Robust Blind Multiuser Detection against Signature Waveform Mismatch Based on Second Order Cone Programming

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***Abstract*— Blind signal detection in multiuser CDMA system is particularly attractive when only the desired user signature is known to a given receiver. A problem common to several existing blind multiuser CDMA detectors is that the detection performance is very sensitive to the Signature Waveform Mismatch (SWM) which may be caused by channel distortion. In this paper we consider the design of a blind multiuser CDMA detector that is robust to the SWM. We present a convex formulation for this problem by using the Second Order Cone (SOC) programming. The resulting SOC problem can be solved efficiently using the recently developed interior point methods. Computer simulations indicate that the performance of our new robust blind multiuser detector is superior to those of many existing methods.**

***Index Terms*— Blind multiuser detection, robust multiuser detection, second order cone programming.**

I. Introduction

A commonly encountered problem in Code-Division Multiple-Access (CDMA) systems is the so called the near-far effect whereby weaker users are dominated by stronger users (interferers). It is well known that in such circumstances the traditional matched filter single-user detection is not effective, and multiuser detection should be used [1], [8], [9]. While in a standard multiuser detector all user signature and timing information must be known [1] to the receiver, a recent work [2] presented a simple blind near-far resistant “multiuser” detector which requires only the desired users’ waveform. Some further work along this line have been reported in [3], [4], [6], [7].

A problem common to several existing blind multiuser CDMA detectors is that their performance tend to be negatively affected by the Signature Waveform Mismatch (SWM) caused by channel distortion. Since channel distortion exists in most environments where CDMA is deployed (e.g., cellular mobile telephony), it is essential for the blind multiuser receivers to mitigate the SWM effect when we design a practical CDMA detector with near-far resistance [2], [10]. One possible approach to deal with the SWM problem is to allow the use of training sequences during transmission such that the channel distortion can be periodically estimated by the receiver. The identified channel response (distortion) can be used to design compensating measures against signature waveform mismatch. However, in a mobile communication environment where channel distortion varies quickly, training based approaches may consume too much channel capacity. An alternative means of mitigating SWM is to design a blind multiuser detector which has strong robustness to SWM. In reference [2] (see also [5]), a particular mechanism was presented for the design of robust blind multiuser CDMA detectors which calls for the minimization of the detector’s output energy. Moreover, two gradient descent algorithms (the Stochastic Gradient (SG) algorithm and the Least Squares (LS) algorithm) were proposed in [2] for achieving the Minimum Output Energy (MOE) under the constraint that the so-called “surplus energy” created by SWM is bounded. However, constraining the “surplus energy” is an indirect and heuristic way to achieve receiver robustness. A more natural (and perhaps also more desirable) formulation is to directly maximize the worst case system performance given a specific bound of SWM. Such is the approach taken in this paper. Another drawback of the two iterative algorithms proposed in [2] is that they require some data-dependent parameters that are not easy to select, and a poor choice could lead to unacceptable performance. The Constrained MOE method (CMOE) [6] first estimates the channel blindly and then minimizes the channel output energy subject to certain constraints aimed at protecting the desired signal that has propagated through the estimated channel. However, the channel identification phase of the CMOE method requires large number of samples and high SNR. Moreover, it requires special techniques to resolve an intrinsic unitary ambiguity matrix.

In this correspondence we present a new formulation for the design of robust blind multiuser CDMA detectors. Our formulation is direct in the sense that it allows explicit control of the amount of required robustness in the detector. Moreover, our optimization formulation is convex since it is based on the Second Order Cone programming (SOC). As such, this new robust blind multiuser detector can be obtained using the highly efficient interior point methods recently developed in the optimization community. Computer simulations indicate that the performance of our new robust blind multiuser detector, when combined with a blind signal separation method (e.g., the JADE algorithm [13]), is superior

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to those that exist in the literature for both non-dispersive and dispersive propagation environment, while the number of required samples is significantly smaller.

II. Problem Description

Consider an antipodal K-user synchronous direct sequence CDMA channel corrupted by some additive and white Gaussian noise $n(t)$. Our notation follows that of [2]:

- σ – the standard deviation of channel $n(t)$;
- $s_k(t)$ – the normalized signature waveform for the k-th user with $\|s_k(t)\| = 1$;
- $\{b_k[i]\}$ – the transmitted BPSK data bits;
- T – bit duration at the transmission rate of $1/T$;

Given the above notations, the received signal can be written as

$$y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + n(t), \qquad t \in [0, T] \tag{1}$$

When we sample the received signal waveform $y(t)$ at the chip rate $1/\Delta$, where $\Delta > 0$ is the chip interval, we obtain the following discrete version of (1):

$$\mathbf{y} = \sum_{k=1}^{K} A_k b_k \mathbf{s}_k + \mathbf{n}, \tag{2}$$

where

$$\mathbf{y} = \begin{pmatrix} y(\Delta) \\ y(2\Delta) \\ \vdots \\ y(N\Delta) \end{pmatrix}, \ \mathbf{s}_k = \begin{pmatrix} s(\Delta) \\ s(2\Delta) \\ \vdots \\ s(N\Delta) \end{pmatrix}, \ \mathbf{n} = \begin{pmatrix} n(\Delta) \\ n(2\Delta) \\ \vdots \\ n(N\Delta) \end{pmatrix}$$

with N being the code spreading factor. Note that $T = N\Delta$ as a result.

Without loss of generality, suppose that the user one is our desired user whose signature waveform is denoted as $\mathbf{s}_1$. Our goal in receiver design is to select a vector $\mathbf{c}_1$ which, upon correlating with the received vector $\mathbf{y}$ and passing through a hard limiter, will recover the data bits $\{b_1[i]\}$ sent by user one. The Minimum Output Energy (MOE) based multiuser detector introduced in [2] can be described as follows:

$$\begin{array}{ll} \text{minimize} & E\,|\langle \mathbf{y}, \mathbf{c}_1 \rangle|^2 = \mathbf{c}_1^T \mathbf{R} \mathbf{c}_1, \\ \text{subject to} & \mathbf{c}_1^T \mathbf{s}_1 = 1, \end{array} \tag{3}$$

where $\mathbf{c_1}$ is the vector to be determined, and $\mathbf{R} = E(\mathbf{y}\mathbf{y}^T) \in \Re^{N \times N}$. In practice we have only finite number of snapshots of the received data. Thus, we need to replace $\mathbf{R}$ in (3) with the sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{N_b} \sum_{n=1}^{N_b} \mathbf{y}[n] (\mathbf{y}[n])^T,$$

where N_b is the number of transmitted data bits and $\mathbf{y}[n]$ is the n-th received data vector. This leads to the following implementable version of (3):

$$\begin{array}{ll} \text{minimize} & \mathbf{c}_1^T \hat{\mathbf{R}} \mathbf{c}_1, \\ \text{subject to} & \mathbf{c}_1^T \mathbf{s}_1 = 1. \end{array} \tag{4}$$

It is well known that the MOE solution to (4) is highly sensitive to SWM and often lead to poor BER performance. To overcome this sensitivity to SWM, Honig *et. al.* [2] introduced the following energy-constrained version of MOE detector:

$$\begin{array}{ll} \text{minimize} & \mathbf{c}_1^T \hat{\mathbf{R}} \mathbf{c}_1, \\ \text{subject to} & \mathbf{c}_1^T \mathbf{s}_1 = 1, \quad \|\mathbf{c}_1 - \mathbf{s}_1\|^2 = \chi, \end{array} \tag{5}$$

where χ is called the surplus energy and is chosen by the user *a priori* so that $\chi_I < \chi < \chi_S$, with χ_I and χ_S being the user-selected lower and upper bounds on the surplus energy. Also, a stochastic gradient algorithm was proposed [2] to solve (5), but rigorous convergence analysis was neither given nor known. In some sense, the formulation (5) attempts to generate a robust solution to (4) which is insensitive to SWM in $\mathbf{s}_1$. This robustness is achieved indirectly by constraining the surplus energy. The main weakness of the formulation (5) is its lack of convexity.

III. SOC Algorithm Development

We now describe a more direct (and arguably a more natural) way to construct a robust solution for the minimum output energy (MOE) formulation (4) under SWM. It turns out that both the objective function and constraints in this new formulation are convex so that a globally optimal solution can be found efficiently.

We model the actual received signature waveform as $\bar{\mathbf{s}}_k = A_k(\mathbf{s}_k + \mathbf{e}_k)$, where $\mathbf{e}_k$ is the mismatch error vector and A_k is the channel gain. Notice that A_k can be easily estimated by matching the channel output power with $\|\mathbf{s}_k\|^2$. In this way, we obtain the normalized received signature waveform:

$$\hat{\mathbf{s}}_k = \mathbf{s}_k + \mathbf{e}_k. \tag{6}$$

Clearly, $\|\mathbf{e}_k\|$ is a measure of the magnitude of signal waveform mismatch. The distortion can be due to asynchronism or multipath fading. For example, in the case of timing asynchronism, we can use Taylor approximation to bound $s(t+\tau) - s(t)$, where τ is the timing offset. Hence, the SWM error is easily bounded by $\|\mathbf{e}\| = \|\hat{\mathbf{s}} - \mathbf{s}\| \le BN\tau$, where B is the upper bound for the derivative of the continuous signature waveform $s(t)$, and N is the spreading factor. In a multipath environment with an M-tap channel response $\mathbf{h}$, the actual received signature waveform is $\hat{\mathbf{s}}_k = \mathbf{s}_k \otimes \mathbf{h}$. Hence, we can obtain the following bound on the mismatch error vector:

$$\begin{array}{rl} \|\mathbf{e}\| = \|\hat{\mathbf{s}}_k - \mathbf{s}_k\| & \le \|\mathbf{s}_k \otimes \mathbf{h} - \mathbf{s}_k\| \\ & \le \|\mathbf{s}_k \otimes (\mathbf{h} - \mathbf{h}_{\text{ideal}})\| \\ & \le \sqrt{M} \|\mathbf{h} - \mathbf{h}_{\text{ideal}}\| \end{array} \tag{7}$$

where $\mathbf{h}_{\text{ideal}}$ denotes the ideal channel response (i.e., delta function). If the channel has a main line of sight component and small multipath components, then $\|\mathbf{h} - \mathbf{h}_{\text{ideal}}\|$ will be small.

We assume the distortion error $\mathbf{e}_1$ in the desired signal waveform can be bounded by some constant $\delta > 0$, that is $\|\mathbf{e}_1\| \le \delta$. The size of δ can be estimated using, for example, (7). The actual received signal waveform $\hat{\mathbf{s}}_1$ can be described as a vector in the set

$$S_1(\delta) = \{ \hat{\mathbf{s}}_1 \mid \hat{\mathbf{s}}_1 = \mathbf{s}_1 + \mathbf{e}_1, \ \|\mathbf{e}_1\| \le \delta \}.$$

Since $\hat{\mathbf{s}}_1$ can be any vector in $S_1(\delta)$, we must ensure that the detector gain for all signals in $S_1(\delta)$ should be greater than 1, that is, $\mathbf{c}_1^T\hat{\mathbf{s}}_1 \geq 1$ for all vectors $\hat{\mathbf{s}}_1 \in S_1(\delta)$. Such a constraint ensures that we can extract the data bits from user one regardless how its signature waveform is distorted, as long as the distortion is bounded by δ. Now suppose that this gain constraint is enforced, then our goal remains to find a vector $\mathbf{c}_1$ that minimizes $\mathbf{c}_1^T\hat{\mathbf{R}}\mathbf{c}_1$. Consequently, a robust version of (4) can be described as follows:

$$\begin{array}{ll} \text{minimize} & \mathbf{c}_1^T\hat{\mathbf{R}}\mathbf{c}_1, \\ \text{subject to} & \mathbf{c}_1^T\hat{\mathbf{s}}_1 \geq 1 \quad \text{for all } \hat{\mathbf{s}}_1 \in S_1(\delta), \end{array} \tag{8}$$

where δ is an upper bound on the norm of the signal mismatch error vector.

For each choice of $\hat{\mathbf{s}}_1 \in S_1(\delta)$, the condition $\mathbf{c}_1^T\hat{\mathbf{s}}_1 \geq 1$ represents a linear constraint on $\mathbf{c}_1$. Since there are infinite number of $\hat{\mathbf{s}}_1$ in $S_1(\delta)$, the constraints in (8) are semi-infinite and linear. To facilitate the computation of optimal $\mathbf{c}_1$, we will convert these semi-infinite linear constraints into a so called second-order cone constraint. This is achieved by considering the worst case performance as follows. Note that the optimal solution of the minimization problem

$$\min_{\hat{\mathbf{s}}_s \in S_1(\delta)} \mathbf{c}_1^T\hat{\mathbf{s}}_1 \qquad \text{or equivalently} \qquad \min_{\|\mathbf{e}_1\| \leq \delta} \mathbf{c}_1^T(\mathbf{s}_1 + \mathbf{e}_1)$$

is given by

$$\mathbf{e}_1 = -\delta \mathbf{c}_1 / \|\mathbf{c}_1\|.$$

This can be easily verified by Cauchy-Schwartz inequality. Therefore, the constraint

$$\mathbf{c}_1^T\hat{\mathbf{s}}_1 \geq 1 \quad \text{for all } \hat{\mathbf{s}}_1 \in S_1(\delta)$$

can be equivalently described by

$$\mathbf{c}_1^T\left(\mathbf{s}_1 - \delta\frac{\mathbf{c}_1}{\|\mathbf{c}_1\|}\right) \geq 1, \quad \text{or} \quad \mathbf{c}_1^T\mathbf{s}_1 - \delta\|\mathbf{c}_1\| \geq 1. \tag{9}$$

Substituting (9) into (8), we obtain a new problem formulation

$$\begin{array}{ll} \text{minimize} & \mathbf{c}_1^T\hat{\mathbf{R}}\mathbf{c}_1, \\ \text{subject to} & \mathbf{c}_1^T\mathbf{s}_1 - \delta\|\mathbf{c}_1\| \geq 1 \end{array} \tag{10}$$

Notice that the constraint in (10) is of the form

$$\|\mathbf{P}\mathbf{c}_1\| \leq \mathbf{p}^T\mathbf{c}_1 + q,$$

for some given $\mathbf{P} \in \Re^{N\times N}$, $\mathbf{p} \in \Re^N$ and $q \in \Re$, which is called a second-order cone constraint.

Next we convert the quadratic objective function of (10) into a linear one. To do so, we first notice that $\mathbf{c}_1^T\mathbf{R}\mathbf{c}_1 = \|\mathbf{L}\mathbf{c}_1\|^2$, where $\mathbf{L}^T\mathbf{L} = \mathbf{R}$ is the Cholesky factorization. Obviously, minimizing the quadratic norm $\|\mathbf{L}\mathbf{c}_1\|^2$ is equivalent to minimizing $\|\mathbf{L}\mathbf{c}_1\|$. Introducing a new variable t and a new constraint $\|\mathbf{L}\mathbf{c}_1\| \leq t$, we can convert (10) into the following:

$$\begin{array}{ll} \text{minimize} & t, \\ \text{subject to} & \|\mathbf{L}\mathbf{c}_1\| \leq t, \quad \|\delta\mathbf{c}_1\| \leq \mathbf{s}_1^T\mathbf{c}_1 - 1. \end{array} \tag{11}$$

The above formulation (11) is now in the standard form of a second-order cone programming (SOCP) [11] problem. This is because the objective function is linear and the two constraints are both second-order cone constraints (which are convex). Such optimization problem can be efficiently solved using primal-dual potential reduction method SeDuMi [14]. The total computational complexity for solving (11) is $O(N^{3.5}\log 1/\epsilon)$. This, plus the complexity of accumulating the sample correlation matrix $\hat{\mathbf{R}}$ and performing its Cholesky factorization, gives the SOC approach an overall complexity of $O(N^{3.5}\log 1/\epsilon + N_bN^2)$, where N_b is the total number of transmitted data bits. The accuracy parameter ϵ can be either fixed (say, $\epsilon = 10^{-4}$) or chosen to vary inversely proportional to the signal to noise ratio (SNR).

IV. Simulations

We now compare the simulation performance of our new SOC method with those of the existing blind linear CDMA receivers which include the classical Matched Filtering method (MF), the standard (non-robust) MOE detector, the two versions of robust MOE methods proposed by [2] (one based on Least Squares approach and the other based on Stochastic Gradient approach), as well as the Constrained Minimum Output Energy (CMOE) Method [6]. Although time synchronism was assumed throughout the algorithm development, we first test the performance of our algorithm on asynchronous CDMA systems. Then we consider multipath propagation CDMA systems.

A. SWM timing asynchronism model

Timing asynchronism is modelled through the presence of Signature Waveform Mismatch (SWM) as in (6), where $\mathbf{e_k} \sim \mathcal{N}(\mathbf{0}, \sigma^{\mathbf{2}})$. We consider CDMA systems using Gold codes of length $N = 31$ with the number of users $K = 7$ and $K = 30$. For the system with $K = 7$ users the Interference to Signal Ratio (ISR) is set to be

$$\text{ISR} = 20\log(A_k/A_1) = 20\ dB, \qquad k = 2, \cdots, K,$$

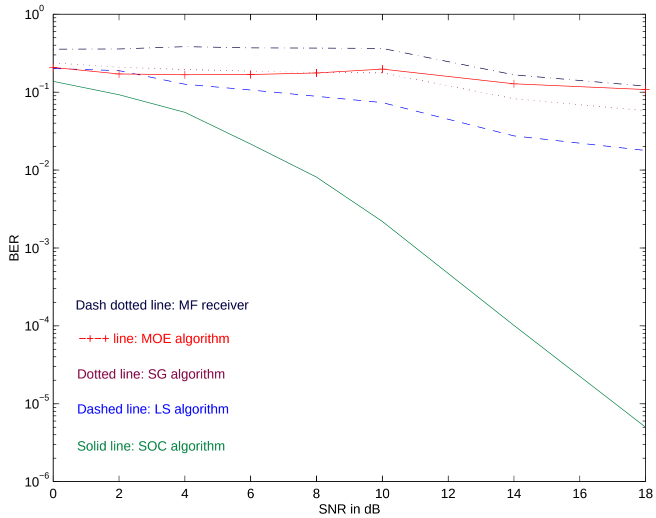
where A_k denotes the received signal amplitude of the k-th user. For the other system with $K = 30$ ISR is taken to be

$$\text{ISR} = 20\log(A_k/A_1) = 10\ dB, \qquad k = 2, \cdots, K.$$

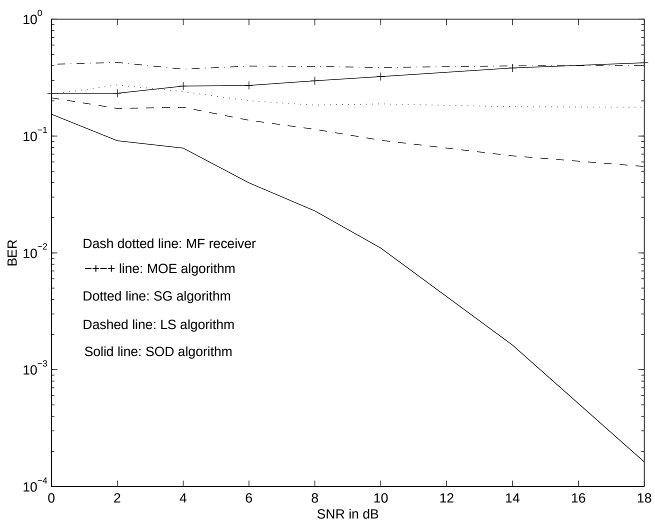
Both cases represent a severe near-far effect.

In our simulations, we test the systems with long sequences of transmitted bits ($N_b = 100$ and $N_b = 400$) to ensure adequate iterative convergence of both LS and SG methods which we shall compare with our SOC method. It is also needed to ensure that the sample covariance matrix $\hat{\mathbf{R}}$ is a close approximation of the true covariance matrix $\mathbf{R}$. At the ℓ-th run, a random distortion with the norm no more than δ is added to every signature waveform $\mathbf{s}_k$ to result in a mismatched waveform $\hat{\mathbf{s}}_k^\ell$. In addition, new additive Gaussian noise vectors $\mathbf{n}^\ell$ as well as a new data sequence $\{b_k^\ell\}$ are generated.

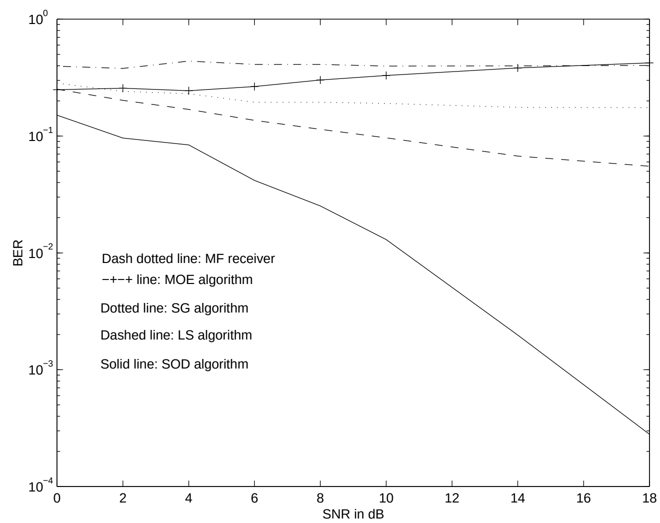
To solve (11) we have used a Matlab-based tool called SeDuMi [14] which is an efficient implementation of a primal-dual interior point method for solving SOC problems. For the LS and the SG methods, we have experimented with various different values of χ (the "surplus energy" in (5)) and have chosen the one which gives the best Averaged BER performance, even though such a luxury is not practically affordable.



(a) $\delta = 0.4$ and $\hat{\delta} = 0.4$.



(b) $\delta = 0.4$ and $\hat{\delta} = 0.6$.

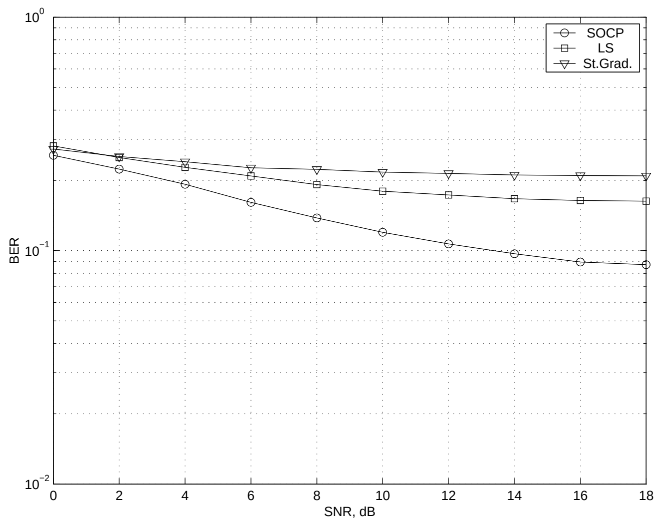


(c) $\delta = 0.4$ and $\hat{\delta} = 0.2$.

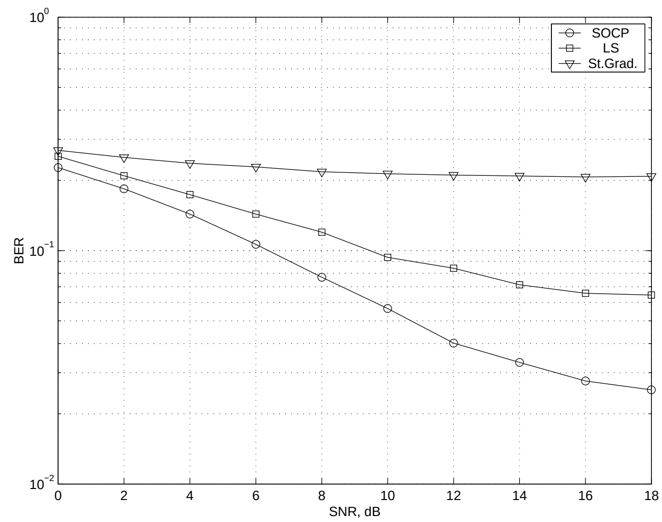
Fig. 1. BER versus SNR, comparison of SOC, LS, SG, MOE and MF detectors.

The results for $K = 7$ users are shown in Fig. 1. We test this system with mismatch $\delta = 0.4$. Note that δ is the upper bound on the SWM realization at each random run. We experimented with different δ values and results seem to be the same qualitatively. It can be seen from Fig. 1 that the SOC method has the best BER performance, followed by LS, SG, MOE and MF. Fig. 1(a) assumes the value of δ is known to the detector and we use this value ($\hat{\delta} = \delta$) in the SOC formulation (11). Figs. 1(b) and 1(c) show the performance of SOC method when δ is over-estimated and under-estimated respectively. It can be seen that the SOC detector is robust to errors in estimating the SWM bound. In all of above reported simulation results, we have chosen the surplus energy χ optimally (by trial and error) and have plotted only the best results. It comes as no surprise that the MF detector has a poor BER performance since it does not deal with the presence of strong co-channel interferences. Notice that the BER for the MOE detector worsens when the SNR increases. This is the case for a non-robust detector like the MOE method because a part of the signal power will be contributing towards the interference when SWM is present, leading to larger interference power and worse BER performance as the signal power increases.

Fig. 2 shows the results for heavily loaded system with $K = 30$ users and severe interference with ISR set to be $10\ dB$. Fig. 2(a) and Fig. 2(b) show the comparison of SOC formulation, LS and SG for data blocks of 100 and 400 bits respectively. At each run a mismatch of norm $\delta = 0.4$ is added to signature waveforms of all users and δ is assumed to be known to the detector. Inaccuracy of estimating the



(a) Data block 100 bits.



(b) Data block 400 bits.

Fig. 2. BER versus SNR, 30 users, ISR $= 10\ dB$, mismatch $\delta = 0.4$.

mismatch does not lead to the significant deterioration of the performance of SOC method, thus, in practice an approximate estimate of δ will be enough. The simulations suggest that the robust SOC formulation is superior to both LS and SG methods.

B. Multipath Propagation

We now test the performance of our algorithm in a multipath propagation scenario. In our simulation, the dispersive channel is modelled as an FIR filter with a tap-spacing equal to the chip rate [12]. The spreading codes are again chosen to be Gold sequences of length $N = 31$. The input signal is a BPSK i.i.d. sequence for every user. Each user's chip sequence is transmitted through a randomly generated multipath fading channel of length $q + 1 = 3$ chip periods. The user of interest is assumed to be the weak user: the channel gains are scaled so that each interfering user is $20\ dB$ stronger than the user of interest. This corresponds to a severe near-far situation.

Comparison of CMOE, SOC and JADE methods:

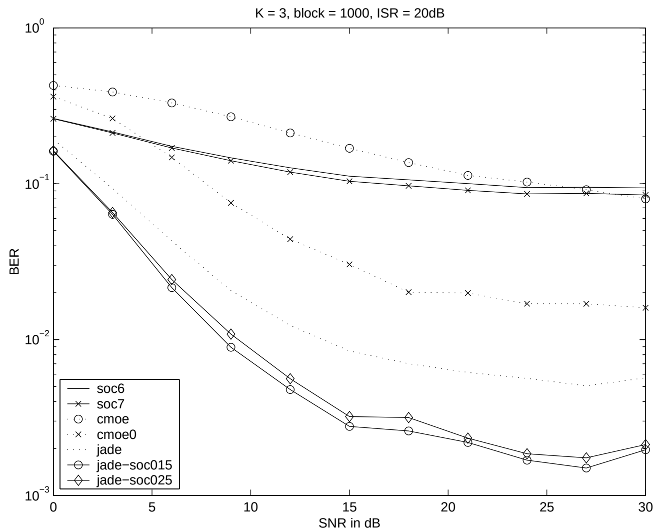
Under the above model, we compared the CMOE and SOC on a CDMA system with $K = 3$ users. The other multiuser detectors considered in the previous section showed a poor performance under the given multipath propagation model.

Notice that an unitary ambiguity matrix exists in the blind identification phase of the CMOE algorithm. Such ambiguity is generic to the multiuser blind identification approaches based on second order statistics, and cannot be resolved unless additional information is available. In our simulation, we have manually resolved this ambiguity using a (unrealistic) training sequence prior to data transmission. Even with this ambiguity removed, the CMOE algorithm still cannot recover the desired (weak) user, regardless of the block size. The BER results, averaged over 4000 Monte Carlo runs, are shown in Fig. 3, with a curve 'cmoe'. At each run, new data sequences of length $5,000$ bits (compared to $1,000$ bits for the other algorithms), new additive white Gaussian noise vectors, as well as new channel realizations are generated. We have experimented with longer data sequences ($10,000$–$20,000$ bits), but the results appear the same qualitatively. We also tested the CMOE algorithm on a CDMA system in which all users had the same power (ISR $= 0\ dB$), and those results are shown with a curve 'cmoe0'. Our simulation results suggest that the performance of the CMOE is seriously affected by the presence of noise and other users in the system. It should be also recognized that in scenarios when the channel length is unknown and has to be estimated, the performance of the CMOE would further degrade.

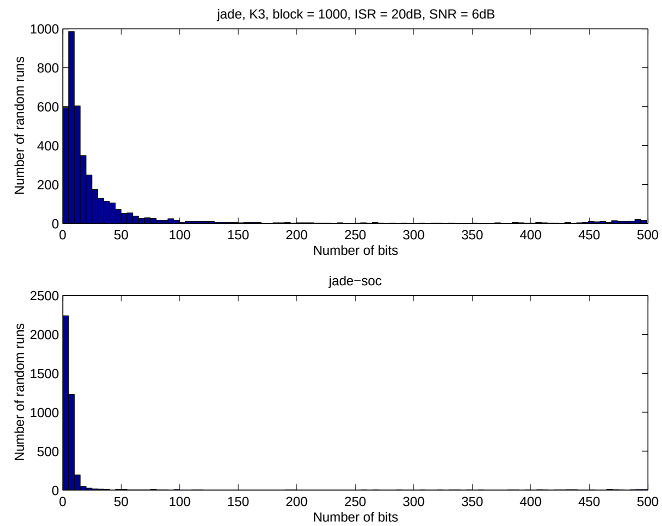
We have also compared our second-order cone method to the JADE [13] algorithm. The curves 'SOC6' and 'SOC7' represent our method when $\hat{\delta}$ parameter is set to be $\hat{\delta} = 0.6$ and $\hat{\delta} = 0.7$, respectively. The curve 'jade' represents the JADE algorithm. For the SOC algorithm, we have used the desired user's Gold code sequence as the nominal signature waveform $\hat{\mathbf{s}}_1$ in the formulation (8). We can see from Fig. 3(a) that this choice of nominal signature waveform is not appropriate, i.e., the actual received signature waveforms do not lie in the $\hat{\delta}$ vicinity of the chosen nominal signature waveforms. In other words, the norm bounded channel distortion model is appropriate for communication systems with a strong line of sight and small multipath components, as derived in (7). For systems with severe presence of multipath components we propose the following algorithm.

C. JADE-SOC Methods

It is possible to combine JADE and SOC methods to achieve a better performance than what is possible by either method individually. Indeed, when operating alone, JADE method may not identify the channel accurately due to the



(a) Probability of error vs. SNR



(b) Histogram

Fig. 3. Comparison SOC, CMOE, JADE and JADE-SOC

combined effect of noise, multiuser interference and short sample size, while SOC method may suffer from a poor choice of nominal signature waveform. However, when operating in tandem (JADE followed by SOC), the robustness of SOC method can be used to mitigate the estimation error found in the estimates of JADE method.

We have tested this combined approach: we first use the JADE algorithm to estimate the received signature waveform of the desired user, and then use it as the nominal signature waveform $\hat{\mathbf{s}}_1$ in our formulation (8) of SOC method. This combined approach (named JADE-SOC algorithm) outperforms the JADE, as shown by the curves 'jade-soc015' and 'jade-soc025' (corresponding to $\hat{\delta} = 0.15$ and $\hat{\delta} = 0.25$ respectively) in Fig. 3. This suggests that the robust SOC blind multiuser detector is able to correct the estimation errors caused by the JADE algorithm.

We can see from Fig. 3(b) that in most of the random runs the received blocks in JADE-SOC algorithm had less than 25 corrupted bits per block. In particular, nearly 2,500 blocks were successfully (error-free) decoded. This is not true for the JADE algorithm. The given histogram Fig. 3(b) compares the performance of JADE-SOC to the JADE when signal-to-noise ratio is set to be SNR $= 6\ dB$. Similar results have been obtained for other SNR values. If we set signal-to-noise ratio to be SNR $= 24\ dB$, we can see from Fig. 4 that the JADE-SOC algorithm is able to successfully decode even those blocks for which the JADE algorithm has a BER up to 40%. This clearly demonstrates of added value of our new robust blind multiuser detector. In contrast, the CMOE algorithm has a poor performance, see Fig. 4(a).

We point out that SOC method can be used in conjunction with any channel identification method (not just the JADE method), blind or nonblind. The extra robustness of SOC method can be expected to partially mitigate the errors found in the channel estimates, thus leading to improved bit error rate performance.

Finally, we remark that, in our simulations, solving each SOC problem (11) with the Matlab tool SeDuMi [14] takes less than a second on a 600 MHz Pentium III PC.

V. Concluding Remarks

In this correspondence we have proposed a new robust blind multiuser detector for synchronous CDMA in the presence of signature waveform mismatch (SWM). Our method is based on a robust formulation of the Minimum Output Energy (MOE) detector using the Second-Order-Cone (SOC) programming technique. The SOC formulation (11) is convex and can be efficiently solved by the recently developed interior point methods. Computer simulations indicate that the new SOC detector has a much better performance when compared to the existing multiuser detectors (robust or otherwise). Simulation results also show that the SOC detector can be used effectively in the dispersive propagation environment, provided that a reasonable estimate of the received signature waveform for the desired user is available.

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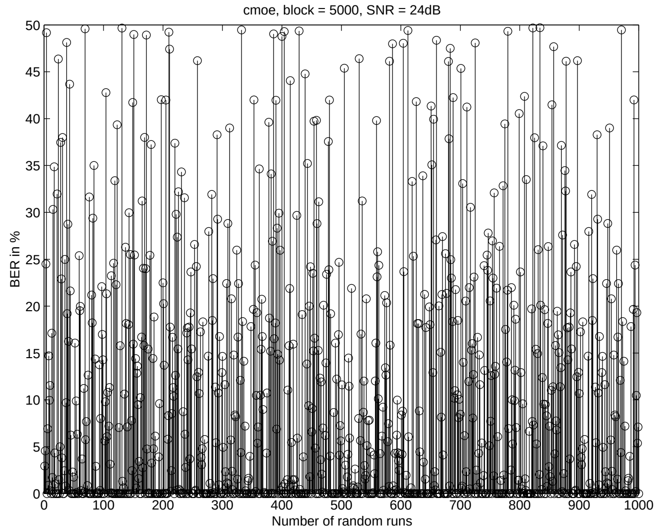
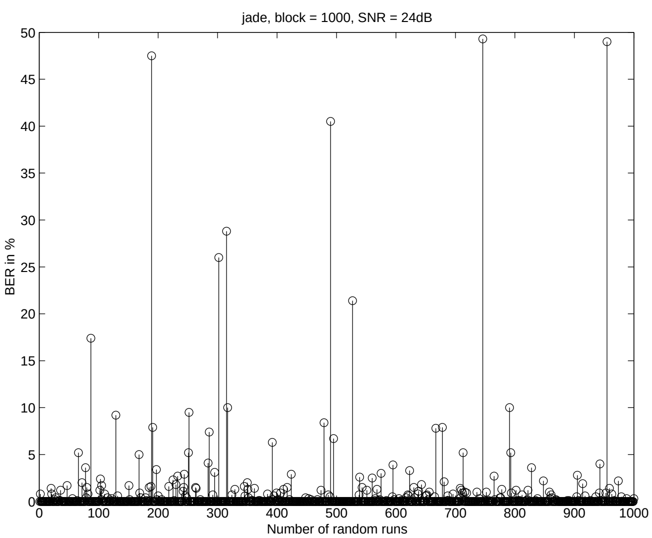
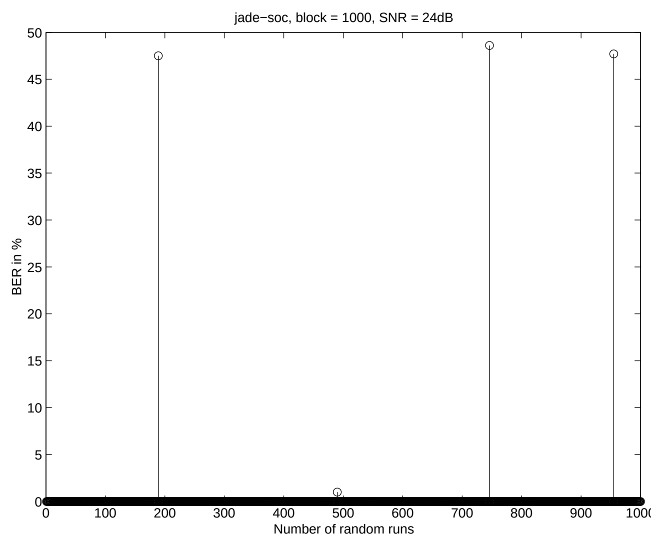


(a) CMOE



(b) JADE



(c) JADE-SOC

Fig. 4. Probability of error for different random runs

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