Lecture 19

Penetration Theory

(unsteady-state mass transfer model / surface renewal model)

$$(N) = C[\beta][k][\Xi](y_0 - y_\delta)$$

1.0 Answers needed:

1.1 what is the correction factor?

1.2 what is [k] in terms of binary k's?

1.3 can also give k_{ij}

2.0 What is the Model? (section 9.1)

2.1 surface renewal idea?

2.2 transient model

2.3 exposure time (t_e)

2.4 Higbie (1935)

all surface elements have same exposure time

$$\psi(t) = \frac{1}{t_e}$$

2.5 Danckwerts (1951)

chance of surface element being replaced with fresh fluid is independent of the thime for which it has been exposed

$$\psi(t) = se^{-st}$$

where *s* is the fraction of the surface area that is replace with fresh fluid in unit time

3.0 Binary Analysis (section 9.2):

3.1 Higbie low mass flux mass transfer coefficient k:

$$k = 2\sqrt{\frac{D}{\pi t_e}}$$

3.2 Danckwerts low mass flux mass transfer coef. k:

$$k = \sqrt{Ds}$$

3.3 Correction factor Ξ :

$$\Xi = \frac{exp(-\Phi^2/\pi)}{\{1 + erf(\Phi/(\sqrt{\pi}))\}}$$

where $\Phi = N_t/(c_t k)$

3.4 ‘A priori’ need for t_e or s

3.5 iterative method needed

4.0 Multicomponent Analysis (sec. 9.3)

4.1 exact analytic solution of this approximate model is not useful (too cumbersome).

4.2 Linearized theory approximation useful

4.3 assuming constant [D], evaluated at average properties

- 4.3.1 form for multicomponent [k]:
  - 4.3.1.1 Higbie model: equation 9.3.33
  - 4.3.1.2 Danckwerts model: equation 9.3.34
- 4.3.2 form for correction factor for both models given in equations 9.3.31 and 9.3.32

4.4 using modal matrix approach equations 8.4.25, 9.3.37-39

5.0 Limitation of Penetration(classical) Theory

5.1 boundary condition (eq. 9.1.6):

$$z \rightarrow \infty \qquad t > 0 \qquad x_i = x_{i\infty}$$

5.1.1 the penetrating, or diffusing component does not see the bulk fluid

5.1.2 strictly true only for short contact times

$$Fo = \frac{Dt}{\delta^2} < 0.20$$

5.2 for long contact times and/or short distances between the interface and the core (δ) of the fluid phase $\Rightarrow$ need improved penetration theory!

5.3 thus is the case for many bubbles, drops, jets:

6.0 Fractional Approach (section 9.4.1)

6.1 define the fractional approach to equilibrium:

$$F \equiv \frac{(x_{10} - \langle x_1 \rangle)}{(x_{10} - x_{1I})}$$

6.2 find the Sherwood Number for spherical particle at time t by taking the driving force to be $(x_{10} - \langle x_1 \rangle)$

$$Sh = \frac{2}{3(1-F)} \cdot \frac{\partial F}{\partial Fo}$$

where $Fo = \frac{Dt}{r_0^2}$

6.3 When substituted and solved to give Sh as a function of time:

$$Sh = \frac{2}{3}\pi^2 \cdot \left(\sum_{m=1}^{\infty} \exp\{-m^2\pi^2 Fo\} / \sum_{m=1}^{\infty} \frac{1}{m^2} \exp\{-m^2\pi^2 Fo\} \right)$$

6.4 Or averaged across time to give:

$\overline{Sh} = -2\ln(1-F)/(3Fo)$ or $\bar{k} = -\ln(1-F)/(a't)$

where a' is the surface area per unit volume and *t* is the contact time.

6.5 Note: this results in Sh at large Fo approaching a limit where k varies with D (as in the film model) as opposed to $D^{1/2}$ as in the classical penetration model.

6.6 This model has also been revised for internal circulation (see eqns. 9.4.9 and 9.4.10)

6.7 The solution for cylinders (jets) is also shown (see eqns. 9.4.12 - 9.4.14)

7.0 Multicomponent Analysis (sec. 9.4.2)

7.1 analysis with constant [D].

7.2 solution using Sylvester's formula: eqn. 9.4.26

7.3 solution using modal matrix of the Fick matrix [D]

7.4 see example 9.4.1

which shows sensitivity to Fo.

this revised method improves the multicomponent analysis significantly

(can even change the direction of the mass transfer)