72 P-17-1

**BELLCOMM, INC.**

## COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE – Description of Apollo Entry Guidance TM – 66-2012-2

FILING CASE NO(S) –

DATE – August 4, 1966

AUTHOR(S) – I. Bogner

FILING SUBJECT(S) –
(ASSIGNED BY AUTHOR(S) – Guidance
Entry

### ABSTRACT

This memorandum is a description of the Apollo entry guidance logic at several levels of detail.

1. A brief outline of the logic.
2. A detailed description based upon a simplified flow chart. This discussion should meet the needs of readers not interested in detailed logic or derivations.
3. A listing of the derived equations used in the guidance. This is followed by a description of the major steps in the exact logic for a nominal AS-202 guided entry.
4. A set of detailed derivations of the equations used in the guidance.

(NASA-CR-110924) DESCRIPTION OF APOLLO
ENTRY GUIDANCE (Bellcomm, Inc.) 72 p

N79-71906

Unclas
00/13 11928

FF No. 602
CR-110924
(NASA CR OR TMX OR AD NUMBER) (CATEGORY)
[redacted]

**BELLCOMM, INC.** TM-66-2012-2

## DISTRIBUTION

### COMPLETE MEMORANDUM TO

CORRESPONDENCE FILES:

OFFICIAL FILE COPY
plus one white copy for each
additional case referenced

TECHNICAL LIBRARY (4)

T. A. Keegan-NASA/MA ← COPY TO
P. D. Schrock-NASA/MAT

A. Cohen-MSC/PD4
T. F. Gibson-MSC/FM2
C. Graves-MSC/FM5
J. P. Mayer-MSC/FM
O. Hill-MSC/FM5

J. O. Cappellari, Jr.
D. R. Hagner
W. G. Heffron
J. A. Hornbeck
B. T. Howard
B. Kaskey
J. Z. Menard
I. D. Nehama
B. G. Niedfeldt
T. L. Powers
I. M. Ross
R. V. Sperry
T. H. Thompson
R. L. Wagner
Central Files
Department 1023

### COVER SHEET ONLY TO

BA-146 (8-64)

**BELLCOMM, INC.**

**SUBJECT:** Description of Apollo Entry Guidance
Case 310

**DATE:** August 4, 1966

**FROM:** I. Bogner

TM-66-2012-2

# TECHNICAL MEMORANDUM

The requirements for entry guidance have been explicitly defined in the Apollo Program Specification (1) as:

1. ". . . (Entry) shall be possible without exceeding 10 g deceleration . . . ".
2. ". . . (The guidance system) shall be capable of guiding the CM during entry to the preselected point of parachute deployment within a 10-NM CEP."
3. . . . The guidance must be capable of reaching any landing site within the specified range, 1,500 NM to 2500 NM.

Additional consideration for crew safety include:

1. Insuring capture and preventing uncontrolled skip-out by precluding exit at super-circular speeds. (The CM normally carries only some two hours of life support supplies.)
2. Guiding along a trajectory for which the heat load is within the capability of the heat shield.

Proper entry initial conditions, which will permit the entry guidance to meet the above crew safety needs, are the responsibility of the transearth midcourse guidance. At the start of entry the spacecraft must find itself somewhere in what is known as an entry corridor. This can be defined as that small range of initial flight path angles, outside of which the spacecraft experiences excessive g-loads or uncontrolled skip, regardless of subsequent guidance.

In accomplishing the above design goals the guidance is constrained by operational and equipment considerations. Two most important constraints are:

1. Due to the communication blackout the guidance must be self-contained.
2. No thrust, except that for attitude control, is permitted. The only control exercised by the guidance is that of roll attitude which permits the vehicle lift to be rotated in any direction about the relative wind axis.

The Instrumentation Laboratory at MIT has designed the guidance to meet the above requirements under a broad spectrum of environmental and equipment constraints. (1)(2)(3)(7) It is the purpose of this memo to assist the reader in gaining an understanding of the MIT-designed entry guidance by describing it at four different levels of detail.* And so, this report has four major sections:

1. A brief look at the guidance for a nominal entry.
2. A descriptive presentation of the guidance logic.
3. A presentation of the derived equations used in the guidance together with the assumptions made in the derivations. This is followed by a walk through the entry guidance logic for a nominal 202 mission.
4. Detailed derivations of the equations used in the guidance.

## I. Brief Description of Entry Guidance

Figure 1 may be used to describe the entry hardware. Prior to entry the command module is properly oriented by means of guidance commands to the yaw, roll and pitch reaction jets. Orientation is such that the torques generated by the aerodynamic forces are zero, i.e., a stable or trim condition is achieved. Thereafter attitude control operates continually to damp out yaw and pitch oscillations. With the vehicle in trim the heat shield is forward, in the proper direction for entry.

---

*This memo is based on the guidance used in mission AS-202 as described in Reference 3. Subsequent missions make use of guidance which is fundamentally the same as that in AS-202 but which may differ in important details.

Vehicle motion and attitude are sensed by an inertial measurement unit consisting of three single-degree-of-freedom gyros and three integrating accelerometers. Sensor outputs are used by the guidance computer to plan the trajectory and then to provide a roll command which positions the lift vector.

Vehicle range control is obtained by rotating the lift vector about the relative wind (drag) axis so that the vertical component of lift is the value called for by the guidance. Cross range control is achieved by reflecting the horizontal component of lift about the vertical axis when the predicted cross range miss exceeds some fraction of the vehicle cross range capability.

The type of guided entry selected by the designers is based on two major considerations: heating and the ability readily to detect an uncontrolled skip. The last point is referred to as the monitoring consideration. For heating, the general rule is that the higher the deceleration, the higher the peak heating rate and the lower the total heat. There is also the correlation between higher deceleration and shorter range.

Figure 2 illustrates a typical entry. The various phases are described below.

## Phase 1: Initial Roll (INITIAL ROLL*)

For mission AS-202 the lift vector is directed down at the start of entry. As the spacecraft penetrates the atmosphere the acceleration builds up, and at 0.2g the lift is directed full up. This phase lasts until a threshold altitude rate is reached, and then trajectory monitoring and planning take place. This sequence of lift down and then lift up has two purposes:

a. to insure that the vehicle does not skip out some time later, but

b. to simultaneously insure that there is enough energy left to permit the vehicle to reach the landing point.

Measured deceleration is used to partially compensate for atmospheric density variations.

(It might be pointed out that, for the 500 series missions, this sequence is changed. In the 500 series, flights an initial test is made to determine the position of the spacecraft

---

*Guidance program Fortran designation.

in the entry corridor. A shallow high-speed entry calls for lift down while a slow steep entry calls for initial lift up.)

## Phase 2: Planning (HUNTEST) and Constant Drag (CONSTD)

When the rate of descent reaches 700 fps, planning starts. A range to the landing site is predicted based upon a constant L/D flight through pull-out to exit. Included in the range prediction are calculations for the ballistic portion and the subsequent second entry. Highly critical planning variables generated in this computation phase are the projected exit velocity and its associated altitude rate. If the predicted range exceeds the desired range, the spacecraft is directed to fly a constant drag trajectory until such time as the predicted miss is less than 25 NM.

In mission AS-202 the constant drag level equals the predicted drag at pull-out but is not less than 40 $\text{fps}^2$. In the 500 series the drag is fixed at 130 $\text{fps}^2$.

There is an override "no liftdown" command if the deceleration exceeds a safe limit.

## Phase 3: Self Generated Reference Trajectory (UPCONTROL)

At some point the constant drag phase reduces the velocity sufficiently to make the trajectory plan acceptable. Assuming the range is such that the ballistic phase is necessary, it becomes necessary to achieve the planned exit velocity and flight path angle. This is perhaps the most critical portion of the flight, for the ballistic range is quite sensitive to both exit velocity and flight path-angle (i.e., exit altitude rate). The order of magnitude is 5 NM/fps for both velocity and altitude rate, although slow, steep exits are less sensitive than fast, shallow exits.

Sensitivity to altitude rate and velocity means sensitivity to navigation errors. This was a major consideration in selecting the scheme used in this portion of entry. The technique claimed to be less sensitive to altitude rate errors is to generate an in flight reference trajectory based upon then existing conditions and the desired exit conditions. Drag is the input or sensed variable. From it, assuming a constant L/D, are computed a reference velocity and altitude rate. The vertical component of lift is then specified in a manner which nulls the deviation between estimated velocity and altitude rate and the reference values. Here again a g-limit test and override command are used.

### Phase 4: Ballistic Lob (KEPL)

Atmospheric exit and reentry are defined as those states at which the sensed acceleration falls below and then exceeds approximately 0.2g. In the time between these conditions no control is exercised save that of roll attitude hold with pitch maintained for proper aerodynamic trim.

### Phase 5: Final Entry (PREDICT3)

It is assumed that the ballistic phase puts the spacecraft close to a nominal range-to-go for this final portion of entry. Steering then takes place along a stored reference trajectory. The predicted range is calculated based upon the velocity and corrected for off-nominal conditions of drag and altitude rate. A vertical component of L/D is computed using a nominal L/D plus the range error divided by $\partial RANGE/\partial(L/D)$, the range - L/D influence coefficient. This phase (and guidance in general) stops when the relative velocity drops below 1,000 fps.

It is the final phase which is called upon to negate the effects of errors in earlier phases. If, for example, the vehicle at the start of second entry deviates as much as 200 NM from the nominal, the guidance is still capable of meeting the terminal error criteria.

## II. Description of Guidance Logic

Having completed the brief description of the guidance, this section examines the individual phases in greater detail. Figure 3, which represents a simplified version of the guidance logic, should be consulted while reading this section.

Guidance computations are made once every two seconds. First the computer performs navigation and targeting computations. Following targeting the computer is switched to one of a number of "modes", the name given a major segment of the guidance computation. The initialization routine sets the mode selector to the mode INITIAL ROLL. There are, as the figure shows, five such modes. Guidance is over when the final phase or mode (PREDICT3) finds that the velocity has dropped below 1,000 fps.

### INITIALIZATION

This set of computations is performed once only to supply the remainder of the guidance with necessary constants and initial data for the program to operate satisfactorily. These include initial position and velocity, target coordinates, and an estimate of the entry range, and cross range miss. From the predicted cross range miss the program decides to roll left (or right) toward the target.

## NAVIGATION

Navigation computations are performed during each pass through the guidance. Outputs of the on-board integrating accelerometers are used to estimate vehicle velocity and position using the "average-g" equations.

## TARGETING

Using vehicle and landing site coordinates this section computes the predicted landing site, the range-to-go and the cross range miss.

## MODE SELECTOR

Preceding entry, the mode selector is set to INITIAL ROLL. Following navigation and targeting the guidance assumes the mode designated. Mode switching is designated within the modes according to set criteria. Available modes are:

1. INITIAL ROLL
2. HUNTEST
3. UPCONTROL
4. KEPL
5. PREDICT3

## INITIAL ROLL

When the sensed acceleration, designated D, reaches 0.2g, the vehicle is command full lift up. This lift is held until the altitude rate, RDOT, decreases in magnitude (from a nominal initial value of about 1,800 fps) to 700 fps. Trajectory planning then begins. The 700 fps occurs near the first deceleration peak. Aerodynamic forces are then considered sufficient to exercise control.

## HUNTEST (PLANNING PHASE)

The guidance computer predicts the velocity, V1, and acceleration AO, at pull-out, i.e., when RDOT will equal zero. It then predicts the exit velocity, VL, and exit flight path angle, GAMMAL.

There are three ways to exit from this area of the guidance computation.

*1. A normal exit implies that the exit conditions VL and GAMMAL are acceptable. Trajectory planning continues by going to RANGE PREDICTION. This prediction involves assuming a ballistic skip and a final atmospheric entry phase.

2. If the predicted exit velocity is low, less than 18,000 fps, the guidance is directed to the final phase, PREDICT3.

3. If the predicted exit velocity is greater than satellite velocity a constant drag phase is specified to permit the vehicle to slow down, until such time as predicted exit conditions are acceptable.

## RANGE PREDICTION

Predicted range to landing, based upon the predicted pull-out and exit conditions V1, VL and GAMMAL, is compared to the required range-to-go.

There are three ways to exit the computation:

*1. The predicted range is within 25 NM of the required range-to-go. Control is transferred to UPCONTROL. Trajectory planning is now complete and the guidance attempts to carry out the plan.

2. The predicted range indicates an overshoot of more than 25 NM. Guidance calls for a constant drag phase until, during some later guidance cycle, the plan becomes acceptable.

3. The predicted range indicates an undershoot of more than 25 NM. It may be recalled that the range prediction is based upon predicted pull-out and exit conditions. Because the plan based upon these conditions is found to be unacceptable, pull-out velocity V1 is properly incremented to produce an increase in range. Exit conditions are then recomputed in HUNTEST. The iteration on V1 continues until a satisfactory plan is generated resulting in a miss of less than 25 NM.

---

*Normal path.

## CONSTANT DRAG

For a nominal speed, short range entry, HUNTEST via RANGE PREDICTION calls for a constant drag trajectory to slow the vehicle down sufficiently to prevent an overshoot. The same is true for a high speed nominal range entry. The actual trajectories for these cases might be quite different. In the former the vehicle might go from the constant drag portion directly to the final atmospheric descent. In the high speed entry the vehicle might experience all the remaining normal phases including a ballistic phase.

## UPCONTROL

With an acceptable plan the guidance flies the vehicle in one of two primary guided phases, pre-up and pull-out to exit. In addition there are a number of contingency paths which can be taken.

*1. Pre-pull-out to pull-out phase. (Pre-up Phase). If V is greater than V1 (the predicted pull-out velocity), the lift is directed to achieve V = V1, D = AO, and RDOT = 0 simultaneously.

*2. Pull-out to exit phase. (Up-Phase). After V1 is reached and until $D \leq Q7$, the guidance checks the drag D. The primary path is such that for $AO > D > Q7$, i.e., pull-out acceleration > present acceleration > exit acceleration, the lift is directed to achieve the conditions corresponding to V = VL, $\gamma$ = GAMMAL and D = Q7 simultaneously.

Contingencies under 2 are:

(a) If $D > AO$, the vehicle is directed to pull full lift up.

(b) During the up-phase, should a negative lift be called for, it is overriden to zero lift if the then present acceleration is greater than 5.44g. In AS-501 this g-limit is included in the pre-up phase also.

3. For $D > Q7$, exit is assumed and control is transferred to the ballistic phase.

---

*Normal path.

4. Transfer to the final atmospheric entry phase is possible without going through the up-phase or the ballistic phase. The specific conditions for this transfer are: $V \leq V1$,

   $RDOT < 0$, and

   $V < VL + 500$

   Specifically, a short entry range may result in a constant drag phase until the velocity has been reduced to the point where only a pre-up phase and a final phase are necessary.

*KEPL (Ballistic Phase)

The roll attitude at the start of KEPL is maintained until the sensed acceleration has built up to 6.5 $\text{fps}^2$.

*PREDICT3 (Final Phase)

*1. Assuming a normal descent, the vehicle is flown to a stored reference trajectory, corresponding to a constant L/D of 0.18. Range-to-go (PREDANGL) is predicted as a function of V, RDOT and D. Lift is directed to null the difference between the range capability and desired range. This is a noteworthy point. Aside from the initial plan in HUNTEST, PREDICT3 is the only phase where the vehicle is guided continuously to null the terminal range error.

   Following this lift calculation control is transferred to the GLIMITER.

2. If the vehicle has passed the landing site the lift is designated as full down and control is transferred to GLIMITER.

3. If the vehicle velocity is less than 1,000 fps the vehicle is directed to hold the last roll command and guidance is considered completed.

*GLIMITER

This portion of the guidance is used only during the final phase. It compares present and predicted accelerations with the maximum permitted in order to determine whether to permit or override the lift specified in the final phase.

---

*Normal path.

*1. If the present acceleration D is less than 5g, the lift specified in PREDICT3 is accepted.

2. If the present acceleration is equal to or greater than 10g full lift up is commanded.

3. If the present acceleration is between 5 and 10g a critical altitude rate X (a positive number) is computed. The rate such that if full lift-up were maintained, the vehicle would eventually experience 10g.

   (a) If the present altitude rate RDOT is equal to or greater than -X, the lift specified in PREDICT3 is accepted.

   (b) If the present altitude rate is less than -X a lift-up is called for proportional to the difference between X and RDOT, plus an additional amount to balance the centrifugal and gravitational accelerations. (In mission AS-501 this last situation, $RDOT \leq - X$, calls for full lift up.)

*LATERAL LOGIC

The lateral logic makes a number of checks.

1. If the lift called for is greater than that corresponding to a roll of 15° either way from full up or down, i.e., $|L/D| > 0.3 \cos 15°$, two options are open:

   (a) If the present direction of actual lift is such as to reduce the cross range miss, a roll of 15° from full up or down toward the target is called for.

   (b) If the present direction of lift is such as to increase the cross range miss, the miss is compared to the remaining lateral range capability. If the predicted miss is greater than half the remaining lateral capability, the roll called for is reflected about the 0°, 180° roll line in order to head the vehicle toward the target. If the predicted miss is less than half the remaining lateral capability, the L/D is met but with the lift pointing away from the target, thus conserving attitude maneuvering fuel.

---

*Normal path.

2. If less lift is called for than that corresponding to a 15° roll from vertical, the logic checks for lateral range capability and then computes and commands the roll angle based on the command lift. Roll-lift switching is pictured in Figure 4.

3. If the target has been passed, all cross range considerations are bypassed. Full lift-up or down as specified in the final phase and g-limiter is passed on to the roll attitude control.

During portions of UPCONTROL and CONSTANT DRAG an acceleration check is made in NEGTEST if lift is negative. Should the acceleration exceed 5.44 g, zero lift is called for and a flag is set preventing reversal of roll angle regardless of lateral range considerations.

## III. Entry Guidance Equations

This third section of the memo describes the major guidance logic events which occur in the AS-202 nominal entry. At each event major routines which are performed are indicated, as are the results of the computations. The important equations used in the guidance together with the assumption implicit in the equations are shown in Figure 5*. MIT/IL Flow charts of the guidance incorporating the above equations appear in Figures 6 through 18, and Table 1 lists the variables and constants used in the flow charts. In the remainder of this section the logic sequence of a nominal entry will be described using the charts. The description considers only the points in time at which major logic changes take place.

Time = 0. seconds. Start of Entry

Initial conditions are (3):

| | Vehicle | Target Site |
|---|---|---|
| Geodetic Latitude | -5.11° | 17.25° |
| Longitude | 134.70° E | 170.° E |
| Altitude | 399,188. ft | |
| Velocity | 28,706. fps | |
| Heading | 58.65° E | |
| Flight-path Angle | -3.57° | |

---

*The less than tenacious reader is urged to spare himself the drudgery of going through the derivations which appear in the Appendix, the last major section of the memo.

Time = 2. seconds. First pass through guidance.

Guidance Sequence: NAVIGATION, TARGETING, INITIAL ROLL, LATERAL LOGIC.

Control: Full lift down.

Time = 94. seconds. Shift to lift up.

Guidance Sequence: NAVIGATION, TARGETING, INITIAL ROLL, LATERAL LOGIC.

At this point the drag exceeded the threshhold and full lift up was called for.

$D = 6.47 > KA = 6.44$ fps$^2$

Control: $L/D = LAD = +0.3$

Time = 128. seconds. Trajectory planning takes place and vehicle begins guidance to pull-out.

Guidance Sequence: NAVIGATION, TARGETING, INITIAL ROLL, HUNTEST, RANGE PREDICTION HUNTEST1, RANGE PREDICTION, UPCONTROL, LATERAL LOGIC

The altitude rate had dropped below 700 fps starting the shift to HUNTEST. $-RDOT = 696.98 < VCONTRL = 700.$ The sequence of calculations at this point is shown on the flow charts. A first pass through HUNTEST, HUNTEST1 and RANGE PREDICTION resulted in an undershoot of 311.156 NM. This called for an iteration, changing pull-out velocity V1 from 26,402.24 fps to 26,902.24 fps. The second pass resulted in an acceptable miss of 15.718 NM permitting control to pass to UPCONTROL.

Control: Fly Along computed reference to pull-out.

Time = 166. seconds. Pull-out is reached and vehicle begins guidance to exit.

Guidance Sequence: NAVIGATION, TARGETING, UPCONTROL, NEGTEST, LATERAL LOGIC.

At this point V has dropped below V1 calling for the shift in control.

Control: Fly along computed reference to exit.

Time = 314. seconds. Vehicle begins the ballistic phase.

Guidance Sequence: NAVIGATION, TARGETING, UPCONTROL, KEPL, LATERAL LOGIC

The drag D (5.956 $fps^2$) has fallen below the exit threshhold Q7 = 6 $fps^2$, and the vehicle is considered in the ballistic phase.

Control: Attitude hold.

Time = 440. seconds. Vehicle reenters atmosphere, and begins guidance along a stored reference.

Guidance Sequence: NAVIGATION, TARGETING, KEPL, PREDICT3, GLIMITER, LATERAL LOGIC.

The drag has built up so that $D > Q7 + KDMIN$ $6.567 > 6. + 0.5$ $fps^2$.

Control: Fly along stored reference.

Time 847.6 seconds. Vehicle reaches a 25,000 ft. altitude, with an actual miss of the target site of 0.917 NM. Parachute is opened.

In order to round out the picture on this particular entry, graphs of a number of key variables are included. Figure 19 shows the continually decreasing velocity as a function of time. Points of note include the reversal in lift prior to pull-out, the start of range control and the location of the heat rate and g-load peaks. Note that the peak altitude for the ballistic portion was under 270,000 feet. Figure 20 illustrates the two heat-rate peaks as well the heat load. Figure 21 shows the vehicle will experience about 45 $fps^2$ and 117 $fps^2$ deceleration peaks (1.4 g and 3.6 g), well within safety bounds. Figures 22-24 are included to give an indication of the ranges and form of altitude rate, and flight path angle and velocity. Of particular interest is the fact that the flight path angle remains significantly less than 0.1 radian most of the time, thus justifying the small angle approximation made throughout the guidance equation deviations.

## IV. Acknowledgements

The writer wishes to acknowledge the assistance of Messrs. W. G. Heffron and R. V. Sperry in the preparation of this memo.

I. Bogner

2012-IB-jdc

I. Bogner

## REFERENCES

1. "Apollo Program Specification" SE 005-001-1, Revision A March 1, 1966.

2. "Apollo Reentry Guidance", D. J. Lickly, H. R. Morth, B. S. Crawford, MIT/IL, Report R416, August 1963. Confidential

3. Guidance and Navigation System Operation Plan, Apollo Mission 202 R-477 (Rev. 2). MIT Instrumentation Laboratory.

4. G. L. Hunt Internal Note No. 65-FM-157, Apollo Reentry Guidance and Navigation Equations and Flow Logic. Manned Spacecraft Center.

5. J. A. Stiles Unpublished Notes. Derivation and Discussion of Equations in MIT Reentry Program. Bell Telephone Laboratories.

6. W. G. Heffron, I. Bogner, "Capabilities of the Entry Guidance Equations for Mission AS-202," TR-66-310-5

7. R. Morth, MIT/IL, Report R-532, Reentry Guidance for Apollo, January 1966.

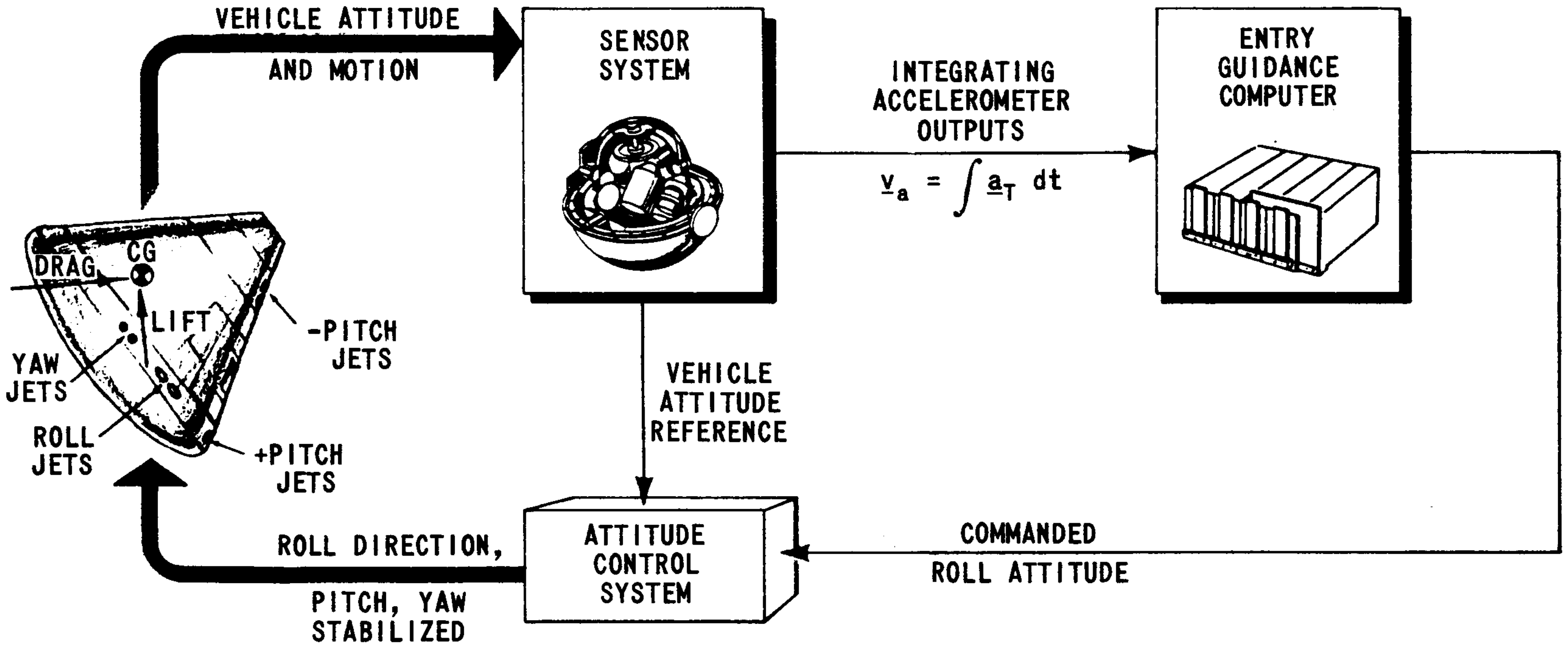


FIGURE 1. ENTRY GUIDANCE SYSTEM

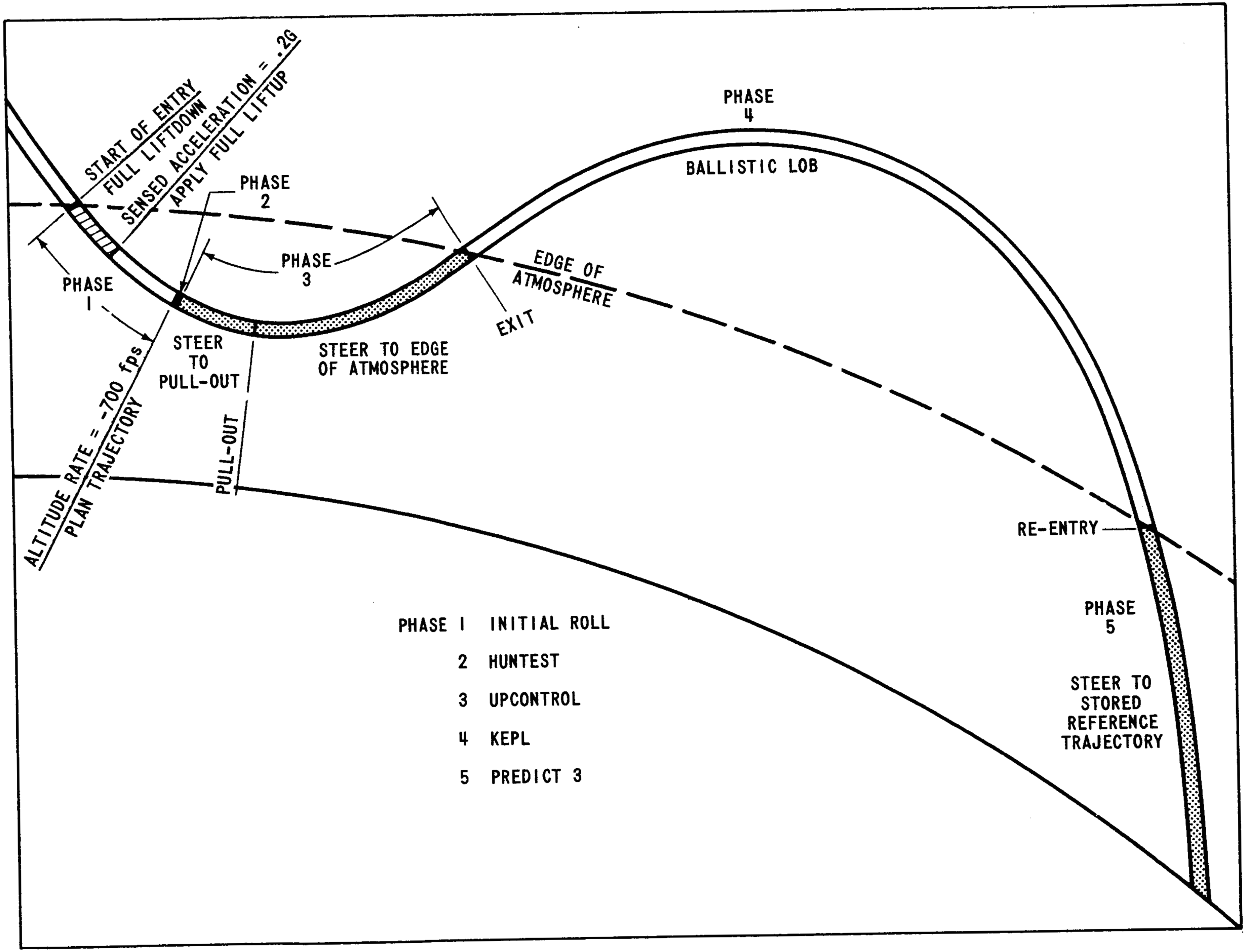


FIGURE 2. TYPICAL ENTRY

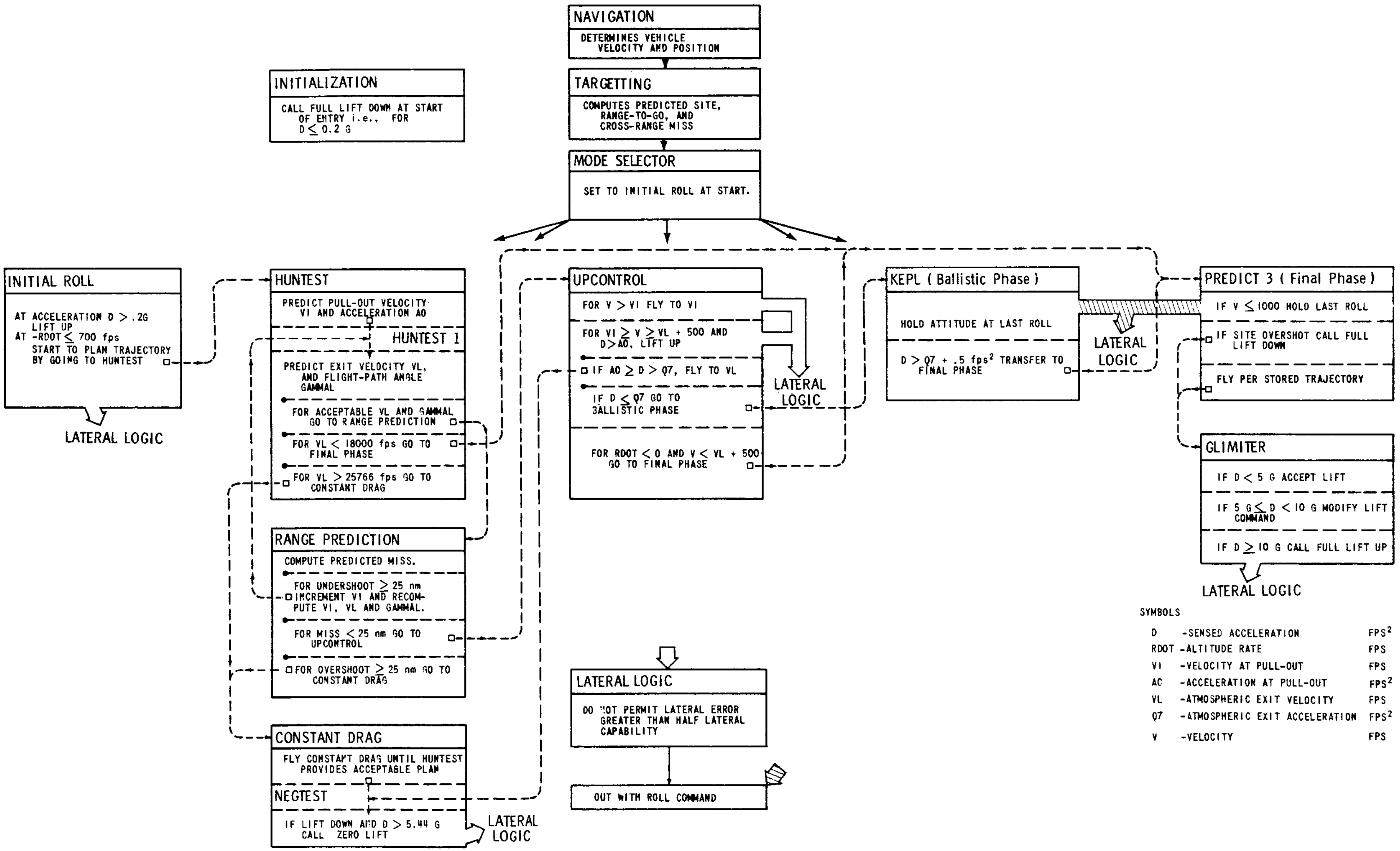
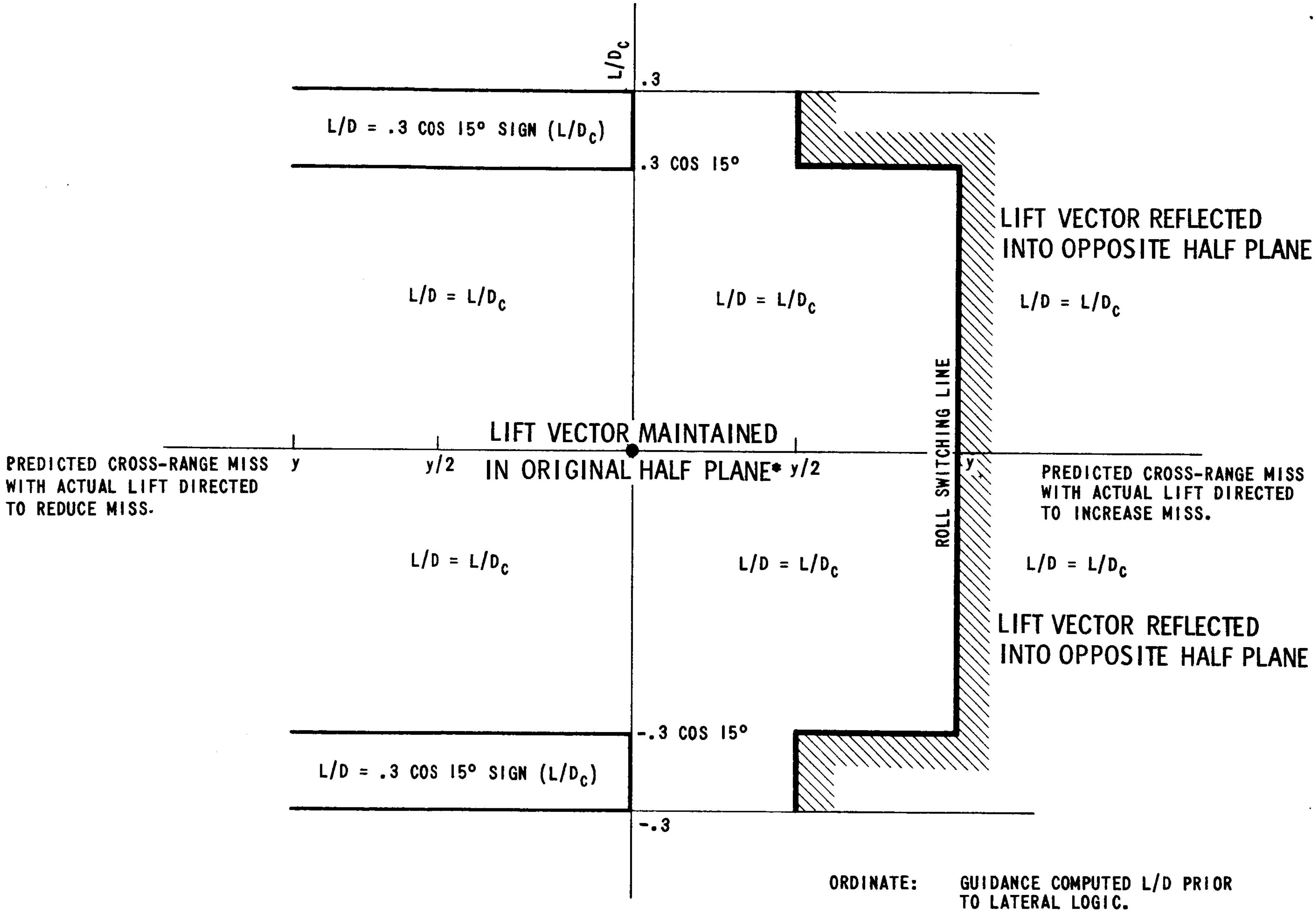


FIGURE 3. SIMPLIFIED FLOW CHART FOR ENTRY GUIDANCE



ORDINATE: GUIDANCE COMPUTED L/D PRIOR TO LATERAL LOGIC.
ABSCISSA: *PRIOR TO LATERAL LOGIC.
y: LATERAL RANGE CAPABILITY.

FIGURE 4. LATERAL LOGIC FOR LIFT-ROLL CONTROL

FIGURE 5a. DERIVED EQUATIONS USED IN ENTRY GUIDANCE

| PHASE | FUNCTION | ASSUMPTIONS MADE | EQUATION | MODE |
|---|---|---|---|---|
| DESCENT TO PULL-OUT | PREDICT PULL-OUT VELOCITY V1. | 1, 2, 3, 5, 6, 7 | V1 = V + KC1 RDOT/LAD | HUNTEST |
| | PREDICT PULL-OUT ACCELERATION AO. | 1, 3, 4, 5, 6 | AO = D + KC2 RDOT$^2$/(2 HS LAD) | HUNTEST |
| | COMPUTE RANGE TO PULL-OUT. | 1, 3, 5, 6, 7, 8 | ASPDWN= -RDOT V ATK/(AO LAD RE) | HUNTEST-RANGE PRED. |
| | COMPUTE REFERENCE ALTITUDE-RATE FOR L/D COMMAND. | 1, 2, 3, 5, 6, 7 | RDTR=(LAD/KC1) (V1-V) | UPCONTROL |
| | COMPUTE REFERENCE ACCELERATION FOR L/D COMMAND. | 1, 3, 4, 5, 6 | DR = AO-KC2 RDTR$^2$/(2 HS LAD) | UPCONTROL |
| PULL-OUT TO EXIT | PREDICT EXIT VELOCITY VL. | 1, 2, 3, 5, 7, 11 | ALP = 2AO HS/(LEWD V1$^2$)<br>FACT1 = V1/(1-ALP)<br>FACT2 = ALP(ALP-1)/AO<br>VL = FACT1 [1-SORT (FACT2 Q7 + ALP)] | HUNTEST |
| | PREDICT EXIT FLIGHT-PATH ANGLE GAMMAL. | 1, 2, 3, 5, 7, 10, 11 13, 14, 15, 16, 17, 18 | DVL = VS1-VL<br>DHOOK = [(1-VS1/FACT1)$^2$-ALP ] /FACT2<br>AHOOK = CHOOK (DHOOK/Q7-1)/DVL<br>GAMMAL1 = LEWD (V1-VL)/VL<br>GAMMAL=GAMMAL1- $\frac{\text{CHIGS DVL}^2\text{(1+AHOOK DVL)}}{\text{DHOOK VL}^2}$ | HUNTEST-HUNTEST1 |
| PULL-OUT TO APOGEE IF NO EXIT | PREDICT APOGEE VELOCITY V2. | 1, 2, 3, 7, 9, 10, 12, 13, 14, 15, 16 | V2 = VL + $\frac{\text{GAMMAL VL}}{\text{LEWD-(3AHOOK DVL}^2\text{+2DVL) [CHIGS/(DHOOK VL)]}}$ | HUNTEST-HUNTEST1 |
| | PREDICT APOGEE ACCELERATION Q7. | 1, 2, 3, 5, 7, 11 | Q7 = [(1-V2/FACT1)$^2$ -ALP] /FACT2 | HUNTEST-HUNTEST1 |

ASSUMPTIONS

1. a) STATIONARY ATMOSPHERE
   b) INVERSE SQUARE GRAVITY
   c) EXPONENTIAL ATMOSPHERE
2. $D >> G \text{ SIN } \gamma$
3. a) $\text{COS } \gamma = 1$
   b) $\frac{d\dot{R}}{dt} = \dot{V} \text{ SIN } \gamma + V\dot{\gamma} \text{ COS } \gamma \approx V\dot{\gamma}$
4. $\frac{H}{HS}\left(\frac{\dot{H}}{H}\right) >> \frac{\dot{V}}{V}$
5. $L >> G\left(\frac{V^2}{GR} - 1\right)$
6. L/D = LAD
7. $R\dot{\theta} = V$
8. $R\dot{\theta} = V$ A CONSTANT
9. $\text{SIN } \gamma = \gamma$
10. L/D = LEWD
11. $\ln(1 + X) = X$
12. $GR = VSAT^2$ A CONSTANT
13. $e^{H/HS} = e^{H1/HS}\left[1 + a\,(VS1-V1)\right]$
14. VS1 = VSAT A CONSTANT
15. $\ln(1 + X) = X - X^2/2$
16. $\dot{R}_{VS1} = 0$
17. V = VL A CONSTANT
18. V = A CONSTANT
19. $VL\big|_{\gamma=0} = VL - \frac{\dot{R}_L}{\left.\frac{d\dot{R}}{dV}\right|_{V=VL}}$
20. IGNORE HIGHER ORDER TERMS IN CALCULATING $d\dot{R}/dV$ IN UP-PHASE
21. $R\dot{\theta} = V = VL$ A CONSTANT
22. $\dot{R} = \dot{R}_L$ A CONSTANT
23. GAMMAL = GAMMAL1
24. VS1 = V1

# FIGURE 5b. DERIVED EQUATIONS USED IN ENTRY GUIDANCE (cont'd)

| PHASE | FUNCTION | ASSUMPTIONS MADE | EQUATION | MODE |
|---|---|---|---|---|
| PULL-OUT TO EXIT OR APOGEE | COMPUTE RANGE FROM PULL-OUT TO EXIT OR APOGEE. | 1, 2, 3, 5, 7, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23 | $\mathrm{ASPUP} = \frac{\mathrm{ATK}}{\mathrm{RE}} (\mathrm{HS/GAMMAL1})\ \mathrm{LOG} \left[\mathrm{AOVL}^2/(\mathrm{Q7\ VI}^2)\right]$ | HUNTEST-RANGE PRED. |
|  | COMPUTE REFERENCE VELOCITY VREF FOR L/D COMMAND. | 1, 2, 3, 5, 7, 11 | $\mathrm{VREF} = \mathrm{FACT1} \left[1 - \mathrm{SQRT}\ (\mathrm{FACT2\ D} + \mathrm{ALP})\right]$ | UPCONTROL |
|  | COMPUTE REFERENCE ALTITUDE RATE FOR L/D COMMAND. | 1, 2, 3, 5, 7, 10, 11, 12, 13, 14, 15, 16, 24 | $\mathrm{RDOTREF} = \mathrm{LEWD}\ (\mathrm{VI}-\mathrm{VREF})$<br>$-\mathrm{CHIGS}\ (\mathrm{VSI}-\mathrm{VREF})^2 \left[\frac{1+\mathrm{AHOOK}\ (\mathrm{VSI}-\mathrm{VREF})}{\mathrm{DHOOK\ VREF}}\right]$ | UPCONTROL |
| CONSTANT DRAG PHASE | COMPUTE REFERENCE ALTITUDE RATE RDOTREF FOR L/D COMMAND. | 1, 7 | $\mathrm{RDOT} = -2\mathrm{HS\ DO/V}$ | CONSTD |
| BALLISTIC PHASE | COMPUTE BALLISTIC RANGE. | 1 | $\mathrm{VBARS} = \mathrm{VL}^2/\mathrm{VSAT}^2$<br>$\mathrm{COSG} = 1 - \mathrm{GAMMAL}^2/2$<br>$\mathrm{E} = \mathrm{SQRT} \left[1+(\mathrm{VBARS}-2)\ \mathrm{COSG}^2\mathrm{VBARS}\right]$<br>$\mathrm{ASKEP} = 2\mathrm{ATK\ SIN}^{-1}\ (\mathrm{VBAR\ COSG\ GAMMAL/E})$ | HUNTEST-RANGE PRED. |
| FINAL PHASE | FLY TO STORED REFERENCE TRAJECTORY. | 1 |  | PREDICT3 |
|  | COMPUTE RANGE FROM REENTRY TO TERMINATION. | 1 | $\mathrm{ASP1} = \mathrm{Q2} + \mathrm{Q3\ VL}$<br>$\mathrm{ASP3} = \mathrm{Q5}\ (\mathrm{Q6}-\mathrm{GAMMAL})$ | HUNTEST-RANGE PRED. |

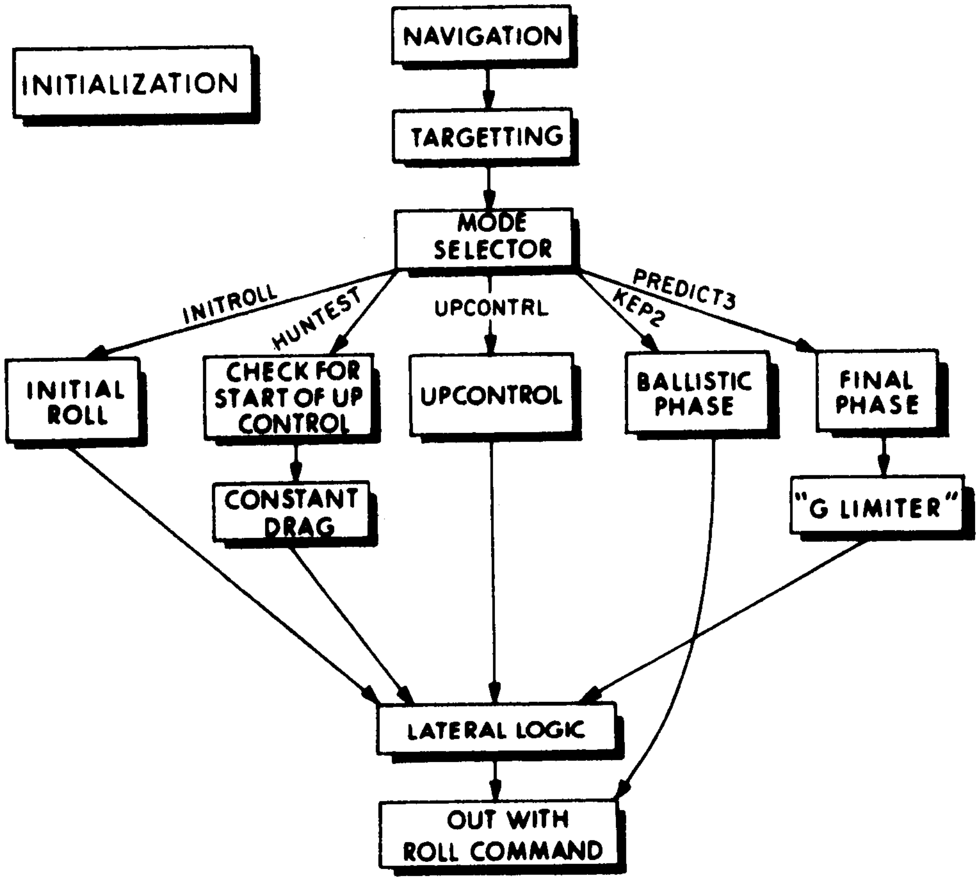


FIGURE 6. RE-ENTRY STEERING

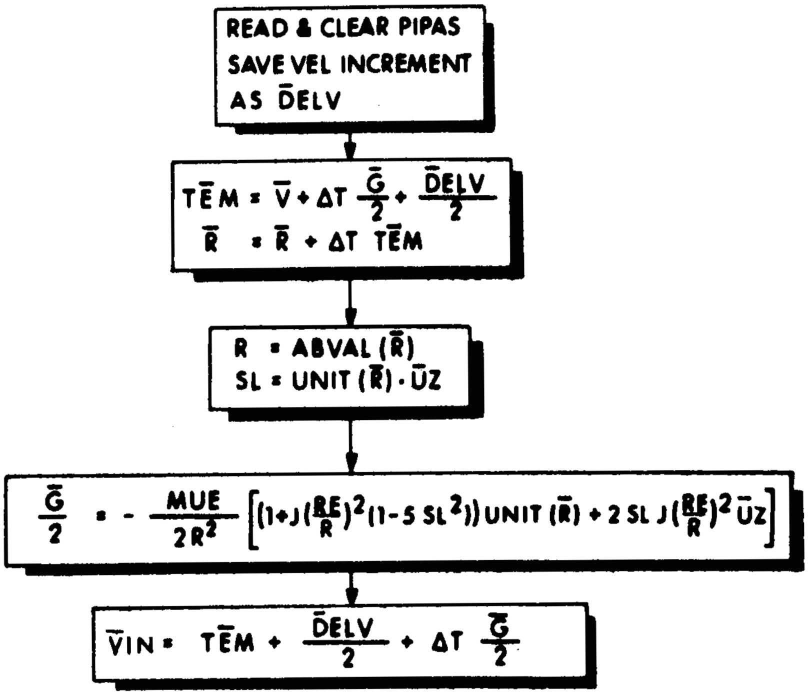


FIGURE 7. RE-ENTRY STEERING - NAVIGATION (AVG. G)

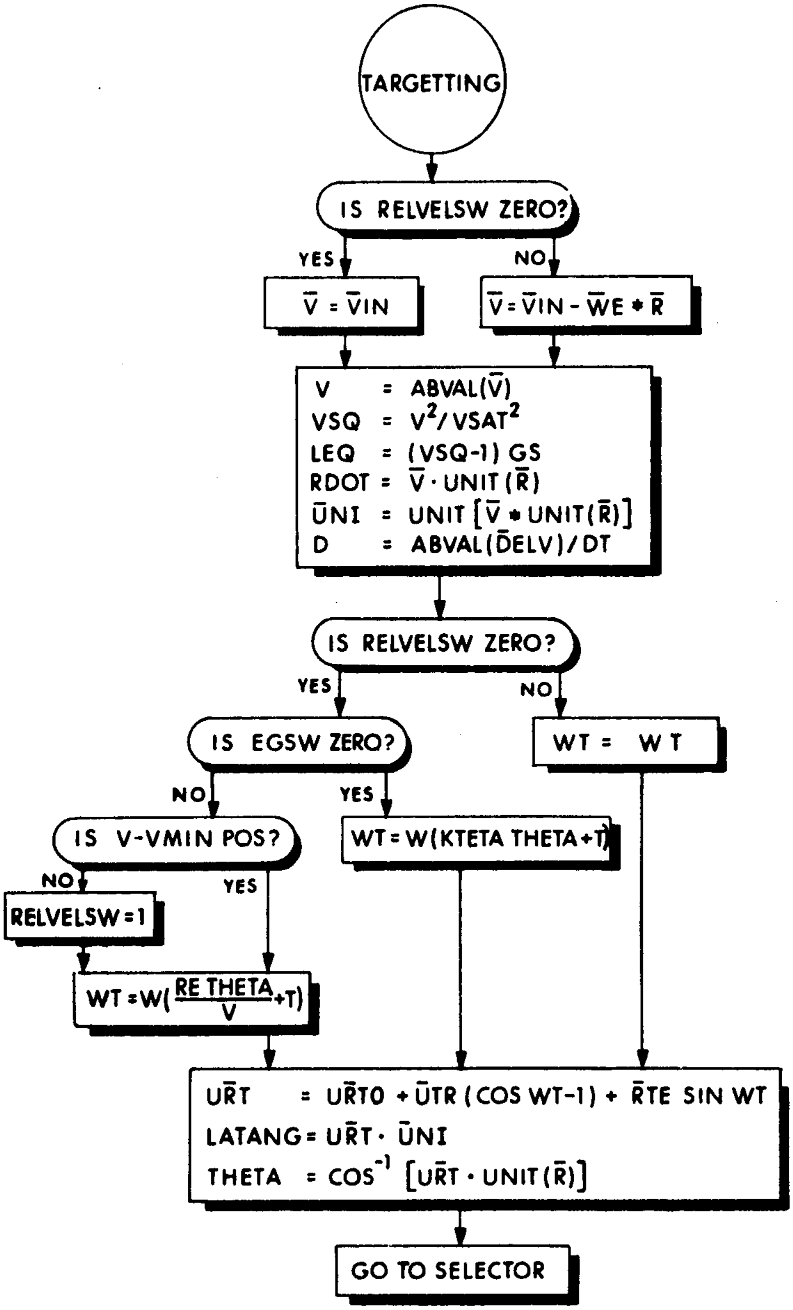


FIGURE 8. RE-ENTRY STEERING - TARGETING

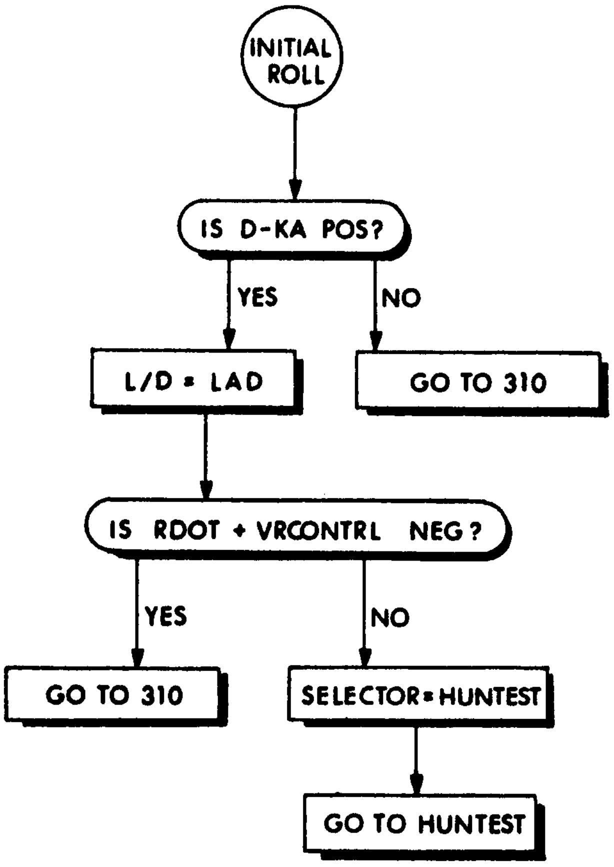


FIGURE 9. RE-ENTRY STEERING - INITIAL ROLL

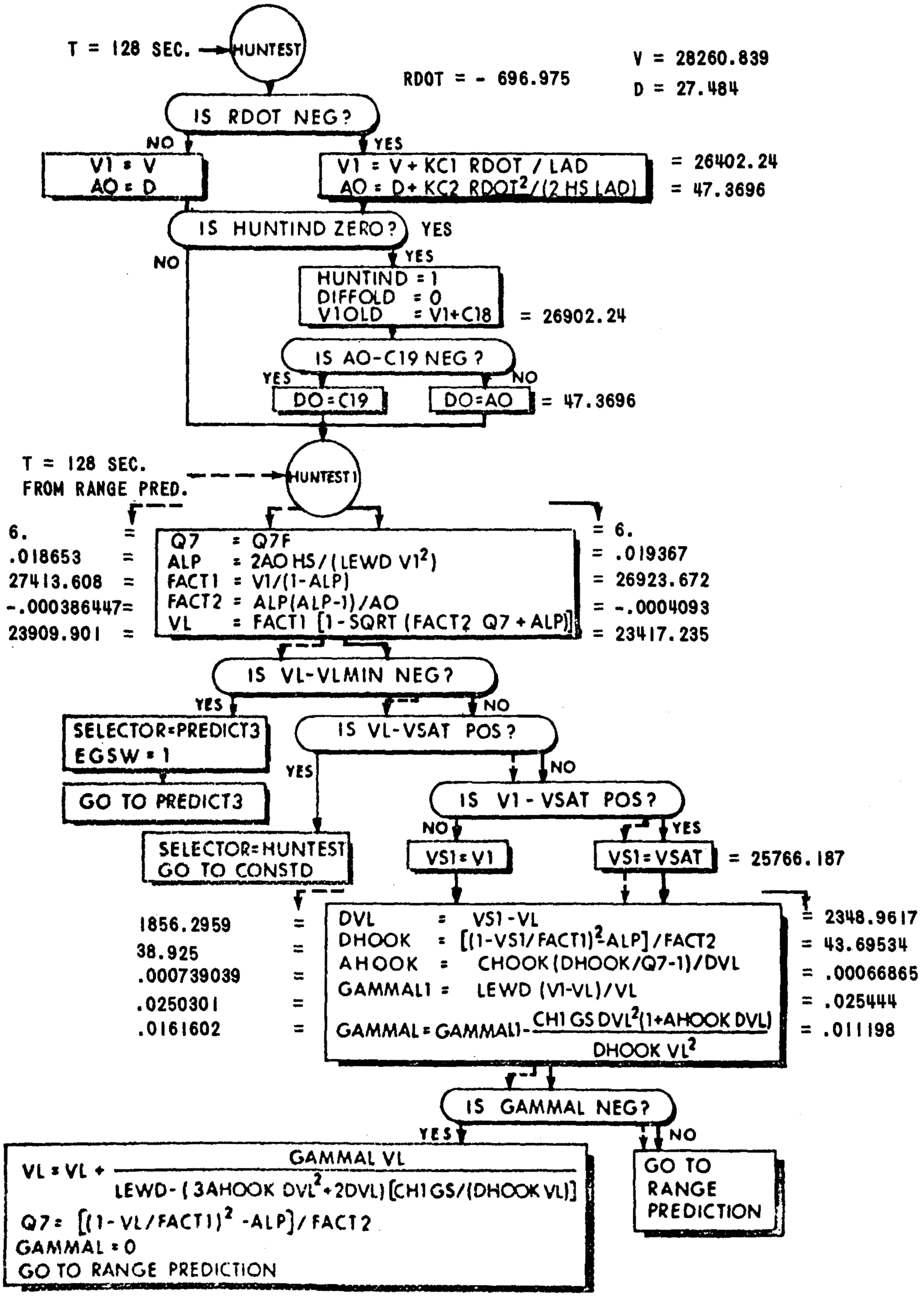


FIGURE 10. RE-ENTRY STEERING - HUNTEST
]

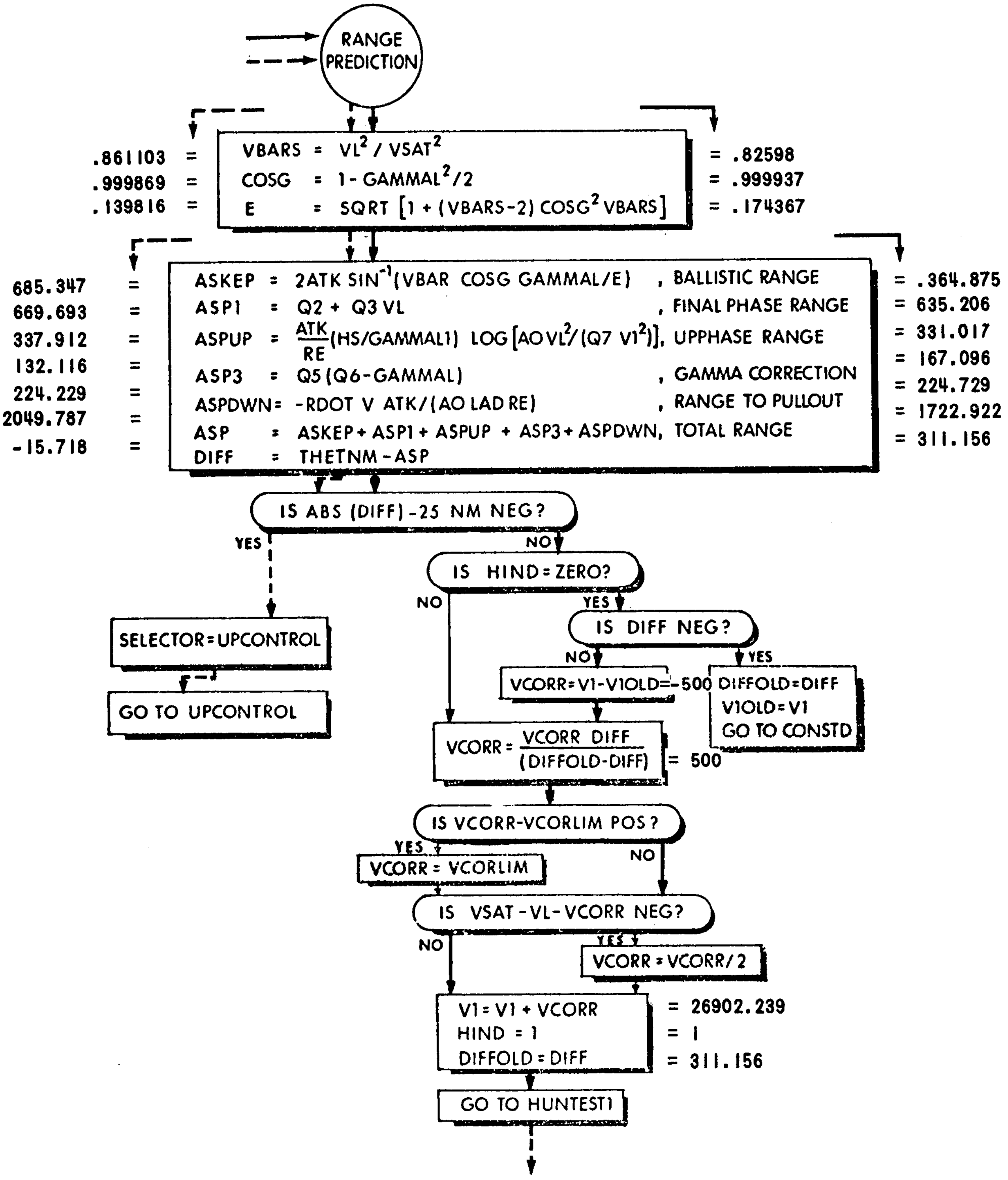


FIGURE 11. RE-ENTRY STEERING - RANGE PREDICTION

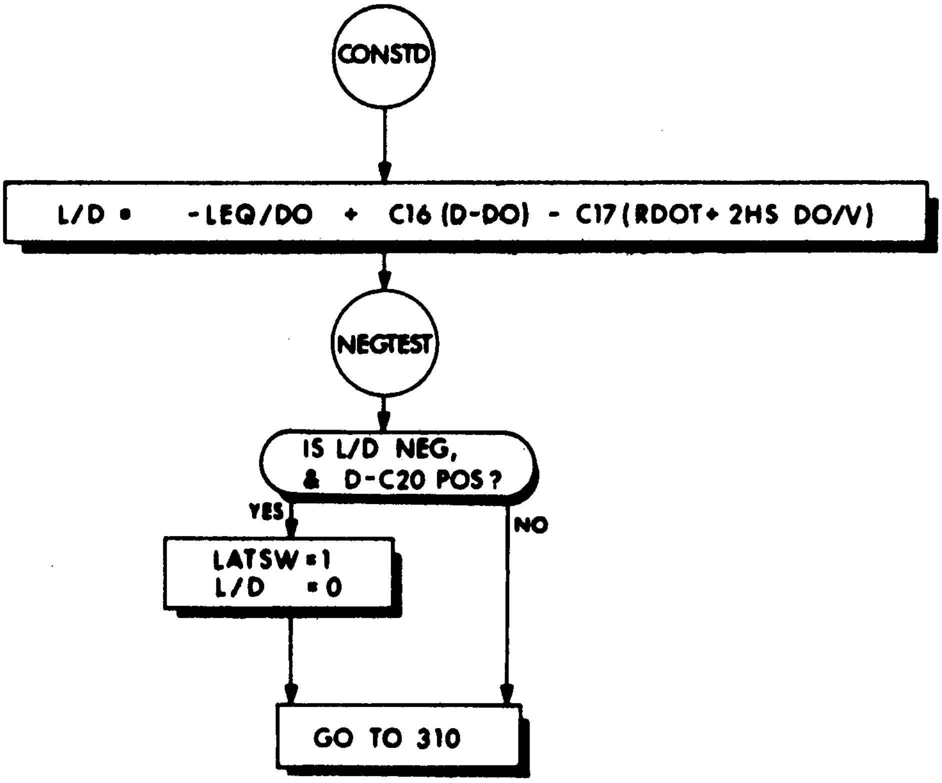


FIGURE 12. RE-ENTRY STEERING - CONSTD

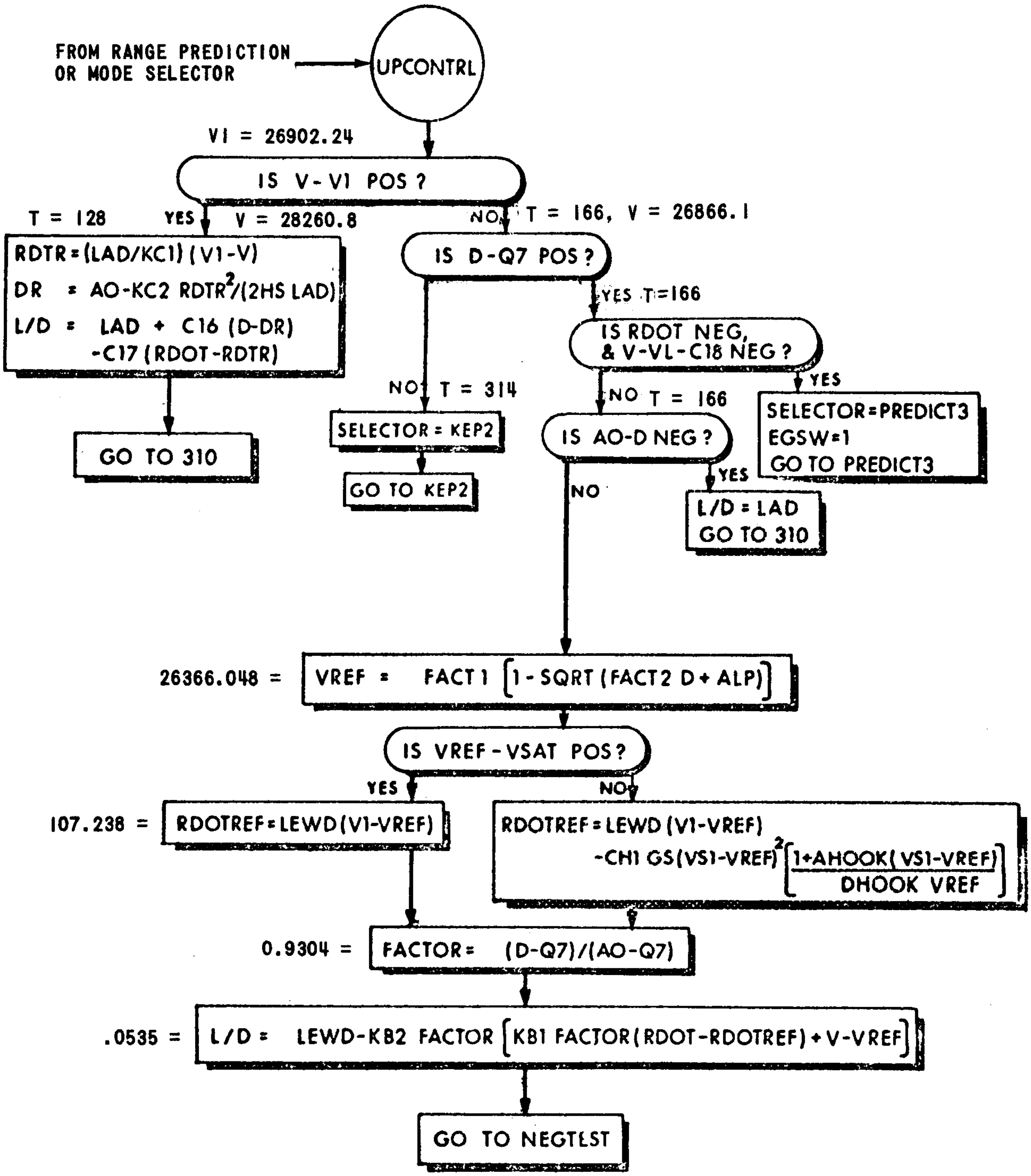


FIGURE 13. RE-ENTRY STEERING - UPCONTROL

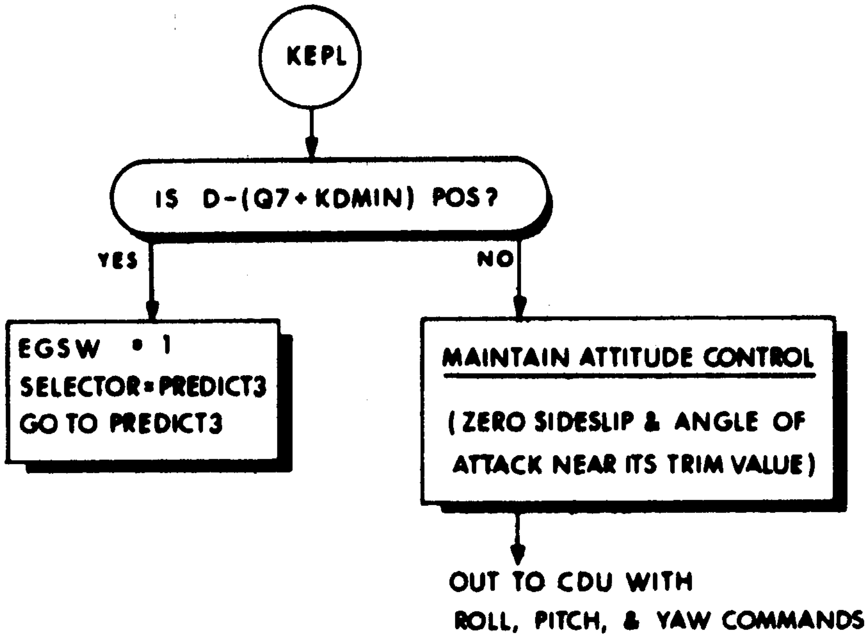


FIGURE 14. RE-ENTRY STEERING - BALLISTIC

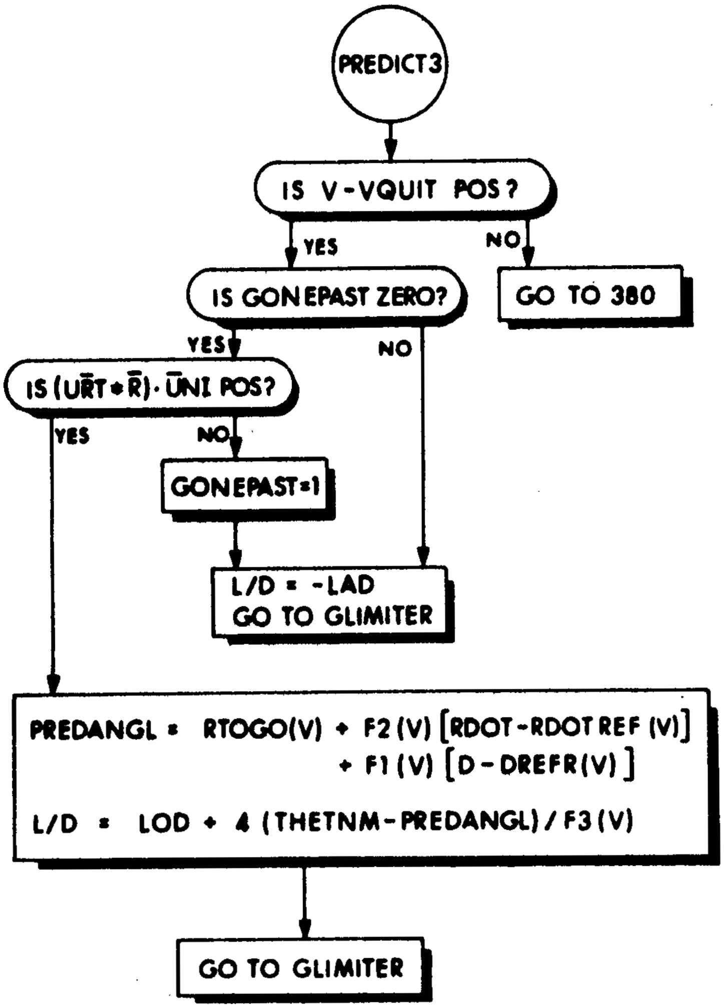


FIGURE 15. RE-ENTRY STEERING - PREDICT 3

| VREF<br>FPS | RDOTREF<br>FPS | DREFR<br>FPSS | DR/DRDOT F2<br>NM/FPS | DR/DA F1<br>NM/FPSS | RTOGO<br>NM | DR/DL/D F3<br>NM |
|---|---|---|---|---|---|---|
| 0 | -331 | 34.1 | 0 | -.02695 | 0 | 1 |
| 337 | -331 | 34.1 | 0 | -.02695 | 0 | 1 |
| 1080 | -693 | 42.6 | .002591 | -.03629 | 2.7 | 6.44 x 2 |
| 2103 | -719 | 60. | .003582 | -.05551 | 8.9 | 10.91 x 2 |
| 3922 | -694 | 81.5 | .007039 | -.09034 | 22.1 | 21.64 x 2 |
| 6295 | -609 | 93.9 | .01446 | -.1410 | 46.3 | 48.35 x 2 |
| 8531 | -493 | 98.5 | .02479 | -.1978 | 75.4 | 93.72 x 2 |
| 10101 | -416 | 102.3 | .03391 | -.2372 | 99.9 | 141.1 x 2 |
| 14014 | -352 | 118.7 | .06139 | -.3305 | 170.9 | 329.4 |
| 15951 | -416 | 125.2 | .07683 | -.3605 | 210.3 | 465.5 |
| 18357 | -566 | 120.4 | .09982 | -.4956 | 266.8 | 682.7 |
| 20829 | -781 | 95.4 | .1335 | -.6483 | 344.3 | 980.5 |
| 23090 | -927 | 28.1 | .2175 | -2.021 | 504.8 | 1385 |
| 23500 | -820 | 6.4 | .3046 | -7.569 | 643.0 | 1508 |
| 35000 | -820 | 6.4 | .3046 | -7.569 | 643.0 | 1508 |

FIGURE 16. FINAL PHASE REFERENCE

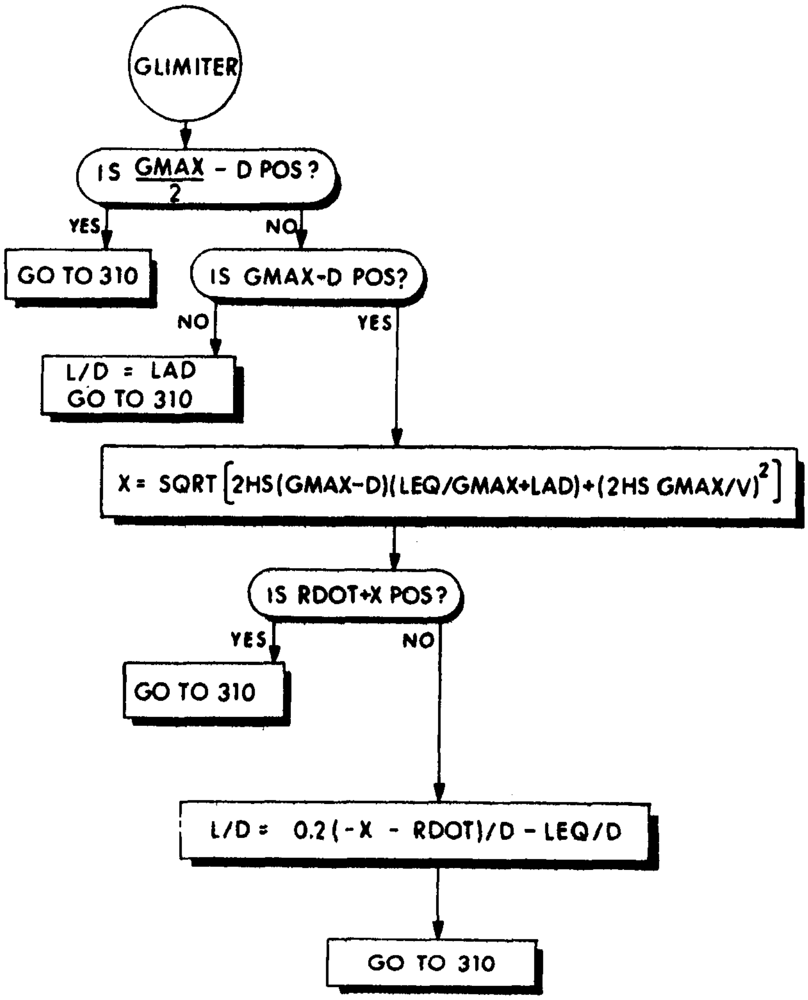


FIGURE 17. RE-ENTRY STEERING - GLIMITER

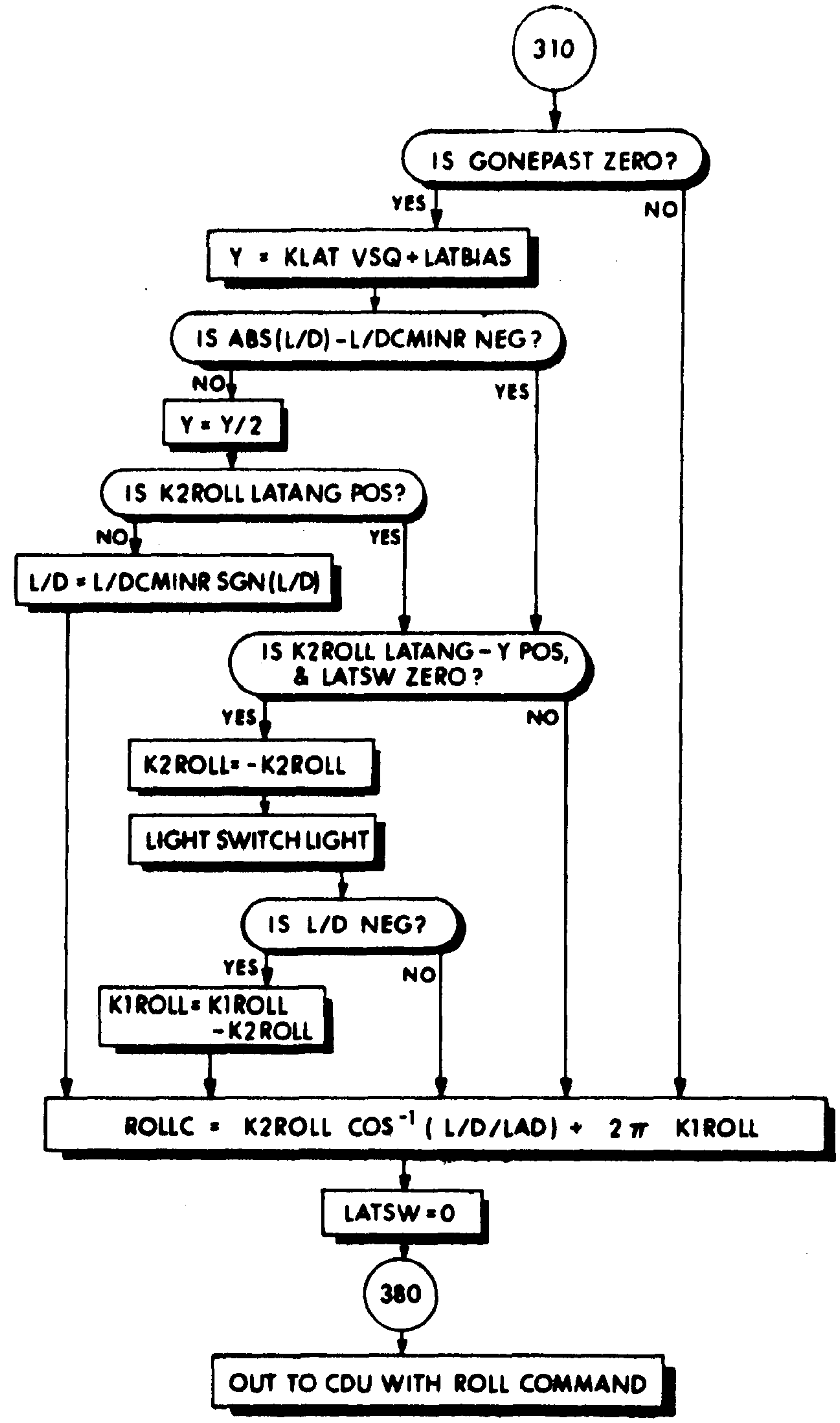


FIGURE 18. RE-ENTRY STEERING - LATERAL LOGIC

# TABLE I-A

## VARIABLES FOR RE-ENTRY CONTROL

| | |
|---|---|
| $U\bar{R}TO$ | INITIAL TARGET VECTOR |
| $\bar{U}Z$ | UNIT VECTOR NORTH |
| $\bar{V}$ | VELOCITY VECTOR |
| $\bar{R}$ | POSITION VECTOR |
| $\bar{R}TE$ | VECTOR EAST AT INITIAL TARGET |
| $\bar{U}TR$ | NORMAL TO $\bar{R}TE$ AND $\bar{U}Z$ |
| $\bar{W}E$ | EARTH RATE VECTOR |
| $U\bar{R}T$ | TARGET VECTOR |
| $\bar{U}NI$ | UNIT NORMAL TO TRAJECTORY PLANE |
| $\bar{D}ELV$ | INTEGRATED ACCELERATION VECTOR |
| $\bar{G}$ | GRAVITY VECTOR |
| AO | INITIAL DRAG FOR UPCONTRL |
| AHOOK | TERM IN GAMMAL COMPUTATION |
| ALP | CONST FOR UPCONTRL |
| ASKEP | KEPLER RANGE |
| ASP1 | FINAL PHASE RANGE |
| ASPUP | UPRANGE |
| ASP3 | GAMMA CORRECTION |
| ASPDWN | RANGE DOWN TO PULL-UP |
| ASP | PREDICTED RANGE = ASKEP+ASP1+ASPUP+ASP3+ASPDWN |
| COSG | COSINE (GAMMAL) |
| D | TOTAL ACCELERATION |
| DO | CONTROLLED CONST DRAG |
| DHOOK | TERM IN GAMMAL COMPUTATION |
| DIFF | THETNM-ASP (RANGE DIFFERENCE) |
| DIFFOLD | PREVIOUS VALUE OF DIFF |
| DR | REFERENCE DRAG FOR DOWNCONTROL |
| DREF | REFERENCE DRAG |
| DVL | VS1 -VL |

## TABLE I-A (Cont'd)

| | | |
|---|---|---|
| E | ECCENTRICITY | |
| F1 | DRANGE/D DRAG | (FINAL PHASE) |
| F2 | DRANGE/DRDOT | (FINAL PHASE) |
| F3 | DRANGE/D(L/D) | (FINAL PHASE) |
| FACT1 | CONST FOR UPCONTRL | |
| FACT2 | CONST FOR UPCONTRL | |
| FACTOR | USED IN UPCONTRL | |
| GAMMAL | FLIGHT PATH ANGLE AT VL | |
| GAMMAL1 | SIMPLE FORM OF GAMMAL | |
| KA | ACCELERATION LEVEL TO ROLL LIFT UP | |
| K1ROLL | INDICATOR FOR ROLL SWITCH | |
| K2ROLL | INDICATOR FOR ROLL SWITCH | |
| LATANG | LATERAL RANGE | |
| LEQ | EXCESS C. F. OVER GRAV = (VSQ-1) GS | |
| L/D | DESIRED LIFT TO DRAG RATIO (VERTICAL PLANE) | |
| PREDANGL | PREDICTED RANGE | (FINAL PHASE) |
| Q7 | MINIMUM DRAG FOR UPCONTROL | |
| RDOT | ALTITUDE RATE | |
| RDOTREF | REFERENCE RDOT FOR UPCONTRL | |
| RDTR | REFERENCE RDOT FOR DOWNCONTRL | |
| ROLLC | ROLL COMMAND | |
| RTOGO | RANGE TO GO | (FINAL PHASE) |
| SL | SIN OF LATITUDE | |
| T | TIME | |
| THETA | DESIRED RANGE (RADIANS) | |
| THETNM | DESIRED RANGE (NM) | |
| V | VELOCITY MAGNITUDE | |
| V1 | INITIAL VELOCITY FOR UPCONTRL | |
| V1OLD | PREVIOUS VALUE OF V1 | |

## TABLE I-A (Cont'd)

| | | |
|---|---|---|
| VCORR | VELOCITY CORRECTION FOR UPCONTRL | |
| VL | EXIT VELOCITY FOR UPCONTRL | |
| VS1 | VSAT OR V1, WHICHEVER IS SMALLER | |
| VBARS | $VL^2/VSAT^2$ | |
| VSQ | NORMALIZED VELOCITY SQUARED = $V^2/VSAT^2$ | |
| WT | EARTH RATE X TIME | |
| X | INTERMEDIATE VARIABLE USED IN G LIMITER | |
| Y | LATERAL MISS LIMIT | |
| SWITCHES | | INITIAL STATE |
| RELVELSW | RELATIVE VELOCITY SWITCH | (0) |
| EGSW | FINAL PHASE SWITCH | (0) |
| HUNTIND | INITIAL PASS THRU HUNTEST | (0) |
| HIND | INDICATES INTERATION IN HUNTEST | (0) |
| LATSW | NO LATERAL CONTROL WHEN ON | (0) |
| GONEPAST | INDICATES OVERSHOOT OF TARGET | (0) |

## TABLE I-B. GUIDANCE CONSTANTS

| | Symbol | Value |
|---|---|---|
| Pacific recovery point: Latitude | | 17.25°N |
| Longitude | | 170.00°E |
| Constant drag gain (on drag) | C16 | 0.1 |
| Constant drag gain (on RDOT) | C17 | 0.00497 |
| Lead velocity for up control start | C18 | 500 ft/s |
| Minimum constant drag | C19 | 40 ft/s/s |
| Minimum D for lift up | C20 | 175 ft/s/s |
| Factor in AHOOK computation | CHOOK | 0.25 |
| Factor in GAMMAL computation | CH1 | 0.75 |
| G-limit | GMAX | 10g |
| Minimum drag for lift up if down | KAFIX | 0.2g |
| Up control gain, optimized | KB2 | 0.0034 |
| Up control gain, optimized | KB1 | 3.4 |
| Factor in V1 computation | KC1 | 0.8 |
| Factor in A0 computation | KC2 | 0.7 |
| Lateral switch gain | KLAT | 0.0075 |
| Increment to Q7 to end kepler | KDMIN | 0.5 ft/s/s |
| Time of flight calculation gain | KTETA | 1000 |
| Max L/D | LAD | 0.3 |
| Lateral switch bias term | LATBIAS | 0.00012 |
| LAD cos (15°) | L/DCMINR | 0.2895 |
| Up control L/D | LEWD | 0.2 |
| Final Phase L/D | LOD | 0.18 |
| Acceptable tolerance to stop range iteration | 25NM | 25 n.m. |
| Final phase range (-23500 Q3) | Q2 | -1002 n.m. |
| Final phase dR/dV | Q3 | 0.07 n.m. /ft/s |
| Interval between steering updates | DT | 2 sec |
| Final phase dR/dGAMMAL | Q5 | 7050 n.m. |
| Final phase initial GAMMAL | Q6 | 0.0349 |
| Minimum drag for up control | Q7F | 6 ft/s/s |
| Limit value of VCORR | VCORLIM | 1000 ft/s |
| Minimum RDOT to close loop | VRCONTRL | 700 ft/s |
| Velocity to switch to relative velocity | VMIN | 12,883 ft/s |
| Minimum VL | VLMIN | 18,000 ft/s |
| Velocity to stop steering | VQUIT | 1,000 ft/s |
| Normalization factor, acceleration | GS | 32.2 ft/s/s |
| Atmosphere Scale Height | HS | 28,500 ft |
| Normalization factor, velocity | VSAT | 25,766.197 ft/sec |
| Nominal earth's radius (entry only) | RE | 21,202,909 ft |
| Range angle to nautical mile factor | ATK | 3437.7468 n.m./rad. |

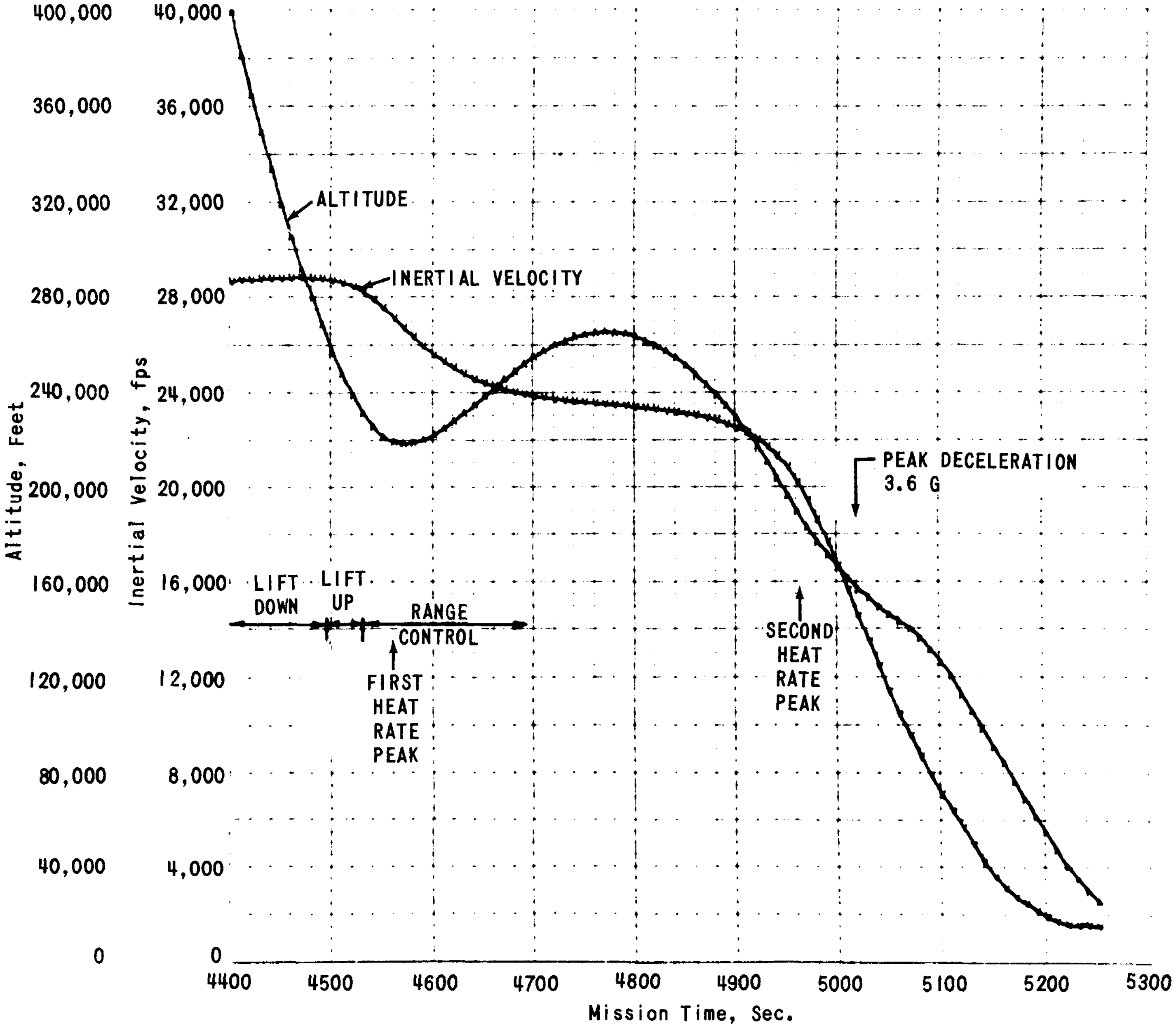


FIGURE 19. ALTITUDE AND INERTIAL VELOCITY FOR THE NOMINAL TRAJECTORY (10 SEC BETWEEN MARKS ON PLOTS)

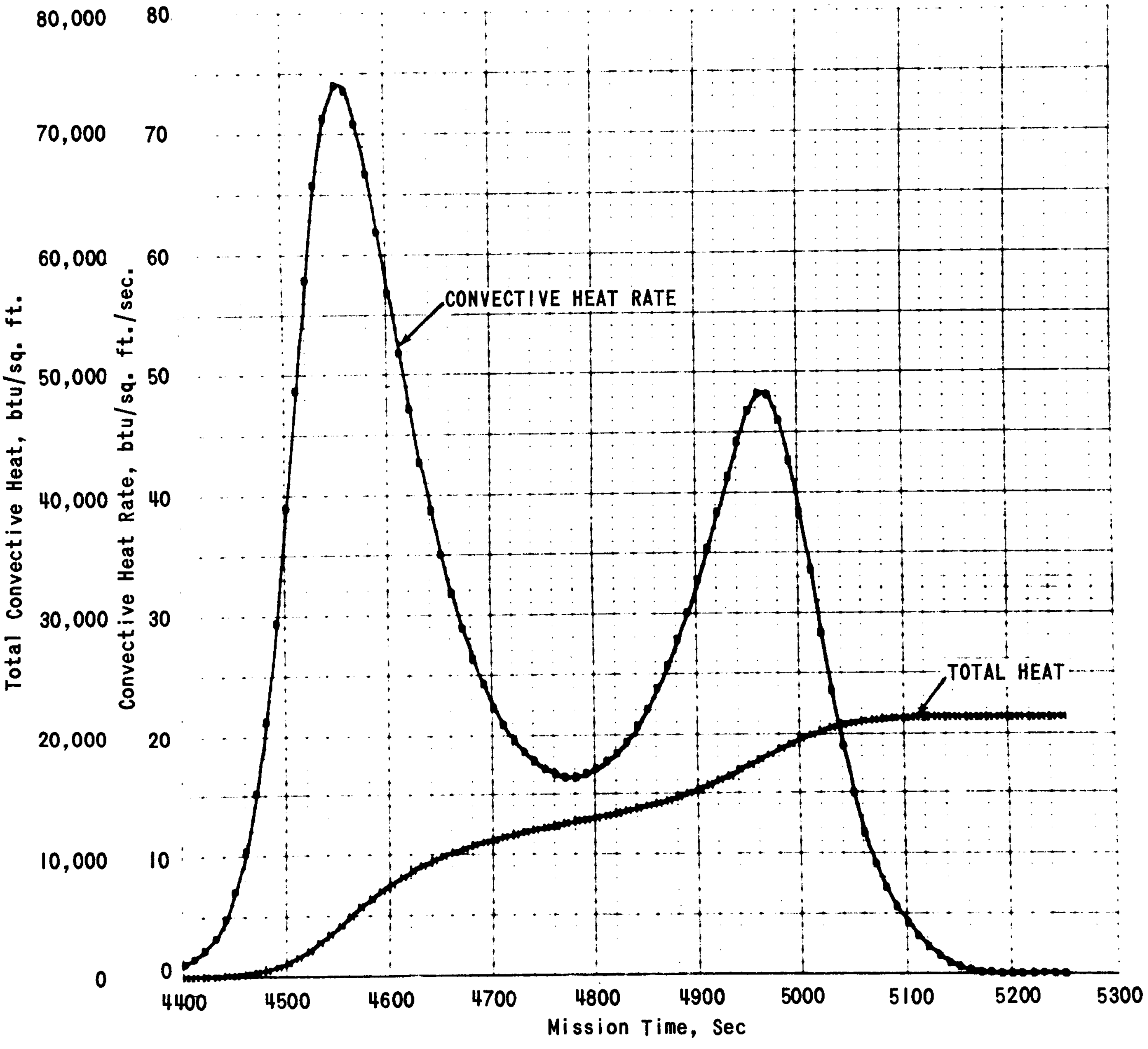


FIGURE 20. HEAT RATE AND HEAT FOR THE NOMINAL TRAJECTORY

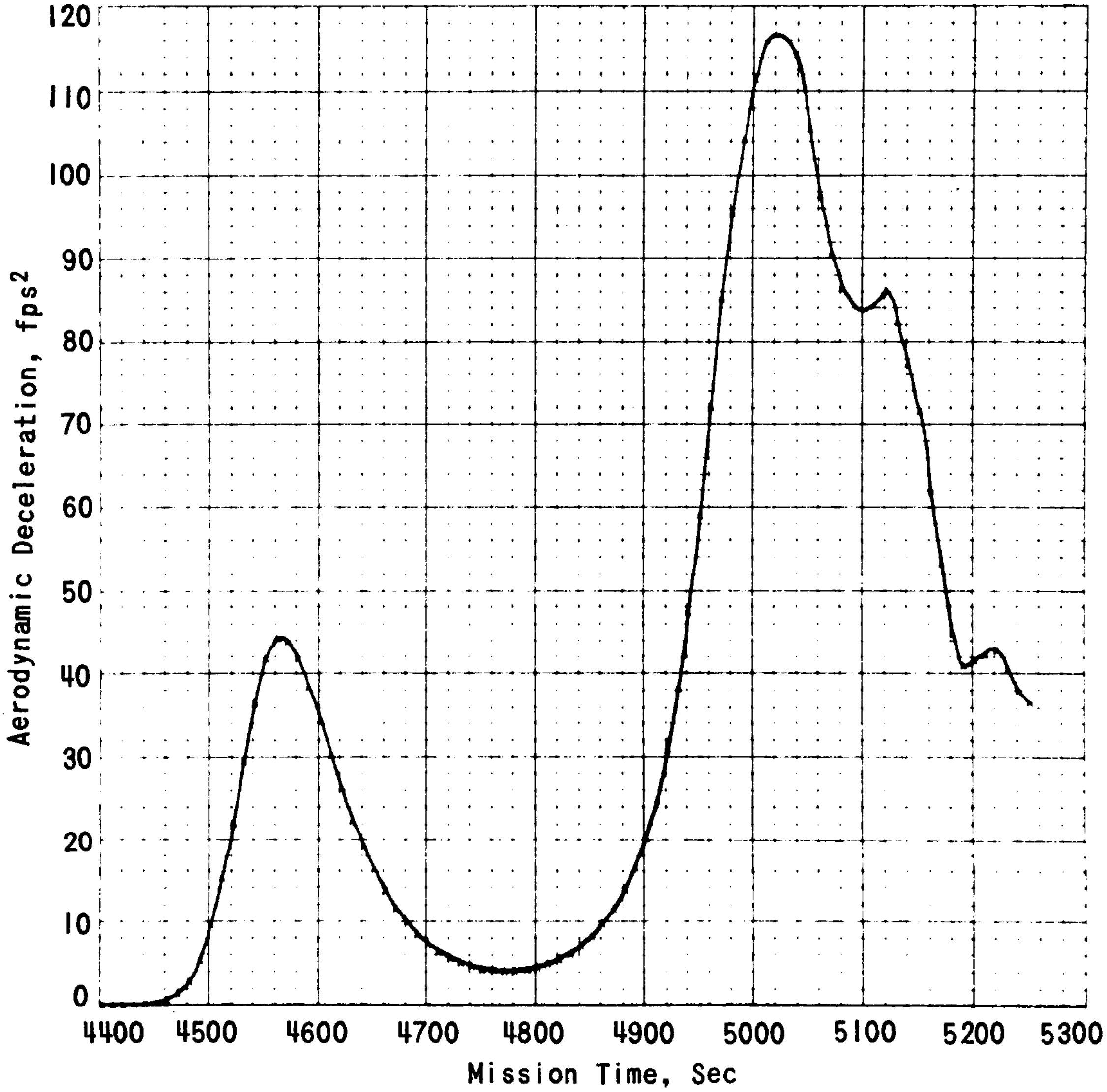


FIGURE 21. AERODYNAMIC DECELERATION VS. TIME, NOMINAL MISSION

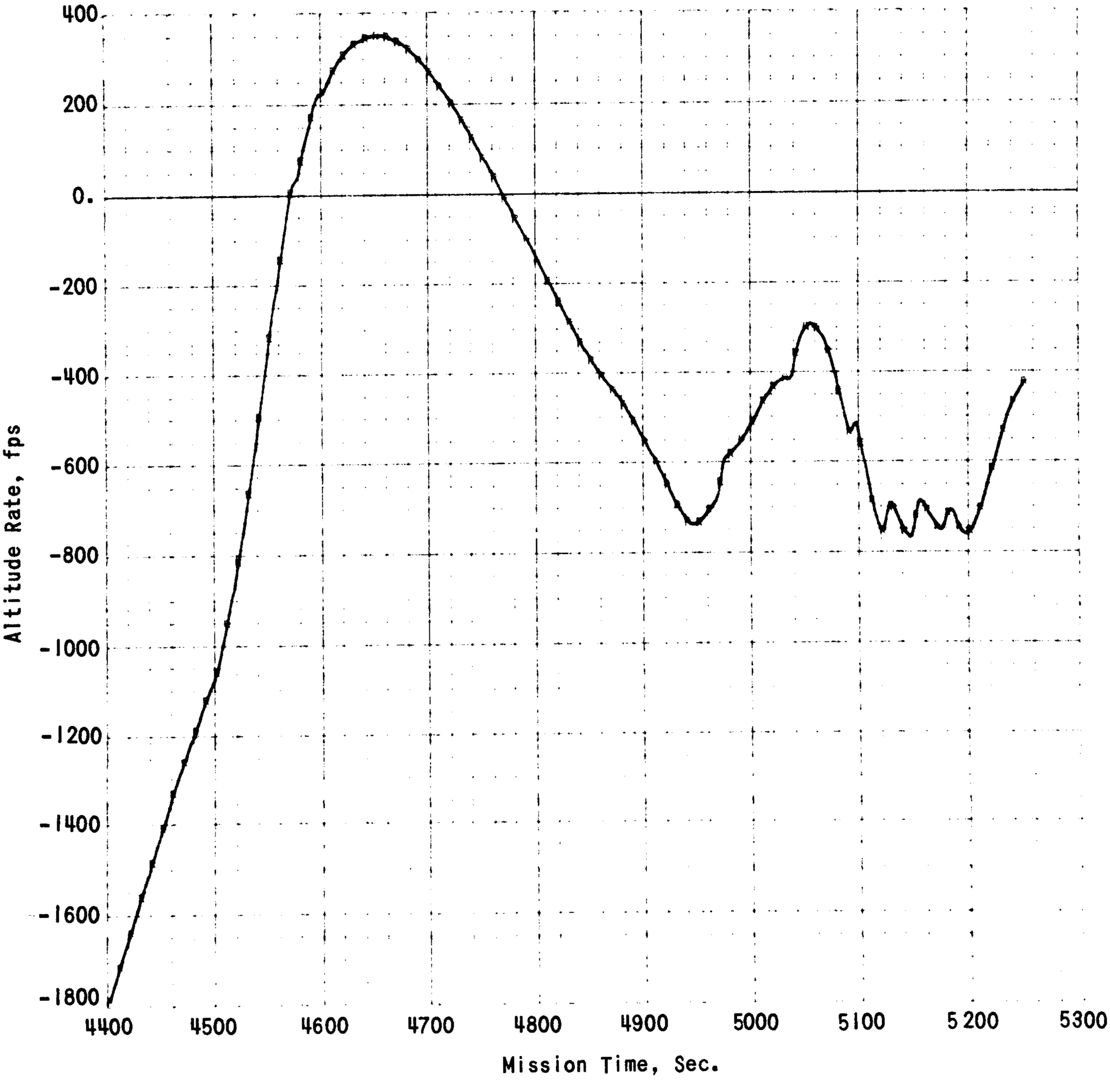


FIGURE 22. ALTITUDE RATE FOR THE NOMINAL TRAJECTORY

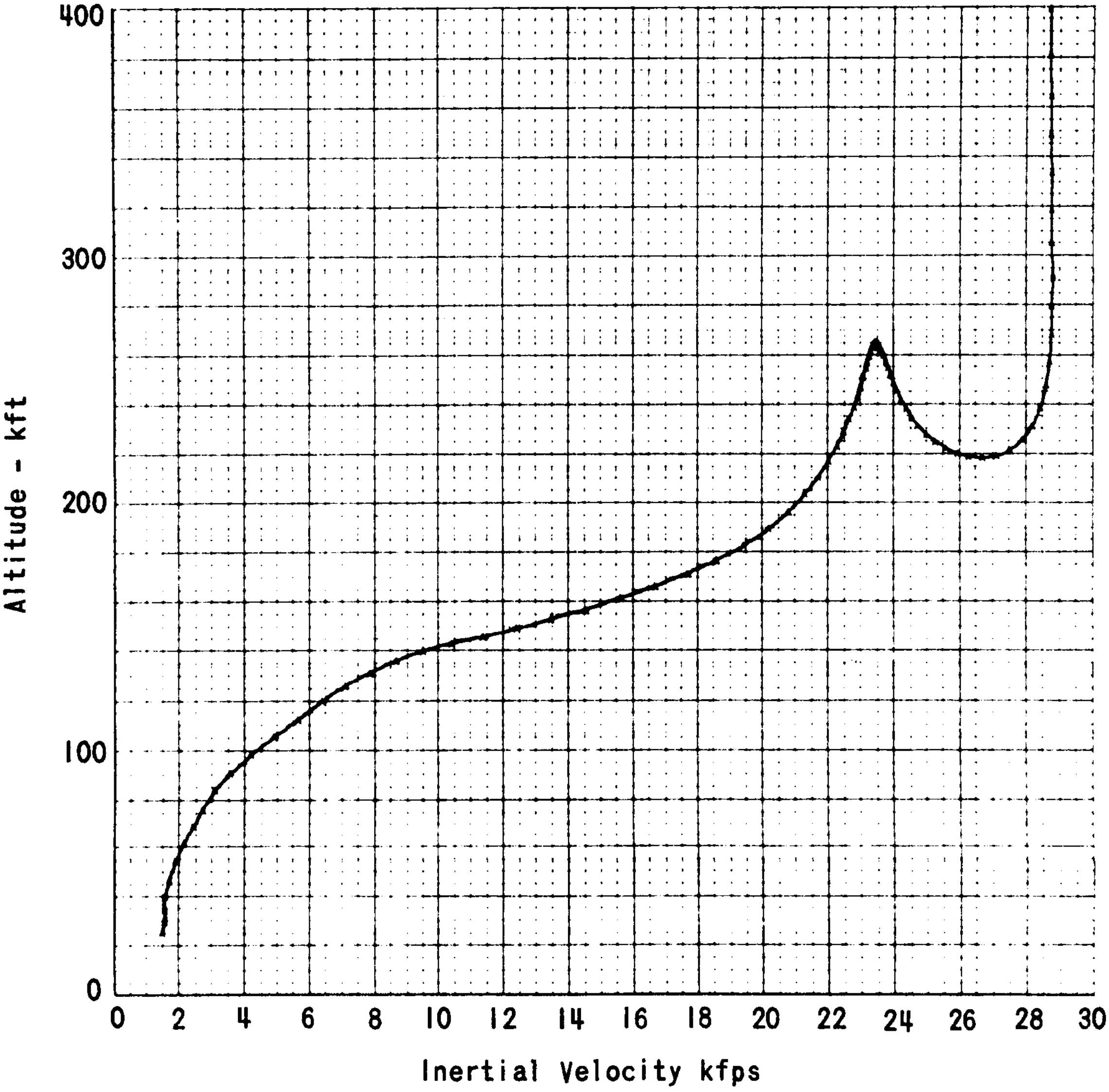


FIGURE 23. ALTITUDE VS. INERTIAL VELOCITY, NOMINAL MISSION

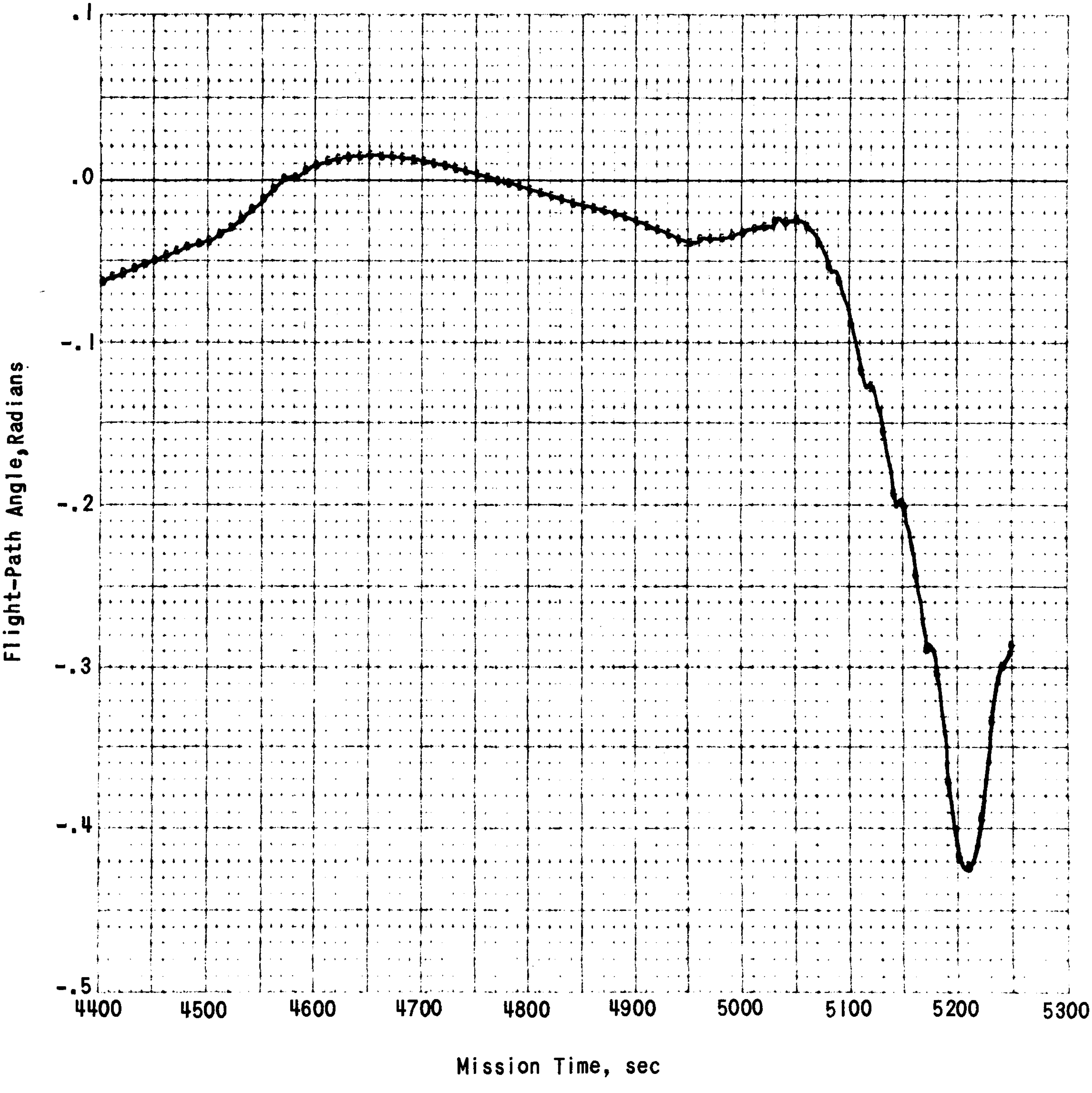


FIGURE 24. FLIGHT-PATH ANGLE VS. TIME, NOMINAL MISSION

# APPENDIX

## Notation

A more complete list is given in Table 1-A following the reentry flow charts.

| | |
|---|---|
| $C_D$ | drag coefficient. |
| $C_L$ | life coefficient. |
| D | drag acceleration, fps$^2$. |
| G | gravitational acceleration, fps$^2$. |
| H | spacecraft altitude, feet. |
| HS | atmosphere scale height, feet. |
| L | vertical component of lift acceleration, fps$^2$. |
| LAD | maximum possible value of L/D |
| LEQ | $V^2/R - G$, excess centrifugal acceleration, fps$^2$. |
| M | spacecraft mass, slugs. |
| R | spacecraft distance from Earth center, feet. |
| RE | Earth radius, feet. |
| S | spacecraft frontal area, feet$^2$. |
| V | spacecraft velocity magnitude, fps. |
| $\gamma$ | spacecraft flight path angle, radians. |
| $\rho$ | atmosphere density, slugs/ft$^3$. |

## Entry Guidance Equations

Following a presentation on the targetting scheme this section describes first the basic differential equations of motion for planar entry. Assumptions are made to permit simplifications in the equations which result in ease of handling. Finally the derived equations are compared to the equations used in the guidance computer. Use is made of material in the listed references.

### A. Targetting

Targetting geometry may be described by referring to Figure A1.

$\overline{V}$ - vehicle inertial velocity vector.

$\overline{R}$ - vehicle position vector.

$\overline{U}NI$ - unit normal vector to plane of motion.

$$\overline{U}NI = \frac{\overline{V} \times \overline{R}}{|\overline{V} \times \overline{R}|}$$

$U\overline{R}TO$ - unit vector along position vector of landing site, corresponding to some reference time.

$\overline{U}Z$ - unit vector along Z axis.

$\overline{U}TR$ - projection of $U\overline{R}TO$ onto the site latitude plane.

$\overline{R}TE$ - (non-unit) east vector on unit sphere.

$\overline{R}TE = \overline{U}Z \times U\overline{R}TO$

$\overline{U}TR = \overline{R}TE \times \overline{U}Z$

The target at any time, measured from the reference time, may be written in terms of $U\overline{R}TO$, $\overline{U}TR$ and $\overline{R}TE$ as

$$U\overline{R}T = U\overline{R}TO + \overline{U}TR(\cos WT-1) + \overline{R}TE \sin WT$$

the argument WT is the earth's sidereal rate times the time between liftoff (T=0) and predicted landing.

During the early portion of the flight the approximation used is

$$WT = W(1000\ THETA + T).$$

THETA is the downrange angle-to-go in radians. Thus, approximately, the formula assumes it takes 1000 seconds to traverse 1 radian range angle. (Satellite velocity gives an equivalant constant of 815 seconds per radian at 250,000 feet altitude.

In the final phase if

$$V > 12{,}883 \text{ fps}$$

the approximation used is

$$WT = \left(\frac{RE \cdot THETA}{V} + T\right)$$

$RE = 21{,}202{,}909.$ feet, the nominal earth radius for entry only. ($R_{earth}$ + 277,171 ft, the "average" entry altitude.)

If $V \leq 12{,}883$ fps the then present target coordinates are used.

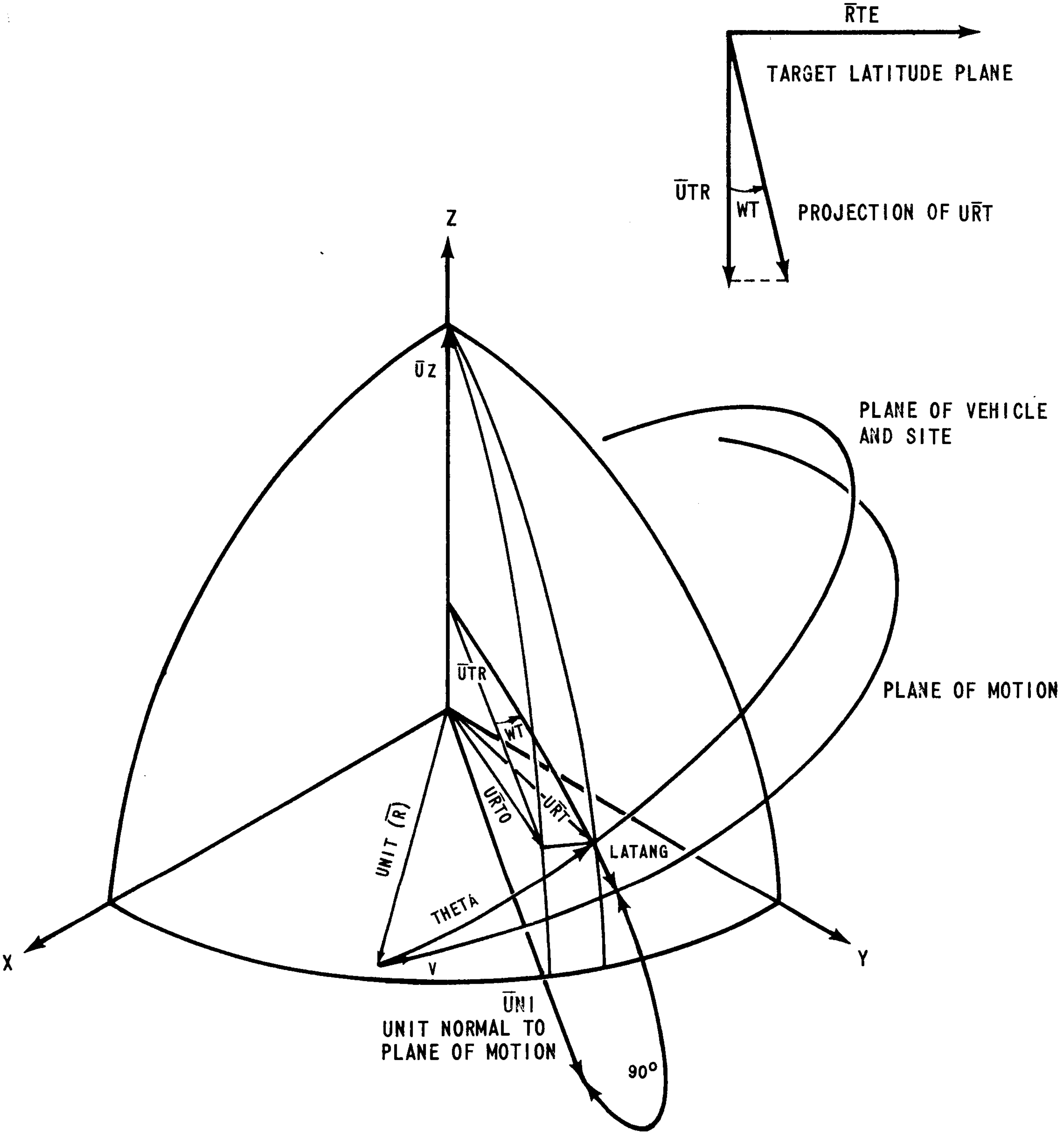


FIGURE A1. TARGETING GEOMETRY

Down range-to-go THETA, and cross range-to-go LATANG are given by

$$\text{THETA} = \cos^{-1}\left[\overline{\text{URT}} \cdot \frac{\overline{R}}{R}\right]$$

and from

$$\sin(\text{LATANG}) \cong \text{LATANG} = \overline{\text{URT}} \cdot \overline{\text{UNI}} = \cos(90 - \text{LATANG})$$

In Figure A1 the LATANG shown is a negative angle. Thus for

LATANG > 0 target is to right of plane of motion.

LATANG < 0 target is to left of plane of motion.

B. Equations of Motion

Basic to the differential equations of motion to be presented are certain assumptions.

- a stationary atmosphere.
- a spherical earth and inverse square gravity.
- an exponential atmosphere.

The two equations for planar motion are

$$\dot{V} = -D - G \sin\gamma \qquad (1)$$

$$V\dot{\gamma} = \left(\frac{V^2}{R} - G\right)\cos\gamma + L \qquad (2)$$

Drag, lift and density are given by

$$D = \frac{1}{2}\rho V^2 \frac{S\,C_D}{M} \qquad (3)$$

$$L = \frac{1}{s}\rho V^2 \frac{S\,C_L}{M} \qquad (4)$$

$$\rho = \rho_o e^{-\frac{H}{HS}} \qquad (5)$$

Additional equations used are:

$$\frac{dH}{dt} = \dot{R} \qquad (6)$$

$$\dot{R} = V \sin\gamma \qquad (7)$$

$$\frac{d\ \mathrm{RANGE}}{dt} = \frac{R\ \dot{\theta}}{RE} \qquad (8)$$

$$\frac{R\dot{\theta}}{RE} = \frac{V\ \cos\ \gamma}{RE} \qquad (9)$$

$$\frac{d\dot{R}}{dt} = \dot{V}\ \sin\ \gamma + V\ \dot{\gamma}\ \cos\ \gamma \qquad (10)$$

From equations 3, 4 and 5 differentiation yields

$$\frac{dD}{dt} = -\ \frac{D}{HS}\ \dot{R} + 2\frac{\dot{V}}{V}\ D \qquad (11)$$

$$\frac{dL}{dt} = -\ \frac{L}{HS}\ \dot{R} + 2\frac{\dot{V}}{V}\ L \qquad (12)$$

For all analysis $\gamma$ is assumed small so that

$\cos\ \gamma = 1$

$\sin\ \gamma = \gamma$ (or zero)

It is always assumed that $G\ \sin\ \gamma$ can be neglected with respect to D, and in many but not all cases gravity is assumed to balance centrifugal force.

To provide a tidy set of equations from which to derive almost all of the guidance equations the above set is put in a matrix form.

$$\frac{d}{dt}\begin{bmatrix} D \\ L \\ R\dot{\theta} \\ \dot{R} \\ \mathrm{RANGE} \\ H \end{bmatrix} = \begin{bmatrix} -\frac{\dot{R}}{HS} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\dot{R}}{HS} & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{RE} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} D \\ L \\ R\dot{\theta} \\ \dot{R} \\ \mathrm{RANGE} \\ H \end{bmatrix} + \begin{bmatrix} \frac{2\ \dot{V}}{V}\ D \\ \frac{2\ \dot{V}}{V}\ L \\ 0 \\ G(\frac{V^2}{GR} - 1) \\ 0 \\ 0 \end{bmatrix} \qquad (13)$$

The above equation may be summarized as

$$\frac{d\,[Y]}{dt} = [A]\ [Y] + [B] \tag{14}$$

where

$$[Y] = \begin{bmatrix} D \\ L \\ R\dot{\theta} \\ \dot{R} \\ \text{RANGE} \\ H \end{bmatrix} = \begin{bmatrix} D \\ L \\ V\cos\gamma \\ V\sin\gamma \\ \text{RANGE} \\ H \end{bmatrix} \tag{15}$$

In some of the work the independent variable is changed. One may make use of the following:

$$\frac{d\,[Y]}{dt} = \frac{d\,[Y]}{d\,(R\dot{\theta})}\ \frac{d\,(R\dot{\theta})}{dt} \tag{16}$$

$$\frac{d\,[Y]}{dt} = \frac{d\,[Y]}{d\,(\dot{R})}\ \frac{d\,(\dot{R})}{dt} \tag{17}$$

where from equation (13)

$$\frac{d\,(R\dot{\theta})}{dt} = -\,D \tag{18}$$

$$\frac{d\dot{R}}{dt} = L \tag{19}$$

## C. Derived Equations

One can now proceed to the equations used in each phase of the guidance.

1. Descent to pull-out (Pre-up Phase). The tasks are:

   - predict pull-out velocity V1.
   - predict pull-out acceleration AO.
   - predict range to pull-out.
   - guide the vehicle along a constant L/D trajectory to the desired pull-out conditions.

A key assumption is that the vector [B] in equation 14 is zero.

(a) To determine V1.

From equations 13 line 3 and equations 16 and 19 one obtains

$$\frac{d\dot{R}}{d(R\dot{\Theta})} = -\frac{L}{D} \quad . \qquad (20)$$

Let

$$\frac{L}{D} = \mathrm{LAD}, \text{ a constant.}$$

and

$$R\dot{\Theta} = V \quad . \qquad (21)$$

Integration to pull-out results in

$$\int_{\mathrm{RDOT}}^{0} d\dot{R} = -\,\mathrm{LAD} \int_{\mathrm{V1}}^{\mathrm{V}} d(R\dot{\Theta}) \quad . \qquad (22)$$

or

$$\boxed{\mathrm{V1} = \mathrm{V} + \frac{\mathrm{RDOT}}{\mathrm{LAD}}} \qquad (23)$$

(b) To determine AO.

From equation 13 line 1

$$\frac{dD}{d\dot{R}} = -\frac{\dot{R}}{HS}\ \frac{1}{L/D} \quad . \qquad (24)$$

With the same assumptions as above, one integrates to pull-out,

$$\int_{D}^{AO} dD = \frac{1}{HS\ LAD} \int_{RDOT}^{0} \dot{R}\, d\dot{R} \qquad (25)$$

to yield

$$\left[ AO = D + \frac{1}{2HS\ (LAD)}\ RDOT^2 \right] \qquad (26)$$

The equations in HUNTEST are

$$V1 = V + 0.8\ \frac{RDOT}{LAD} \qquad (27)$$

$$AO = D + \frac{0.7}{2HS\ (LAD)}\ RDOT^2 \qquad (28)$$

MIT/IL has added the 0.8 and 0.7 factors.

(c) To determine RANGE.

From equation 13 line 5

$$\frac{d\ RANGE}{d\ \dot{R}} = \frac{R\dot{\theta}}{RE}\ \frac{1}{(L/D)\ D}\ . \qquad (29)$$

Additional assumptions are:

$$R\dot{\theta} = V, \text{ a constant.} \qquad (30)$$

$$D = AO, \text{ a constant.} \qquad (31)$$

Integrating equation 29 results in:

$$\int_{0}^{RANGE} d\ RANGE = \frac{V}{RE\ (LAD)}\ \frac{1}{AO} \int_{RDOT}^{0} d\dot{R} \qquad (32)$$

$$RANGE = -\frac{V}{RE\ LAD}\ \frac{1}{AO}\ RDOT \qquad (33)$$

This is the same as that used in the guidance except for ATK, the conversion factor from nm to radians.

(d) To determine L/D. Assume V1 and AO are known. The task of computing the commanded L/D to fly to the desired pull-out is handled as follows:

Equation 23 and 26 may be used to specify the desired or reference values of D and RDOT at each instant prior to pull-out. These are designated DR and RDTR.

$$\left[\mathrm{RDTR} = \mathrm{LAD}(\mathrm{V1}-\mathrm{V})\right] \tag{34}$$

$$\left[\mathrm{DR} = \mathrm{AO} - \frac{1}{2\mathrm{HS}\ (\mathrm{LAD})}\ \mathrm{RDOTR}^2\right] \tag{35}$$

The guidance uses

$$\mathrm{RDTR} = \frac{\mathrm{LAD}}{0.8}\ (\mathrm{V1}-\mathrm{V}) \tag{36}$$

$$\mathrm{DR} = \mathrm{AO}\ \frac{0.7}{2\mathrm{HS}\ (\mathrm{LAD})}\ \mathrm{RDTR}^2 \tag{37}$$

MIT/IL has added the 0.8 and 0.7 factors.

The commanded L/D is then given by

$$\frac{L}{D} = \mathrm{LAD} + \mathrm{C}\ 16\ (\mathrm{D} - \mathrm{DR}) - \mathrm{C}\ 17(\mathrm{RDOT} - \mathrm{RDTR}) \tag{38}$$

where C16 and C17 are gain constants added by MIT/IL.

2. Pull-out to Exit Phase (Up Phase)

Assume pull-out conditions V1 and AO are known. The tasks are as follows:

- predict the exit velocity and flight-path angle VL and GAMMAL.
- predict the range to exit.
- guide to the exit condition.

(a) To determine VL.

Assume

$$[B] = [0]$$

Again from equation 13 line 4 and equation 16 and 19

$$\frac{d\dot{R}}{d(R\dot{\Theta})} = -\frac{L}{D} \tag{39}$$

Now assume additionally

$$\frac{L}{D} = \text{LEWD, a constant.} \qquad (40)$$

$$R\dot{\Theta} = V \qquad (41)$$

Integrating from pull-out to V, and $\dot{R}$

$$\int_{0}^{\dot{R}} d\dot{R} = -\text{LEWD} \int_{V1}^{V} dV \qquad (42)$$

$$\dot{R} = \text{LEWD}\ (V1 - V) \qquad (43)$$

From equation 13 line 6

$$\frac{dH}{d(R\dot{\Theta})} = -\frac{\dot{R}}{D} \qquad (44)$$

Combine equations 43 and 44 and make use of 41.

$$\frac{dH}{dV} = -\text{LEWD}\ \frac{(V1 - V)}{D} \qquad (45)$$

Use is also made of equation 5 in 45.

$$\int_{V1}^{VL} \frac{V1 - V}{V^2}\ dV = 1 \int_{H1}^{HL} \frac{K1}{\text{LEWD}}\ e^{-\frac{H}{HS}}\ dH. \qquad (46)$$

$$K1 = \frac{1}{2}\ \rho_o\ \frac{SC_D}{M} \qquad (47)$$

Equation 46 integrates to form

$$1 - \frac{V1}{VL} + \ln \frac{V1}{VL} = \frac{K1}{\text{LEWD}}\ HS \left(e^{-HL/HS} - e^{-H1/HS}\right) . \qquad (48)$$

If Q7 is defined as the drag acceleration at exit H = HL, equation 48 becomes:

$$-(1 - \frac{V1}{VL} + \ln \frac{V1}{VL}) = \frac{HS}{\text{LEWD}}\ \frac{AO}{V1^2} \left[1 - \frac{Q7}{AO}\ \frac{V1^2}{VL^2}\right] \qquad (49)$$

By expressing

$$\ln \frac{V1}{VL} = \ln \left(1 + \frac{V1 - VL}{VL}\right) \cong \frac{V1 - VL}{VL} \qquad (50)$$

(expanding and ignoring higher than first order terms), one arrives at

$$\left(\frac{V1}{VL} - 1\right)^2 = ALP \left(1 - \frac{Q7}{AO} \; \frac{V1^2}{VL^2}\right) \quad , \qquad (51)$$

where ALP is given by

$$\left[ALP = \frac{2AO \quad HS}{LEWD \; V1^2}\right] \quad . \qquad (52)$$

Equation 51 is a quadratic in V1/VL which when solved yields:

$$VL = \frac{V1}{1-ALP}\left[1 - \sqrt{\left[\frac{ALP(ALP-1)}{AO}\right] Q7 + ALP}\;\right] . \qquad (52)$$

The negative square root is used because VL<V1. (Exit occurs after pull-out).

By defining

$$FACT1 = \frac{V1}{1-ALP} \qquad (53)$$

$$FACT2 = \frac{ALP(ALP-1)}{AO} \qquad (54)$$

the equation for VL may be written as

$$\left[VL = FACT1 \left[1 - \sqrt{FACT2 \cdot Q7 + ALP}\;\right]\right] \quad . \qquad (55)$$

Equations 52 and 55 are used in HUNTEST.

(b) To determine GAMMAL.
Assume [B] ≠ [0]
From equation 13 line 4

$$\frac{d\dot{R}}{d(R\dot{\theta})} = -\frac{L}{D} + \frac{G}{D}\left(1 - \frac{V^2}{GR}\right) \qquad (56)$$

Further new assumptions are

$$R\dot{\theta} = V \qquad (57)$$

$$GR = VSAT^2\text{, a constant.} \qquad (58)$$

$$L/D = LEWD\text{, a constant.} \qquad (59)$$

Recalling $D = K1\ V^2 e^{-H/HS}$

$$\frac{d\dot{R}}{dV} = -\ LEWD + \frac{G}{K1\ e^{-H/HS}} \left( \frac{1}{V^2} - \frac{1}{VSAT^2} \right) \tag{60}$$

To make a useful substitution for $e^{-H/HS}$, one first writes it as

$$e^{H/HS} = e^{(H1 + \Delta H)/HS} = e^{H1/HS}\ e^{\Delta H/HS}$$

where H1 is the altitude where V = VS1*. One then assumes

$$e^{\frac{\Delta H}{HS}} = 1 + a(VS1 - V) \tag{61}$$

where the slope a is to be determined. The resulting approximation then takes the form

$$e^{H/HS} \cong e^{H1/HS} \left[ 1 + a\ (VS1 - V) \right] \tag{62}$$

Equation 62 is substituted into 60 to yield

$$\frac{d\dot{R}}{dV} = -\ LEWD - \frac{G\ VS1^2}{D_{VS1}} \left[ 1 + a(VS1 - V) \right] \left[ \frac{1}{VSAT^2} - \frac{1}{V^2} \right] . \tag{63}$$

Assume

$$VS1 = VSAT. \tag{64}$$

Equation 63 is now integrated

$$\int_{\dot{R}_{VS1}}^{\dot{R}} d\dot{R} = \int_{VS1}^{V} \ldots\ldots\ dV \tag{65}$$

---

*H1 and VS1 are the names given altitude and velocity at the time the GAMMAL computation is made. The guidance directs that VS1 use the minimum of V1 and VSAT.

Following the integration the assumption is made that

$$\ln\left[1 - \left(\frac{VS1 - V}{VS1}\right)\right] = -\left(\frac{VS1 - V}{VS1}\right) - \frac{1}{2}\left(\frac{VS1 - V}{VS1}\right)^2 \qquad (66)$$

After much manipulation the result takes the form

$$\dot{R} = \dot{R}_{VS1} + LEWD\ (VS1 - V) - \frac{G}{D_{VS1}V}\left[1 + a(VS1 - V)\right](VS1 - V)^2 \ . \qquad (67)$$

The assumption is made that

$$\dot{R}_{VS1} = 0. \qquad (68)$$

Equation 67 then becomes

$$\dot{R} = LEWD\ (VS1 - V) - \frac{G}{D_{VS1}V}\left[1 + a(VS1 - V)\right](VS1 - V)^2 \ . \qquad (69)$$

From equation 10 with assumptions noted earlier

$$d\dot{R} = V d\gamma \ . \qquad (70)$$

Assuming

$$V = VL \quad \text{a constant,} \qquad (71)$$

equation 70 can be integrated as follows

$$\int_0^{GAMMAL} d\gamma = \frac{1}{VL}\int_0^{\dot{R}_L} d\dot{R} \qquad (72)$$

The lower limits of zero are consistent with the assumption of equation 68. Applying equation 69 to 72 with the note that at

$\dot{R} = \dot{R}_L$ , $V = VL$.

$$\text{GAMMAL} = \frac{\text{LEWD (VS1 - VL)}}{\text{VL}} - \left\{ \frac{G}{D_{\text{VS1}} \text{V VL}} \left[ 1 + a(\text{VS1} - \text{V}) \right] (\text{VS1} - \text{V})^2 \right\}_{\text{V=VL}} \tag{73}$$

The first term on the right in equation (73) is GAMMAL1 in HUNTEST.

To evaluate the term in braces assume V is constant. Then

$$D_{\text{VS1}} \, e^{-\frac{\Delta H}{HS}} = \text{Q7} \tag{74}$$

Using approximation in equation 62,

$$D_{\text{VS1}} = \text{Q7} \left[ 1 + a(\text{VS1} - \text{VL}) \right] \tag{75}$$

From this

$$a = \frac{D_{\text{VS1}} - \text{Q7}}{\text{Q7 (Vs1 - VL)}} \tag{76}$$

DVS1 remains to be evaluated.

Note that the quadratic equation in (51) which yields 52 and 55 relates the velocity VL and acceleration Q7 to the pull-out conditions V1 and AO. The equation clearly applies to any intermediate point between V1 and VL. One can therefore adapt equation 55 to the form

$$\text{VS1} = \text{FACT1} \left[ 1 - \sqrt{\text{FACT2} \cdot D_{\text{VS1}} + \text{ALP}} \right], \tag{77}$$

and therefore

$$D_{\text{VS1}} = \frac{\left(1 - \frac{\text{VS1}}{\text{FACT1}}\right)^2 - \text{ALP}}{\text{FACT2}} \; . \tag{78}$$

Make the following definitions:

$$D_{\text{VS1}} \overset{\Delta}{=} \text{DHOOK} \tag{79}$$

$$\text{DVL} \overset{\Delta}{=} \text{VS1 - VL} \tag{80}$$

The guidance equations write equation 76 as

$$\text{AHOOK} = \left( \frac{\text{DHOOK}}{\text{Q7}} - \frac{\text{Q7}}{\text{DVL}} \right) \text{CHOOK} \quad , \qquad (81)$$

where

$$\text{CHOOK} = .25 \quad . \qquad (83)$$

One can then write the calculation for GAMMAL

$$\begin{aligned} \text{GAMMAL} &= \text{GAMMAL1} \\ &- \text{CH1} \; \frac{\text{GS}}{\text{DHOOK}} \frac{\text{DVL}^2}{\text{VL}^2} \; [1 + \text{AHOOK DVL}] \end{aligned} \qquad (83)$$

where

$$\text{G} \overset{\Delta}{=} \text{GS}$$

$$\text{CH1} = 0.75$$

CH1 above is an additional factor used by the guidance equations.

If GAMMAL is negative the inference is that the vehicle will not exit. It is therefore necessary to determine the V = V2 at apogee, i.e., at GAMMAL = 0. This is then used as the VL. The result is obtained by a first order Taylor expansion.

$$\text{V2} = \text{V}\Big|_{\gamma=0} = \text{VL} - \frac{\dot{\text{RL}}}{\left.\dfrac{d\dot{\text{R}}}{d\text{V}}\right|_{\text{V=VL}}} \qquad (85)$$

$\frac{d\dot{R}}{dV}$ is obtained not from equation 63 but by differentiating 69.

$$\begin{aligned} \frac{d\dot{\text{R}}}{d\text{V}} &= - \text{LEWD} + \frac{\text{G}}{\text{D}_{\text{VS1}}\text{VL}} \left[ 2\,(\text{VS1} - \text{VL}) + 3\,a(\text{VS1} - \text{VL})^2 \right] \\ &+ \frac{\text{G}}{\text{D}_{\text{VS1}}\text{VL}^2} \left[ (\text{VS1} - \text{VL})^2 + a(\text{VS1} - \text{VL})^3 \right] \end{aligned} \qquad (86)$$

The third term on the right is neglected. Using equation 86 and equation 87,

$$\dot{R}_L \quad = \quad \text{VL GAMMAL} \tag{87}$$

$$\left[ V2 = VL + \frac{\text{VL GAMMAL}}{\text{LEWD} - \left[ 3a(VS1 - VL)^2 + 2(VS1 - VL) \right] \left[ \frac{G}{D_{VS1} VL} \right]} \right] \tag{88}$$

With appropriate substitutions except as noted below

$$V2 = VL + \frac{\text{VL GAMMAL}}{\text{LEWD} - [3 \text{ AHOOK DVL}^2 + 2\text{DVL}] \left[ \frac{GS}{\text{DHOOK}} \quad \frac{CH1}{VL} \right]} \tag{89}$$

The guidance introduces CH1 = 0.75 which would not follow directly from 88.

A new acceleration at this V2 is computed in the manner discussed prior to equation 77.

$$\left[ Q7 \quad = \quad \frac{(1 - V2/FACT1)^2 - ALP}{FACT2} \right] \tag{90}$$

(c) To determine RANGE.
The assumptions are

$$R\dot{\Theta} \quad = \quad V \quad = \quad VL, \text{ a constant.} \tag{91}$$

$$\dot{R} \quad = \quad \dot{R}_L, \text{ a constant.} \tag{92}$$

from equation 13 line 5 and 6

$$\frac{dRANGE}{dt} \quad = \quad \frac{dRANGE}{dH} \quad \frac{dH}{dt} \quad = \quad \frac{1}{RE} \quad VL \tag{93}$$

$$\frac{dH}{dt} \quad = \quad \dot{R}_L \tag{94}$$

$$\frac{dRANGE}{dH} \quad = \quad \frac{VL}{RE} \quad \frac{1}{\dot{R}_L} \tag{95}$$

Integrating from pull-out to exit

$$\text{RANGE} = \frac{\text{VL}}{\text{RE}} \frac{1}{R_L} \Delta\text{H} \tag{96}$$

To express the increment in altitude in terms of V1, A0, Q7, VL consider

$$\text{A0} = \frac{1}{2} \rho \text{V1}^2 \frac{C_D S}{M} e^{\frac{-\text{H1}}{\text{HS}}}$$

$$\text{Q7} = \frac{1}{2} \rho \text{VL}^2 \frac{C_D S}{M} e^{\frac{-\text{HL}}{\text{HS}}}$$

$$e^{\frac{\text{HL}-\text{H1}}{\text{HS}}} = e^{\frac{\Delta\text{H}}{\text{HS}}} = \frac{\text{A0}/\text{V1}^2}{\text{Q7}/\text{VL}^2} \tag{97}$$

$$\Delta\text{H} = \text{HS} \ln\left(\frac{\text{A0}/\text{V1}^2}{\text{Q7}/\text{VL}^2}\right) \tag{98}$$

Combining 96, 98, 87 and 73

$$\text{RANGE} = \frac{\text{HS}}{\text{RE GAMMAL}} \ln\left(\frac{\text{A0}/\text{V1}^2}{\text{Q7}/\text{VL}^2}\right) \tag{99}$$

The assumption is made that

$$\text{GAMMAL} = \text{GAMMAL1} \tag{100}$$

$$\boxed{\text{RANGE} = \frac{\text{HS}}{\text{RE GAMMAL1}} \ln\left(\frac{\text{A0}/\text{V1}^2}{\text{Q7}/\text{VL}^2}\right)} \tag{101}$$

(d) To determine L/D.
From the work earlier we have

$$\text{VL} = \text{FACT1} \left(1 - \sqrt{\text{FACT2} \cdot \text{Q7} + \text{ALP}'}\right) . \tag{55}$$

As previously noted this holds for any pair of V and D along this phase. The V may be designated as the desired or reference VREF.

$$\boxed{\text{VREF} = \text{FACT1} \left(1 - \sqrt{\text{FACT2} \cdot \text{D} + \text{ALP}'}\right)} \tag{102}$$

A reference altitude rate is obtained from equation 69 assuming

$$\text{VS1} = \text{V1} \tag{103}$$

and

$$\text{V} = \text{VREF} \tag{104}$$

Appropriate substitutions are made for $D_{VS1}$, G and a, and CH1 is included.

$$\left[\text{RDOTREF} = \text{LEWD (V1 - VREF)} - \text{CH1 GS(V1 - VREF)}^2 \left(\frac{1 + \text{AHOOK (V1 - VREF)}}{\text{DHOOK} \qquad\qquad \text{VREF}}\right)\right] \qquad (105)$$

The guidance defines a variable gain

$$\text{FACTOR} = \frac{\text{D - Q7}}{\text{AO - Q7}} \qquad (106)$$

and commands an L/D according to the deviations from the referenced V and $\dot{R}$.

$$\frac{L}{D} = \text{LEWD - KB2 FACTOR} \left[\text{KB1 FACTOR (RDOT - RDOTREF)} + \text{(V - VREF)}\right] \qquad (107)$$

$$\text{KB1} = 3.4$$

$$\text{KB2} = .0034 \qquad (108)$$

KB1 and KB2 are MIT/IL specified gain constants.

3. Constant Drag Phase.

During the descent to pull-out, trajectory planning may predict an overshoot of 25 nm or more. The assumption is then that the anticipated pull-out velocity is too large and must be reduced. The guidance calls for flying a constant-drag trajectory until, at some subsequent pass through the guidance calculations, the predicted miss becomes less than 25 nm. The value of reference drag DO, is designated in the planning phase (HUNTEST) as the minimum of AO and C19. Thus DO is set equal to AO but not less than 40 $\text{fps}^2$.

From equation 13 line 1

$$\frac{dD}{dt} = -\frac{D}{HS}\dot{R} + 2\frac{\dot{V}}{V}D$$

Assume, in equation 13 line

$$\frac{d(R\dot{\theta})}{dt} = \dot{V} = -D$$

Along the constant-drag trajectory

$$D = DO \text{ and } \dot{D} = 0. \qquad \text{Hence}$$

$$\dot{H} = \dot{R} = -\frac{2HS}{V} DO \tag{109}$$

This is the reference altitude rate in the guidance. Equation 13 line 4 reads

$$L = \frac{d\dot{R}}{dt} - G\left(\frac{V^2}{GR} - 1\right) \tag{8}$$

and from 109

$$\frac{d\dot{R}}{dt} = -2\frac{HS}{V^2} DO^2 . \tag{110}$$

The lift is then

$$L = -2\frac{HS}{V^2} DO^2 - G\left(\frac{V^2}{GR} - 1\right) \tag{111}$$

The guidance, defining $G\left(\frac{V^2}{GR} - 1\right)$ as LEQ, commands L/D as

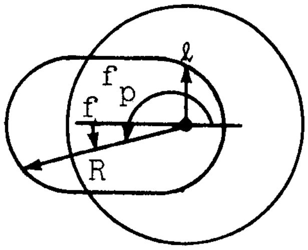
$$\left(\frac{L}{D}\right) = -LEQ + C16\ (D-DO) - C17\left(RDOT + \frac{2HS\ DO}{V}\right) \tag{112}$$

It may be seen that the first term on the right of equation 111 has been neglected.

## 4. Range Prediction

Range calculations for all but the ballistic phase are included in the sections on those phases. This section describes the Keplerian range computation. It then describes the method for incrementing pull-out velocity V1, should an excessive undershoot be predicted.

### (a) Ballistic Phase. To determine ASKEP.



The technique is to compute 2f, where f is the true anomaly has 180°. The polar form of the conic path is given by

$$R = \frac{\ell}{1 + e \cos f_p} \tag{113}$$

or

$$R = \frac{\ell}{1 - e \cos f} \tag{114}$$

From previous work

$$\dot{R} = V \sin \gamma$$

and

$$R\dot{f}_p = V \cos \gamma \quad .$$

The angular momentum and semi-latus rectum for the polar conic are given by

$$h = R^2\dot{f}_p = RV \cos \gamma \tag{115}$$

and

$$\ell = \frac{h^2}{\mu} = \frac{h^2}{GR^2} \quad . \tag{116}$$

Combine 115 and 116 to form

$$\frac{\ell}{R} = \frac{V^2 \cos^2 \gamma}{GR} \quad .$$

Make the following definitions:

$$GR = VSAT^2$$

$$\overline{V} = V/VSAT$$

The equation for $\ell/R$ may then be written as

$$\frac{\ell}{R} = \overline{V}^2 \cos^2 \gamma \tag{117}$$

This will be used later.

Differentiating equation 113 results in

$$\dot{R}\ (1 + e \cos f_p) = eR\ \dot{f}_p \sin f_p \quad .$$

Make appropriate substitutions to yield

$$V \sin \gamma \frac{\ell}{R} = eV \cos \gamma \sin f_p .$$

Therefore

$$\sin f_p = \frac{\ell}{eR} \tan \gamma$$

From 113

$$\cos f_p = - \frac{1}{e} \left(1 - \frac{\ell}{R}\right)$$

The above two equations may be combined.

$$e^2 = \left(\frac{\ell}{R}\right)^2 (1 + \tan^2 \gamma) - 2 \left(\frac{\ell}{R}\right) + 1 \quad .$$

By making use of equation 117 this reduces to

$$e^2 = 1 - 2\, \bar{V}^2 \cos^2 \gamma + \bar{V}^4 \cos^2 \gamma \quad . \tag{118}$$

From equation 114

$$\cos f = \frac{1}{e} \left(1 - \frac{\ell}{R}\right) \quad .$$

It then follows, that

$$\sin f = \frac{\sqrt{e^2 - \left(1 - \frac{\ell}{R}\right)^2}}{e} \quad .$$

Eliminating e and $\frac{\ell}{R}$ in the numerator by using equations 117 and 118, one arrives at

$$f = \sin^{-1} \left[\frac{\bar{V}^2 \cos \gamma \sin \gamma}{e}\right] \tag{119}$$

Equation 119 is used to compute ASKEP when the following substitution is made:

$$\bar{V}^2 = \text{VBARS} = \frac{\text{VL}^2}{\text{VSAT}^2} \quad ,$$

and the factor of 2 is used to give the full ballistic range rather than the range from apogee.

(b) To determine VCORR.
The first pass through the planning stage HUNTEST, generates a V1OLD

$$\text{V1OLD} = \text{V1} + \text{C18}$$

$$\text{V1OLD} = \text{V1} + 500.$$

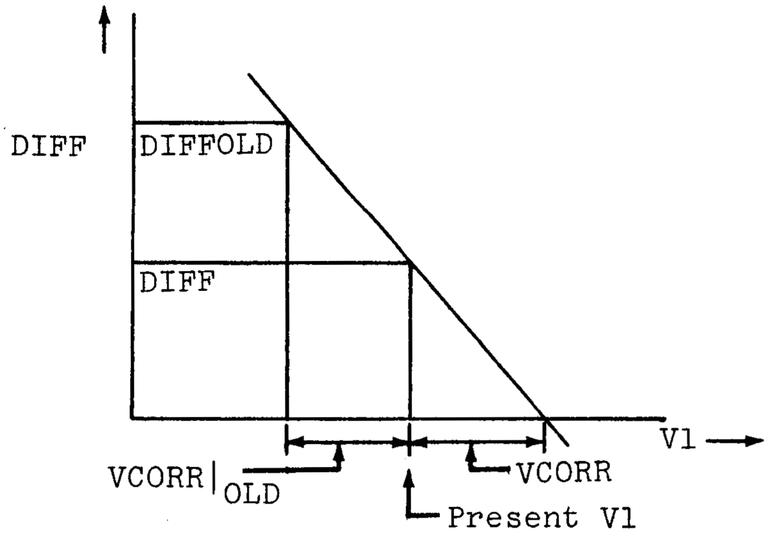
For the first iteration, the increment in V1, called VCORR, is taken as 500 fps. Checks are made on VCORR as follows:

a) VCORR is limited 1,000 fps.

b) If VL + VCORR > VSAT the increment is cut in half. Subsequent passes through the loop use

$$\text{VCORR} = \frac{\text{VCORR}\big|_{\text{OLD}}\ \text{DIFF}}{\text{DIFF}\big|_{\text{OLD}} - \text{DIFF}} \qquad (120)$$

This is a direct linear extrapolation to a DIFF (miss) of zero.



5. Final Phase

(a) To determine RANGE-TO-GO.
The range to go during the final phase is obtained from a series expansion about the stored reference.

$$\text{RANGE} = \text{RANGE}\Big|_{\text{ref.}} + \frac{\partial\text{RANGE}}{\partial\dot{R}}\Big|_{\text{ref.}} \Delta\dot{R} \qquad (121)$$

$$+ \frac{\partial R}{\partial D}\Big|_{\text{ref.}} \Delta D$$

This is called PREDANGL in the guidance.

(b) To determine L/D.
The increment in range as a function of vertical component of lift as given by

$$\Delta\text{RANGE}\Big|_{\Delta(\frac{L}{D})} = \frac{\partial\text{RANGE}}{\partial(\frac{L}{D})} (\Delta\frac{L}{D}) \qquad (122)$$

The error between range capability PREDANGL and required range THETNM then is nulled.

$$\text{PREDANGL} + \Delta\text{RANGE}\Big|_{\Delta(\frac{L}{D})} = \text{THETNM}.$$

$$\Delta(\frac{L}{D}) = \frac{\text{THETNM} - \text{PREDANGL}}{\frac{\partial\text{RANGE}}{\partial(\frac{L}{D})}} \qquad (123)$$

This increment is added to the reference $\frac{L}{D}$ to give the command value.

(c) To determine Range Prediction.
This range prediction is needed for trajectory planning. Range is expanded in a series about the nominal with V and GAMMAL as the parameters.

$$\text{RANGE (V, GAMMAL)} = \text{RANGE}\Big|_{\substack{\text{ref.}\\ \text{VREF}=23,500\\ \text{GAMMAL}=2°}} + \frac{\partial\text{RANGE}}{\partial\ V}\Big|_{\text{ref.}} \Delta V \qquad (124)$$

$$+ \frac{\partial\text{RANGE}}{\partial\text{GAMMAL}}\Big|_{\text{ref.}} \Delta\text{GAMMAL}$$

This reduces to

$$\mathrm{RANGE} = \left\{ \mathrm{RANGE}\Big|_{\mathrm{ref.}} - \frac{\partial \mathrm{RANGE}}{\partial V}\Big|_{\mathrm{ref.}} V_{\mathrm{ref}} \right\} + \frac{\partial \mathrm{RANGE}}{\partial V}\Big|_{\mathrm{ref.}} \mathrm{VL}$$

$$+ \frac{\partial \mathrm{RANGE}}{\partial \mathrm{GAMMAL}}\Bigg|_{\mathrm{ref.}} \left(\mathrm{GAMMAL}\Big|_{\mathrm{ref.}} - \mathrm{GAMMAL}\right) \qquad (125)$$

$$\mathrm{RANGE} = \mathrm{Q2} + \mathrm{Q3\ VL} + \mathrm{Q5}\ (\mathrm{Q6} - \mathrm{GAMMAL}) \qquad (126)$$

This is the form of the equation in RANGE PREDICTION.

## 6. Glimiter

The section checks existing and predicted accelerations for the case where sensed acceleration is between 5 and 10 g. A calculation is made of the altitude rate which will result in 10g acceleration if full lift up is commanded. From earlier work

$$\frac{d\dot{R}}{dt} = G\left(\frac{V^2}{GR} - 1\right) + L$$

$$\dot{R} = \frac{dH}{dt}$$

$$\mathrm{LEQ} = G\left(\frac{V^2}{GR} - 1\right)$$

$$\frac{d\dot{R}}{dt} = \frac{d\dot{R}}{dH}\,\frac{dH}{dt}$$

combining the above leads to

$$\dot{R}d\dot{R} = (\mathrm{LEQ} + L)\ dH\ . \qquad (127)$$

Assume V is constant and replace L by its function of altitude and integrate.

$$\int_{\dot{R}1}^{\dot{R}2} \dot{R}d\dot{R} = \int_{H1}^{H2} \left(\mathrm{LEQ} + L1\ e^{-\frac{(H-H1)}{HS}}\right)dH$$

$$\dot{R}2^2 \quad -\dot{R}1^2 \quad = 2 \text{ LEQ } (H2 - H1) - 2HS\ (L2 - L1)$$

$$\dot{R}2^2 \quad -\dot{R}1^2 \quad = 2 \text{ HS LEQ } \ln\left|\frac{L1}{L2}\right| - 2HS\ (L^2 - L1) \tag{128}$$

This equation enables $\dot{R}2$ to be predicted from $\dot{R}1$ in going from L1 to L2.

The maximum acceleration maybe obtained by differentiating the equation for D and setting it to zero. This was shown in the section on constant drag to result in:

$$\dot{R} = \dot{H} = -\frac{2HS}{V}\ D$$

Thus when D is specified as the maximum acceleration GMAX the corresponding $\dot{R}$ is given by

$$\dot{R}2 = -\frac{2HS}{V}\ GMAX \quad . \tag{129}$$

Equation 129 is combined with 128,

$$\dot{R}1^2 = \left(\frac{2HS\ GMAX}{V}\right)^2 - 2HS\ LEQ\ \ln\left(\frac{L}{LMAX}\right) + 2HS\ (LMAX - L) \quad . \tag{130}$$

For a constant $\frac{L}{D}$

$L = (L/D)\ D$

$LMAX = (L/D)\ GMAX$

$L/LMAX = D/GMAX$

Using the above equations in equation 130 results in

$$\dot{R}1^2 = \left(\frac{2HS\ GMAX}{V}\right)^2 - 2HS\ LEQ\ \ln\left|1 + \frac{L-LMAX}{LMAX}\right| + 2HS\left|\left(\frac{L}{D}\right) GMAX - \left(\frac{L}{D}\right)D\right| \quad . \tag{131}$$

Approximating the ln term by the first term in its series expansion

$$\dot{R1}^2 = \left(\frac{2HS\ GMAX}{V}\right)^2 + 2HS\ LEQ\ \left(1 - \frac{D}{GMAX}\right) + 2HS\left(\frac{L}{D}\right)\ (GMAX - D) \quad .$$

Finally rearranging terms

$$\dot{R1}^2 = 2HS\ (GMAX - D)\left[\frac{LEQ}{GMAX} + \frac{L}{D}\right] \quad . \quad + \left(\frac{2HS\ GMAX}{V}\right)^2 \qquad (132)$$

The above equation with $\frac{L}{D}$ set to LAD = 0.3 is the form for computing the altitude rate which results in GMAX. (The guidance uses $X = +\sqrt{\dot{R1}^2}$ ).

If the then present altitude rate RDOT is such that

$$- X \geq \dot{R}$$

the guidance calls for a lift up proportional to the difference plus sufficient L/D to balance the LEQ term. In subsequent missions this proportional scheme is eliminated in favor of full lift up.