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FRAGMENTATION HIERARCHY OF BRIGHT SUNGRAZING COMETS AND THE BIRTH AND ORBITAL EVOLUTION OF THE KREUTZ SYSTEM. I. TWO-SUPERFRAGMENT MODEL

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ABSTRACT

A back-and-forth orbit integration technique, developed for our previous investigation of the splitting of the parent of the sungrazers C/1882 R1 and C/1965 S1, is now applied in an effort to understand the history and orbital evolution of the Kreutz sungrazer system, starting with the birth of two subgroups, which show prominently among the bright members and whose inception dates back to the progenitor's breakup into two superfragments. The integration technique is used to reproduce the motion of comet C/1843 D1—the second brightest sungrazer known and presumably the most massive surviving piece of superfragment I—from the motion of C/1882 R1—the brightest sungrazer on record and arguably the most massive surviving piece of superfragment II. Running the orbit of C/1882 R1 back to A.D. 326, the progenitor comet is found to have split at a heliocentric distance of 50 AU and nearly 30 yr before perihelion. The superfragments acquired separation velocities of $\sim$8 m s^{-1} in opposite directions. Using the same technique, we show next that (1) the motions of two additional sungrazers, C/1880 C1 and C/1887 B1, are matched extremely well if these objects shared a common parent with C/1843 D1, and (2) C/1963 R1 (Pereyra), the second brightest subgroup I member on record, is more closely related to subgroup II objects (such as C/1882 R1 and C/1965 S1) than to C/1843 D1. This finding raises serious doubts about the major role of the subgroups in the system's orbital history and offers an incentive for considering an alternative dynamical scenario. The fragmentation models for C/1963 R1 and two additional bright sungrazers, C/1945 X1 and C/1970 K1, suggest that (1) these comets may have been the most massive pieces of the fragment populations formed from their respective disintegrating parents, and (2) the course of evolution of the Kreutz system at the upper end of the mass spectrum may be better ascertained from the distribution of the sungrazers' arrival times than from the sources of subgroups. If so, the fragment hierarchy should be determined primarily by the cascading nature of the fragmentation process, which was recently shown by Sekanina to control the evolution of minor fragments as well. The sungrazer system's estimated age is in any case very short, less than 1700 yr.

Subject headings: comets: general — comets: individual (X/1106 C1, C/1843 D1, C/1880 C1, C/1882 R1, C/1887 B1, C/1945 X1, C/1963 R1, C/1965 S1, C/1970 K1) — methods: data analysis

1. INTRODUCTION

In a series of recent papers (Sekanina 2002a, 2002b, 2003; Sekanina & Chodas 2002a, 2002b), we have addressed a variety of issues pertinent to the origin, history, and aging of the system of sungrazing comets, first investigated systematically by Kreutz (1888, 1891, 1901) and usually referred to as the Kreutz sungrazers. Approaching the Sun's photosphere in most instances to within 1 solar radius (1 $R_\odot = 0.0046524$ AU) at perihelion, these objects have long been known to move about the Sun in extraordinarily elongated paths, reaching some 120–200 AU at aphelion and typically requiring 500–1000 yr to complete one revolution, as established from analysis of the five members with the best-determined orbits. One of the techniques developed and applied by us provides a numerical demonstration that the motion of one member of the Kreutz system can be accurately derived from the known motion of another member when the two are direct products of their common parent object (Sekanina & Chodas 2002a, 2002b). This technique starts from the orbit of the brighter fragment: its motion is integrated numerically (with the planetary perturbations and the relativistic effect accounted for) back in time to the instant of an assumed breakup, when the momentum of the fainter fragment is slightly changed (i.e., its derived separation velocity is added); the motion of the fainter fragment is then integrated forward to the time for which the observed set of orbital elements is available for comparison. The aim is to match the observed orbit as closely as possible, a task addressed by a least-squares optimization procedure incorporated in the approach.

This back-and-forth orbit integration technique was successfully applied first to the sungrazing pair C/1882 R1 (Great September Comet) and C/1965 S1 (Ikeya-Seki) to describe the conditions at their separation (Sekanina & Chodas 2002a) and subsequently to examine the source and history of C/1970 K1 (White-Ortiz-Bolelli), the most recent sungrazer discovered from the ground, and to establish the relationship between two additional Kreutz objects, C/1843 D1 (Great March Comet) and C/1880 C1 (Great Southern Comet; Sekanina & Chodas 2002b).

This same technique is now employed as a tool for exploring the formation and dynamical evolution of the Kreutz system by studying the hierarchy of bright sungrazers, the objective of this investigation. The aim is to explain their inception by invoking the least possible number of fragmentation events.

2. THE SUBGROUPS

The presence of two distinct subgroups among the bright sungrazing comets was first proposed by Hasegawa (1966) and independently by Kresák (1966). The subgroups were investigated in detail by Marsden (1967, 1989). A consensus has developed that any two members of the same subgroup are likely to be related to one another more closely than a member of one subgroup to a member of the other subgroup, even though no rigorous model has ever been offered to justify this belief.

In a classification, based solely on orbital similarity, subgroup I includes C/1843 D1, C/1880 C1, C/1887 B1 (Great Southern Comet), and C/1963 R1 (Pereyra), while subgroup II consists of C/1882 R1, C/1945 X1 (du Toit), and C/1965 S1. The sungrazer C/1970 K1 appears to represent an extension to subgroup II; Marsden (1989) classified this object as the only member of subgroup IIa.

Among the bright sungrazers the subgroups are quite prominent, as they are separated by major gaps in both the angular elements (the argument of perihelion ω, the longitude of the ascending node Ω, and the inclination i) and the perihelion distance q. The differences in the orbital elements between the subgroups are much greater than the scatter among most members in either subgroup. Specifically, the differences in the sense subgroup II minus subgroup I are about $+0.5\ R_\odot$ in the perihelion distance, $-20°$ in the nodal longitude, $-15°$ in the argument of perihelion, and $-2\overset{\circ}{.}5$ in the inclination. All four members of subgroup I have perihelia closer to the photosphere than $\frac{1}{5}\ R_\odot$, whereas the three subgroup II objects approach it only to about $\frac{2}{3} R_\odot$ and C/1970 K1 to approximately 9/10 $R_\odot$.

Marsden (1967) examined the spatial distribution of the sungrazers with the best-determined orbits and found that their osculating elements could be closely approximated as a function of the mean longitude of Jupiter, $\Lambda_{\rm J}$, at the times the comets were at perihelion:

$$\begin{pmatrix} \omega \\ \Omega \\ i \\ q \end{pmatrix} = \begin{pmatrix} a_\omega \\ a_\Omega \\ a_i \\ a_q \end{pmatrix} + \begin{pmatrix} b_\omega \\ b_\Omega \\ b_i \\ b_q \end{pmatrix} \sin(\Lambda_{\rm J} + \beta), \tag{1}$$

where $a_\omega, \ldots, b_q$, and $\beta \simeq 100°$ are the empirically derived coefficients. From the equations for the variation of arbitrary constants Marsden showed that the dominant effect—the indirect perturbations by Jupiter—should satisfy this type of relationship when the planet's orbit is approximated by a circle, except that the angle β should be 90° smaller for the perihelion distance than for the angular elements. In addition, the integration of the Jovian indirect attraction over one revolution about the Sun yielded values for the amplitudes $b_\omega, \ldots, b_q$ that were 7–18 times smaller than the empirically derived ones. Thus, the indirect planetary perturbations account at best for only a small fraction of the observed differences and their variations are inconsistent with expectation based on the theory.

3. ORBITAL EVOLUTION OF THE KREUTZ SYSTEM

The set of 16 sungrazers discovered with the space-borne *SOLWIND* and the *Solar Maximum Mission* (*SMM*) coronagraphs and, especially, the collection of hundreds of Kreutz system minicomets found in images taken with the coronagraphs on board the *Solar and Heliospheric Observatory* (*SOHO*) suggest that the discrimination into the two subgroups is, in comparison with the bright sungrazers, fairly inconspicuous among these intrinsically fainter (and presumably much less massive) members. Although this smear is likely to be caused in part by the sizable uncertainties in the orbits of these minor objects (on account of a large pixel size of the coronagraphic images and a short orbital arc covered by the astrometric observations), the existence of intrinsic systematic variations in the orbits cannot be denied (Sekanina 2002a); the majority of the coronagraphically detected objects apparently belong to subgroup I.

Orbital differences of about the same magnitude as those between the subgroups, although not as strongly correlated, have also been exhibited by some of the pairs of nearly simultaneously arriving *SOHO* sungrazers. Because of the timing coincidence, these differences cannot possibly be due to the indirect planetary perturbations, and the only plausible explanation is offered by differential momenta acquired by the components of the pair during their fragmentation at a large heliocentric distance (Sekanina 2000, 2002a, 2002b). For example, a separation velocity of $\sim$5 m s^{-1} imparted to a fragment at the aphelion point of a sungrazing path 170 AU from the Sun can trigger perturbing effects of up to 22° in the argument of perihelion, up to 27° in the longitude of the ascending node, up to 5° in the inclination, and up to 0.7 $R_\odot$ in the perihelion distance (see Table 8 of Sekanina 2002a). It is noted that the orbital differences between the two subgroups (§ 2) do not exceed 75% of these perturbing effects and that therefore, in principle, a single fragmentation event involving separation velocities of several meters per second could fully explain the birth of the Kreutz subgroups.

An event of this nature can bring about dramatic perturbations of the orbit orientation and perihelion distance because of an extremely low near-aphelion orbital velocity of the sungrazers, on the order of only 20 m s^{-1}. Separation velocities of several meters per second then represent a relatively large fraction of the orbital velocity, with the necessary result of a major orbital transformation. Significantly, fragmentation far from the Sun should not affect very much the time of the next perihelion passage, so that the fragments will arrive nearly simultaneously, consistent with the excessively large number of close *SOHO* sungrazer pairs and clusters (Sekanina 2002b).

The behavior of minor sungrazers in the *SOHO* set provides information that is instrumental for our understanding of the evolution of the entire Kreutz system. In particular, none of the *SOHO* minicomets have ever been observed all the way to perihelion, as they all vanish while on their approach to the Sun. As a result, their existence as separate objects could not predate the previous perihelion passage, when they must have been embedded in their parent bodies, sungrazers of much larger dimensions. With evidence based on the pairs and clusters, this argument offers a strong case for *runaway fragmentation*, a process proceeding throughout the orbit about the Sun, predominantly at very large heliocentric distances.

On the other hand, at least six of the eight bright Kreutz comets are known to have survived their returns to the Sun intact, even though two (or three) of them were reported to have split near perihelion. One of the eight (C/1887 B1) was observed as a headless tail when receding from the Sun, as its nucleus disintegrated shortly after perihelion (Sekanina 1984; see also § 9). In addition, one of the eight (C/1945 X1) was seen only on its approach to the Sun (Paraskevopoulos 1945), in spite of intensive postperihelion searches (van Biesbroeck 1946a, 1946b). The motion of this sungrazer was studied by Marsden (1967, 1989) and is reexamined here in § 10.

The process of terminal erosion was recently shown to offer a satisfactory interpretation of the *SOHO* sungrazers' light curves (Sekanina 2003). This study led to a conclusion that, on its approach to the Sun, a sungrazer should be at least 1 km in diameter to survive the return, the exact value depending on the perihelion distance. Although the nuclear dimensions of the bright sungrazers are not well known, the sizes of the six observed survivors can be estimated at several to several tens of kilometers across (Sekanina 1997, 2002a), thus satisfying the erosion constraint. Unfortunately, one cannot be more specific because at the time of observation the orbital motion of minor sungrazers, such as those in the *SOHO* set, carries no recognizable "memory" of the experienced fragmentation events other than the most recent one. Given the high probability that many *SOHO* sungrazers underwent a number of such events, the identity of their parent(s) at the time of the previous perihelion passage cannot be individually extracted from the *SOHO* orbital data.

4. BIRTH SCENARIO FOR THE SUBGROUPS

In conformity with the argument in § 3, it is next assumed that the origin of the subgroups dates back to a prime event of the progenitor sungrazer's breakup into two giant superfragments (or primary fragments), which subsequently continued to split repeatedly into an enormous number of fragments of the second, third, etc., generations. This scenario does not necessarily exclude the potential existence of another fragmentation episode involving the progenitor and preceding the prime breakup event, even though the known sungrazers offer no compelling evidence for any such precursor episode.

4.1. *The Limitations*

An inclusive quantitative investigation of a specific birth scenario for the subgroups along these lines can be undertaken only if very restricted conditions apply, requiring that there be at least a fair chance for the major residual masses of the two superfragments to exist among the known bright sungrazers from the 19th and 20th centuries and that enough information be available on the orbital motions of these residual masses to warrant a meaningful analysis.

In a recent paper on the *SOHO* sungrazers, the first author (Sekanina 2002a) proposed that much of the mass of one of the superfragments may still be locked in C/1882 R1, a member of subgroup II and intrinsically by far the brightest known sungrazer. He speculated that the most significant fraction of the mass of the other superfragment was preserved in C/1843 D1, a member of subgroup I and the second brightest known sungrazer. Accounting for the orbital perturbations as a product of a momentum change in the motions of the superfragments upon the progenitor's splitting but ignoring the indirect planetary perturbations, Sekanina (2002a) concluded that the predecessors of comets C/1843 D1 and C/1882 R1 did not separate from one another near perihelion, but on their way toward the Sun at the end of the 3rd century A.D., at a heliocentric distance of $\sim$56 AU, with either object acquiring a velocity of $\sim$13 m s^{-1} relative to their center of mass. This educated guess would put the age of the Kreutz system at only some 1700 yr, which would make it much younger than previously thought.

Upon breakup, all fragments are bound to acquire differential momenta relative to the parent mass. By assuming that the most massive (and, presumably, the brightest) fragment is subject to no extra momentum, one introduces a slight, but not necessarily trivial, error, whose effect, propagating through the model calculations, cannot unfortunately be avoided given that the momentum partitioning function is unknown. While the choice of C/1882 R1 as an excellent approximation to the subgroup II superfragment seems fully justified, C/1843 D1 is a weaker substitute for the subgroup I superfragment because of its lesser brightness. In addition, it was shown by us to be, together with C/1880 C1, a product of a breakup episode that occurred long after the prime fragmentation event (Sekanina & Chodas 2002b; see also § 8). The parent of C/1843 D1 and C/1880 C1 would unquestionably be a better approximation to the subgroup I superfragment than is C/1843 D1, if only its orbit were known. On the other hand, since C/1880 C1 was much fainter (and, presumably, less massive) than C/1843 D1, the parent's motion must have been nearly identical with that of C/1843 D1.

Accordingly, in our search for the prime fragmentation event we see no alternative to accepting C/1882 R1 and C/1843 D1, the two most prominent known sungrazers, as plausible substitutes for the superfragments. In our subgroup birth scenario, we approximate the motion of the subgroup I superfragment, called from now on superfragment I, by that of C/1843 D1 and the motion of the subgroup II superfragment, or superfragment II, by that of C/1882 R1. This study of the nature of a relationship between the two sungrazers should at least answer the question of plausibility of their common source.

4.2. *Input and Output*

To be able to apply the back-and-forth orbit integration technique (§ 1), we employ the set of orbital elements for the center of mass of the nucleus components of C/1882 R1, derived by us elsewhere (Sekanina & Chodas 2002a), as the reference orbit to initiate the search for the fragmentation solution. We note that the presumed identity of C/1882 R1 with the dominant fragment of comet X/1106 C1 makes it possible to determine the sungrazer's orbital period (and the eccentricity) with exceptionally high accuracy. For C/1843 D1, Kreutz's (1901) set of elements, derived on the assumption that the orbital period is equal to 800 yr, was previously converted by us to equinox J2000.0 (Sekanina & Chodas 2002b) and is employed in our computations as the set of "proxy" observations (see Sekanina & Chodas 2002a). The exception, of course, is the forced eccentricity, which is not used in the optimization procedure. No nongravitational effect has been incorporated in the calculations, as its magnitude is known for neither sungrazer. Besides, we showed that because of a steep fall of the nongravitational force with increasing heliocentric distance, its relatively minor effect in a sungrazing orbit can always be satisfactorily approximated by incorporating it into the separation velocity (Sekanina & Chodas 2002a). The sets of orbital elements used for the two sungrazers are listed in Table 1.

An iterative least-squares differential correction optimization procedure, which is linked with a sophisticated orbit integration code to constitute the employed method of computer search, provides fragmentation solutions as sets of four parameters: the time of fragmentation $t_{\rm frg}$ and the three components of the separation velocity $V_{\rm sep} = |\boldsymbol{V}_{\rm sep}|$ in the right-handed RTN coordinate system, which is centered on the reference object and is referred to the parent's orbit plane and defined by the orthogonal directions radial away from the Sun, V_R, transverse in the orbit plane, V_T, and normal to the plane, V_N. The gravitational interaction of the fragments immediately following their separation is ignored because observations provide no information on these effects. The resulting separation

TABLE 1
ADOPTED ORBITAL ELEMENTS FOR COMETS C/1882 R1 AND C/1843 D1 (EQUINOX J2000.0)

Orbital Element	Comet C/1882 R1[a]	Comet C/1843 D1[b]
Time of perihelion passage T (ET)	1882 Sep 17.72410 $\pm$ 0.00004	1843 Feb 27.91434 $\pm$ 0.00120
Argument of perihelion ω (deg)..............	69.5851 $\pm$ 0.0018	82.8063 $\pm$ 0.0600
Longitude of ascending node Ω (deg).....	347.6559 $\pm$ 0.0022	3.7283 $\pm$ 0.0735
Orbital inclination i (deg)........................	142.0109 $\pm$ 0.0005	144.3893 $\pm$ 0.0091
Perihelion distance q (AU)......................	0.0077508 $\pm$ 0.0000007	0.0054897 $\pm$ 0.0000161
Orbital eccentricity e................................	0.99991034 $\pm$ 0.00000016	(0.9999363)
Orbital period P (yr)................................	803.7 $\pm$ 2.2	(800)
Epoch (ET)...	1882 Oct 2.0	1843 Mar 21.0[c]

[a] Planetary perturbations and relativistic effect included (for details see Sekanina & Chodas 2002a).
[b] As derived by Kreutz 1901, with a forced orbital period of 800 yr (parenthesized). Planetary perturbations and relativistic effect not included.
[c] Standard, 40 day osculating epoch nearest the midtime of astrometric observations adopted.

velocity is therefore the relative velocity after the interaction has ceased. When one of the fragments is considerably brighter (and presumably more massive) than the other, it is taken as the reference object and the separation velocity is simply the velocity of the fainter (less massive) fragment relative to the reference fragment. In the case of the prime fragmentation event that involves the two superfragments, however, a different scenario is proposed. Whereas superfragment II (with the assigned orbit of C/1882 R1) is employed as the reference object, the modeled separation velocity of superfragment I relative to superfragment II, $\boldsymbol{V}^*_{\rm sep} = \{V^*_R, V^*_T, V^*_N\}$, can be interpreted as the difference between the separation velocities of the two superfragments relative to the progenitor (Fig. 1), $\boldsymbol{V}^*_{\rm sep} = \boldsymbol{V}^{\rm I}_{\rm sep} - \boldsymbol{V}^{\rm II}_{\rm sep}$, where $\boldsymbol{V}^{\rm I}_{\rm sep} = \{V^{\rm I}_R, V^{\rm I}_T, V^{\rm I}_N\}$ and $\boldsymbol{V}^{\rm II}_{\rm sep} = \{V^{\rm II}_R, V^{\rm II}_T, V^{\rm II}_N\}$. In this scenario, the progenitor ceases to exist at time $t_{\rm frg} = t^*$ of the breakup event. If the momentum partitioning coefficient, determining the fraction of the separation velocity acquired by superfragment I, is f $(0 < f < 1)$, i.e., $\boldsymbol{V}^{\rm I}_{\rm sep} = f\boldsymbol{V}^*_{\rm sep}$, then the relative velocity of superfragment II is $\boldsymbol{V}^{\rm II}_{\rm sep} = (f-1)\boldsymbol{V}^*_{\rm sep}$. Assuming that the velocity is rotational in nature and that the progenitor comet splits into nearly equally massive superfragments, one implies that $f \simeq \frac{1}{2}$ and $\boldsymbol{V}^{\rm I}_{\rm sep} \simeq -\boldsymbol{V}^{\rm II}_{\rm sep}$. On the other hand, if superfragment I is less massive than superfragment II, then $\frac{1}{2} < f < 1$. A range of orbital solutions for the progenitor can be examined as a function of f.

The solutions are optimized in units of the rms errors of the orbital elements, so that the residuals are dimensionless quantities. An important feature of the optimization procedure is the option to solve for any combination of fewer than the four parameters, so that a total of 15 different versions are available. This option proves very helpful to the user, especially in an early phase of the optimization process, before the solution settles around the resulting values of the parameters, or when the convergence is slow. In our computations described below, this feature has been used extensively to much advantage. In particular, it has often been possible to include the fragmentation time $t_{\rm frg}$ as a variable only after the separation velocity components were significantly constrained.

Our experience with the Kreutz system suggests that separation velocities of fragments never exceed $\sim$10 m s^{-1} (Sekanina & Chodas 2002a). Accordingly, we ignore as unacceptable all solutions that require separation velocities greater than this limit, a condition that provides a very tight constraint. For the prime event it implies that $f \leq 10/|\boldsymbol{V}^*_{\rm sep}|$, with $|\boldsymbol{V}^*_{\rm sep}|$ in m s^{-1}.

As in our study of C/1882 R1 and C/1965 S1 (Sekanina & Chodas 2002a), the present search for a fragmentation solution uses one of the two weighting systems: system I, in which the contributions from the individual orbital elements are weighted by the rms errors of the elements of C/1882 R1, or a generally preferable system II, in which the weighting is determined by the errors of the elements of C/1843 D1. As before, this approach allows us to test the degree of stability of the solution.

4.3. *Birth of the Subgroups: Computations and Results*

Using weighting system II, we have been unable to find an acceptable fragmentation solution for the pair of C/1882 R1 and C/1843 D1 on the assumption that the progenitor's breakup occurred at any time during or after the 5th century A.D. This negative result (1) agrees with expectations based on the preliminary estimate by Sekanina (2002a) (§ 4.1), (2) is consistent with the previous suggestions (Marsden 1967; Sekanina & Chodas 2002a) that comet X/1106 C1 could be the common parent of C/1882 R1 and C/1965 S1, and (3) makes sense in light of what has been stated in § 3 about the orbital perturbations due to momentum transfer at separation. Indeed, a time span of less than two revolutions about the Sun

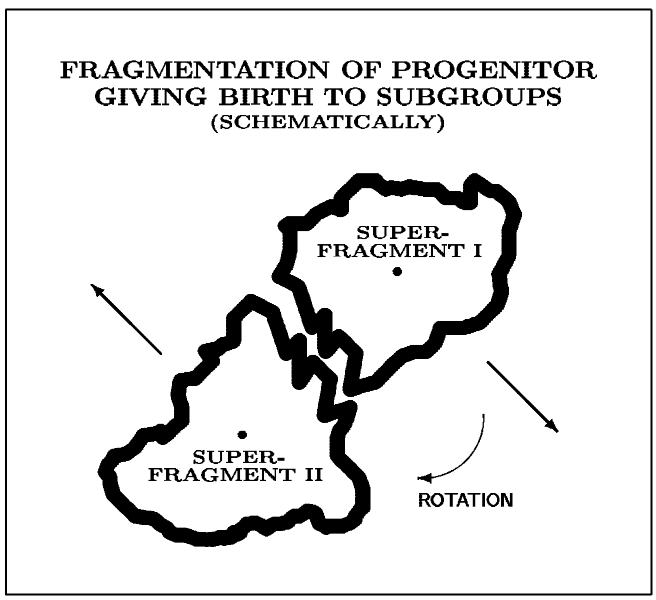

FIG. 1.—Schematic outline of the superfragments separating from the split progenitor, which gives birth to the subgroups. The separation velocities are assumed to be rotational in nature.

TABLE 2

TIME AND ORBIT LOCATION OF THE PRIME FRAGMENTATION EVENT AND COMPUTED ELEMENTS FOR SUPERFRAGMENT I WITH RESIDUALS (C/1843 D1 RELATIVE TO C/1882 R1; EQUINOX J2000.0)

Fragmentation Parameter	Value
Prime fragmentation event:	
Date (ET, old style)	326 Mar 28 $\pm$ 92
Years before perihelion[a] in A.D. 356	30.01 $\pm$ 0.25
Heliocentric distance (AU)	50.33 $\pm$ 0.26
Distance from ecliptic (AU)	-29.26 ± 0.15

Orbital Element	Computed Set of Orbital Elements for Superfragment I (C/1843 D1)	Residual: Observed Minus Computed	
		In Absolute Units	In Dimensionless Normalized Units[b]
T (days)	1843 Feb 27.91434 ET	0.00000	0.000
ω (deg)	82.7826	+0.0237	+0.395
Ω (deg)	3.7593	-0.0310	-0.421
i (deg)	144.3888	+0.0005	+0.055
q (AU)	0.0054895	+0.0000002	+0.011
e	0.99993339	...	...
P (yr)	748.1	...	...
Epoch	1843 Mar 21.0 ET	...	...
$[\mathcal{P}_0]$	...	...	0.3364
rms error	...	...	$\pm$0.580

[a] Defined as an average of the perihelion times of superfragments I and II (Table 4).

[b] Weighting system II (units are the errors of the orbital elements of comet C/1843 D1 from Table 1); $[\mathcal{P}_0]$ is the dimensionless sum of squares of the normalized residuals as defined by eq. (5) of Sekanina & Chodas 2002a.

is not long enough to accommodate both the sizable differences between the angular elements and the nearly 40 yr gap between the perihelion times of the two sungrazers (Table 1).

Searching next for the potential solutions with fragmentation times fixed at every 10 yr between A.D. 200 and 400, we have noticed that the fit was improving until the year 330, then rapidly deteriorated as the postulated fragmentation time was approaching perihelion in the year 356. A subsequent attempt at solving simultaneously for both the separation velocity and the breakup time proved successful, yielding the results presented in three tables. Table 2 presents information on the prime fragmentation event itself, the computed orbital elements for superfragment I, and the residuals from the optimized four-parameter solution. Table 3 shows the dependence of the separation velocity components and the progenitor's orbital elements on the value adopted for the momentum partitioning coefficient f. Table 4 lists the orbital elements of the two superfragments computed for their first two returns after separation.

Broad ramifications of the results from Tables 2, 3, and 4 are discussed in § 11, after the fragmentation scenarios for all bright sungrazers have been fully described. Here we only comment on some implications and details of the tabulated data. First of all, it should be mentioned that the parameters of the fragmentation solution were checked by independently applying weighting system I. This test supports the results in Tables 2, 3, and 4, confirming the fragmentation event in the year 326 and the separation velocities in Table 3. The formal errors were now somewhat greater, about $\pm$150 days and up to ± 0.16 m s^{-1}, respectively. The normalized rms error of the solution was $\pm$18.8 units because of the much lower errors of the orbital elements of C/1882 R1 (Table 1) on which weighting system I was based. Similarly, the computed sets of elements for the progenitor comet before the event and for the superfragments at their subsequent returns were close to the respective sets in Tables 3 and 4.

From Table 3 we conclude that the celebrated Aristotle comet of 372 B.C.E. was obviously not a member of the Kreutz system, contrary to many speculations in the literature. Historical records do not show any comet in 315–314 B.C.E, the predicted time of the progenitor's arrival to perihelion. Since the observing conditions are more favorable when the perihelion occurs in October or February than in May or July, the comet would have more probably been recorded if the momentum partitioning coefficient was between 0.52 and 0.61 than between 0.45 and 0.52. The absence of chronicle reports may favor f close to, and perhaps slightly below, $\frac{1}{2}$. The relatively narrow span of the predicted perihelion times in the 4th century B.C.E. expands rapidly to $\sim$19 yr in the 11th century B.C.E., when no meaningful comparison with historical records is possible. There apparently was a comet observed by the Chinese in 1055 or 1030 B.C.E. (Ho 1962; Hasegawa 1980), but no details are available about its identity. Continuing orbital integration back in time indicates a strong divergence and nonlinearity of the results, the computed perihelion times spanning nearly 140 yr for the first return in the second millennium B.C.E. and nearly 400 yr for the return in the third millennium B.C.E.

Turning now to the superfragments, their predicted perihelion times (Table 4) are the end of March and early April 356 (old style = Julian calendar), that is, during the time of the year that does not offer favorable observing conditions for the comets to become prominent twilight objects in the northern hemisphere. Not surprisingly, there are no historical records of a comet in A.D. 356. Superfragment II, subjected to a momentum change in the general direction of the orbital motion relative to the center of mass, arrived at the 356 perihelion first, about $6\frac{1}{2}$ days before superfragment I. However, the orbital period of superfragment II was somewhat longer than that of superfragment I, so the order in which the pair arrived at the next perihelion was interchanged: superfragment I reached it in the year 1100, superfragment II—as the celebrated comet

TABLE 3

DEPENDENCE OF SEPARATION VELOCITIES ACQUIRED BY SUPERFRAGMENTS I AND II DURING PRIME FRAGMENTATION EVENT ON PARTITIONING COEFFICIENT f AND COMPUTED SETS OF PROGENITOR COMET'S ORBITAL ELEMENTS (EQUINOX J2000.0)

	$f = 0.500$		$f = 0.608$		$f = 0.547$		$f = 0.456$	
FRAGMENTATION PARAMETER OR ORBITAL ELEMENT	I	II	I	II	I	II	I	II
Separation velocity component (m s^{-1}):								
Radial, V_R	$+2.97 \pm 0.01$	-2.97 ± 0.01	$+3.61 \pm 0.01$	-2.33 ± 0.01	$+3.25 \pm 0.01$	-2.70 ± 0.01	$+2.71 \pm 0.01$	-3.24 ± 0.01
Transverse, V_T	-6.59 ± 0.07	$+6.59 \pm 0.07$	-8.01 ± 0.09	$+5.17 \pm 0.06$	-7.21 ± 0.08	$+5.97 \pm 0.07$	-6.01 ± 0.07	$+7.17 \pm 0.08$
Normal, V_N	-3.93 ± 0.04	$+3.93 \pm 0.04$	-4.78 ± 0.04	$+3.08 \pm 0.03$	-4.30 ± 0.04	$+3.56 \pm 0.03$	-3.58 ± 0.03	$+4.28 \pm 0.04$
Total, V_{sep}	8.23 ± 0.06	8.23 ± 0.06	10.00 ± 0.08	6.45 ± 0.05	9.00 ± 0.07	7.46 ± 0.06	7.50 ± 0.06	8.96 ± 0.07
Computed Sets of Orbital Elements for the Progenitor Comet in 4th Century B.C.E.								
T (ET; old style)	B.C.E. 314 May 6.2		B.C.E. 315 Oct 10.4		B.C.E. 314 Feb 2.6		B.C.E. 314 Jul 24.4	
ω (deg)	68.78		68.69		68.72		68.87	
Ω (deg)	346.42		346.35		346.38		346.52	
i (deg)	141.62		141.66		141.64		141.62	
q (AU)	0.00735		0.00759		0.00744		0.00729	
e	0.9999107		0.9999083		0.9999099		0.9999111	
P (yr)	746.8		752.6		750.9		742.3	
Epoch (ET; o. s.)	B.C.E. 314 Apr 17.0		B.C.E. 315 Sep 29.0		B.C.E. 314 Jan 27.0		B.C.E. 314 Jul 6.0	
Computed Sets of Orbital Elements for the Progenitor Comet in 11th Century B.C.E.								
T (ET; o. s.)	B.C.E. 1040 Feb 16.7		B.C.E. 1053 May 3.8		B.C.E. 1048 Sep 26.8		B.C.E. 1034 Jun 19.5	
ω (deg)	68.00		68.43		66.38		66.17	
Ω (deg)	345.43		345.92		343.30		343.01	
i (deg)	141.48		141.65		140.81		140.69	
q (AU)	0.00700		0.00765		0.00766		0.00752	
e	0.9999183		0.9999089		0.9999084		0.9999081	
P (yr)	792.6		768.8		764.5		739.6	
Epoch (ET; o. s.)	B.C.E. 1040 Feb 7.0		B.C.E. 1053 Apr 16.0		B.C.E. 1048 Oct 7.0		B.C.E. 1034 Jun 16.0	
Computed Sets of Orbital Elements for the Progenitor Comet in 17th–19th Centuries B.C.E.								
T (ET; o. s.)	B.C.E. 1836 Jul 28.9		B.C.E. 1800 Sep 18.5		B.C.E. 1766 Jan 1.5		B.C.E. 1698 Apr 4.4	
ω (deg)	68.15		68.42		64.77		64.85	
Ω (deg)	345.47		345.82		341.18		341.37	
i (deg)	141.59		141.63		140.30		140.20	
q (AU)	0.00649		0.00745		0.00819		0.00861	
e	0.9999276		0.9999143		0.9998984		0.9998874	
P (yr)	848.6		810.0		724.2		668.7	
Epoch (ET; o. s.)	B.C.E. 1836 Aug 5.0		B.C.E. 1800 Sep 25.0		B.C.E. 1766 Jan 10.0		B.C.E. 1698 Apr 3.0	
Computed Sets of Orbital Elements for the Progenitor Comet in 23rd–27th Centuries B.C.E.								
T (ET; o. s.)	B.C.E. 2677 Jul 6.4		B.C.E. 2605 Jan 7.8		B.C.E. 2444 Feb 26.7		B.C.E. 2299 Nov 12.0	
ω (deg)	67.83		68.14		64.47		64.75	
Ω (deg)	344.96		345.40		340.68		341.30	
i (deg)	141.50		141.61		140.12		140.19	
q (AU)	0.00603		0.00670		0.00875		0.00942	
e	0.9999365		0.9999283		0.9998867		0.9998714	
P (yr)	926.6		901.7		679.2		626.3	
Epoch (ET; o. s.)	B.C.E. 2677 Jul 11.0		B.C.E. 2605 Jan 15.0		B.C.E. 2444 Feb 18.0		B.C.E. 2299 Nov 24.0	

TABLE 4

Sets of Orbital Elements Computed for Superfragments I (C/1843 D1) and II (C/1882 R1 = X/1106 C1) at First Two Returns to Sun after Their Separation in a.d. 326 (Equinox J2000.0)

Orbital Element	First Return to Sun		Second Return to Sun	
	Superfragment I	Superfragment II	Superfragment I	Superfragment II[a]
T (ET; old style)	356 Apr 4.47	356 Mar 28.91	1100 Mar 27.99	1106 Jan 26.50
ω (deg)	76.367	68.416	80.183	68.200
Ω (deg)	355.650	345.914	0.420	345.805
i (deg)	143.417	141.659	143.972	141.703
q (AU)	0.005710	0.008387	0.005219	0.007979
e	0.9999306	0.9998986	0.9999386	0.9999065
P (yr)	746.1	752.2	784.4	787.8
Epoch (ET; o. s.)	356 Mar 15.0	356 Mar 15.0	1100 Mar 29.0	1106 Jan 17.0

[a] This set of orbital elements is identical with the set in col. (3) of Table 2 of Sekanina & Chodas 2002a; the apparent disagreements in the eccentricity and orbital period are due to the choice of different osculation epochs.

X/1106 C1—years later. The actual orbital period of superfragment I was almost exactly 744 yr, so in 1100 it arrived at the same time of the year as in 356: under identical observing conditions and again undetected.

We conclude that this proposed birth scenario for the subgroups is plausible and that this conceptual model for the Kreutz system evolution should be explored further. To demonstrate that Tables 2, 3, and 4 describe the results of a meaningful hypothesis, it is indeed desirable to show that the motions of all known members of the sungrazer system can be modeled in a consistent manner as products of subsequent fragmentation events.

To address this issue, different techniques need to be employed for (1) the bright sungrazers, whose birth can individually be traced with some confidence to their common parent(s), and (2) the minor, coronagraphically discovered fragments, whose motions preserve the memory of only the most recent period of their fragmentation evolution and whose sources can only be explored by means of statistical, Monte Carlo techniques. In the following, we provide a comprehensive account of the orbital history of the bright sungrazers and leave the problem of minor fragments for a future investigation.

5. COMET C/1965 S1 (IKEYA-SEKI)

The separation of this sungrazer from its common parent with C/1882 R1 was investigated elsewhere (Sekanina & Chodas 2002a). Here we only summarize our results: (1) this fragmentation event was found to have occurred 18 ± 7 days after perihelion, on or about 1106 February 13 (old style), at a heliocentric distance of 0.75 ± 0.19 and 0.39 ± 0.10 AU below the ecliptic, and (2) the two fragments separated from each other in, or close to, the parent's orbital plane at a rate of about 7 ± 1 m s^{-1}, with Ikeya-Seki moving nearly in the antisolar direction relative to C/1882 R1.

6. COMET C/1970 K1 (WHITE-ORTIZ-BOLELLI)

The birth of this object was examined extensively in our previous study (Sekanina & Chodas 2002b). We concluded that the direct parent of White-Ortiz-Bolelli was neither C/1882 R1 nor C/1965 S1 nor their parent, X/1106 C1. We proposed that, instead, C/1970 K1 separated from an unknown parent fragment, which itself broke off from the 1106 comet at the same time as, or shortly before, C/1882 R1 and C/1965 S1 were born. This postulated direct parent of White-Ortiz-Bolelli subsequently split into two at a heliocentric distance of some 150 AU around the mid-18th century, with C/1970 K1 separating from the rest of the mass at a rate of 3–5 m s^{-1} in the general direction of the Sun and to the north of the parent's orbital plane.

We calculated that the other fragment reached perihelion a few months later than White-Ortiz-Bolelli, between 1970 June and August. It was presumed to have been missed on account of unfavorable observing conditions.

7. COMET C/1963 R1 (PEREYRA)

No scenario was proposed for the birth of comet Pereyra in our previous investigations, although we considered it at one point a potential parent candidate for C/1970 K1 (Sekanina & Chodas 2002b). Establishing Pereyra's history and source of origin is critical because it is one of only three sungrazers, next to C/1882 R1 and C/1965 S1, whose orbital period has been determined with high accuracy (Marsden, Sekanina, & Everhart 1978), as shown in Table 5. Pereyra also played an essential role in Marsden's (1989) intriguing scenario, in which this comet was required to make one revolution about the Sun in a time span only slightly longer than that needed by the sungrazers C/1843 D1 and C/1880 C1 to make two complete revolutions.

7.1. *Premise of Common Origin with C/1843 D1*

Based on the orbital and intrinsic brightness data, the most logical fragmentation scenario for the birth of comet Pereyra should be its common origin with C/1843 D1. The two sungrazers are the second and third brightest, and their near-perihelion osculating orbits differ by only $3\overset{\circ}{.}5$ in ω, $4\overset{\circ}{.}4$ in Ω, $0\overset{\circ}{.}2$ in i, and 0.1 $R_\odot$ in q. However, they arrived at perihelion 120 yr apart, which requires a separation to have occurred fairly near the Sun. We explored hundreds of scenarios with assumed fragmentation times of up to ~100 days from perihelion, some even farther away.

We began with a large set of potential events during the return of 1100, both before and after perihelion, but found a very persistent problem: a failure of all solutions to converge, even when the number of unknowns to solve for was strongly constrained. The convergence was especially poor in the perihelion time. In addition, there were always large residuals, several degrees or more, in at least some of the angular elements, and the separation velocities required were entirely outside a plausible range, typically hundreds of meters per second.

We hoped that the 120 yr long gap between the perihelion passages of the two objects could be more easily spanned on the assumption that the two sungrazers separated from each

TABLE 5

Adopted Orbital Elements for Comets C/1880 C1, C/1887 B1, and C/1963 R1 (Pereyra) (Equinox J2000.0)

Element	Comet C/1880 C1[a,b]	Comet C/1887 B1[a,c]	Comet C/1963 R1[d]
T (ET)	1880 Jan 28.09679	1887 Jan 11.934 ± 0.069	1963 Aug 23.95638 ± 0.00491
ω (deg)	85.1285	83.513 ± 0.525	86.16006 ± 0.03348
Ω (deg)	6.4762	4.585 ± 0.646	7.93929 ± 0.04894
i (deg)	144.5226	144.383 ± 0.083	144.58207 ± 0.00552
q (AU)	0.0055347	0.004834 ± 0.000091	0.0050649 ± 0.0000146
e	(0.9999358)	(1.0)	0.9999458 ± 0.0000005
P (yr)	(800)	...	903 ± 13
Epoch (ET)	1880 Feb 15.0[e]	1887 Jan 29.0[e]	1963 Sep 8.0

[a] Planetary perturbations and relativistic effect not included.
[b] As derived by Kreutz 1901, with a forced orbital period of 800 yr (parenthesized).
[c] As derived by Sekanina (Marsden & Roemer 1978), with use of a custom-made technique; parabolic approximation.
[d] As derived by Marsden et al. 1978; planetary perturbations included.
[e] Standard, 40 day osculating epoch nearest the midtime of astrometric observations adopted.

other during the previous return to the Sun, in A.D. 356. Unfortunately, this did not turn out to be the case. Only several of the solutions showed a tendency to converge, but they all provided inferior orbital solutions and required unacceptably high separation velocities of several tens of meters per second or more. After exhausting all possible avenues that we could come up with, we reluctantly abandoned the idea of a direct relationship between Pereyra and C/1843 D1.

7.2. *Premise of Common Origin with C/1965 S1*

The failure to link C/1963 R1 to C/1843 D1 eliminated what we considered the most attractive scenario for explaining the motion of Pereyra, and there were very few choices left. A potential link of C/1963 R1 to C/1880 C1 or C/1887 B1 is of little interest because neither of these comets was bright enough to serve as Pereyra's parent.

The rest of the major sungrazers consist of subgroup II or IIa members. A search for scenarios that could link Pereyra to a subgroup II object would contradict the currently recognized status of the subgroups (§ 2) and question their significance in the Kreutz system evolution. Yet, unless one is prepared to concede that Pereyra is unrelated to all other known bright sungrazers, an exploration of a potential cross link between the subgroups is necessary. Conducting such a search is also in the interest of completeness of this investigation.

Our first object of interest was Ikeya-Seki, which passed through perihelion only about 2 yr after Pereyra did. The required separation velocity should be much lower than in a scenario in which Pereyra would share a common parent directly with C/1882 R1.

The most surprising result of this search was a rapidly converging solution, which fitted Pereyra's orbit at better than a 2 σ level and implied a very recent fragmentation event, in the mid-19th century, with an uncertainty of about ±14 yr. Unfortunately, the required radial component of the separation velocity came out to be completely unacceptable, $\sim$95 m s^{-1}. It turned out that the separation of 2 yr between the perihelion times of the two comets was still too much to accommodate, given the very recent birth date.

7.3. *Adopted Birth Scenario*

Our experience with the search for a fragmentation event involving C/1963 R1 and C/1965 S1 is reminiscent of the C/1970 K1 versus C/1965 S1 case (Sekanina & Chodas 2002b; see also § 6). Use of the same argument, sharing a common parent with a missing hypothetical comet that passed through perihelion under unfavorable circumstances, was explored next in an effort to find an acceptable fragmentation solution similar to that for White-Ortiz-Bolelli.

To begin with, we assumed that Pereyra's precursor was released from X/1106 C1 (=superfragment II) at the same time as Ikeya-Seki (Sekanina & Chodas 2002a) and the parent of C/1970 K1 (see the E_1-type scenarios in Sekanina & Chodas 2002b). We varied all three separation velocity components in order to make the hypothetical parent of Pereyra pass through perihelion between late July and early August of 1963, that is, shortly before Pereyra's perihelion, which was on 1963 August 23. With the normal component having hardly any effect, the arrival time to perihelion was found to be most sensitive to the radial component, varying at a rate of $\sim$46 days per 1 cm s^{-1}. The rate of change with the velocity's transverse component was about a factor of 10 lower. All scenarios in which the perihelion time of Pereyra's precursor occurred a few weeks prior to Pereyra's arrival led to four-parameter solutions requiring essentially the same separation velocity for Pereyra, $\sim$10 m s^{-1}, and placed the fragmentation event in the mid-19th century at a heliocentric distance somewhat greater than 100 AU. This makes Pereyra the youngest fragment among the bright sungrazers, with an approximate age only slightly exceeding 100 yr at the time of its 1963 perihelion.

The adopted fragmentation constants for the precursor and the results from a representative four-parameter solution for Pereyra are listed in Table 6, which also shows the quality of fitting the comet's observed motion. The values of the normalized orbital residuals indicate that, with the exception of the eccentricity, the agreement is always better than 3 σ. The discussion of the implications of this unexpected, apparently direct relationship of a member of subgroup I with members of subgroup II is postponed to § 11, along with a critical assessment of the role of the precursors.

8. GREAT SOUTHERN COMET C/1880 C1

This sungrazer was already shown to share a common parent, now called superfragment I, with C/1843 D1 (Sekanina & Chodas 2002b). However, the 1843 osculating value for the orbital period of C/1843 D1 was in that paper assumed to be exactly 800 yr, yielding the previous perihelion passage to have taken place in the year 1048, more than 50 yr much too early relative to the time derived from the present model (Table 4). Although we showed earlier (Sekanina & Chodas 2002a) that the fragmentation time relative to perihelion is insensitive to the adopted orbital period within broad limits, it

TABLE 6

Parameters of Fragmentation Events Leading to Birth of C/1963 R1 (Pereyra) from Superfragment II (X/1106 C1) and Computed Elements for C/1963 R1 with Residuals (Equinox J2000.0)

Fragmentation Parameter	Birth of Precursor	Birth of Pereyra
Time and location of fragmentation event:		
Calendar time (ET)	1106 Feb 13.5	1847.13 ± 0.51
Time after perihelion of X/1106 C1	18 days	741 yr
Heliocentric distance (AU)	0.75	112.73 ± 0.27
Distance from ecliptic (AU)	−0.39	−65.15 ± 0.15
Separation velocity (m s^{-1}):		
Radial component, V_R	+6.87	+2.83 ± 0.02
Transverse component, V_T	+0.70	−7.32 ± 0.73
Normal component, V_N	0.00	−6.75 ± 0.59
Total, V_{sep}	6.91	10.35 ± 0.64

Orbital Element	Computed set of orbital elements for C/1963 R1	Residual: Observed Minus Computed — In Absolute Units	Residual: Observed Minus Computed — In Dimensionless Normalized Units[a]
T (days)	1963 Aug 23.95638 ET	0.00000	0.000
ω (deg)	86.07839	+0.08167	+2.439
Ω (deg)	8.08186	−0.14257	−2.913
i (deg)	144.58194	+0.00013	+0.024
q (AU)	0.0050461	+0.0000188	+1.288
e	0.9999439	+0.0000019	+3.800
P (yr)	852.8	...	...
Epoch	1963 Sep 8.0 ET	...	...
$[\mathcal{P}_0]$	...	...	30.5338
rms error	...	...	±3.907

[a] Weighting system II (units are the errors of the orbital elements of comet C/1963 R1 from Table 5); $[\mathcal{P}_0]$ has the same meaning as in Table 2, except that it also includes the residual of optimized eccentricity.

is prudent to verify that our former scenario for the pair of C/1843 D1 and C/1880 C1 has not been invalidated.

To this end, we made an attempt to reproduce the orbital elements of C/1880 C1 (Table 5) from the motion of C/1843 D1 (Table 1) with the eccentricity adjusted to fit the perihelion time in the year 1100 (Table 4). Since the rms errors for the orbital set of C/1880 C1 were not listed by Kreutz (1901), we have used weighting system I, based on the orbital errors of C/1843 D1. The eccentricity, being a forced element, has again been excluded from the optimization procedure.

The results of the optimized four-parameter solution are summarized in Table 7. Comparison with our previous findings (Sekanina & Chodas 2002b) shows that the time of fragmentation relative to perihelion has not changed at all, although it now appears to be better determined. The separation velocity has increased nominally from 7.4 to 8.0 m s^{-1}, thus remaining within its error of ±0.8 m s^{-1} (see Table 10 of Sekanina & Chodas 2002b). The separation velocity vector has shifted by not more than a few degrees. The normalized rms error of the solution is well within a 1 σ limit, so the two-superfragment model clearly yields for C/1880 C1 (whose orbit is identical with that for fragment Ia, its direct parent; see §§ 9.3 and 11.2 and Fig. 2) a very satisfactory result.

9. GREAT SOUTHERN COMET C/1887 B1

The peculiar appearance of C/1887 B1 was already alluded to in § 3. The comet consisted of nothing but a long, straight, sharp, and narrow tail, unquestionably of dust nature (Sekanina 1984). Absolutely no head or condensation was detected during the period of 10 days, 1887 January 20–30, when the object was under observation. Several very discordant sets of orbital elements were published in the literature, all suffering from the absence of a point to bisect (e.g., Oppenheim 1889; Kreutz 1901; Marsden 1967). The first author eventually derived an orbit using a custom-designed technique. Although the resulting set of elements was referred to long ago (Marsden & Roemer 1978) and subsequently used in an investigation of the sungrazer's tail (Sekanina 1984), no description of the orbit determination technique has ever been published. Since we strive here to present sources of orbital information for each of the eight bright sungrazers, it is appropriate to provide a brief account of the computer approach employed in deriving the arguably best available orbital set for C/1887 B1.

9.1. *Custom-made Orbit Determination Technique*

A total of 21 tail-axis orientation data have been collected in the literature (Finlay 1887; Todd 1887; Thome 1887a, 1887b; Tebbutt 1887a, 1887b; Cruls 1887), which are of two types: (1) astrometric estimates of the tail's densest part, that is, its (sunward) tip, and (2) brief descriptions and/or charts of the tail train in the star field. The location of the tail's tip was determined either by reading the circles of a telescope or by approximating its position by the position of a field star. Thome (1887a, 1887b) pointed out that he moved the telescope sunward along the tail axis until the tail faded away. In any case, the positional accuracy of the measured tip was extremely poor along the tail, with an estimated uncertainty of at least ~0°.3, but it was much (nearly a factor of 10) better in the direction perpendicular to the tail. Consequently, there was a good chance to improve the quality of the comet's orbit

TABLE 7

PARAMETERS OF FRAGMENTATION EVENT GIVING BIRTH TO C/1880 C1 FROM COMMON PARENT WITH C/1843 D1 AND COMPUTED ELEMENTS FOR C/1880 C1 WITH RESIDUALS (EQUINOX J2000.0)

Fragmentation Parameter	Value
Time and location of fragmentation event:	
Date (ET)	1100 Aug 2
Days after perihelion[a]	127 ± 10
Heliocentric distance (AU)	2.77 ± 0.15
Distance from ecliptic (AU)	-1.57 ± 0.09
Separation velocity (m s^{-1}):	
Radial component, V_R	$+6.75 \pm 0.19$
Transverse component, V_T	$+2.44 \pm 0.52$
Normal component, V_N	-3.53 ± 0.23
Total, V_{sep}	8.00 ± 0.25

Orbital Element	Computed Set of Orbital Elements for C/1880 C1	Residual: Observed Minus Computed	
		In Absolute Units	In Dimensionless Normalized Units[b]
T (days)	1880 Jan 28.09679 ET	0.00000	0.000
ω (deg)	85.1084	+0.0201	+0.335
Ω (deg)	6.4987	−0.0225	−0.306
i (deg)	144.5229	−0.0003	−0.033
q (AU)	0.0055355	−0.0000008	−0.050
e	0.99993442	...	...
P (yr)	775.5	...	...
Epoch	1880 Feb 15.0 ET	...	...
[$\mathcal{P}_0$]	...	...	0.2095
rms error	...	...	± 0.458

[a] Perihelion passage occurred on 1100 March 27.99 ET (Table 4).

[b] Weighting system I (units are the errors of the orbital elements of comet C/1843 D1 from Table 1); [$\mathcal{P}_0$] has the same meaning as in Table 2.

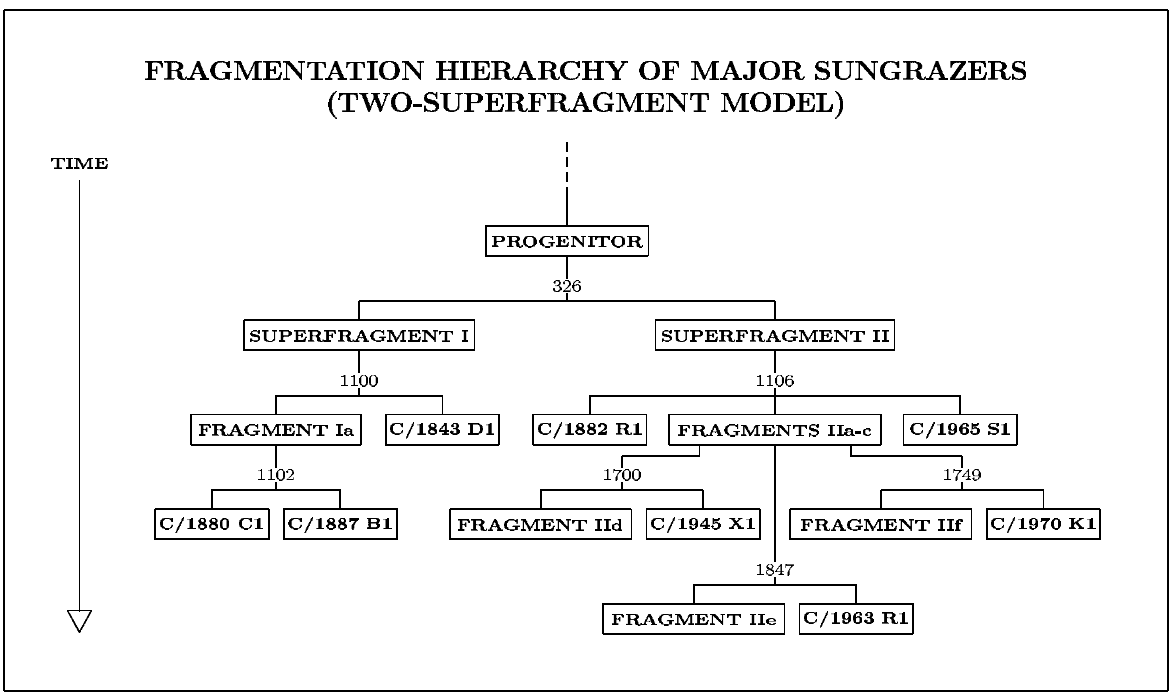

FIG. 2.—Fragmentation hierarchy of the bright sungrazers observed from the ground between 1843 and 1970. Fragments Ia and IIa–f have not been observed. The year of each fragmentation event is identified at the point of branching. Time increases from top downward.

determination by requiring that it crossed the great circle fitted through the observed positions of the tail axis at the reported times. This is like determining an orbit only from declinations or only from right ascensions. By itself, such a solution would of course be indeterminate, unless it is strengthened by an additional constraint. In this case, the orbit was required to satisfy the spatial orientation of the common line of apsides of the Kreutz system (see, e.g., Marsden 1967, 1989; Sekanina 2002a). With this constraint, only three independent elements were to be solved for in the case of a parabolic approximation, namely, the time of perihelion passage, the perihelion distance, and one of the three angular elements.

9.2. *Technique's Description and Equations*

Of the 21 observations available, two points on the tail axis were reported in eight cases, three points in 11 cases, and five points in two cases. In 18 cases, one of the reported points was the sunward tip, close to a hypothetical position of the undetected nucleus. The equatorial coordinates, $\{\alpha_k, \delta_k\}$, of each set of the n tail-axis points ($k = 1, \ldots, n$) were fitted (by least squares, if more than two) by the equation of the great circle,

$$\tan\delta_k = A\cos\alpha_k + B\sin\alpha_k, \tag{2}$$

where

$$A = -\cot\delta_{\rm pole}\cos\alpha_{\rm pole},$$
$$B = -\cot\delta_{\rm pole}\sin\alpha_{\rm pole}, \tag{3}$$

and $\{\alpha_{\rm pole}, \delta_{\rm pole}\}$ are the equatorial coordinates of the great circle's pole.

Because of the observational uncertainties, the great circle does not pass exactly through the nucleus with the equatorial coordinates $\{\alpha, \delta\}$ but misses it at a distance ϵ, which is expressed by

$$\sin\epsilon = \sin\delta\sin\delta_{\rm pole} + \cos\delta\cos\delta_{\rm pole}\cos(\alpha - \alpha_{\rm pole}). \tag{4}$$

Since this distance is always very small, one can approximate $\sin\epsilon \simeq \epsilon$, and since the offset residual "observed minus computed" $O - C = -\epsilon$, the equations of condition can be written thus:

$$\xi\Delta x + \eta\Delta y + \zeta\Delta z + \epsilon = 0, \tag{5}$$

where

$$\begin{pmatrix}\xi\\ \eta\\ \zeta\end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix}\cos\delta_{\rm pole}\cos\alpha_{\rm pole}\\ \cos\delta_{\rm pole}\sin\alpha_{\rm pole}\\ \sin\delta_{\rm pole}\end{pmatrix}, \tag{6}$$

Δ is the comet's geocentric distance at time t, and Δx, Δy, and Δz are the differential increments of the comet's heliocentric equatorial coordinates $x(t)$, $y(t)$, and $z(t)$. Before Δx, Δy, and Δz are expressed in terms of the differential increments of the orbital elements, $\Delta T, \ldots, \Delta q$, the relationships among the three angular elements need to be introduced from the condition that they satisfy the prescribed orientation of the line of apsides. Taking, for example, the longitude of the ascending node as an independent variable and the argument of perihelion and the inclination as dependent variables, one finds

$$\cos\omega = \cos B_\pi \cos(L_\pi - \Omega),$$
$$\cot i = \cot B_\pi \sin(L_\pi - \Omega), \tag{7}$$

where L_π and B_π are, respectively, the adopted standard values of the ecliptical longitude and latitude of perihelion. The final form of the equations of condition for parabolic motion is then

$$\left(\xi\frac{\partial x}{\partial T} + \eta\frac{\partial y}{\partial T} + \zeta\frac{\partial z}{\partial T}\right)\Delta T + \left(\xi\frac{\widehat{\partial x}}{\partial\Omega} + \eta\frac{\widehat{\partial y}}{\partial\Omega} + \zeta\frac{\widehat{\partial z}}{\partial\Omega}\right)\Delta\Omega$$
$$+\left(\xi\frac{\partial x}{\partial\ln q} + \eta\frac{\partial y}{\partial\ln q} + \zeta\frac{\partial z}{\partial\ln q}\right)\Delta\ln q + \epsilon = 0, \tag{8}$$

where

$$\frac{\widehat{\partial x}}{\partial\Omega} = \frac{\partial x}{\partial\Omega} + \frac{\partial x}{\partial\omega}\frac{\partial\omega}{\partial\Omega} + \frac{\partial x}{\partial i}\frac{\partial i}{\partial\Omega}, \tag{9}$$

and similarly for $\widehat{\partial y/\partial\Omega}$ and $\widehat{\partial z/\partial\Omega}$. The expressions for $\partial\omega/\partial\Omega$ and $\partial i/\partial\Omega$ follow directly from equation (7), whereas $\partial x/\partial T$, $\partial x/\partial\omega$, $\partial x/\partial\Omega$, $\partial x/\partial i$, $\partial x/\partial\ln q$, etc., are given by the well-known formulae of the orbital differential correction method. The routine iterative least-squares technique applied to the normal equations formed from equation (8) allows one to optimize the solution for the elements T, Ω, and q, with the elements ω and i derived from equation (7).

9.3. *Orbital and Fragmentation Solutions*

The described orbit determination method was used to calculate the motion of the comet's undetected nucleus from the tail data. The critical entries derived from the 21 observations are listed in Table 8. The residuals $O - C$ between the observed and the calculated positions of the tail axis have been determined in a two-step iteration process, with the perihelion longitude L_π and latitude B_π revised from their subgroup-independent values of, respectively, 282°.7 and +35°.2 in the first iteration to 282°.55 and +35°.35 (equinox J2000.0) after it became apparent that the comet is a member of subgroup I. Six of the 21 data points that left residuals greater than 5′ were removed and are parenthesized in Table 8.

The resulting set of orbital elements, presented in Table 5, was next employed to investigate candidate scenarios for an assumed common origin of C/1887 B1 with C/1843 D1 or C/1880 C1. Even though the premise of C/1887 B1 separating directly from the precursor of C/1843 D1 offered slightly better solutions in terms of formal errors, all such solutions required separation velocities exceeding 15 m s^{-1}. The problem was with the normal component for assumed fragmentation events occurring nearer the Sun, but with the radial component for episodes farther away.

The premise of a common parent for C/1887 B1 and C/1880 C1 led to slightly worse, but still excellent, solutions, given the orbital uncertainties of C/1887 B1. The best among these solutions suggested the fragmentation event occurring in 1101 March, about 1 yr after perihelion, but they implied a separation velocity that exceeded 12 m s^{-1}. For later separation times the fit deteriorated only marginally, while the required separation velocity was dropping rapidly. Our preferred solutions are those with the separation times in 1102 December with a velocity below 7 m s^{-1}. A representative fragmentation scenario is described in Table 9, which shows that the sum of squares of the dimensionless normalized residuals, $[\mathcal{P}_0]$, is only 0.12, a factor of ∼40 lower than would be the value corresponding to a 1 σ residual in each of the tested elements.

The separation of C/1887 B1 from its common parent with C/1880 C1, which itself broke off from a precursor it shared

TABLE 8

Tail Axis of Comet C/1887 B1 and Orbital Residuals (Equinox J2000.0)

Date (UT)	Tail's Pole Position α_{pole}	δ_{pole}	Residual[a] $O - C$ (arcmin)	Observer and Site
1887 Jan 20.490..............	2 33.87	+6 59.0	(−14.5)	Todd (Adelaide)
1887 Jan 21.490..............	2 32.38	+10 52.7	−2.8	Todd (Adelaide)
1887 Jan 22.046..............	2 29.83	+13 23.1	+2.8	Thome (Cordoba)
1887 Jan 22.490..............	2 31.21	+14 24.5	(+11.3)	Todd (Adelaide)
1887 Jan 22.817..............	2 34.98	+14 54.1	+1.4	Finlay (Cape of Good Hope)
1887 Jan 23.046..............	2 33.81	+16 01.2	−1.3	Thome (Cordoba)
1887 Jan 23.827..............	2 31.16	+19 02.6	+4.5	Finlay (Cape of Good Hope)
1887 Jan 24.532..............	2 27.55	+21 51.2	(+8.2)	Todd (Adelaide)
1887 Jan 24.834..............	2 30.67	+22 22.2	+0.1	Finlay (Cape of Good Hope)
1887 Jan 24.946..............	2 33.26	+22 26.6	(−9.2)	Cruls (Rio de Janeiro)
1887 Jan 25.046..............	2 38.58	+21 03.9	(+25.6)	Thome (Cordoba)
1887 Jan 25.490..............	2 34.13	+23 43.6	+0.5	Todd (Adelaide)
1887 Jan 25.844..............	2 34.08	+24 48.4	−1.0	Finlay (Cape of Good Hope)
1887 Jan 26.046..............	2 34.78	+25 18.2	−2.0	Thome (Cordoba)
1887 Jan 26.490..............	2 35.92	+26 26.0	−3.8	Todd (Adelaide)
1887 Jan 27.501..............	2 36.71	+29 01.0	+0.5	Todd (Adelaide)
1887 Jan 27.834..............	2 33.68	+30 18.3	−1.4	Finlay (Cape of Good Hope)
1887 Jan 28.477..............	2 39.35	+31 19.4	−3.9	Tebbutt (Windsor)
1887 Jan 28.848..............	2 37.11	+32 23.8	+1.7	Finlay (Cape of Good Hope)
1887 Jan 29.834..............	2 41.05	+34 19.0	+4.8	Finlay (Cape of Good Hope)
1887 Jan 30.522..............	3 00.86	+34 49.5	(−35.6)	Tebbutt (Windsor)

Note.—Units of right ascension are hours and minutes, and units of declination are degrees and arcminutes.

[a] Parenthesized entries were not incorporated in the least-squares differential correction optimization process.

TABLE 9

Parameters of Fragmentation Event Giving Birth to C/1887 B1 from Common Parent with C/1880 C1 and Computed Elements for C/1887 B1 with Residuals (Equinox J2000.0)

Fragmentation Parameter	Value
Time and location of fragmentation event:	
Date (ET)	∼1102 Dec
Days after perihelion in late March of 1100....	∼1000
Heliocentric distance (AU)	∼10.9
Distance from ecliptic (AU)	∼−6.2
Separation velocity (m s^{-1}):	
Radial component, V_R......................................	+2.46 ± 0.01
Transverse component, V_T...............................	+1.92 ± 0.63
Normal component, V_N....................................	+5.86 ± 0.28
Total, V_{sep}...	6.64 ± 0.31

Orbital Element	Computed Set of Orbital Elements for C/1887 B1	Residual: Observed Minus Computed: In Absolute Units	In Dimensionless Normalized Units[a]
T (days)..	1887 Jan 11.934 ET	0.000	0.000
ω (deg)	83.597	−0.084	−0.160
Ω (deg)...	4.410	+0.175	+0.271
i (deg)...	144.396	−0.013	−0.157
q (AU)..	0.004833	+0.000001	+0.011
e..	0.9999473	. . .	. . .
P (yr)..	878.7	. . .	. . .
Epoch	1887 Jan 29.0 ET	. . .	. . .
[$\mathcal{P}_0$]...	. . .	. . .	0.1238
rms error..	. . .	. . .	±0.249

[a] Weighting system II (units are the errors of the orbital elements of comet C/1887 B1 from Table 5); [$\mathcal{P}_0$] has the same meaning as in Table 2.

TABLE 10

Residuals for Astrometric Observations of C/1945 X1 (du Toit) from Three Orbital Solutions (Equinox J2000.0)

Time of Observation (UT)	Residuals[a] from Orbital Solutions (arcsec)					
	Based on Eight Points		Based on Seven Points		Based on Six Points	
	R.A.	Decl.	R.A.	Decl.	R.A.	Decl.
1945 Dec 11.04687......	−1.9	+1.1	−0.3	+0.5	0.0	+0.1
1945 Dec 12.04691	+4.0	+0.5	(+6.8)	+0.8	(+6.1)	−0.1
1945 Dec 13.07189	−1.7	−2.8	+0.9	−1.9	0.0	(−3.2)
1945 Dec 15.06885	−0.3	+1.1	−0.6	+0.7	0.0	0.0

[a] Residuals in R.A. include cos (decl.) factor; rejected data are parenthesized.

with C/1843 D1, completes the fragmentation hierarchy of superfragment I. Both C/1880 C1 and C/1887 B1 belong to a third generation of fragments.

10. COMET C/1945 X1 (DU TOIT)

This is the only major sungrazer that was observed only before perihelion, photographically on five consecutive nights between 1945 December 11 and 15, by the discoverer at the Boyden Observatory in Bloemfontein, South Africa. The plates had not been properly measured and reduced until 7 yr later, and the astrometric observations, including a reconstructed one from December 14 (which had been found inferior by Marsden 1967 and was subsequently lost), were eventually published by Marsden (1989). The parabolic orbit most consistent with four of the five positions was computed by Marsden (1967); four alternative solutions were presented by him more recently (Marsden 1989).

10.1. *Orbit Uncertainties*

We closely confirmed Marsden's (1967) parabolic approximation, based on the positions from December 11, 12, 13, and 15, yet we were interested in further experimenting with the observations using additional constraints. Our orbit determination code automatically accounts for the planetary perturbations and the relativistic effect, although both are negligible over the observed orbital arc of C/1945 X1. The second and third columns of Table 10 list the residuals from our eight-point (four-position) solution. They never deviate more than $0\farcs1$ from Marsden's (1967) residuals, a difference so small that it can readily be attributed to rounding-off errors during the transformation of the positions from equinox B1950.0 to J2000.0. The December 12 residual in right ascension, $4''$, is somewhat worrisome (relative to the others), and we were curious to see what effect the elimination of this data point would have. A new fit, now based on seven data points, shows this residual in the fourth column of Table 10 to climb up to almost $7''$, but the orbital elements change by less than 0.001 days in T, $\sim 0\fdg5$ in ω, $\sim 0\fdg7$ in Ω, much less than $0\fdg1$ in i, and not even 0.01 $R_\odot$ in q. Similarly, a six-point solution could be derived, if one should be concerned with the December 13 residual in declination, although this would be hard to justify; besides, the effects on the elements are then even smaller.

Of greater importance are effects resulting from forcing the orbital period (or the eccentricity). We found that replacing a parabolic approximation with an 800 yr period ellipse leads to differences of 0.02 days in T, $1\fdg3$–$1\fdg4$ in ω and Ω, almost $0\fdg2$ in i, and more than 0.03 $R_\odot$ in q. These differences are equivalent to more than 20 σ in T, between 1.3 and almost 5 σ in the angular elements, and about 3 σ in q. In Table 11 we list the elements from the seven-point parabolic and 950 yr elliptical solutions, as well as the first- and second-order variations in the elements valid in the eccentricity range from about 0.99990 to 0.99995.

Independent information on true uncertainties involved in the orbital elements of C/1945 X1 is provided by checking the orientation of the line of apsides. The differences between the Kreutz system's standard coordinates for the direction to perihelion, $L_\pi = 282\fdg7$, $B_\pi = +35\fdg2$, and the coordinates for this sungrazer listed in Table 11 amount to almost 1° in both the longitude and latitude. These differences have nothing in common with the eccentricity-dependent variations in the elements of C/1945 X1 because the longitudes and latitudes derived from the 800 yr elliptical solution differ from those derived from the parabolic approximation by less than $0\fdg2$. Thus, there appears to be some evidence for additional uncertainties involved in the comet's derived orbit, which are not described by the formal errors of the elements and whose source is rather unclear.

This suspicion is amply supported by Marsden's (1989) calculation of alternative orbits, which were constrained to satisfy the standard perihelion direction of the Kreutz system, as in the case of C/1887 B1 (see § 9.3). These sets yielded for the angular elements values that are smaller than the values in Table 11 by 4°–6° in ω, 6°–8° in Ω, and $0\fdg2$–$0\fdg6$ in i. One of Marsden's alternative solutions, orbit D, based on the assumption of a barycentric orbit identical (except for T, of course) to that of C/1965 S1, left fairly small residuals (although not nearly as small as those in Table 10). From these results, C/1945 X1 is very probably a member of subgroup II, even though in terms of ω and Ω from Table 11 this sungrazer is only about twice as close to subgroup II as it is to subgroup I.

The root of this problem rests with the small number of astrometric data available and the short orbital arc covered. Indeed, it is not even clear which of the images yielded the best astrometric data. Marsden (1989) pointed out that it may have been the December 13 position (and perhaps also the December 11 one) that was inferior rather than the December 14 one. In any case, we need to keep these uncertainties in mind during our search for the source and the likely evolutionary path of C/1945 X1.

10.2. *Premise of Common Origin with C/1965 S1*

The orbital similarities suggest that one should begin with investigating a possible common origin of C/1945 X1 and C/1965 S1. While this scenario requires very low,

TABLE 11

Adopted Sets of Orbital Elements for Comet C/1945 X1 (du Toit) (Equinox J2000.0; Epoch 1945 Dec 11.0 ET)

			Variations[a] in $X = \{T, \omega, \Omega, i, q, L_\pi, B_\pi\}$	
Element	Parabolic Approximation	Forced Elliptical Solution	$\partial X/\partial e$	$\partial^2 X/\partial e^2$
T (ET)	1945 Dec 27.96604 $\pm$ 0.00094	1945 Dec 27.98358 $\pm$ 0.00088	−234.953	+57828.9
ω (deg)	72.6110 $\pm$ 0.8192	73.7629 $\pm$ 0.8182	−15478.5	+5522350
Ω (deg)	351.9335 $\pm$ 1.0895	353.2181 $\pm$ 1.0898	−17280.2	+6406130
i (deg)	141.9094 $\pm$ 0.0480	142.0613 $\pm$ 0.0350	−1923.23	−812644
q (AU)	0.0075481 $\pm$ 0.0000479	0.0074194 $\pm$ 0.0000507	+1.68419	−42.1839
e	1.0	0.999923226	...	...
P (yr)	...	950	...	...
L_π (deg)	283.632	283.486	+1945.1	−466410
B_π (deg)	+36.066	+36.178	−1483.1	+238060

[a] For forced eccentricity between about 0.99990 and 0.99995, a correction to element $X = \{T, \omega, \Omega, i, q, L_\pi, B_\pi\}$ in the third column is given by $\Delta X = (\partial X/\partial e)\Delta e + (\partial^2 X/\partial e^2)(\Delta e)^2$, where $\Delta e = e - 0.999923226$; e.g., $e = 0.99992045$ and $\Delta e = -0.000002776$ for $P = 900$ yr.

submeter-per-second transverse and normal components of the separation velocity over an extremely wide range of postulated fragmentation times, the problem is once again the velocity's radial component. The temporal separation of 20 yr between the arrivals of the two comets to the Sun demands that this component be at least a few tens of meters per second, whereas the near-optimized solutions require it to exceed 100 m s^{-1}. This result also automatically eliminates any direct link between C/1945 X1 and C/1882 R1 but leaves open the possibility of an indirect association, via yet another undetected fragment, similar to the adopted scenarios for C/1963 R1 and C/1970 K1.

10.3. *Adopted Birth Scenario*

Since C/1945 X1 passed through perihelion in late December, an undetected fragment sharing the direct parent with it would have to arrive at the Sun either 5–7 months earlier or 5–7 months later. Although we explored both options, we report only on one particular case of the first type because the required separation velocity was lower then.

The postulated parent was assumed to have separated from X/1106 C1 at the time of birth of C/1882 R1 and C/1965 S1 and in the same direction relative to C/1882 R1 as C/1965 S1 and the parents of C/1963 R1 and C/1970 K1. The separation velocity components of the parent of C/1945 X1 were dictated by the required arrival time in 1945 May–August. The representative case in Table 12 implies a perihelion passage on 1945 July 7. This parent then was assumed to split again into two, the undetected fragment and C/1945 X1.

A search for an optimized three-parameter solution was conducted by varying the parent's fragmentation time. The step gradually increased from 1 to 50 yr between 1107 and 1200, then remained constant at 50 yr until 1900. As $t_{\rm frg}$ increased, the fit to the orbital elements derived from the observations was steadily improving, but unfortunately the required separation velocity kept climbing, reaching ~7 m s^{-1} at $t_{\rm frg} = 1700$, ~10 m s^{-1} at 1750, ~15 m s^{-1} at 1800, ~28 m s^{-1} at 1850, and ~80 m s^{-1} at 1900. The preferred scenarios were those with the fragmentation time generally in the period of 1650–1750, which offered an acceptable compromise between the quality of fit (systematically deteriorating prior to 1650) and the realistically low separation velocity. This optimization process was applied to both the parabolic approximation and the forced elliptical orbit presented in Table 11 in the role of the proxy observations. A representative solution, summarized in Table 12, shows that there is not much difference between the two versions.

At first sight, the residuals are not entirely satisfactory, reaching ~5 σ, but comparison with one of Marsden's (1989) alternative solutions for C/1945 X1, detailed in a footnote to Table 12, suggests that our scenario yields a set of orbital elements that is still within the overall limits of this comet's orbital uncertainty. We believe that no more specific conclusion can be made.

11. DISCUSSION AND SUMMARY

An unprecedented, concerted effort to understand the observed motions of all eight bright sungrazers discovered from the ground between 1843 and 1970 has led to the proposed model for the origin and evolution of the Kreutz system. The applied methodology illustrates the potential of the back-and-forth orbit integration technique, a novel approach to comet fragmentation studies, and offers an avenue for addressing other similar topics, such as the formation of new near-Sun comet groups.

11.1. *Constraining the Solutions*

The modus operandi of our approach is based on a requirement that the relationships among the eight sungrazers be interpreted in the most focused manner, with the least number of fragmentation events assumed to have taken place in the shortest possible span of time. The multitude of solutions is constrained in two ways: (1) by requiring that no scenario be accepted without proper screening by an optimization procedure, and (2) by introducing a condition that the differential momenta acquired by the products of a fragmentation event be realistically low, with the corresponding separation velocities not exceeding ~10 m s^{-1}.

Because the partitioning function of the momentum distribution is not known for any fragmentation event, it is arbitrarily assumed that during any such episode the brightest fragment acquires no extra momentum whatsoever, that is, its motion is identical with the motion of the parent's center of mass before the event. This approximation is unquestionably a source of unchecked errors that propagate throughout the modeled fragmentation hierarchy. The introduced effect is believed to be usually insignificant (perhaps a fraction of 1 m s^{-1} or so in the separation velocity), except in instances of two or more nearly equally massive fragments at the upper end of the mass spectrum. This may have been the case with C/1882

TABLE 12

PARAMETERS OF FRAGMENTATION EVENTS LEADING TO BIRTH OF C/1945 X1 (DU TOIT) FROM SUPERFRAGMENT II (X/1106 C1) AND COMPUTED ELEMENTS FOR C/1945 X1 WITH RESIDUALS (EQUINOX J2000.0)

Fragmentation Parameter	Birth of Precursor Comet	Birth of Comet du Toit: Fitting Parabolic Approximation	Birth of Comet du Toit: Fitting Forced Elliptical Solution
Time and location of fragmentation event:			
Calendar time (ET)	1106 Feb 13.5	∼1700	∼1700
Time after perihelion of X/1106 C1	18 days	∼600 yr	∼600 yr
Heliocentric distance (AU)	0.75	∼160	∼160
Distance from ecliptic (AU)	−0.39	∼−90	∼−90
Separation velocity (m s^{-1}):			
Radial component, V_R	+5.43	+6.93 ± 0.01	+6.94 ± 0.01
Transverse component, V_T	+0.56	+0.71 ± 0.32	+0.54 ± 0.40
Normal component, V_N	0.00	−0.38 ± 0.24	−0.54 ± 0.21
Total, V_{sep}	5.46	6.98 ± 0.04	6.98 ± 0.04

Orbital Element	Fitting Parabolic Approximation: Computed Set of Orbital Elements for C/1945 X1	Residual $O-C$: In Absolute Units	Residual $O-C$: In Normalized Units[a]	Fitting Forced Elliptical Solution: Computed Set of Orbital Elements for C/1945 X1[b]	Residual $O-C$: In Absolute Units	Residual $O-C$: In Normalized Units[a]
T (days)	1945 Dec 27.96604 ET	0.00000	0.000	1945 Dec 27.98358 ET	0.00000	0.000
ω (deg)	69.2518	+3.3592	+4.101	69.6840	+4.0789	+4.985
Ω (deg)	347.1625	+4.7710	+4.379	347.7075	+5.5106	+5.057
i (deg)	141.9892	−0.0798	−1.662	142.1129	−0.0516	−1.474
q (AU)	0.0075479	+0.0000002	+0.004	0.0074191	+0.0000003	+0.006
e	0.99992175	...	...	0.99992309	...	...
P (yr)	947.4	...	...	947.4	...	...
Epoch	1945 Dec 11.0 ET	...	...	1945 Dec 11.0 ET	...	...
$[\mathcal{P}_0]$	...	...	38.7561	...	...	52.5962
rms error	...	...	±4.402	...	...	±5.128

[a] Weighting system II (units are the errors of the respective orbital elements of comet C/1945 X1 from Table 11); $[\mathcal{P}_0]$ has the same meaning as in Table 2.

[b] This set's residuals from the alternative orbits of Marsden 1989 are very different. For example, relative to his orbit B they amount to +0.008 days in T, $-1^\circ.32$ in ω, $-1^\circ.83$ in Ω, $-0^\circ.43$ in i, −0.00041 AU in q, and −0.00005 in e.

R1 at the time of its most recent perihelion passage (see § 4 of Sekanina & Chodas 2002a).

11.2. *Fragmentation Hierarchy of Major Sungrazers*

The birth of subgroups is proposed here to be the result of a progenitor's single breakup event that occurred at a heliocentric distance of 50 AU on the incoming branch of the orbit, in A.D. 326, less than 1700 years ago (§ 4.3). This episode was the beginning of a fragmentation process, which gave birth to the eight known major sungrazers and which is depicted as a pyramid-shaped construct in Figure 2. The first products were two superfragments, both of which were subsequently subjected to further splitting. A short implied age of the Kreutz system is one of the most significant findings of our study.

We calculate that the superfragments passed through perihelion within a week of each other in A.D. 356. However, there is no evidence that any of the other known bright sungrazers observed between 1843 and 1970 had been born before the superfragments returned to the Sun around 1100. The superfragments may not have split for nearly 800 yr, but if they had, their products did not survive or have all been missed.

The second fragmentation event, for which we have evidence among the known members of the Kreutz system, occurred in 1100, more than 4 months after the perihelion passage of superfragment I. This splitting gave birth to C/1843 D1 and to fragment Ia, the common parent of C/1880 C1 and C/1887 B1. These two sungrazers were born probably in late 1102.

Superfragment II, which is presumed to be identical with X/1106 C1, was shown in our earlier paper (Sekanina & Chodas 2002a) to have split into C/1882 R1 and C/1965 S1 about 18 days after perihelion. It was tempting to explore whether the motions of additional fragments could be matched on the assumption that their roots too were somehow related to that same event. In our previous study (Sekanina & Chodas 2002b), we already found this premise to be attractive in that it led us to a two-step birth scenario for C/1970 K1. Our effort now resulted in a finding that the birth of C/1945 X1 and, quite unexpectedly, C/1963 R1, a subgroup I member, could be described in the same fashion. Accordingly, our model postulates that X/1106 C1 split simultaneously not into two or three, but five pieces, which included fragments IIa, IIb, and IIc, in addition to C/1882 R1 and C/1965 S1. Fragment IIa is proposed to have subsequently split into fragment IId and C/1945 X1 around 1700 (±50 yr), fragment IIb into fragment IIe and C/1963 R1 in 1847, and fragment IIc into fragment IIf and C/1970 K1 in 1749 (see Fig. 2). The origin of all eight bright sungrazers observed between 1843 and 1970 has thereby been accounted for. Fragments IId, IIe, and IIf have been missed, and thus far we have suggested that this was so because of unfavorable observing conditions. We return to this issue in § 11.5.

11.3. *The Progenitor Comet*

Our proposed scenario for the evolution of the major members of the Kreutz system would not offer a compelling picture if the question of why the progenitor comet split at a large heliocentric distance, rather than in the immediate proximity of the Sun, remains unanswered. To address this problem, a plausible conceptual hypothesis is required for the progenitor's early evolution that had led to the first fragmentation event.

Bailey, Chambers, & Hahn (1992) showed that a temporary sungrazing state is a product of long-term indirect planetary perturbations of comets in orbits that initially had perihelion distances less than about 2 AU and whose planes were nearly perpendicular to the ecliptic. The essentially gradual, correlated changes in the orbital elements that were triggered in this way caused the perihelion distance to reach its critical minimum at intervals on the order of 1000 revolutions about the Sun. Because the conditions on the initial orbital size and orientation are so soft, they are satisfied by a large number of comets. Numerous examples of the proposed orbital evolution are indeed presented by Bailey et al. (1992), who also emphasize that the timescale of this cycle is much shorter than that of the dynamical ejection process.

Since the decrease in the perihelion distance is fairly smooth, the stress buildup caused by the Sun's tidal force in the comet's nucleus from one return to the next is gradual. However, the tidal stress is not the only force affecting the nucleus. The comet's rotation (especially a rapid one) and both diurnal and seasonal heating and cooling of the nucleus surface due to the variable insolation also generate stresses, and these, while presumably small, are acting during much of, or throughout, the orbit about the Sun. The tidal stress is known to vary as the square of the characteristic nucleus dimension (diameter) and inversely as the cube of its distance from the perturbing body. Thus, a comet with a 5 times larger nucleus should split at a nearly 3 times larger heliocentric distance, given the same mechanical strength of the material.

The exceptionally large size of the progenitor comet explains why we observe only one Kreutz system, even though there is an enormous supply source of candidate seed objects. With the nucleus diameter of C/1882 R1 estimated at $\sim$50 km (Sekanina 2002a), the progenitor's maximum dimension must have been close to 100 km. Because tidal splitting is nucleus size dependent, cometary nuclei much smaller than the Kreutz progenitor (but of the same tensile strength) are less likely to crack or break up in the Sun's corona. Larger (but not *very* large) comets would not split into fragments massive enough to survive another perihelion passage as independent objects, but only into minor fragments, such as the *SOHO* sungrazers, which succumb to the rampant erosion process. A recent detailed study of this process (Sekanina 2003) shows that a surviving fragment needs to be initially at least $\sim$1 km in diameter. Thus, it is only a very few parent sungrazers—the ones with *abnormally* large nuclei—that are capable of producing an authentic Kreutz-type system.

If the fragments' separation velocities are rotational in nature (see § 11.4), then the enormous size of the progenitor readily explains their fairly high values, on the order of several meters per second. For example, estimating the progenitor's maximum dimension at between 75 and 100 km, a splitting into two superfragments along the short axis (Fig. 1), with either of them acquiring a separation velocity of 8.2 m s^{-1} (Table 3) at its center of mass, would imply 4.0–5.3 hr for the progenitor's rotation period, in a plausible range. A conservation of angular momentum law may contribute dramatically to a rapid spin-up of smaller fragments of higher generations and thus to a major role of rotational bursting in the fragmentation process.

An important point is that the tensile strength of a comet's nucleus is not only extremely low (e.g., Sekanina 1982, 1996; Greenberg, Mizutani, & Yamamoto 1995) but necessarily also uneven throughout the interior, as dictated by the processes of comet formation. Location-dependent damage caused by the solar tides means that the nucleus first becomes riddled with cracks and fractures. Its disruption into two (or more) parts can only be completed after its strength has failed at all points along a cross-sectional area running through the interior. This apparently can happen anywhere in the orbit, judging from available information on cometary splitting (Sekanina 1997 and references therein). Of particular interest to us are (1) the history of D/Shoemaker-Levy 9, whose fragmentation hierarchy was described in detail by Sekanina, Chodas, & Yeomans (1998), and (2) the existence of pairs and clusters in the population of *SOHO* sungrazer minicomets (e.g., Sekanina 2000, 2002b). Both cases are excellent examples of a cascading sequence of discrete fragmentation events that make up a seemingly spontaneous continuation of the tidally triggered process. Thus, for sungrazers, the tidal effect always plays some role in their fragmentation, direct or indirect, regardless of whether a breakup occurs at perihelion or far from the Sun.

The orbits, computed for the progenitor comet in the first to third millennium B.C.E. with different momentum partitioning coefficients f (Table 3), provide only limited evidence on the trends in the perihelion distance that are expected from the Bailey et al. (1992) scenario. Two of the four tabulated cases, $f = 0.456$ and 0.547, show a systematic decrease in the perihelion distance with time, at average rates of, respectively, 0.15 and 0.09 $R_\odot$ per revolution, but for $f = 0.500$ there is a systematic increase at an average rate of 0.095 $R_\odot$ per revolution. A rate of change for a comet with an initial perihelion distance of 2 AU over a time interval of 1000 revolutions is, on the average, $\sim$0.4 $R_\odot$ per revolution, but its value near a minimum is of course expected to be much smaller.

We have consulted rather extensively several of the more recent sources of ancient and medieval bright comets, especially those with a suspected sungrazer type of orbit (Ho 1962; Hasegawa 1980; Hasegawa & Nakano 2001; England 2002; Strom 2002), with the aim of identifying either the progenitor or the superfragments with a historically recorded comet, but with the exception of X/1106 C1 we have found no correlation.

The final issue in this category concerns the potential existence of an extended Kreutz system. Specifically, are there other "branches" of the Kreutz system, which contain products of fragmentation events located temporally and/or spatially so unfavorably that we cannot detect them? For example, if the superfragments split during their return in the year 356, under what circumstances could all the fragments be missed? In addition, was the progenitor's breakup in 326 indeed its first fragmentation episode? These and similar questions are also fueled by possible Kreutz system members, which were observed (but poorly documented) in earlier times and ignored in this study. Especially intriguing is the possibility that four comets appearing near the end of the 17th century, C/1668 E1, C/1689 X1, C/1695 U1, and X/1702 D1, which were examined by both Kreutz (1901) and Marsden (1967, 1989), might somehow be related to the Kreutz system. Unfortunately, none of them have ever been proved a sungrazer, and our feeling about these and earlier candidate objects, discussed by Hasegawa & Nakano (2001), England (2002), and Strom (2002), is that the quality of available information does not warrant a type of study that the sungrazers from 1843 to 1970 have been afforded in this paper.

11.4. *Separation Velocities and Angular Momentum*

If the separation velocities are rotational in nature and if the progenitor's angular momentum has at least approximately been conserved during fragmentation, the vectorial distribution of separation velocities should exhibit a concentration toward a great circle, whose poles coincide with the progenitor's rotation poles. A strong effect of this kind was detected by Sekanina et al. (1998) for D/Shoemaker-Levy 9, and a good match was also found from analysis of three fragmentation solutions for the nuclei of comet 141P/Machholz 2 (Sekanina 1999).

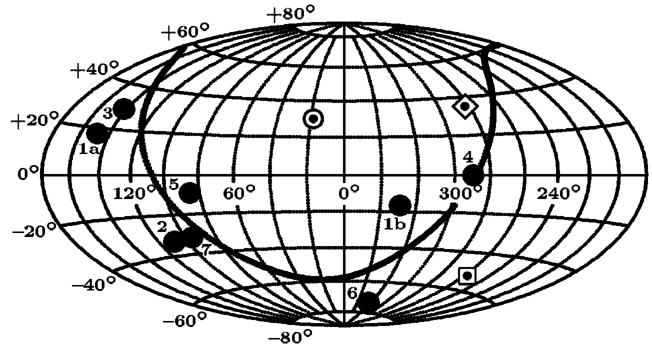

FIG. 3.—Separation velocity vector directions (*filled circles*) in a grid of ecliptical longitudes (increasing horizontally from right to left) and latitudes. The sungrazer identification is as follows: 1a—superfragment I, 1b—superfragment II (=X/1106 C1), 2—C/1965 S1, 3—C/1963 R1, 4—C/1970 K1, 5—C/1880 C1, 6—C/1887 B1, 7—C/1945 X1. The thick curve is the great circle fitted by least squares through the points 1a–7. The optimized position of the progenitor's nominal rotation pole is depicted by the circled dot. The direction to the Kreutz system's perihelion is represented by a diamond and the direction to the north orbital pole of C/1882 R1 by a square. The equinox is J2000.0.

Here we study the vectorial distribution in the ecliptical coordinate system, and our task is to determine longitude L_V and latitude B_V of the directions, in which the sungrazers' separation velocity vectors point. These directions are given by the ecliptical components of the separation velocities, $\{V_X, V_Y, V_Z\}$, which in turn are related to their RTN components, $\{V_R, V_T, V_N\}$. The transformation is accomplished by applying the equations

$$\begin{pmatrix} V_X \\ V_Y \\ V_Z \end{pmatrix} = \begin{pmatrix} P_x & Q_x & R_x \\ P_y & Q_y & R_y \\ P_z & Q_z & R_z \end{pmatrix} \begin{pmatrix} \cos u_{\rm frg} & -\sin u_{\rm frg} & 0 \\ \sin u_{\rm frg} & \cos u_{\rm frg} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_R \\ V_T \\ V_N \end{pmatrix}, \tag{10}$$

where $P_x, \ldots, R_z$ are the ecliptical components of the unit vectors $\boldsymbol{P}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$ directed, respectively, to the perihelion point, the point at true anomaly of $+90°$, and the northern pole of the parent's orbit at the fragmentation time $t_{\rm frg}$, and $u_{\rm frg}$ is the true anomaly at $t_{\rm frg}$. The vectors $\boldsymbol{P}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$ are expressible in terms of the parent's orbital elements ω, Ω, and i, referred to the osculation epoch at fragmentation, $t_{\rm frg}$. The ecliptical longitude and latitude are then calculated from the standard formulae: $L_V = \arctan(V_Y/V_X)$ and $B_V = \arcsin[V_Z/(V_X^2 + V_Y^2 + V_Z^2)^{1/2}]$.

The results of our analysis are summarized in Figure 3, which shows a fair degree of concentration of the points to a great circle. The corresponding coordinates, $L_{\rm rot}$, $B_{\rm rot}$, of the progenitor's rotation pole are

$$\begin{aligned} L_{\rm rot} &= 18° \pm 24°, \\ B_{\rm rot} &= 30° \pm 13°. \end{aligned} \tag{11}$$

Because the sense of rotation is unknown, it is unclear whether this is the northern pole (from which the comet is seen to rotate counterclockwise) or the southern pole.

Also plotted in Figure 3 are the standard direction to perihelion, $L_\pi = 282\overset{\circ}{.}7$, $B_\pi = +35\overset{\circ}{.}2$, and the direction to the north orbital pole of comet C/1882 R1, $L_{\rm orb} = 258°$, $B_{\rm orb} = -52°$. The rotation axis thus subtends an angle of $77° \pm 25°$ with the line of apsides, whereas the nucleus obliquity of C/1882 R1 can be estimated at $132° \pm 25°$ if equation (11) refers to the north pole, but at $48° \pm 25°$ if to the south pole.

11.5. *Missing or Disintegrating Fragments?*

It is reasonable to expect, as we did in our previous paper (Sekanina & Chodas 2002b) in the case of C/1970 K1, that one of two comets of common origin is missed because of poor observing conditions. Because of the spatial orientation of the Kreutz system's line of apsides, it is notoriously known (e.g., Marsden 1967) that even a bright sungrazer reaching perihelion between mid-May and mid-August will be missed by ground-based observers, unless it can be detected in daylight.

This kind of argument was behind our suggestion that C/1970 K1, whose perihelion passage occurred on 1970 May 14, was a member of a sungrazer pair, the missed component of which followed C/1970 K1 closely, within 10–12 weeks. (The first 3 weeks of June should probably be excepted because of the *Soyuz 9* mission with cosmonauts on board.) As the orbit of C/1970 K1 is not particularly close to those of C/1882 R1 or C/1965 S1, the extra momentum acquired during a fragmentation event far from the Sun should have delivered the needed orbital perturbation.

Since it was now necessary to use the same argument to explain the birth of C/1945 X1 and, especially, C/1963 R1, one can rightfully question the likelihood of unfavorable observing conditions intervening in not one but three sungrazer pairs. Fortunately, there is an alternative explanation.

In a recent paper interpreting the light curves of 27 *SOHO* sungrazers, Sekanina (2003) showed that the distribution of their initial masses (that is, the masses at heliocentric distances before the erosion process sets in) satisfies a power law, such that N, their number with an initial mass greater than $\mathcal{M}$, is given by

$$N = \left(\frac{\mathcal{M}_0}{\mathcal{M}}\right)^{\nu}, \tag{12}$$

where $\mathcal{M}_0$ is the initial mass of the most massive fragment and ν is the power law's exponent that slightly exceeds $\frac{1}{2}$.

Thus, if the parent fragment did not split into two but disintegrated instead into a population of subfragments, of which the observed sungrazer (C/1945 X1, C/1963 R1, and C/1970 K1, respectively, in the three cases) was the most massive one, then there is no need for either a missing fragment or its arrival time constraint.

A chance of detecting from the ground the second largest piece in a population of such subfragments is small especially after perihelion (the cases of C/1963 R1 and C/1970 K1) because the erosion process is strongly mass sensitive, as demonstrated dramatically by the *SOHO* sungrazers. However, at least some of the nebulous objects described by several observers (e.g., Schmidt 1882; Barnard 1883; Hartwig 1883; Markwick 1883) as traveling with C/1882 R1 might have been fitting examples of sizable fragments that had separated from their parent comet years before perihelion.

11.6. *Role of the Subgroups*

The true meaning of the subgroups is critical for understanding the dynamical evolution of the Kreutz system. The traditional view, illustrated, for example, by Marsden's (1989) plot of the perihelion distance versus the longitude of the ascending node in his Figure 7, is that the timescale of interaction between the subgroups is longer than the fragmentation timescale within either subgroup. When we began this investigation, we largely subscribed to this concept (§ 2), even though we were open-minded about the length of time involved.

In accordance with this philosophy, we strived (and succeeded) to develop our model of two superfragments, an obvious parallel to the concept of two subgroups. While the nearly 1700 yr that we found to have elapsed since the birth of the superfragments are a much shorter period of time than previously estimated, we expected that during their return to the Sun in the 4th century, the superfragments would have continued to split into smaller pieces. In support of our initial belief that the formation of the subgroups was a key development in the dynamical evolution of the Kreutz system, we spent more computer time on the unsuccessful search for a common-origin scenario of C/1843 D1 and C/1963 R1 (§ 7) than on any other issue investigated in this paper.

Although our findings on C/1963 R1 do not explicitly invalidate the two-superfragment model, they demonstrate the ease of transition between the subgroups and thus appear to deal a blow to the paradigm of their historical importance. Deemphasis of the subgroups' role has, in addition, profound implications that open new opportunities for modeling the evolution of the Kreutz system, as briefly outlined in § 12.

12. CONCLUSIONS AND FUTURE WORK

The proposed two-superfragment model is self-consistent and offers a plausible fragmentation history of the Kreutz system. The implied short time of less than 1700 yr (or a little more than two revolutions about the Sun) for the age of bright sungrazers observed between 1843 and 1970 and the ease of transiting from one subgroup to the other are in line with the independent conclusions based on a recent analysis of the *SOHO* sungrazers' light curves (Sekanina 2003).

While our findings differ from those of Marsden (1967, 1989), it is difficult to compare the two models because they are based on mutually incompatible concepts and criteria and employ different initial conditions. Examples include diverse views of (1) the relative roles of tidal and nontidal fragmentation, (2) the constraints on the separation velocities involved in breakup events, and (3) the range of plausible limits on the sungrazers' orbital periods.

To illustrate the point, let us focus for a moment on the third issue. Marsden's scenario works only on the assumption that the orbital period of C/1843 D1 was in a range of 350–380 yr. With the Kreutz system's allowed age of many revolutions and the actual history of C/1843 D1 nearly unconstrained, the short orbital period is plausible in the framework of Marsden's hypothesis. However, in the context of our conceptual model, the orbital period of C/1843 D1 is dictated by the conditions during the prime fragmentation event. The computed value of the period has changed very little with time and has, on the average, amounted to 743.5 yr during the two revolutions. Thus, an orbital period of less than 400 yr can by no means be accommodated by our scenario because our prime result, the

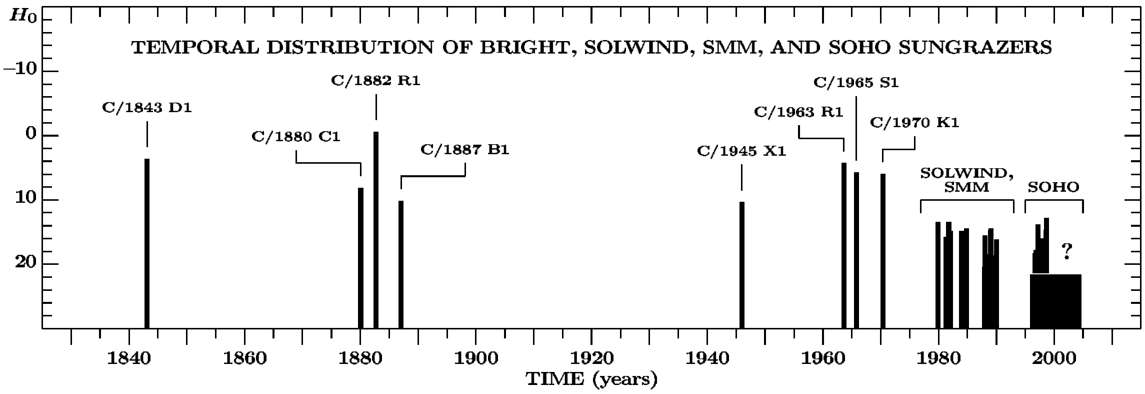

Fig. 4.—Distribution of arrival times for the bright sungrazers since the beginning of the 19th century and for the sungrazers observed with the space-borne *SOLWIND*, *SMM*, and *SOHO* coronagraphs. The ordinate is an estimated visual magnitude normalized to 1 AU from the Sun and Earth. The question mark indicates that from 1999 on the *SOHO* sungrazers have not yet been photometrically analyzed; their detection threshold is near normalized magnitude 22.

birth of the C/1843 D1 and C/1882 R1 precursors from a single parent in a single fragmentation event, would then be invalidated.

The deemphasized role of the subgroups in our scenario necessarily invites increased emphasis on other observed features of the Kreutz system (and tidally triggered multiple comets in general). With our future work in mind, we list three most critical aspects: (1) the distribution of the sungrazers' arrival times, (2) evidence for tidal splitting in the immediate proximity of perihelion, and (3) evidence for secondary, runaway fragmentation throughout the orbit about the Sun.

A plot of the temporal distribution of known sungrazers since the beginning of the 19th century is presented in Figure 4. It shows the two well-known clusters of bright members separated by some 80–90 yr. Judged in terms of the subgroup membership, the clusters are unimportant because the sungrazers in the first trio belong, respectively, to subgroups I, II, and I, whereas in the second trio to subgroups I, II, and IIa. However, if, following our arguments in § 11.6, the subgroup membership is deemed inconsequential, an intriguing coincidence emerges between the cluster spacing and the behavior of the tidally split nucleus of C/1882 R1, which shortly after perihelion appeared as the celebrated "string of pearls:" the difference in the osculating orbital period between any two neighboring nuclear components, for which Kreutz (1891) calculated orbits, was in the range of 80–100 yr. Could it be that the spacing of the clusters in Figure 4 is a diagnostic signature of a tidal, near-perihelion breakup of comet X/1106 C1? If this sungrazer had split into several major pieces, as C/1882 R1 was observed to do, can we expect another cluster to arrive between, say, 2050 and 2070 and yet another one again a century or so later? The analogy with the phenomena in C/1882 R1 can further be extended by pointing out that the space between the "pearls" was filled with material, which also enveloped the entire feature. After stretching along an orbital arc, is one of these densely filled regions of X/1106 C1 observed nowadays as the population of *SOHO* sungrazers? In addition, can we expect the influx of these minicomets to show quasi-periodic, long-term spatial density variations and therefore their gradually increasing rate as we approach the next presumed cluster of major sungrazers?

These and many further questions are waiting for answers, some of which can be offered after an investigation of a new, alternative model for the Kreutz system has been initiated. We like to call it a "cascading model" because the motion of a major sungrazer will be explained as a combined product of momentum transfer during a tidally triggered primary breakup at perihelion and more than one additional nontidal fragmentation event far from the Sun. Nontidal episodes will presumably account for the "structure" of each of the bright sungrazers' clusters, as well as for the orbits of the individual objects. Differences relative to the two-superfragment model will stem primarily from the fact that the progenitor comet will now be identical to X/1106 C1, thus limiting the age of the Kreutz system to a mere 900 yr! In addition, the common-origin paradigm for sungrazers that span the subgroup boundaries will have to cover not only the trio of C/1963 R1, C/1965 S1, and C/1970 K1, but also the cluster of C/1880 C1, C/1882 R1, and C/1887 B1.

A unified concept of the orbital evolution of major (bright) and minor (*SOHO* type) sungrazers in terms of their progressive fragmentation is unquestionably a strength of the proposed cascading model; only the methodology is to be different. On the other hand, a large number of free parameters (relative to the two-superfragment model) is perceived as the model's weakness, and, of course, it will take a major effort to prove its merits.

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