

Four Aspects of Strategic Change: Contributions to Children's Learning of Multiplication

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This study reports a longitudinal investigation of French 2nd graders' acquisition of single-digit multiplication skill. Speed, accuracy, and strategy use were assessed 3 times within the year when children learned multiplication. The data showed that improvements in speed and accuracy that generally accompany learning can reflect at least 4 types of specific strategic changes: introduction of new strategies, increasing use of the most efficient existing strategies, more efficient execution of each strategy, and more adaptive choices among strategies. The data also showed substantial continuities in learning: At all 3 points of measurement, children used multiple strategies, used retrieval most often on the same classes of problems, and used repeated addition on the most difficult problems. Stable individual differences were also apparent. The findings supported a number of predictions of Siegler and Shipley's (1995) adaptive strategy choice model. Implications for understanding learning, arithmetic, and strategy choice processes are discussed.

Among the most striking features of children's thinking is its variability. In such diverse domains as arithmetic, tic-tac-toe, serial recall, and moral reasoning, children know and use multiple strategies. These diverse approaches allow them to adapt flexibly both to inherent task characteristics, such as problem difficulty, and to transitory situational demands, such as needs to answer especially quickly or especially accurately in a particular context (e.g., Bisanz & LeFevre, 1990; Colby, Kohlberg, Gibbs, & Lieberman, 1983; Crowley & Siegler, 1993; Geary & Burlingham-Dubree, 1989; Goldman, Mertz, & Pellegrino, 1989; McGilly & Siegler, 1990).

Recognition of cognitive variability may be especially important for understanding how children learn. The most general features of learning are self-evident: With experience, performance becomes faster and more accurate. Underlying these global changes, however, are a host of more specific changes involving which strategies are used, how

often they are used, how they are executed, and how they are chosen.

The purpose of this article is to illustrate how trial-by-trial assessments of strategy use can allow identification of the distinct contributions of each of these aspects of strategic change and thus yield a more differentiated picture of learning than would otherwise be possible. We longitudinally studied French second graders' acquisition of proficiency in single-digit multiplication. Performance was observed at three points during the school year, spanning the period from when children's knowledge of multiplication was quite rudimentary to when it was quite advanced. By examining changes in which strategies they used, when they used them, and how they executed them, we were able to obtain a detailed depiction of knowledge acquisition in this area. The data also allowed us to test theoretical predictions concerning both continuities and changes during the learning process.

Four Dimensions of Strategic Change

It seems useful to distinguish among four dimensions of strategic competence, changes in any one of which can yield overall improvements in speed and accuracy: (a) which strategies are used, (b) when each strategy is used, (c) how each strategy is executed, (d) how strategies are chosen. The contributions to learning of each of these aspects of strategic competence can be illustrated in the context of children's single-digit addition.

Which strategies are used involves the specific goal-directed procedures that are used to solve problems. In many domains, multiple strategies are used over a prolonged period of development. To add, young elementary school children sometimes count from one, sometimes count from the larger addend, sometimes retrieve the answer from memory, sometimes decompose problems into simpler forms (e.g., $9 + 6 = 10 + 6 - 1$), and sometimes say "I

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don't know." Individual children on average use three or four of these strategies within a single session (Siegler, 1987).

Changes in which strategies children use include both acquisition of new strategies and abandonment of old ones. For example, between kindergarten and second grade, most children begin to use the strategy of counting from the larger addend and stop using the strategy of counting from one (Siegler, 1987; Svenson & Sjöberg, 1983). Global improvements in speed and accuracy in part reflect acquisition of new strategies that allow faster and/or more accurate performance and abandonment of old strategies that produce especially slow and/or inaccurate performance. In the addition example, counting from the larger addend (the min strategy) tends to be faster and more accurate than counting from one. Thus, acquisition of the min strategy contributes to overall increases in speed and accuracy.

When each strategy is used involves both the relative frequencies of each strategy and the types of problems on which the strategy is used. Even preschoolers moderately often retrieve answers to addition problems with sums of 10 or less (Geary & Brown, 1991; Geary & Burlingham-Dubree, 1989; Siegler & Robinson, 1982). Over the next few years of schooling, retrieval comes to be used more consistently on those problems and is extended to larger size problems as well. This produces considerable gains in overall speed and accuracy. Even when children use the same set of strategies earlier and later in learning, increasing frequency of use of the fastest and most accurate approaches can contribute to general increases in speed and accuracy.

How each strategy is executed involves both quantitative and qualitative features of performance. Within any given strategy, problem-solving experience typically leads to improved execution. For example, in Siegler (1987), the speed and accuracy with which children executed the min strategy increased between kindergarten and first grade and again between first and second grade. Even if early and later in learning, children use the same set of strategies and use them equally often on the same problems, speed and accuracy will improve if children execute the strategies more efficiently.

How strategies are chosen involves decisions about which strategy to use on each problem. When children know multiple strategies for solving a given class of problems, they must decide which one to use each time they encounter an example. At times, Strategy A is faster or more accurate for some problems, but Strategy B is faster or more accurate for others. If children learn to choose Strategy A more consistently on the first group of problems and Strategy B more consistently on the second, the result will be faster and a more accurate performance, even if across all problems they continue to use the same strategies, use them equally often, and execute them equally efficiently.

These four dimensions of strategic change provide a conceptual framework for thinking about sources of the global improvements in speed and accuracy that characterize learning. In the next section, we describe what is known about their contributions to the content of interest in the present study—children's learning of multiplication.

Previous Findings on Strategy Use in Multiplication

Much less is known about strategic development in multiplication than in addition. However, a few facts are known.

North American third and fourth graders have been found to use several multiplication strategies: retrieval, repeated addition (adding one of the multiplicands the number of times indicated by the other), writing the problem (and answering without any adding), and counting sets of objects (e.g., solving 3×4 by writing three sets of four hatch marks and adding or counting them; Brownell & Carper, 1943; Cooney, Swanson, & Ladd, 1988; Siegler, 1988).

With learning, the relative frequencies of use of these strategies change. Children increase the proportion of trials on which they retrieve the answer and decrease their reliance on repeated addition and saying "I don't know" (Brownell & Carper, 1943; Cooney et al., 1988). There is also movement away from saying "I don't know" and toward children at least trying to solve each problem (Cooney et al., 1988).

Previous research also has indicated that children make highly adaptive choices between whether to use retrieval or a backup strategy. The more difficult the problem, the more frequent the use of backup strategies (Lemaire, 1993; Siegler, 1988). This pattern of choices is highly adaptive because it enables subjects to use the faster retrieval approach on problems where it yields correct answers and to use the slower backup strategies on problems where they are needed to produce accurate performance. It is unknown whether the adaptiveness of these choices among strategies improves with experience or is always at the same, high level.

These findings point to one source of the global improvement in speed and accuracy in multiplication: changes in when the strategies are used. Using retrieval on more trials increases both speed and accuracy. Saying "I don't know" less often also increases accuracy. It is unknown, however, whether change in the frequency of existing strategies is the whole story or whether children also acquire new strategies, improve the speed and accuracy with which they execute existing strategies, and/or choose increasingly adaptively among the strategies.

A Computer Simulation of Strategic Development

Recently, Siegler and Shipley (1995) developed a computer simulation of strategic development, the adaptive strategy choice model (ASCM). This model yields a number of specific predictions about the four dimensions of strategic change described earlier, as well as about relations between earlier and later performance of individual children. Although the simulation was developed to model the development of children's addition, its predictions seem equally applicable to multiplication (and, we suspect, to many other domains as well).

ASCM's basic organization is schematically outlined in Figure 1. Strategies operate on problems to yield data about the particular answer that was generated, the speed with

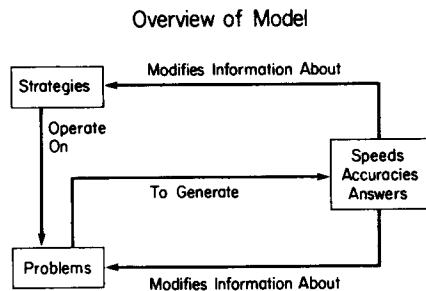


Figure 1. Overview of adaptive strategy choice model (ASCM). Reprinted from "Variation, Selection, and Cognitive Change," by R. S. Siegler and C. Shipley (1995) in G. Halford and T. Simon (Eds.), *Developing Cognitive Competence: New Approaches to Process Modeling*, Hillsdale, NJ: Lawrence Erlbaum Associates. Copyright in press by Erlbaum. Reprinted by permission.

which the answer was generated, and whether the answer was correct. This information about particular answers, speeds, and accuracies feeds back to databases about the effectiveness of strategies and the difficulty of problems.

The simulation's data on each strategy are represented at three levels: effectiveness on problems in general (i.e., speed and accuracy averaged over all problems), effectiveness on problems with particular features (e.g., problems in which the first number is smaller than the second), and effectiveness on individual problems (e.g., $3 + 6$). When a problem is presented, these three types of information are combined to generate the strength of each strategy on that problem. Also entering into the computation of the strategies' strengths is novelty information, information on how often the strategy has been used in the past. The novelty information allows newly formulated strategies to be tried even when existing strategies work quite well. Thus, the system is self-modifying in that the speed, accuracy, and novelty information associated with each strategy changes dynamically as the system gains experience solving problems, leading to increasing use of the more effective strategies and decreasing use of the less effective ones.

ASCM's operation on each trial can be divided into two phases: strategy choice and strategy execution. In the strategy-choice phase, strategies are retrieved with probability proportional to their strength relative to the strength of all strategies (with each strategy's strength reflecting its accuracy, speed, and novelty on problems in general, problems with features in common with the newly presented one, and the specific problem that was presented). Once a strategy is chosen, the model attempts to execute it. If a strategy other than retrieval is chosen, that strategy is executed. If retrieval is chosen, ASCM operates identically to its predecessor, the distributions of associations model (Siegler, 1988; Siegler & Shrager, 1984). First, it chooses a potential answer from the set of answers associated with the problem. The probability of a given answer being chosen is proportional to the strength of its association with the problem relative to that of other answers (exactly paralleling the way in which strategies are chosen). If the associative strength of the potential answer exceeds the confidence criterion (a thresh-

old for whether the potential answer is sufficiently promising to be stated), then that answer is stated. If not, the simulation either tries to retrieve again or chooses a strategy other than retrieval.

An aspect of the simulation that is critical for predicting changes and continuities in strategy use is the assumption that stated answers become associated with the problem on which they were stated. The increment is greater for correct answers than for incorrect ones (reflecting a belief that reinforcement works), but in each case, the strength of association increases following statement of that answer. This, together with the fact that the backup strategies produce the correct answer more often than any single erroneous answer, leads to the correct answer becoming increasingly strong relative to all other answers and also to common errors on the problem becoming increasingly strong relative to less frequent ones. Also critical, retrieval produces learning in the same way as any other strategy. When an answer is retrieved, information on the speed and accuracy of the process feeds back to the databases on the strategy and the problem, just as information produced through backup strategies does.

Predictions About Strategic Development

The simulation predicts a number of changes and continuities concerning when strategies will be used, how they will be executed, and how they will be chosen. It also makes predictions about relations between individual children's earlier and later performance. It does so as follows:

Predictions About Frequencies of Strategy Use

One phenomenon that already has been documented in multiplication learning is progressively increasing use of retrieval. Within ASCM, as the associative strength between a problem and its correct answer grows relative to associations between the problem and incorrect answers, the probability of retrieving the correct answer grows. The probability of its associative strength exceeding the confidence criterion grows through the same process. This increasing success of retrieval also leads to it increasingly often being tried before other strategies, thus adding further to its likelihood of use.

A second aspect of strategy use predicted by ASCM is increasing use of more effective backup strategies relative to less effective ones. ASCM includes retrieval and two backup strategies: the min strategy (adding by counting-on from the larger addend) and the sum strategy (counting from one). As the simulation uses the backup strategies, it learns that the min strategy is usually faster and more accurate. This leads to the min strategy being chosen increasingly and the sum strategy decreasingly (until improved capability at retrieving leads to both alternatives decreasing in use). A similar trend toward using more effective backup strategies should be evident in multiplication: An increasing proportion of backup strategy trials should involve repeated addi-

tion and a decreasing percentage should involve saying "I don't know."

Predictions About Strategy Execution

When children can choose among alternative ways of executing a given strategy, they should increasingly choose the ones that are fastest and that yield the most accurate results. In the case of multiplication, there are two distinct ways to execute repeated addition: adding the smaller multiplicand the number of times indicated by the larger, and adding the larger multiplicand the number of times indicated by the smaller. Children find it easier to add the larger multiplicand the number of times indicated by the smaller (see Siegler, 1988, Table 3). For example, in Siegler (1988), children asked to add four 7s erred on 9% of trials, but the same children asked to add seven 4s erred on 32% of trials. Thus, ASCM predicts a change over the course of the year in the way that children execute repeated addition: On an increasing percentage of repeated-addition trials, they should add the larger addend the number of times indicated by the smaller.

ASCM also predicts changes in the way that retrieval is executed. These changes should be reflected in the relative frequency of different types of errors on retrieval trials. At the outset of the simulation's run, a number of potential answers are weakly associated with the problem. As the simulation gains experience solving problems, it progressively strengthens the answers that are generated, correct and incorrect. Errors that are frequently generated through backup strategies should become increasingly prevalent relative to other errors.

The primary error generated by the most common backup strategy, repeated addition, is operand errors (errors that are multiples of one of the multiplicands, as in $8 \times 7 = 48$). It is easy to see how repeated addition would generate such errors; all that is necessary in this case just mentioned is to accidentally skip a number and therefore add six 8s. Thus the prediction is that operand errors should become increasingly frequent relative to other errors.

Because ASCM encodes featural information, it can learn about general characteristics of the answers that accompany problems as well as specific answers to the problems. Among these general features is odd-even status. When two odd numbers are multiplied, the answer is always odd; when two even numbers or an odd and an even number are multiplied, the answer is always even. Few adults and even fewer elementary school children know this rule explicitly. In one study in which subjects were asked about it, fewer than 10% of adults and 5% of children said that they knew it (Lemaire & Fayol, in press). Yet the performances of both adults and children seem to be affected by implicit knowledge of the pattern. On verification tasks, both children and adults more quickly reject errors that have incorrect odd-even status for the pair of multiplicands (e.g., $5 \times 4 = 21$) than ones that have the correct status ($5 \times 4 = 22$) (Krueger, 1986; Krueger & Hallford, 1984; Lemaire & Fayol, in press).

Early in learning to multiply, children also tend to produce errors that reflect odd-even status (Siegler, 1988). However, their errors tend to conform to the odd-even pattern of addition rather than multiplication. For example, on problems with one odd and one even operand, such as 5×4 , their errors tend to be odd numbers. Odd and even producing odd is the correct pattern for addition but not for multiplication. The hypothesis that follows from ASCM is that with multiplication experience, retrieval errors should increasingly conform to the odd-even status characteristic of multiplication and decreasingly to that characteristic of addition.

Predictions About Strategy Choices

At all points in development, the more difficult the problem, the more likely that a backup strategy will be used. The reason that ASCM generates this pattern of strategy choices is that the same factors that determine problem difficulty—the number and difficulty of operations needed to execute backup strategies correctly—also determine probability of use of retrieval. The strong correlations should be present when speed and accuracy on all trials are considered and also when speed and accuracy on retrieval trials alone are considered (because within the model, the determinants of percent use of retrieval, percent correct on retrieval trials, and mean solution times on retrieval trials are the same).

A second prediction about strategy choices is that the correlation between each problem's difficulty and children's percent use of backup strategies on the problem should increase with experience. As ASCM gains experience solving problems, it estimates increasingly accurately the likely effectiveness of each strategy on each problem. Children should show a similar pattern. Given that strategy choices are highly adaptive even early in learning, this is a quite nonintuitive prediction. No directly relevant data have been collected previously. Ironically, the high quality of early strategy choices may have discouraged investigators from asking whether, with age and experience, the choices become even more adaptive. When correlations involving relatively early performance average around .70 to .80, as they have in previous studies examining correlations between percent correct and percent use of retrieval on each problem (e.g., Geary & Brown, 1991; Geary & Burlingham-Dubree, 1989; Siegler, 1988), further increases seem unlikely. Nonetheless, the model predicts that such increases should occur. Again, the pattern should be present when speed and accuracy on each problem are computed only on retrieval trials, as well as when they are computed for all trials.

A third prediction regarding strategy choices is that the same structural features of problems should at different times of measurement predict when each backup strategy is most often chosen. They should be the factors inherently related to the effectiveness of that strategy relative to other strategies. In multiplication, the most frequent backup strategy is repeated addition. The best predictor of its frequency should be related to its advantages relative to other backup

strategies and to the inherent difficulty of executing it correctly. Both the size of the product, which incorporates the number of operations that must be executed and their difficulty, and the size of the smaller addend, which incorporates the more influential of these factors (the number of operations), fit this prediction.

Predictions About Stability of Individual Differences

Within ASCM, more accurate early execution of backup strategies has the seemingly paradoxical effect of producing more rapid decrease in use of backup strategies. The reason is that the more accurate execution of the backup strategies leads to stronger associations between each problem and its correct answer, and to weaker associations between the problem and incorrect answers, which leads to more frequent use of retrieval. For the same reason, early accurate execution of backup strategies leads to faster and more accurate later retrieval. This implies that those children who early in learning execute backup strategies accurately should later in learning more often use retrieval and do so more quickly and accurately.

A second prediction about relations between earlier and later strategy use is that individual children's early accuracy of retrieval should predict their later speed, accuracy, and frequency of use of retrieval. Within ASCM, just as accurate execution of backup strategies builds strong associations between the problem and the correct answer, and avoids creation of strong competing associations between the problem and incorrect answers, so does accurate use of retrieval.

This Study

To test these predictions, we conducted a longitudinal investigation of 20 French children during second grade, the year during which they are supposed to learn the 100 basic multiplication facts. The longitudinal design was essential for testing the predictions regarding stability of individual performance and in general allowed more sensitive tests of the predictions than would have been possible in a cross-sectional design in which different children, exposed to different teaching techniques, were tested at each grade.

Testing French children allowed us to determine whether the simulation's predictions applied to children outside the United States. All previous tests had been conducted with U.S. children. One of the interesting points in testing French children is that, compared with educational practices in the United States, the French educational system more strongly discourages use of backup strategies. Within this highly centralized system, rapid, accurate, and consistent use of retrieval is an explicitly stated formal goal of mathematics instruction in the second grade, the year that single-digit multiplication is first taught. Most teachers insist that children answer by retrieval almost from the beginning of instruction (Fayol, 1994, personal communication). Teachers in the U.S. often exert pressure in the same direction, but the pressure appears less strong and to be introduced some-

what later in instruction. These instructional differences raised the possibility that French children might not use backup strategies after the very first phases of instruction or, if they did use them, might choose among them less adaptively because they lacked sufficient experience with them. In sum, studying French children allowed a test of the generality of the strategy choice model.

Method

Participants

All 22 children in a second-grade class in a French upper class urban public school (4 girls and 18 boys) took part in this study. At the beginning of the testing, the mean age of the children was 97 months (range = 90–104 months). The results are based on the 20 children who participated in all three testing sessions; 1 child moved midway through the year, and another missed 3 months of school due to illness. The experimenter was a 28-year-old French male PhD student.

Problems

The problems were the 81 combinations of multiplicands (1–9) \times multipliers (1–9). They were randomly divided into two sets, with the constraint that one of each pair of inverse problems (e.g., 4×6 and 6×4) be in the first 41 problems and the other in the second 40. The two sets were presented in different sessions that were separated by 2 or 3 days. Half of the children received the problems in one order, and the others, in the opposite order.

Procedure

The procedure was the same as in Siegler (1988, Experiment 1). Each child was brought individually from the classroom to a nearby vacant room. The child was seated at a table directly across from the experimenter. On the child's side of the table were sheets of blank paper, stacked in a pile, with a pencil on top of the stack. Before each session the child was told (in French):

We are going to do some multiplication problems. I'll read the problems and when you have the answer, tell me what it is. You can do anything you want to get the right answer. There are several pieces of blank paper in front of you on the desk. You can use them or use your fingers or count in your head to find the right answer. I'm going to videotape you so that you'll be able to watch yourself on TV.

Before being presented the first problem, children watched themselves on TV; when the initial excitement diminished, the experimenter presented the problems. During the experiment, the TV screen was removed from children's visual fields so that they could concentrate on the problems. Problems were presented orally in the form "How much is N times M ?" (or simply by saying " $N \times M$?"). The same procedure was followed in all three testing sessions.

Each child's behavior was recorded by a GR-45S JVC camera. Solution times were obtained through the use of a GP-P50S JVC digitizer that printed digital times across the top right of the taped scene. The times were coded to the nearest .1 s. The experimenter classified strategy use on each trial both by making notes at the time of testing and by reviewing the videocassettes at a later time.

Initial classifications were followed unless the subsequent review contradicted them, in which case the videorecorded behavior provided the basis of classification. Following Siegler's (1988) procedure, use of backup strategies was inferred from overt behavior. On trials in which no overt behavior (e.g., subvocal counting, use of fingers, or writing) was evident, children were classified as having retrieved the answer. Two raters who independently classified strategy use on 100 trials agreed on 98% of them.

The three sessions were administered in January, April, and June of the second-grade year. At the time of the first testing, children had received only about a week of formal instruction in multiplication per se, though considerable instruction in tasks relevant to multiplication, such as being asked to add four 6s or three 7s, had been given. The results indicated that at this first testing, many children could already retrieve answers to the easier problems, which suggested that they had received some multiplication instruction outside of the school setting. By April, the children had been taught at school all problems with products of 45 or less. By June, they had been given extensive experience with all single-digit multiplication problems.

Results

Results are reported in six sections. The first provides an overview of the children's performance. The second through fifth focus on changes over the three sessions in the particular strategies that children used, in when they used them, in how they executed them, and in how they chose among them. The final section focuses on relations between early and later performance of individual children. All statistical comparisons that are reported are significant beyond the $p < .05$ level, unless otherwise noted.

Overview

A few summary statistics may provide a sense of the general level of performance. Children answered correctly 45% of problems in the first session, 70% in the second, and 88% in the third. Median solution times were 9.9 s in the first session, 5.5 s in the second, and 3.3 s in the third.

To solve the problems, children used five strategies: retrieval, repeated addition, saying "I don't know," counting-sets-of-objects (by writing groups of tally marks on a piece of paper and then counting the tally marks), and writing the problem (and then answering without visible counting or adding). Across the three sessions, retrieval was used on 63% of trials, repeated addition on 20%, saying "I don't know" on 16%, and counting sets of objects and writing the problem on 1% each. The diversity of strategy use was not produced by one child using one strategy and another child using a different one. All 20 children used at least two strategies in each of the three sessions, and the mean was slightly more than three strategies per child per session.

Changes in Which Strategies Were Used

The five strategies described in the previous paragraph were used by at least one child in each of the three sessions. However, changes occurred both in number of children

using each strategy and mean number of strategies used by each child.

Mean number of strategies used by each child first increased and then decreased: 3.1, 3.7, and 2.4 in Sessions 1, 2, and 3, respectively. Differences between each pair of testing sessions were significant, $t_s(19) > 3.47$. The initial increase and later decrease in number of strategies used by each child fit the general pattern observed by Siegler and Jenkins (1989) for domains in which a single strategy potentially works best on all problems.

Examining the number of children who used each strategy in a given session revealed the sources of these changes. Retrieval and repeated addition were used by all children in all sessions. Saying "I don't know" was used by all 20 children in Session 1, 19 of the 20 in Session 2, and only 11 of 20 in Session 3. Finally, number of children using the two relatively rare strategies—writing the problem and counting sets of objects—first increased and then decreased. The strategy of writing the problem was used by 7, 10, and 5 children in Sessions 1 to 3, and counting sets of objects was used by 1, 4, and 2 children in the three sessions. Thus the increase in mean number of strategies between Sessions 1 and 2 was due to children beginning to use the two relatively rare strategies, and the decrease between Sessions 2 and 3 was due to children ceasing to use these strategies and also ceasing to say "I don't know" as they became more skilled at multiplication.

Changes in When Strategies Were Used

Table 1 indicates the frequency of use of each strategy in each testing session (for strategies used on more than 1% of trials). Across the three sessions, use of retrieval increased and use of both repeated addition and saying "I don't know" decreased.

More specific examination indicated that the path of learning differed greatly for problems of varying difficulty. We divided the 81 problems into four groups on the basis of their products: problems with the 20 lowest products (8 or less) were labeled *easy problems*, problems with the next 20 lowest products (9–18) were labeled *relatively easy*, those with the next 20 lowest products (20–36) were labeled *relatively hard*, and those with the 21 highest products (36–81) were labeled *hard problems*. (To maintain essentially equal numbers of problems in each quartile, one

Table 1
Percentage of Use, Mean Solution Time, and Mean Percentage of Errors for Each Strategy

Strategy	Strategy use			RT (s)			Error (%)		
	1	2	3	1	2	3	1	2	3
Retrieval	38	62	92	3.9	2.8	2.9	27	12	9
Repeated addition	30	22	6	18.8	14.7	11.8	40	35	29
"I don't know"	32	15	2	—	—	—	100	100	100
Total	—	—	—	9.9	5.5	3.3	55	30	12

problem with a product of 36, 9×4 , was arbitrarily placed in the third quartile, and the other two, 4×9 and 6×6 , in the fourth.)

Figure 2 illustrates the results of this analysis. In Session 1, retrieval was the dominant approach on the easiest problems, repeated addition and retrieval were the most common on the next easiest problems, repeated addition and saying "I don't know" were the most common approaches on the relatively hard problems, and saying "I don't know" was the most common approach on the hardest problems. By Session 2, retrieval was the dominant approach on both the easy and the relatively easy problems, retrieval and repeated addition were the most common on relatively hard problems, and all three strategies were common on the most difficult problems. By Session 3, retrieval had become by far the most common approach on all problems.

The progressive extension of retrieval to problems with increasingly large products raised the issue of which factor best predicted frequency of retrieval on each problem. Within ASCM, the associative strength between each problem and its answer is hypothesized to be the best predictor. To test this prediction, we conducted a stepwise multiple-regressor analysis with five predictors. One was the associative strength measure obtained by Siegler (1988) with a sample of U.S. children. (This measure was obtained by asking children to state the first answer they thought of after each problem was presented, with the requirement that they always answer within 4 s.) The other four predictors were structural features of problems that had been hypothesized in previous studies to influence multiplication performance: the product, the smaller multiplicand, the larger multiplicand, and the square of the sum. The results showed that in all three sessions, a problem's associative strength was the best predictor of percent retrieval use on the problem, $r_s(79) = .78, .94, \text{ and } .76, p_s < .01$, in the first, second, and third sessions, respectively. A problem's associative strength was also the best predictor in each session of percent correct retrieval use on the problem, $r_s(79) = .75, .95, \text{ and } .75, p_s < .01$, in the three sessions. These results indicate that the associative strength variable is a very strong predictor of both frequency of retrieval and frequency of correct retrieval on each problem, better even than the product. They also indicated that measures of associative strength obtained from U.S. children accurately predict French children's strategy use as well.

Changes in Strategy Execution

As shown in Table 1, for both repeated addition and retrieval, speed and accuracy of execution increased substantially over the course of the school year. More specific changes in execution of each approach were also apparent.

Changes in execution of repeated addition. Over sessions, children became increasingly likely to execute repeated addition by adding the bigger multiplicand the number of times indicated by the smaller. They did this on 46%, 69%, and 80% of trials in the first, second, and third sessions, respectively. The percentage of this type of exe-

cution increased significantly both from the first to the second session and from the second to the third, $t_s(14) > 5.87$. This was an adaptive change, because children are more accurate in executing repeated addition in this way than in the opposite manner (Siegler, 1988).

Changes in execution of retrieval. As shown in Table 1, retrieval came to be executed both more quickly and more accurately over sessions. Errors decreased from 27% to 12% to 9% of trials; the changes between Sessions 1 and 2 and Sessions 2 and 3 were both significant, $t_s(19) > 3.24$. Similarly, mean solution times changed from 3.9 to 2.8 to 2.9 s; the decrease from the first to the second and third sessions was significant, $t_s(19) > 6.08$.

Changes in strategy execution were even more dramatic on the 32% of problems for which retrieval was used on all three presentations of a given problem (thus controlling for changes in the problems on which retrieval was used). On these problems, errors decreased from 23% to 2% over the three sessions, and mean solution times decreased from 3.8 to 1.8 s. It was not coincidental that this increase was greater than that on retrieval trials as a whole. Changes across sessions in mean accuracy and solution times on retrieval trials will in general understate the true amount of improvement, because early in learning, retrieval is used only on easy problems, whereas later it is extended to increasingly difficult ones.

The quality of children's errors on retrieval trials also improved over sessions. As shown in Table 2, we distinguished among five types of errors. *Operand errors* are erroneous answers that are correct for at least one single-digit multiplication problem that shares one (or both) multiplicands with the presented problem (e.g., $3 \times 6 = 21$; $4 \times 7 = 24$). *Table errors* are incorrect responses that are correct for one or more single-digit multiplication problems that do not share a multiplicand with the problem that was presented (e.g., $8 \times 7 = 54$). *Operation errors* are correct answers for some problem involving the same operands but a different arithmetic operation; for multiplication, they usually are correct answers to addition problems (e.g., $9 \times 8 = 17$). *Operand repetition errors* are answers in which one of the multiplicands is restated as the answer (e.g., $3 \times 4 = 4$). Finally, *non table errors* are incorrect answers that do not fall into any of the other categories (e.g., $3 \times 4 = 17$).

A 3 (session) \times 5 (error type) within-subject analysis of variance (ANOVA) was conducted on percentage of retrieval errors that fell into each category. A significant main effect was observed for error type, $F(4,68) = 28.39, p < .001$. In general, operand errors (those that were multiples of one or both multiplicands) were more common than other types of errors. Of greater interest, however, was the significant session by error type interaction, $F(8,152) = 18.54, p < .001$. As shown in Table 2, the percentage of retrieval errors that were operand errors increased greatly over the three sessions, whereas operation errors, which usually involved stating the answer to the corresponding addition problem, and operand repetition errors, which involved repeating one of the multiplicands as the answer, almost disappeared.

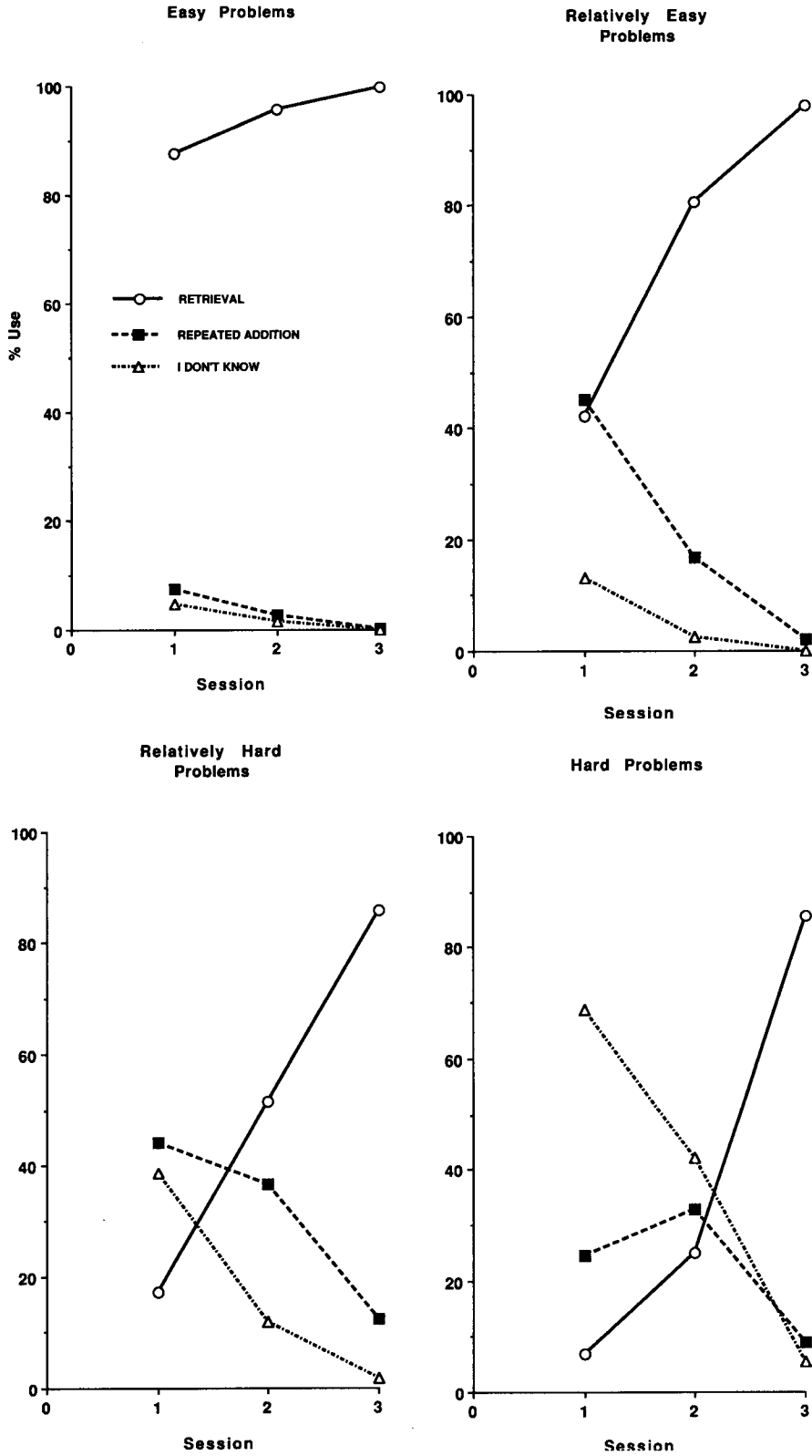


Figure 2. Percentage use of main strategies in each testing session (*easy problems* = problems with products less than 9; *relatively easy problems* = problems with products 9–18; *relatively hard problems* = problems with products 20–36; *hard problems* = problems with products 36 and greater).

Table 2
Percentage of Each Type of Error on Retrieval Trials in Each Session

Error type	Session 1	Session 2	Session 3
Operand	16	54	81
Table	5	16	7
Operation	35	17	2
Operand-repetition	30	4	2
Nontable	14	9	8

Children's errors on retrieval trials also changed considerably over sessions in their odd-even status. The percentage of children's errors consistent with the odd-even pattern characteristic of addition declined from 81% to 75% to 60% over the three sessions. The decline was significant both from the first to the third session, $t(29) = 3.19$, and from the second to the third session, $t(32) = 2.15$. Over the same period, percentage of errors consistent with the odd-even pattern characteristic of multiplication increased from 39% to 48% to 66%. The increase from the first to the third session was significant, $t(27) = 3.26$, as was the increase from the second to the third session, $t(30) = 2.70$.

Changes in Adaptiveness of Strategy Choices

Choices between backup strategies and retrieval. Children's choices of whether to retrieve or use a backup strategy were greatly affected by problem difficulty in all three sessions. Percent use of retrieval on each problem and mean solution time on that problem were highly correlated in all three sessions, $r_s(79) = -.82, -.81, \text{ and } -.83$. Percent use of retrieval on each problem also correlated strongly with percent errors on that problem in all three sessions, $r_s(79) = .90, .92, \text{ and } .75$. The lower correlation in Session 3 reflected both variables having more restricted ranges than in earlier sessions. In Sessions 1 and 2, percent correct and percent retrieval both were as low as 0% for some problems and as high as 100% on others. In contrast, by Session 3, children answered correctly on at least 50% of trials on each problem and used retrieval on at least 65% of trials on each problem. On fully one third of problems (27 of 81), both percent correct and percent retrieval were 100%, and on more than half (42 of 81), both were at least 90%.

Examination of the data indicated that the high correlations in Session 1 were due primarily to performance on $N \times 1$ and $1 \times N$ problems. These problems can be solved by application of the simple rule "When a number is multiplied by 1, that number is the answer." Thus, performance that was classified as retrieval on such problems, because no overt activity was evident, may have actually reflected application of the "1s rule" rather than retrieval of an answer to the specific problem.

This analysis led us to recompute the correlations on the 64 problems that did not involve 1 as a multiplicand. On these problems that could not be solved by applying the 1s rule, correlations between mean RT and percent use of retrieval on each problem increased across the three ses-

sions, $r_s(62) = .50, .75, \text{ and } .80$ for Sessions 1, 2, and 3, respectively. The increase from Sessions 1 to 2 and from Session 2 to 3 were both significant, $t_s(62) > 7.00$. The correlation between percent correct on each problem and percent use of retrieval on that problem also increased on these problems from $r(62) = .80$ in the first session to $r(62) = .90$ in the second, before dropping to $r(62) = .69$ in the third. As in the analysis of all problems, the decrease in correlation from the second to the third session was attributable to ceiling effects leading to restrictions of variance on both variables in Session 3 (Figure 3). In general, however, the predicted increases over sessions in the correlations between problem difficulty and percent use of retrieval were apparent on problems that did not involve 1 as a multiplicand and that therefore could not be solved by applying the 1s rule.

Similar changes in correlations were present when speed and accuracy on each problem were calculated solely on the basis of performance on retrieval trials on that problem. Correlations between mean RT on retrieval trials on each problem and percent use of retrieval on that problem were $r_s(30, 56, 62) = -.34, -.37, \text{ and } -.65$ for the three sessions, respectively. The Session 3 correlation was significantly higher than that in either of the two earlier sessions. The corresponding correlations between percent correct on retrieval trials and percent use of retrieval were $r_s(30, 56, 62) = .39, .77, \text{ and } .57$.¹ The correlation increased significantly from Session 1 to Session 2, before ceiling effects on both percent correct and percent use of retrieval led to a significant decrease in Session 3.

Choices among backup strategies. The main backup strategies that children used were repeated addition and saying "I don't know".² They used them on 95% of trials on which they used any backup strategy. In principle, children could have attempted repeated addition on any problem, but they often chose not to do so. ASCM's prediction was that their choice should depend on the relative advantages of the two strategies. Because saying "I don't know" was always quick and easy but never yielded a correct answer, the likelihood of using repeated addition rather than saying "I don't know" would be expected to vary with the likelihood of correctly executing repeated addition. In particular, children would be expected to choose repeated addition most

¹ As in previous studies (e.g., Siegler, 1988), only problems on which retrieval was used by at least 3 children were included in these correlations. The reason is that including problems on which fewer than 3 children contribute to the mean solution time or percentage correct on retrieval trials results in highly unstable data. Illustratively, if only 2 children retrieved answers on a problem, a change of a single answer could change the percentage correct on retrieval trials on that problem from 0% to 50%.

² Within ASCM, backup strategies are defined as all approaches other than retrieval. Therefore, to test ASCM's predictions regarding choices among strategies and relations between early and later performance, saying "I don't know" was considered a backup strategy. The logic is that the same variables and processes that govern choices among more prototypic strategies also govern choices of whether, for example, to use repeated addition or to say "I don't know."

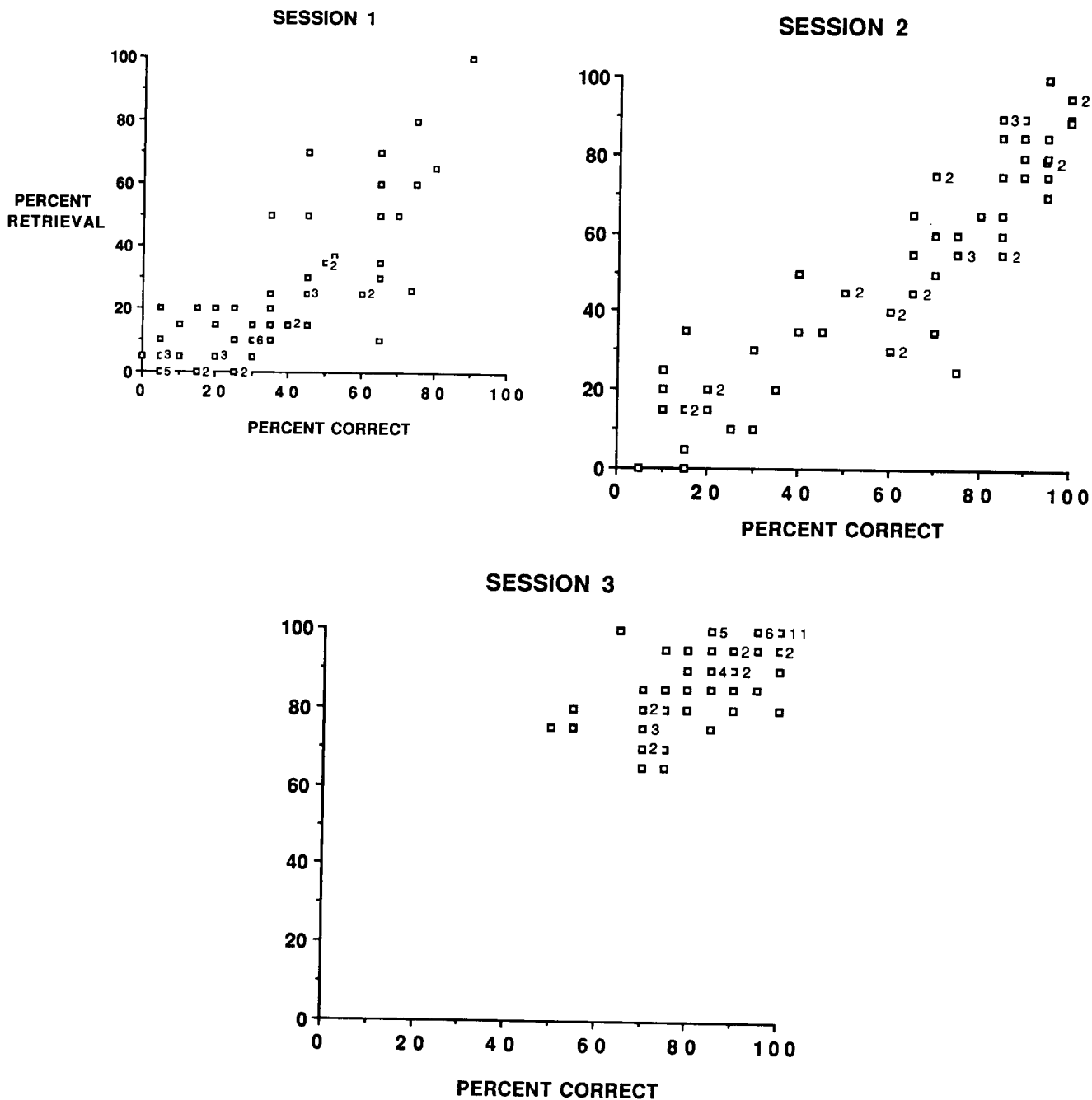


Figure 3. Scatter plots for relations between percentage use of retrieval on each problem and percentage correct on retrieval trials on that problem for Sessions 1, 2, and 3. Note the attenuation of variance on both variables in Session 3. The numbers within the scatter plots refer to the number of observations at the data point immediately to the left of the number.

often when it, unlike the alternative, would be likely to produce the correct answer. Conversely, they would be expected to say "I don't know" primarily on problems where neither strategy was likely to produce the correct answer.

To test this hypothesis, we needed to identify both the predictors of the accuracy of repeated addition and the

predictors of when repeated addition was most often used. First, we conducted a stepwise regression analysis to determine predictors of accuracy on repeated addition trials. The independent variables were each problem's first multiplicand, second multiplicand, smaller multiplicand, larger multiplicand, and product. The dependent variable was the problem's percent correct on those trials where children

used repeated addition. Because of the limited number of uses of repeated addition on many problems and the need to have at least three uses of the strategy to avoid extreme variability of the dependent measure, uses of repeated addition in all three sessions were considered together.³

The analysis showed that two variables accounted for almost identical percentages of variance in accuracy on different problems. Size of the product accounted for 42% of the variance, and smaller multiplicand size for 41%. The predictive value of both variables made sense within ASCM. Product size reflects both the number of addition operations and the size of the numbers being added. Smaller multiplicand size reflects the number of operations that need to be performed.

A second stepwise regression analysis was conducted to determine whether the same variables predicted frequency of use of repeated addition, relative to saying "I don't know." In this regression, the dependent variable was percent use of repeated addition on each problem divided by the combined frequency of repeated addition and saying "I don't know" on that problem. The independent variables were the same as in the analysis of accuracy of repeated addition.

As expected, the best predictors of frequency of use of repeated addition on each problem closely resembled the best predictors of the accuracy with which the strategy could be executed. In Session 1, the product was the best predictor, accounting for 51% of the variance in percent use of repeated addition on each problem. Smaller multiplicand size was the next strongest predictor, accounting for 43% of the variance. In Session 2, size of the smaller multiplicand was the best predictor, accounting for 48% of the variance, and product size was the next best, accounting for 44%. In Session 3, product size was the best predictor, accounting for 29% of variance, and smaller multiplicand the next best, accounting for 26%. (The fall-off in percent variance accounted for was likely due to both repeated addition and saying "I don't know" not occurring very often in the third session; only 24 of the 81 problems met the standard for inclusion in the analysis of the two strategies together being used by at least 3 children.)

Thus at all points in learning, children used retrieval most often on problems on which that fast and easy approach was likely to yield a correct answer; used repeated addition most often on problems that they could solve by that approach but were not so easy that they could solve them through retrieval; and said "I don't know" most often on the difficult problems that they were unlikely to solve correctly by either approach.

Stability of Individual Differences

The longitudinal design used in this study allowed us to examine a number of relations predicted by ASCM about relations between earlier and later performances of individual children. One prediction was that early accurate execution of backup strategies should lead to later accurate retrieval. This prediction was borne out. A child's percent

correct on backup strategy trials in Session 1 was a significant predictor of the child's percent correct on retrieval trials in Session 2, $r(18) = .69$ and in Session 3, $r(18) = .62$. Percent correct on backup strategy trials in Session 2 also correlated significantly with percent correct on retrieval trials in Session 3, $r(18) = .59$.

The second prediction about individual differences was that early incorrect use of backup strategies should lead to more frequent later use of backup strategies (rather than retrieval). Again, the prediction was borne out. Each child's percent errors on backup strategy trials in Session 1 was significantly correlated with that child's percent backup strategy use both in Session 2 and in Session 3, $rs(18) = .63$ and $.67$, respectively. The child's percent errors on backup strategy trials in Session 2 was also significantly correlated with the child's percent use of backup strategies in Session 3, $r(14) = .61$.

ASCM's third prediction about stability of individual differences was that early percent correct on retrieval trials should correlate positively with later percent correct on retrieval trials. This prediction also proved accurate. The correlations of each child's percent correct retrieval in earlier and later sessions were $rs(18) = .71$, $.72$, and $.60$ between Sessions 1 and 2, 1 and 3, and 2 and 3, respectively.

General Discussion

This study of multiplication learning reflected a convergence of several influences: the general conceptual framework that distinguished among four aspects of strategic change, the specific theory of performance and change embodied in the ASCM simulation, the longitudinal design in which individual children were followed over a substantial period of learning, and the strategy assessment method that allowed identification of which strategy was used on each trial. Together, they made possible a much more precise depiction of cognitive change than is ordinarily obtained. In this concluding section, we discuss general implications of the data for understanding of learning, for understanding of arithmetic, and for understanding of strategy choice.

³ The reason why percent correct on repeated addition trials was calculated over all three sessions and percent use of repeated addition was calculated separately for each session was that percentage correct has greater variability. For example, percent correct on repeated-addition trials, is computed only on the subset of trials on which repeated addition is used; frequency of use of repeated addition is computed over all trials. Thus, if repeated addition was used on 2 of the 20 trials, percentage correct on repeated addition trials would be calculated only over those 2 trials, whereas percent use of repeated addition would be calculated over all 20 trials. Given the low frequency of use of repeated addition on many problems, it was necessary to consider its use over all three sessions to obtain stable estimates of its accuracy on each problem.

Implications for Understanding Learning

The improvements in speed and accuracy that generally characterize learning were evident in the current study. From the first to the last session, percent errors decreased from 55% to 12% of trials, and mean solution time per problem decreased from 9.9 to 3.3 s.

The conceptual framework pointed to four potential sources of this improvement. Three of them proved influential in the present context: changes in the relative frequency of use of the strategies, improvements in the efficiency with which the strategies were executed, and more finely calibrated choices among the strategies.

First consider the contribution of changes in relative frequency of use of the strategies to the global improvements in speed and accuracy. In the first session, children used each of the three main strategies on between 30% and 40% of the trials. By the last session, they used the fastest and most accurate approach, retrieval, on more than 90% of trials. Such increased frequency of the fastest and most accurate strategy would have led to considerably faster and more accurate overall performance even if no other strategic changes had occurred.

However, the speed and accuracy with which each strategy was executed also increased substantially over sessions. The improved execution of the strategies reflected qualitative as well as quantitative changes. Repeated addition provided the clearest evidence for such qualitative changes in strategy execution. Part of the improvement in the speed and accuracy of this strategy was due to children increasingly adding the larger addend the number of times indicated by the smaller, rather than vice versa. Even within a given way of executing the strategy, however, execution became faster and more accurate as the children gained experience solving multiplication problems. Thus, as with the data as a whole, improvements in speed and accuracy within a given strategy involved qualitative changes in what was done as well as quantitative changes in how well it was done.

A third contributor to learning was increasingly apt choices among the strategies. Over the three sessions, children used retrieval on the easier problems and backup strategies on the more difficult ones with increasing precision. Even in Session 1, they recognized where the 1s rule was applicable and applied it consistently. Subsequently, as they gained experience with multiplication, they came to choose increasingly adaptively for the full set of single-digit problems. The choices were highly adaptive at all points in learning, but they became increasingly so with experience.

This depiction of the contribution of strategic changes to learning is quite different than most. Usually, when the contribution of strategic change to learning is recognized at all, it is viewed solely in terms of acquisition of new, more effective approaches. Ironically, in the present study, this was the one type of change that was not observed; instead, the substantial improvements in speed and accuracy derived from the other three sources of strategic change. Without question, acquisition of new strategies plays an important part in many cognitive changes, but even when the set of

strategies remains constant, strategic changes still contribute substantially to learning.

Most analyses of learning focus on changes; indeed, the phrase "change in behavior" is a prominent part of many definitions of learning. The present longitudinal design, however, revealed many continuities in learning as well. Among the most prominent continuities were the particular strategies that were used in the three sessions, the identity of the best predictor of how often each strategy was used on each problem, the high correlations between problem difficulty and frequency of use of retrieval, and the individual differences in children's proficiency on the task. The general conceptual framework and the ASCM model pointed to places where such continuities might be expected to be present, and the data set indicated that such continuities did in fact characterize learning in this domain. The more general implication of ASCM is that similar continuities should be present in the many other domains in which people use multiple strategies and obtain extensive problem solving experience.

Implications for Understanding Arithmetic

The results provided both specific and general support for ASCM as a model of children's arithmetic. At a general level, they showed that the model was applicable to multiplication as well as addition and to children trained in the French educational system as well as to those trained in the U.S. The extension to French children demonstrated that even in the face of an educational system that strongly discourages use of any procedure other than retrieval, children generate the same type of adaptive strategy choices that are seen among children exposed to other educational approaches.

At a specific level, the model's predictions were consistently borne out. Among the predictions that were supported were that strategy choices would be highly selective at all points in learning, that with experience they would become increasingly effective, that the predictors of frequency of use of each strategy would be the same factors that determine the speed and accuracy of that strategy relative to alternative strategies, that children would increasingly use more effective forms of backup strategies, that high frequency errors would become increasingly prevalent relative to low frequency ones, that early effective execution of backup strategies would predict later frequency of use of retrieval, and that early frequency and effectiveness of retrieval would predict later frequency and effectiveness of retrieval. Some of these predictions had been demonstrated previously for addition but not for multiplication; others had not previously been demonstrated for either (such as the increasingly adaptive quality of strategy choices as children gain experience with the strategies, the movement toward more effective forms of backup strategies, and early frequency and effectiveness of retrieval predicting later frequency and effectiveness of retrieval).

One especially important prediction that was borne out was that in solving arithmetic problems, children acquire

featural information as well as global and local information. ASCM incorporates three levels of information: information averaged over all problems (global information), information about individual problems (local information), and information about problems that have particular characteristics (featural information). Before this study, however, no empirical support had been obtained for the hypothesis that problem-solving experience leads to children obtaining increasingly useful featural information. The data on odd-even status of retrieval errors provided relevant data. Over the course of the three sessions, children's errors on retrieval trials increasingly often fit the odd-even pattern characteristic of multiplication and decreasingly often fit the pattern characteristic of addition. The pattern suggested that children were coding odd-even status of multiplicands and products, albeit unconsciously, and that they learned that two even multiplicands or an odd and an even multiplicand would produce an even product, whereas two odd multiplicands would produce an odd product.

The results also cast light on an issue that has been discussed extensively in the arithmetic literature but without clear resolution: whether the associative networks for addition and multiplication are progressively integrated or progressively differentiated. On the one hand, considerable evidence for progressive integration of these networks has been obtained (Campbell & Graham, 1985; Hamann & Ashcraft, 1985; Koshmider & Ashcraft, 1991; Lemaire, Barrett, Fayol, & Abdi, 1994; Lemaire, Fayol, & Abdi, 1991; Miller & Paredes, 1990). For example, Lemaire et al. (1994) found that with age and experience in school, cross-operation interference increased. The manifestation of such cross-operation interference involved the time required to reject erroneous answers to addition problems that would be correct answers for multiplication (e.g., $8 + 4 = 32$). Rejecting such answers took longer than rejecting other erroneous answers (e.g., $8 + 4 = 18$). Lemaire et al. found that in third grade such cross-operation interference occurs only on small-size problems, but by fifth grade it is present on large as well as small problems.

On the other hand, evidence such as that found in the present study points toward increasing differentiation of multiplication from addition. Over the three sessions, percentage of cross-operation errors on retrieval trials greatly decreased. Moreover, errors on retrieval trials increasingly reflected the odd-even characteristics of multiplication and decreasingly those characteristic of addition, again pointing to greater differentiation of the two operations.

There may be a simple, theoretically meaningful way of explaining both types of findings. Associative networks for addition and multiplication may be linked increasingly closely, but control processes may allow increasingly effective differentiation on tasks where there is sufficient time for the control processes to operate. Findings of increasing percentages of cross-operation errors, which have been interpreted as indicating increasing integration of addition and multiplication networks, have been obtained when task instructions required subjects to respond quickly. Findings of decreasing percentages of cross-operation errors, interpreted as indicating increasing differentiation of the two

networks, have been obtained when task instructions did not mention speed. Thus, both integration and differentiation may be increasing, with the effect on performance depending on whether sufficient time is available for control processes to operate.

Direct evidence for the impact of temporal demands on operation-interference effects comes from Lemaire et al. (1991). In this study, adults and children were presented verification tasks for both addition and multiplication. When the answer was presented simultaneously with the equation, or 100 ms after it, cross-operation interference effects were observed. However, when the answer was presented 300 or 500 ms after the equation, no such interference effects were present. Presumably, this lack of interference effects at the longer delays occurred because on those trials, subjects had sufficient time to inhibit incorrect activated candidates through use of control processes.

At a general level, the present results point to the importance of children's problem-solving experience as a shaper of their arithmetic competence, above and beyond the instruction that they receive. It seems extremely unlikely that any teacher tells students that multiplying odd numbers results in the product being odd, though adding them results in the sum being even. Yet the pattern of changes in children's errors on retrieval trials indicate that they progressively learn this. Similarly, it seems unlikely that teachers encourage children to execute repeated addition by adding the larger multiplicand the number of times indicated by the smaller. This is especially true of the French system, in which use of all strategies other than retrieval is strongly discouraged. Again, however, children increasingly move toward this approach. Certainly, no one tells children that they should use retrieval on the easiest problems, that they should use repeated addition on the problems with middling levels of difficulty, and that they should say "I don't know" on the hardest problems. Yet they show this pattern of strategy choices at all points in learning. Instruction is an important influence on learning of arithmetic, but many characteristics of arithmetic learning reflect the nature of the system doing the learning rather than the instruction per se.

Implications for Understanding Strategy Choices

Prior studies have demonstrated adaptive choices between retrieval and backup strategies in a variety of domains, including addition, subtraction, multiplication, time telling, word identification, and spelling (Geary & Brown, 1991; Geary & Burlingham-Dubree, 1989; Siegler, 1986, 1988; Siegler & McGilly, 1989). However, these studies were limited in two ways. First, the strategy choices were examined at a point when the subjects were already quite proficient on the types of problems they were presented. Even the youngest children studied, preschoolers, typically answered about 70% of the problems accurately on the small-number addition problems given to them (Geary & Burlingham-Dubree, 1989; Siegler & Robinson, 1982; Siegler & Shrager, 1984). Thus little was known about whether strategy choices were adaptive even at points when children

have little proficiency in the domain. Second, and related, changes in the adaptiveness of strategy choices were not examined; the focus was on adaptiveness at the particular point of observation rather than on whether that adaptiveness changed with experience. Ironically, the high degree of adaptiveness of the strategy choices in all past experiments may have discouraged systematic tests of changes in that adaptiveness.

In the present experiment, children were studied at an early point in learning. In Session 1, they answered correctly on only 45% of total problems and on only 35% of problems that did not have 1 as a multiplicand. On problems where both multiplicands were 2 or more, children's choices of whether to state a retrieved answer or use a backup strategy did become more adaptive as they gained experience. Even very early in learning, the choices were quite adaptive, but they became more finely calibrated to problem difficulty with increasing problem solving experience. This was exactly the prediction that followed from ASCM: adaptive choices from early in learning and choices becoming even more adaptive with learning.

The present findings also suggest a general method for predicting where alternative strategies will be used most often. This method involves identifying the types of problems on which each strategy would have the greatest advantage over the other in speed, accuracy, or both and predicting that the strategy will be used most often on those problems. In the present context, retrieval would be used most often on the problems with the greatest associative strengths, because it would usually generate correct answers on these problems and because it is the fastest strategy that can yield a correct answer. Repeated addition would be used most often on problems on which it could be executed accurately but that were too difficult to allow retrieval of the correct answer. Saying "I don't know" would be used most often on problems that were too difficult for children to be likely to generate a correct answer regardless of which strategy they used. The underlying logic has much in common with Anderson's (1990) method of rational analysis: identify alternative approaches, identify the relevant environments to which the alternative approaches can be applied, predict that each approach will be used most often in the environments in which it is most effective. The present approach complements rational analysis in emphasizing not just the enduring factors that lead certain factors to best predict the types of problems on which given strategies will most often be used but also in emphasizing how improvements in knowledge and strategy execution lead to changes in the strategy choices that are made.

Conclusion

The global changes in speed and accuracy that are characteristic of learning can reflect at least four, more specific, strategic changes: introduction of new strategies, shifts toward greater use of the more efficient existing strategies, improved execution of the strategies, and more adaptive choices among the strategies. In this study, three of these

factors contributed not only to a large variety of changes but also to a number of continuities over the course of learning—continuities in the use of multiple strategies, in the adaptive quality of choices among them, in the characteristics of problems that elicited most frequent use of each strategy, and in individual differences in children's strategy use and strategy execution. Together, the present conceptual framework, method of assessing strategy use on a trial-by-trial basis, and model of strategy choice provide a base for in-depth analyses of learning, not only in the context of multiplication but in a wide variety of other domains as well.

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