# Study of the Electric Guitar Pickup

Independent Study
Thomas Withee
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Prof. Errede

# The Guitar Pickup

## Introduction:

The guitar pickup is a fairly simple device used in the electric guitar. At the base of the strings on the guitar, the pickup is used to convert the vibrations of the strings into electric signals. These electric signals are then converted to sound via the standard methods (not important to this study). The pickup itself is in essence an inductor. It consists of six cylindrical magnets, one placed directly under each string, that are held in place by a metal or plastic frame. There is copper wire wrapped about a thousand times around this frame, with the beginning and end of the wire set up to be the leads of the pickup. Because the guitar strings are coils in a magnetic field; when they vibrate, they induce an EMF in the coils of the string. This EMF then creates a magnetic dipole in the string. These small vibrating magnetic dipoles then induce an EMF in the wire wrapped around the magnets. This EMF oscillates at the same frequency of the string and therefore carries the vibrational (and thus sound) information of the strings. This idea is more clearly illustrated in Appendix A, as is the physical properties of the guitar.

The problem that arises from this simple inductor is the wire coiled around the magnets has inherent resistance in the wire and there is a capacitance between the tightly wound wires. To see what kind of resistance, capacitance and inductance the pickup has is the goal of this study. Prof. Errede, using equipment from his 398 EMI lab, acquired several pickups and placed them in a lab setup, see Appendix B. This setup was used to measure the reactance of the pickup at 120 Hz, 1 kHz and 10 kHz, the DC resistance, and the capacitance above 10 kHz. Also, with the wavetek and DAQ program, the real, imaginary, magnitude and phase of the voltage, current and impedance was measured.

Using this data taken from the guitar pickup, it was determined that because of the resonance behavior, the pickup acted like and LC circuit, of some kind. So, a LR_C, L_CR, LR_CR were examined using the lab equipment (the notation here is LR or CR implies inductor or capacitor in series, and _ indicates parallel, so the LR_C is a capacitor in parallel with an inductor resistor series). Studying this data (see the LR_C, L_CR and LR_CR and one of the pickup excel files) that the pickup behaved more like and LR_CR. So, Prof. Errede asked that the theory be worked out to see if the effective L, C and Rs of the pickup could be determined.

<u>Theory and its application:</u>

The theory worked out in Appendix C, provided some very interesting insights. The theory presented in this report is only for the LR_CR circuit as the LR_C and L_CR models don't fit the pickup. The theory provides the impedance, the real part or resistance of the impedance, the reactance of the impedance, the resonant frequency, the phase angle, the magnitude of the impedance and the power.

Using the measured values of the reactance and capacitance, and the assumed values of the resistors, the derivation for the reactance yields values for the inductance. However, as only three values of reactance could be taken for a given pickup, this data is not terribly useful. Also, finding these values was an afterthought of the project, as it was assumed that the inductance might be frequency dependent. But, the inductance turned to not be frequency dependent. Also, this data is not terribly reliable as this method was checked on the LR_CR circuit and the calculated values did not agree with the measured values (see the pickup and LR_CR excel files).

From the resonant frequency, based on the fact that the frequency is real, a limit on the resistance in parallel with both the inductor and capacitor was derived (see appendix C for the derivation and notation); $r = Y/2 + R^2/2Y$ and $R = X/2 + r^2/2X$. This is an important derivation as it puts a range on the values that r and R can assume. Unfortunately, this turns out to be a large range and not helpful in determining the resistor values.

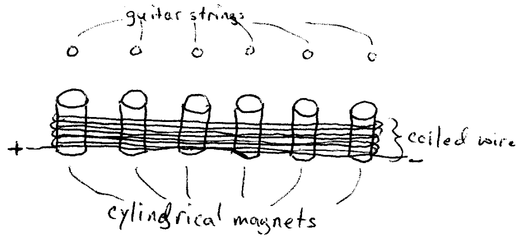
More importantly, it turned out, was examining the power in the pickup. The power, as seen in Appendix C, is based on the real part and magnitude of the impedance. Since values for this were measured, the calculated power and the measured power can be compared. From the derivation of the power, it was found that the average power, $\langle P \rangle$, was proportional to the real part of the impedance divided by the magnitude of the impedance squared; $\langle P \rangle \sim re(Z)/|Z|^2$, the average power goes like one over resistance. This became even more interesting when the $re(Z)/|Z|^2$ was expanded and found to be (see appendix C for notation) $re(Z)/|Z|^2 = R/(X^2 + R^2) + r/(Y^2 + r^2)$. This factor of the circuit, $re(Z)/|Z|^2$, could be applied to each branch of the circuit and added, superposition. In other words, take the $re(Z)/|Z|^2$ of the CR branch and the $re(Z)/|Z|^2$ of the LR branch and add them. Furthermore, taking the limit of $re(Z)/|Z|^2$ for the circuit as frequency goes to zero gives 1/r, and taking the limit of $re(Z)/|Z|^2$ for the circuit as frequency goes to infinity gives 1/R. This was checked and found to be true for both the pickups and the LR_CR circuit. Using this information, the guessed values of the resistors were confirmed.

Conclusion:

This project was set up to get a better understanding of the guitar pickup and its physical properties. From the theory developed this semester, a few important things about the pickup were learned and an idea about LR_CR circuits. First and foremost, that in general or to within first order in frequency, the pickup behaves like an LR_CR circuit. The data supports this to a certain degree and for most purposes it is applicable. This means that the magnets effect the inductance linearly. The inductance might be frequency dependent but not dependent enough to show up in the calculations. Of course, this was based on the assumption that the resistors and capacitance remain constant. It is a reasonably valid assumption that the capacitance remains constant as it is only based on geometry. Also, the power derivations provided accurate depictions of the pickups resistances as they would for an LR_CR circuit. In addition, the ability to use superposition on the impedance of the branches can prove useful in other circuit applications.

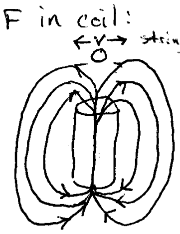
# Appendix A

## Guitar Pickup Structure

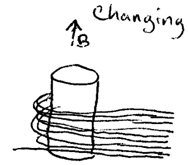



magnets are centered under strings and are vertically placed based on diameter of string. These drawings are not to scale

EMF in coil:



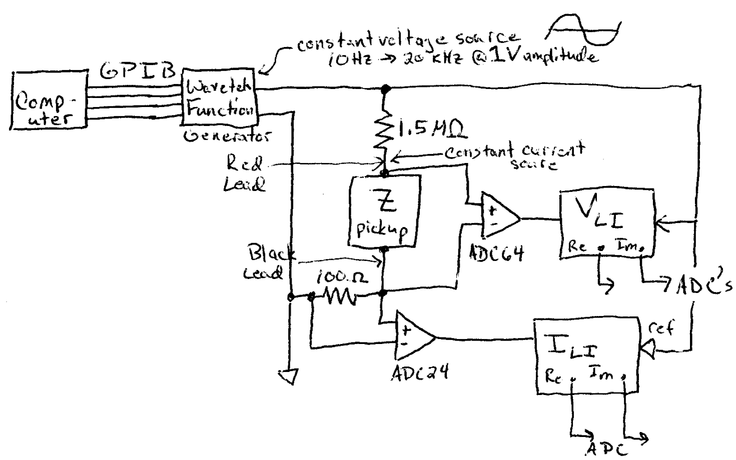

string vibrates in B-field of magnet. This ~~induces~~ induces Emf in string. The induced EMF changes sinsoidally at vibrating frequency. Also, there is a B-field induced in the string. →




Changing B-field induces EMF in the coiled wire. This EMF carries the signal from the vibrating string.

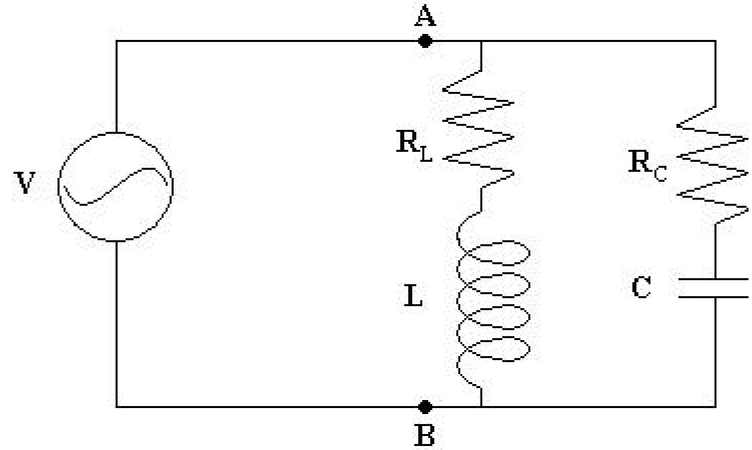
# Appendix B

## Lab Setup



# Appendix C
# Study of the Guitar Pickup

I.) Proposed Circuit Representation:




This model was chosen to simulate the guitar pickup for many reasons. First, the guitar pickup is in essence an inductor. The pickup itself is basically six magnets wrapped a thousands times while copper wiring. So, the inductor is essential. Furthermore, the resonance curves show that the voltage has a sharp peak a certain frequencies and then decays. A standard graph for a LC circuit. In addition, the curves of the pickup were compared to LC, LR||C, and CR||L as well and it was found that both resistors were needed. Also, in the lab setup, a wavetek sinusoidal voltage was used to simulate the vibrating guitar string. This leaves us with the following:

$V = V'\sin(wt) = V' + 0j$
$L = L$
$r = R_L$
$R = R_C$
$C = C$
w=2pf where f is the frequency
Let $j^2 = -1$

A.) Impedance across AB=Z

Let $Y = wL$
$X = 1/wC$

Now the impedance can be calculated as follows:

$1/Z = 1/(r + jY) + 1/(R - jX)$

$Z=(r + jY)(R - jC)/( R + r + jY - jX)$

Rationalize:

$$Z=\frac{(rR + YX) + j(Yr – XR)}{(r + R) + j(Y –X)} * \frac{(R + r)-j(Y –X)}{(R + r)-j(Y –X)}$$

$$Z=\frac{\{(rR+YX)(r+R)+(Yr–XR)(Y–X)\}+j\{(Yr+XR)(r+R)–(rR+YX)(Y–X)\}}{(r+R)^2 + (Y–X)^2}$$

This gives a real and imaginary part, Resistance and Reactance, $Z=R'+jX'$.

$$\text{Resistance} = R' = \frac{(rR+YX)(r+R)+(Yr–XR)(Y–X)}{(r+R)^2 + (Y–X)^2} = \frac{r(X^2 + R^2) + R(Y^2 + r^2)}{(r+R)^2 + (Y–X)^2}$$

$$\text{Reactance} = X' = \frac{(Yr+XR)(r+R)–(rR+YX)(Y–X)}{(r+R)^2 + (Y–X)^2} = \frac{Y(X^2 + R^2) – X(Y^2 + r^2)}{(r+R)^2 + (Y–X)^2}$$

B.)The resonant frequency can be found by setting the Reactance = 0.

$$Y(X^2 + R^2) – X(Y^2 + r^2) = 0$$

$$wL((wC)^2 + R^2) = ((wL)^2 + r^2)/(wC)$$

$$w\{wL((wC)^2 + R^2)\}= w\{((wL)^2 + r^2)/(wC)\}$$

$$w^2Lr^2 + L/C^2 = r^2/C + w^2L^2/C$$

$$w^2(LR^2 - L^2/C) = r^2/C – L/C^2$$

$$w^2 =\frac{(r^2/C-L/C^2)}{R^2L - L^2/C}=\frac{(L/C - r^2)}{(LC)^2(L/C - R^2)}$$

$$\text{so, } w =\frac{1}{LC}\frac{v(L/C - r^2)}{v(L/C - R^2)}$$ Note,v indicates square root

because w is real, we can solve for C and L separately,

solve for C

$$w^2Lr^2 + L/C^2 = r^2/C + w^2L^2/C$$

$$C^2(w^2L^2) - C(w^2L^2+R^2) + L = 0$$

$$C=\frac{(w^2L^2+R^2)\pm v[(w^2L^2+R^2)^2-4w^2L^2r^2)]}{2w^2L^2r^2}$$

solve for L

$$w^2Lr^2 + L/C^2 = r^2/C + w^2L^2/C$$

$$L^2(w^2/C) – L(1/C^2 + w^2r^2) + R^2/C^2$$

$$L=\frac{(1/C^2+w^2r^2)\pm v[(1/C^2+w^2r^2)-4(w^2R^2/C^2)]}{2w^2/C}$$

the value under the square root has to be real, therefore

$$(w^2L^2+R^2)^2 = 4w^2L^2r^2 \qquad (1/C^2+w^2r^2)^2 = 4\, w^2R^2/C^2$$

$$r = \frac{wL}{2} + \frac{R^2}{2wL} \qquad R = \frac{1}{2wC} + \frac{r^2wC}{2}$$

this gives $r = \frac{Y}{2} + \frac{R^2}{2Y}$ and $R = \frac{X}{2} + \frac{r^2}{2X}$

so that r only depends on L and R, while R only depends on C and r!

C.) Phase Angle, F , and magnitude of Impedance, |Z|

$$\tan F = X'/R' = \frac{Y(X^2 + R^2) - X(Y^2 + r^2)}{r(X^2 + R^2) + R(Y^2 + r^2)}$$

$$|Z|^2 = (R'^2 + X'^2)^2 = \frac{(r(X^2 + R^2) + R(Y^2 + r^2))^2 + (Y(X^2 + R^2) - X(Y^2 + r^2))^2}{((r+R)^2 + (Y-X)^2)^2}$$

$$= \frac{r^2(X^2+R^2)^2+R^2(Y^2+r^2)^2+2Rr(X^2+R^2)(Y^2+r^2)+Y^2(X^2+R^2)^2+X^2(Y^2+r^2)^2-2XY(X^2+R^2)(Y^2+r^2)}{((r+R)^2 + (Y-X)^2)^2}$$

$$= \frac{(X^2 + R^2)^2(Y^2 + r^2) + (Y^2 + r^2)^2(X^2 + R^2) + 2(X^2 + R^2)(Y^2 + r^2)(rR - XY)}{((r+R)^2 + (Y-X)^2)^2}$$

$$= \frac{(X^2 + R^2)(Y^2 + r^2)\{(Y^2 + r^2) + (X^2 + R^2) + 2(rR - XY)\}}{((r+R)^2 + (Y-X)^2)^2}$$

$$= \frac{(X^2 + R^2)(Y^2 + r^2)\{(r+R)^2 + (Y-X)^2\}}{((r+R)^2 + (Y-X)^2)^2}$$

$$|Z|^2 = \frac{(X^2 + R^2)(Y^2 + r^2)}{(r+R)^2 + (Y-X)^2}$$

so, $|Z| = v[(X^2 + R^2)(Y^2 + r^2)]/[(r+R)^2 + (Y-X)^2)]$

D.) Power in the circuit:

let average power = <P>

from above: $Z = R' + jX'$, $|Z| = v(R'^2 + X'^2)$, $|V| = v(V'^2 + 0^2) = V'$

$\cos F = R'/v(R'^2 + X'^2)$, $|I| = |V|/|Z|$

by definition, $\langle P\rangle = |I|\,|V| \cos F$

so, $\langle P\rangle = |V|/|Z| * |V| * R'*|Z| = |V|^2R'/|Z|^2 = (|V|/|Z|) * (|V|/|Z|) * R' = |I|^2R'$

$= V'^2R'/(R'^2 + X'^2)$

therefore, $$\langle P\rangle/|V|^2 = R'/|Z|^2 = \frac{[r(X^2 + R^2) + R(Y^2 + r^2)][(r+R)^2 + (Y-X)^2]}{[(r+R)^2 + (Y-X)^2][(X^2 + R^2)(Y^2 + r^2)]}$$

$$= R'/|Z|^2 = \frac{r(X^2 + R^2) + R(Y^2 + r^2)}{(X^2 + R^2)(Y^2 + r^2)} = \frac{r}{Y^2 + r^2} + \frac{R}{X^2 + R^2}.$$

lim $r/(Y^2 + r^2) + R/(X^2 + R^2) = 1/r$ (as w? 0) lim $R/(X^2 + R^2) + r/(Y^2 + r^2) = 1/R$ (as w? 8 )

lim $r/(Y^2 + r^2) = 1/r$ (as w? 0) lim $R/(X^2 + R^2) = 1/R$ (as w? 8 )