1. $\frac{-5}{8}+(-\frac{3}{10})$
2. $-\frac{7}{10}+\frac{11}{15}$
3. $4\frac{3}{8}-7\frac{5}{12}$
4. $9\frac{5}{18}-(-4\frac{7}{12})$

Section 2-3—Multiplying Rational Numbers

Background:

1. A positive number multiplied or divided by a positive number equals a positive number.

2. A positive number multiplied or divided by a negative number equals a negative number.

3. A negative number multiplied or divided by a negative number equals a positive number.

4. If two numbers with the same sign are multiplied or divided then the answer is a positive number.

5. If two numbers with different signs are multiplied or divided then the answer is a negative number.

Procedure and Examples:

A. Multiplying or Dividing Integers—
   1. Ignore all positive and negative signs. Multiply or divide the numbers according to the problem to get a number.
   2. Start positive and every time there is a "- " in the problem flip the sign from positive to negative or from negative to positive

A. Examples: Multiply or Divide Integers—
   1. $4 \cdot (-6)$
   2. $-3(-10)$
   3. $\frac{-24}{-8}$
   4. $14 \div (-2)$
   5. $-16 \div 2$
   6. $5\ (-2) \cdot 3 \cdot (-4) \cdot (-1)$

B. Multiplying Decimals—
   1. For each number pretend as though the decimal point has been erased
   2. Find the longest (the number with the most digits) absolute value of each number
   - Find the absolute value of each number (what are the numbers without any negative signs
   - Decide which absolute value is the longest
   3. Write the longest number found in step 1 without the decimal point
   4. Write the shortest number found in step 1 below the number written in step 3

5. Multiply the numbers together
6. For both numbers in the original problem count the total number of digits that are on the right side of the decimal point. This is the quantity of digits that should be on the right side of the decimal point in the answer found in step 5
7. Place the decimal point in the number found in step 5 so that the quantity of digits to the right of the decimal point found in step 6 matches the number of digits. For example:

- If total number of digits to the right of the decimal point in step 6 is four, then the total number of digits to the right of the decimal point in the number found in step 5 should also be four.

8. Follow the step 2 in multiplying and dividing integers to find the sign of the answer

B. Examples: On a separate sheet of paper, find the product—
1. -4.5(-3.07)
2. (-2)(5.20)
3. -8.1(2.3)(-0.0001)

C. Multiplying Fractions—
1. Ignore all positive and negative signs
2. Write each number as a fraction or an improper fraction (there should be no whole numbers or mixed numbers)
3. Cancel any numerator with any denominator. Repeat this step if possible.
4. Multiply the numerators together to find the numerator of the answer
5. Multiply the denominators together to find the denominator of the answer
6. Follow the step 2 in multiplying and dividing integers to find the sign of the answer

C. Examples: On a separate sheet of paper, simplify the following expressions—
1. $\frac{-12}{15} \bullet \frac{24}{-9}$
2. $8 \bullet \frac{2}{3}$
3. $\frac{22}{81} \bullet \frac{36}{55}$
4. $5\frac{2}{5} \bullet 2\frac{4}{9}$
5. $3\frac{1}{9} \bullet \frac{3}{49}$

D. Evaluating expressions—

1. Replace each variable with its value in parentheses
2. Use order of operations for multiplying, dividing, adding and subtracting integers to find the answer

D. Examples: On a separate sheet of paper, evaluate the expressions if a = 12.1, b = 3.7 and c = -4.02—

1. abc
2. ab + c
3. $\frac{4ab}{c}$
4. $\frac{4ab^2}{c-a}$

E. Simplifying Expressions—

1. For each term circle the term and the operation before the coefficient, but after the pervious term.
2. Use the order of operations and the directions for adding, subtracting, multiplying and dividing integers to find the new coefficient
3. Write the new coefficient
4. Add or multiply the variables together to find the new variable name
   - Remember that $x \cdot x = x^2$
   - Remember that $x + x = 2x$
5. Write this behind the coefficient

E. Examples: On a separate sheet of paper, simplify the following expressions—

1. -4a(3) – 5a
2. (-2b)(-5a) + 7(-3a)
3. -8(7x + x)
4. (5j)(-2k) - (-3m)(4n)

Section 2-4—Dividing Rational Numbers

Background:

1. A positive number multiplied or divided by a positive number equals a positive number.

2. A positive number multiplied or divided by a negative number equals a negative number.

3. A negative number multiplied or divided by a negative number equals a positive number.

4. If two numbers with the same sign are multiplied or divided then the answer is a positive number.

5. If two numbers with different signs are multiplied or divided then the answer is a negative number.

Procedure and Examples:

A. Dividing Decimals—

1. Move the decimal point in the divisor (the number that is behind the division sign) to the end of the number.
2. Move the decimal point in the dividend (the number that is before the decimal point) the same number of places as in step 1
3. Write the new number that is before the division sign inside the division box.
4. Write the new number that is after the division sign outside the division box
5. Rewrite the decimal point in the same location but directly above the division box
6. Decide if the divisor (the number outside the division box) can be divided into the first digit of the dividend (the number inside the division box).
7. If it cannot be divided, then put an x in the same location but directly above the division box. If the divisor can be divided into the dividend, then go to step 10.
8. Now decide if the divisor (the number outside the division box) can be divided into the first two digits of the dividend (the number inside the division box). Repeat step 7.
9. Continue step 8 until the divisor can be divided into the dividend.
10. If it can be divided, then find the maximum number of times that the divisor will divide into the dividend. Write this maximum number above the division box.
11. Multiply the number written in step 10 by the divisor. Write this answer below the dividend and subtract.

12. Bring down the next number in the dividend.
13. Find the maximum number of times that the divisor will divide into this new dividend. Write this maximum number above the division box.
14. Multiply the number written in step 13 by the divisor. Write this answer below the new dividend and subtract.
15. Repeat steps 12 through 14 until there are no numbers left in the original dividend to bring down.
16. The answer above the division box is the quotient (the answer).
17. To find the sign of the quotient, start positive and every time there is a "- " in the problem flip the sign from positive to negative or from negative to positive

A. Examples: On a separate sheet of paper, divide the following decimals, rounding to the nearest hundredth if necessary—

1. 0.276 ÷ (-7)
2. -812 ÷ (-8)
3. 783.9 ÷ 9
4. -19.8 ÷ (0.32)

B. Dividing Fractions—

1. Ignore all positive and negative signs
2. Write each number as a fraction or an improper fraction (there should be no whole numbers or mixed numbers)
3. Write the first fraction, followed by a multiplication sign and the reciprocal of the fraction behind the division sign.
4. Cancel any numerator with any denominator. Repeat this step if possible.
5. Multiply the numerators together to find the numerator of the answer
6. Multiply the denominators together to find the denominator of the answer
7. Follow the step 2 in multiplying and dividing integers to find the sign of the answer

B. Examples: On a separate sheet of paper, divide the following fractions—

1. $\frac{-36}{-18} \div \frac{42}{-14}$
2. $\frac{-3}{8} \div \frac{-1}{-3}$
3. $-4\frac{4}{9} \div 8$
4. $-2\frac{3}{11} \div (-2\frac{1}{22})$

5. $4\frac{1}{12} \div (-7\frac{7}{8})$

C. Simplifying algebraic expressions—

1. If there is only one term in the numerator of the fraction, then reduce the coefficients of the numerator and denominator
2. If there are two terms in the numerator of the fraction, then rewrite the fraction as two fractions
   - Write the first term that is in the numerator over the denominator
   - Write the "+" or "-" that is between the terms in the numerator
   - Write the second term that is in the numerator over the denominator
3. In the first fraction divide the denominator into the numerator and simplify
4. Write the "+" or "-" that was written between the two fractions in step 1
5. In the second fraction divide the denominator into the numerator and simplify

C. Examples: On a separate sheet of paper, simplify the following algebraic expressions—

1. $\frac{-39b+65}{13}$
2. $\frac{-45-9x}{-3}$
3. $\frac{54-9x}{-3}$
4. $\frac{-27h}{9}$
5. $\frac{-42f}{-14}$

D. Evaluating algebraic expression given a value--

1. Write the expression
2. Replace each variable with the value of the variable in parentheses
3. Simplify using the order of operations

B. Examples: On a separate sheet of paper, evaluate the following algebraic expression if q = -4, r = 3, x = 4.1, and y = 3.2. Round to the nearest tenth if necessary —

1. $\frac{7q}{r+4}$
2. $r(q \div 2r)$
3. $r + \frac{q}{2} \cdot 6$
4. $\frac{xy}{8}$

5. $\frac{4(2y+10)}{0.3}$