ON THE LIFE AND WORK OF ILYA PIATETSKI-SHAPIRO *

Ilya Piatetski-Shapiro, an extraordinary scientist-refusenik from the Soviet Union, died on February 22, 2009 in Tel-Aviv, after a long debilitating illness.

During a career that spanned sixty years, he made major contributions to applied science, as well as theoretical mathematics. These contributions range from cell biology, geophysics, automata, and homogeneous networks, to digital computers. In the last forty years, his research focused on pure mathematics; in particular, analytic number theory, group representations and algebraic geometry. His main contribution and impact was in the area of automorphic forms and L-functions.

He was a member of the Israel Academy of Sciences, since 1978, and the recipient of many prizes, including the Israel Prize (1981) and the Wolf Prize (1990). Most striking was the fact that he continued to perform research at the highest level until the end of his life, in spite of a deteriorating condition, that left him severely handicapped, and often deprived him of speech. Even when he could barely move, he traveled the world, attending conferences, in order to exchange thoughts with colleagues about their latest researches.

Ilya Piatetski-Shapiro was born March 30, 1929 in Moscow, Russia. Both of his parents were from traditional Jewish families. His father was from Berdichev, a small city in the Ukraine, with a largely Jewish population. His mother was from Gomel, a similar small city in Belarus. Both parents' families were middle-class, but they sank into poverty after the October revolution of 1917.

In 1952, Piatetski-Shapiro won the Moscow Mathematical Society Prize for a Young Mathematician. His winning paper contained a solution to the problem of the French analyst Raphael Salem on sets of uniqueness of trigonometric series. A subset E of the unit circle is called a set of uniqueness if any trigonometric series that converges outside E to zero must have zero coefficients. The problem of characterization of sets of uniqueness and non-uniqueness goes back to Riemann, Cantor and Men'shov. Piatetski-Shapiro

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took this problem when he was a third year student of the Moscow State University, attending a course of Bari on problems of trigonometric series. He introduced the new notion of H^r sets, which have played an important role in this classical topic, and, in particular, in the characterization of which certain Cantor sets, are sets of uniqueness.

Piatetski-Shapiro was surprised by the award, which was announced one week before Stalin's death, because of the surge in anti-Jewish activities sanctioned by the authorities at that time. Indeed, his application to enter a graduate program at Moscow University was rejected by the University Communist Party Committee, despite a very strong recommendation by his mentor Alexander O. Gelfond. It was only by Gelfond's persistence that Piatetski-Shapiro was ultimately admitted to the Moscow Pedagogical Institute, where he later received his Ph.D., at the age of 25, under the supervision of A.D. Buchstab.

Another early work, of Piatetski-Shapiro, is now known as the Piatetski-Shapiro Prime Number Theorem (published in 1953), which asserts that for $1 < c < 12/11$, there are infinitely many primes, which can be written as the integer part of n^c, where n is itself an integer, and, moreover, the number $\pi_c(x)$ of integers $n \leq x$, for which the integer part $[n^c]$ is prime, is asymptotically $\pi_c(x) \sim x/(c \log x)$ as $x \to \infty$ (the admissible value for c has been improved a bit over the years).

After receiving his Ph.D., Piatetski-Shapiro was sent to teach at Kaluga, a small town, about 100 miles away from Moscow. Three years later, when he returned to Moscow, he gave lectures on Siegel domains and Siegel integrals. These originated from Shavarevich suggestion to Piatetski-Shapiro to translate, from English to Russian, Siegel's lecture notes from the Institute of Advanced Study on automorphic functions. Thus, influenced by Igor Shafarevich, Piatetski-Shapiro turned to the theory of automorphic functions on locally symmetric spaces, which was to be his main interest throughout his career.

His contact with Shaferevich, who was a professor at the Steklov Institute, broadened Piatetski-Shapiro's mathematical outlook and directed his attention to modern number theory and algebraic geometry. This led, after a while, to a joint paper on algebraic surfaces that has influenced many mathematicians. Piatetski-Shapiro valued Shafarevich's friendship, even when years later Shafaravich published his controversial essays.

Piatetski-Shapiro made significant contributions to our understanding of locally symmetric spaces. First, he established several fundamental results

about complex homogeneous domains, that he considered as a tool to investigate automorphic forms. For some symmetric domains, Siegel used their realization as tube domains (Siegel domains of the first kind in the terminology of Piatetski-Shapiro) in order to describe the decomposition of automorphic forms in Fourier series, and give an estimate of the dimensions of spaces of such forms. Such realizations exist only for certain domains (domains with Shilov boundary of minimal dimension). Piatetski-Shapiro has found a remarkable generalization of tube domains - Siegel domains of the second kind - that exist for all complex symmetric domains, and allow to prove the same results about automorphic forms as for Siegel domains of the first kind.

Another fundamental contribution of Piatetski-Shapiro pertains to the question of arithmeticity, where he gave the general formulation of what is an arithmetic lattice. Much later, he constructed, with Gromov, the first examples of non-arithmetic lattices acting on higher-dimensional hyperbolic space.

In 1958, Piatetski-Shapiro joined Israel M. Gelfand's department at the Institute of Applied Mathematics in Moscow. Since that time and till emigration from the Soviet Union, Piatetski-Shapiro also actively worked in applied mathematics. He collaborated with specialists in various fields (crystallography, seismology, neurophysiology, morphology, cell biology, to mention a few) on interpretation of experimental results and developing of mathematical models consistent with these results. Together with his students, Piatetski-Shapiro developed general models of behavior of finite automata in randomized environments. By the 1960's, he was recognized as a Soviet star. In 1965, he was appointed to an additional professorship, at the prestigious Moscow State University. His reputation spread internationally; in 1962 and 1966, he was invited to address the quadrennial International Mathematical Congresses.

In the 1970s, there was a growing emigration of Soviet Jews to Israel. At first, Piatetski-Shapiro rejected any thought of leaving his country, in spite of persistent anti-semitism. However, in 1973, he was badly shaken by the divorce of a close Jewish friend, who was married to a non-Jew. The couple had a genuine love for each other, but divorced, so their son could get a new identity card stamped Russian rather than Jew. That year, Piatetski-Shapiro arranged for his wife and son to leave Russia. The result was that he was fired from his Moscow State University professorship. In 1974, he applied for an exit visa to Israel.

As a refusenik, he lost access to mathematical libraries and other aca-

demic resources. He continued his researches nevertheless, and colleagues took books from the library for him. His plight as a mathematician, with such restrictions on his researches, attracted much attention in the U.S. In 1976, a presentation was made to the Council of the National Academy of Sciences urging the use of their good offices to get Piatetski-Shapiro an exit visa. One of the supporting statements was made by Andre Weil, (a leading founder of the Bourbaki), declaring that Ilya Piatetski-Shapiro was one of the "top half dozen" mathematicians in the Soviet Union. Later that year, Piatetski-Shapiro obtained an exit visa. He was welcomed warmly upon arrival in Israel, and accepted a professorship at Tel Aviv University. Starting in 1977, Ilya divided his time between Tel-Aviv University and Yale, directing doctoral dissertations in both places.

Piatetski-Shapiro's main focus of work during the last few decades was in the field of automorphic forms and L-functions. Together with Gelfand, who was one of the most influential figures in Piatetski-Shapiro's mathematics, they played a key role in developing modern automorphic form theory, in terms of infinite dimensional representation theory of Adele groups, associated with linear algebraic groups, defined over a number field. He initiated a long-running and fundamental search for understanding L-functions associated to automorphic forms, by means of integral representations. He contributed decisively to the theory of global integrals of Rankin-Selberg type and of Shimura type (for $GL(n) \times GL(m)$, with Jacquet and Shalika, and for classical groups with Rallis, Ginzburg and Gelbart). With Cogdell, he established his celebrated converse theorem for $GL(n)$. He envisioned a proof of the existence of the Langlands functorial lift from classical groups to general linear groups, through the converse theorem and the analytic properties of L-functions, to be derived from the global integrals. These efforts, combined with Shahidi's work on L-functions, culminated in the proof of the existence of this functorial lift for generic representations (Cogdell, Kim, Piatetski-Shapiro and Shahidi). In parallel, Piatetski-Shapiro, together with Howe, Soudry, made inspirational contributions to the theory of the theta correspondence of automorphic forms, in reductive dual pairs, and their relations to special values of L-functions and functorial lifting. In these studies, he found these extraordinary cuspidal representations, which do not satisfy the Ramanujan hypothesis; he called them CAP representations. This great body of work continues to be a vivid and active domain of research today, where Piatetski-Shapiro's vision and ideas are the main guidance and light.

Ilya Piatetski-Shapiro advised many doctoral students, and taught and

collborated with mathematicians from all generations and continents. Through his deep ideas, far reaching mathematical insight and vision, he has inspired the work of many, who, today, are passing his legacy on to the next generation of young mathematicians.

In later years, his former student, James Cogdell, was the only person outside Piatetski-Shapiro's immediate family, who could understand his speech, and Piatetski-Shapiro continued to work with Cogdell, until 2009. Piatetski-Shapiro's wife Edith took care of him, with great devotion, to the end, traveling with him wherever he wished to go, and helping him to maintain warm connections with a wide variety of friends in the mathematical community.

Ilya Piatetski-Shapiro is survived by his wife Edith, his daughter Shlomit (Shelley), his son Gregory, and his step-daughter, Vera Lipkin. He was buried in Tel-Aviv on February 24, 2009. May his memory be blessed.