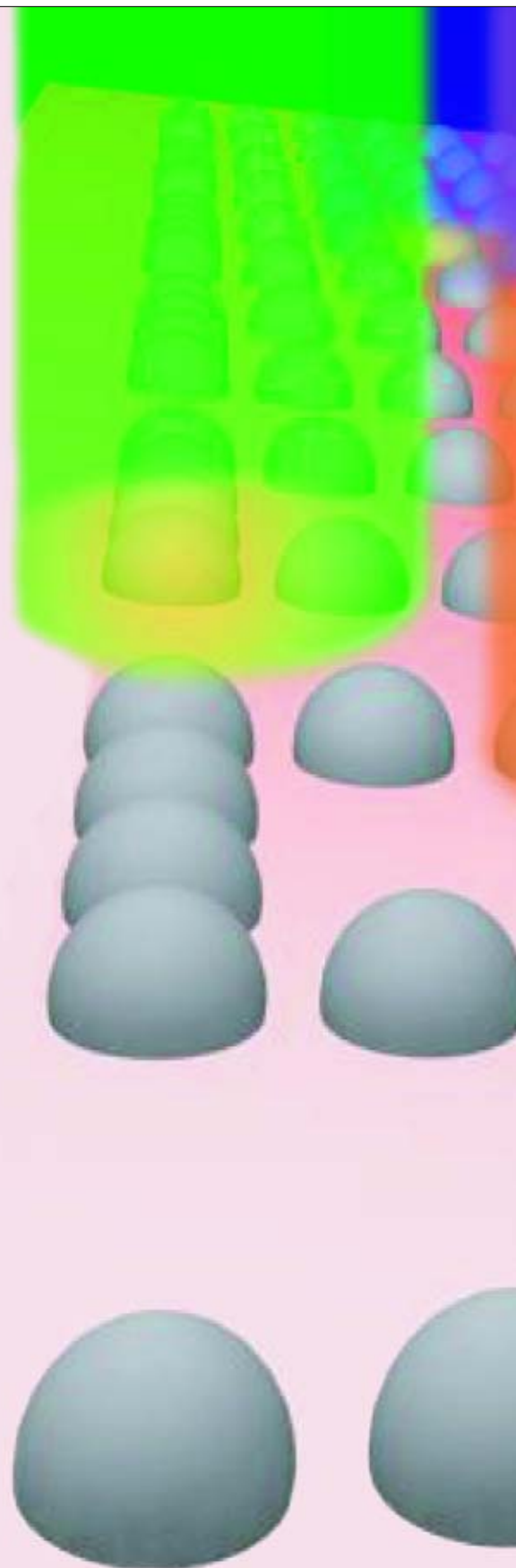
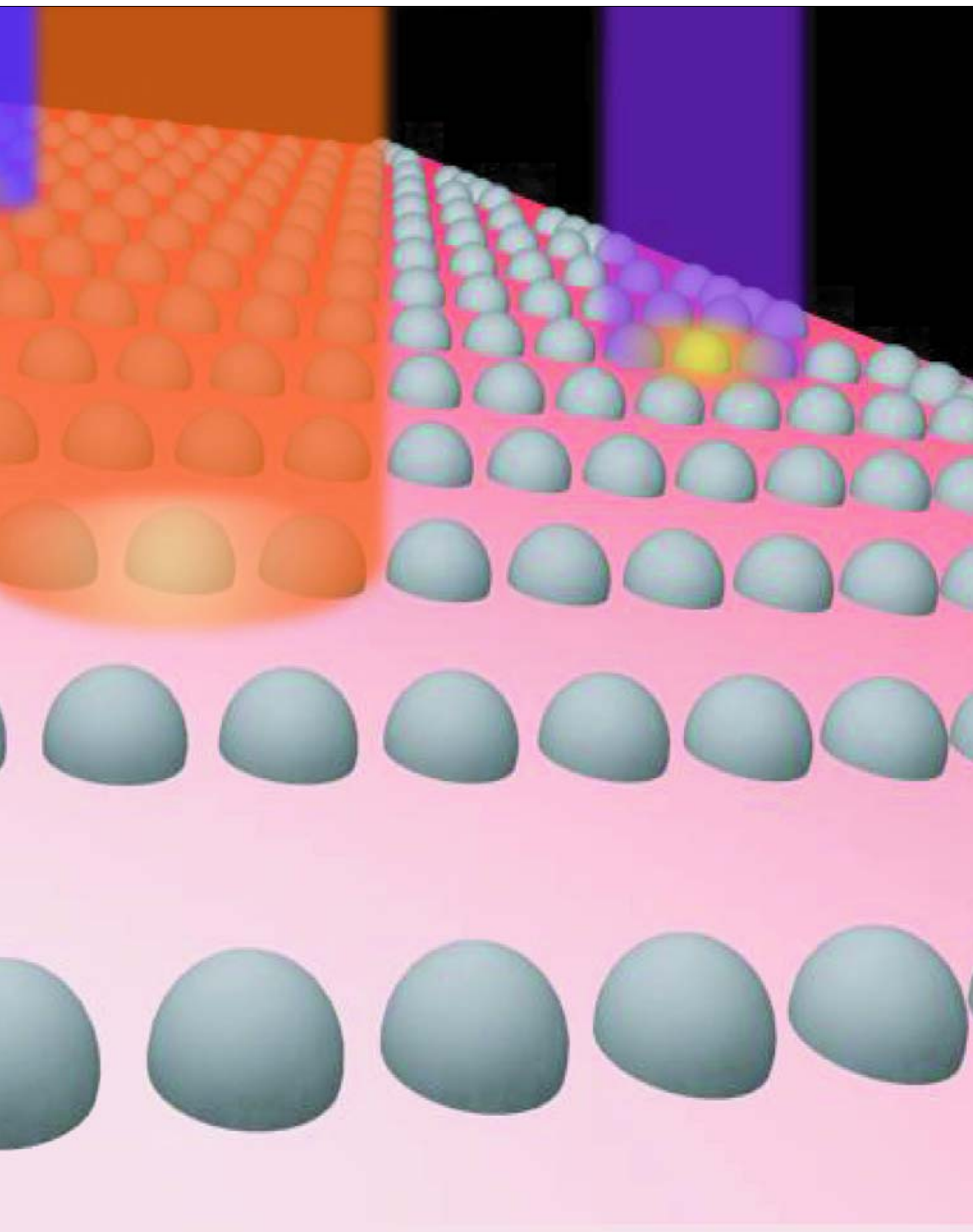


# Quantum Information Processing Based on Optically Driven Semiconductor Quantum Dots

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It may one day be possible to use arrays of quantum dots driven by ultrafast laser pulses to carry out quantum information processing. The article reviews recent experimental progress in the area of coherent optical control of single dots and outlines a plan for further research.





For decades we have enjoyed the exponential rate of growth in computing power and information processing speed predicated by Moore's law: the density of computer chips has, in fact, doubled every year. But this amazing rate of growth is expected to encounter its Malthusian limit when component size reaches the quantum limit, usually held to be the size of an atom.

The situation is different in III-V semiconductors, which have a conduction band electron with an effective mass that is less than one tenth that of the free electron mass. As a result, the size of a semiconductor quantum dot is around 10 nm in the quantum limit. The increased size makes it easier to address and manipulate individual quantum dots compared to atomic systems. A paradigm shift toward direct use of the quantum properties of information processing devices is now underway.

Today's information processing devices in regular computers also depend on the quantum properties of electrons and photons, and quantum theory applies here as well. But these devices are based on aggregates of large numbers of quantum particles. Clearly, quantum effects manifest themselves before the single wave length limit is reached. Implementation in the future of devices that operate in the quantum limit will involve the capacity to prepare, manipulate and measure *single* particles. Recent seminal advances in the theory of quantum information and computation have stimulated many avenues of experimental exploration in this regard.

In this article we will concentrate on recent progress in the coherent optical control of one or two excitations in single semiconductor nanodots. The approach we will describe exploits the vast technology base of the electronics and photonics industries. Because of space limitations, we are unable to do justice here to the tremendous advances achieved recently in the context of approaches such as optical control of trapped ions and neutral atoms, the sophisticated application of linear optics, nuclear magnetic resonance and small superconductors.

The qubit—the basic unit of quantum information—consists of two distinctive orthogonal states analogous to



Scientist aligns system for quantum gate measurement.

the 0 and 1 of the classical bit. A curious property of the qubit is that it can exist in a linear combination state of the two basis states, a characteristic which provides it with the potential to contain vastly more information than a classical bit can. There are, however, obstacles to the complete realization of this quantum potential. These obstacles involve, in particular, preparing qubits in desirable initial states and measuring the states of qubits, which necessarily reduces the states to 0 or 1.

The advantages of quantum information processing over classical information processing derive from clever schemes of control of the evolution of the qubits from the initial states to the final measurements. The quantum algorithms that have stimulated this field include: Shor's algorithm for quantum Fourier transform, the best known application of which is the factorization of large numbers; Grover's search algorithm; and enhanced communication protocols and simulations of quantum processes.

The power of quantum algorithms derives from two sources. The first is the superposition of the binary states that creates the opportunity for parallel processing. For example, if the computer of  $N$  qubits is initially set with each qubit at the  $|0\rangle$  state, a unitary transformation on each qubit can change its state to a

superposition state with half the probability in each basis state, such as the state  $(|0\rangle + |1\rangle)/\sqrt{2}$ . This particular transformation is called a Hadamard gate, a single qubit gate. The state for  $N$  qubits has an equal probability distribution over all  $2^N$  possible states. A unitary operation on all  $N$  qubits thus offers the opportunity for massive parallel processing of  $2^N$  states.

When the state of two qubits is not simply a product of each, the two states are correlated and the collective state is said to be entangled. The second source of quantum power derives precisely from the entanglement of two or more qubits. An example of an entangled state is the singlet  $(|10\rangle - |01\rangle)/\sqrt{2}$ , where the first digit denotes the state of one qubit and the second digit that of the other qubit. In this well known example, if the first qubit is measured and found to be in state 1, the second is bound to be in state 0, and vice versa.

Such perfect correlation can be used for parallel processing of branching or logic decisions. Entangled states have also been used for quantum teleportation, a technique for moving quantum states between a sender and a recipient, even in the absence of a quantum communication channel.

Entanglement is the key to quantum cryptography as well as to distributed quantum computation.

Any quantum algorithm can be decomposed into elementary gates that operate on one or two qubits at a time only. Single-bit rotations and a non-trivial two-bit gate, such as a controlled-NOT (CNOT) gate, would make up a set of universal gates.

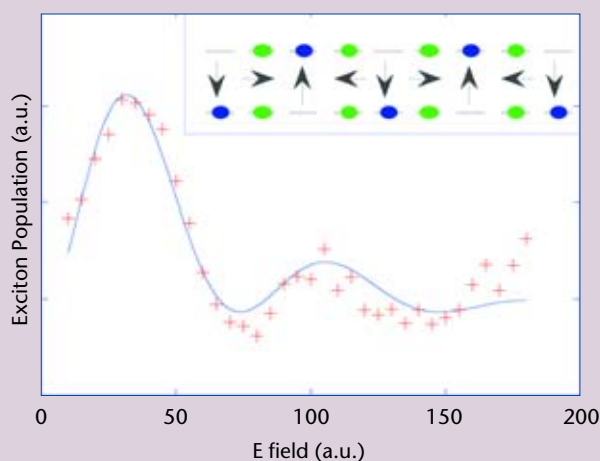
### Basic logic gates based on exciton qubits

Circularly polarized light can excite an electron and a hole trapped in a nanodot. Experiments on exciton qubits have been carried out in naturally formed quantum dots. When very thin (3 nm) GaAs quantum wells with AlGaAs barriers are grown, the monolayer spatial fluctuations of the interfaces result in formation of islands, about 30 nm across, which localize the excitons. The spin-polarized exciton transition can be used as a qubit, where the presence (absence) of the exciton corresponds to the state  $|1\rangle$  ( $|0\rangle$ ). When the temperature is cooled to  $\leq 10$  K by use of liquid helium, exciton qubits are naturally initialized to the ground state  $|0\rangle$ . Because of their short lifetime, about 100 ps, they serve primarily as prototypes for more robust systems. In self-assembled quantum dots, their lifetime could be lengthened to the nanosecond range.

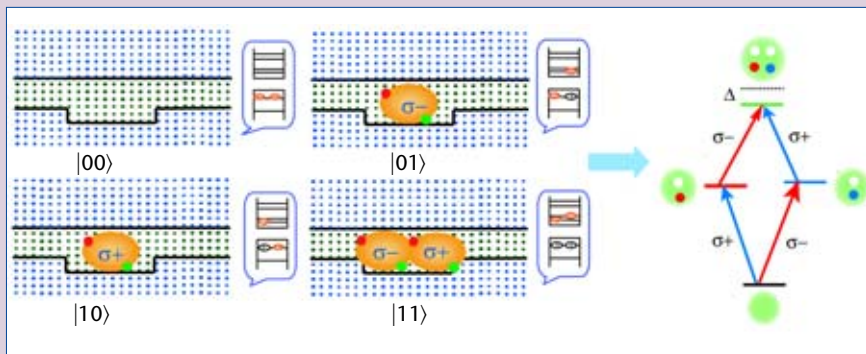
### Single-qubit gate

Rabi oscillations have long been observed in atomic and molecular systems. Under a strong and resonant coherent driving field, the population of the atom goes through oscillations as a function of pulse area, as illustrated in the inset in Fig. 1. The light-atom interaction is characterized by the product of the dipole moment of the transition and the electric field amplitude ( $\mu \cdot E(t) = \hbar R(t)$ ) which, in turn, defines the Rabi frequency ( $R(t)$ ). (Here for simplicity we ignore the phase of the electric field.) The pulse area is the time integration of Rabi frequency ( $\Theta(t) = \int_{-\infty}^{\infty} R(t) dt$ ). The state at time  $t$  is  $|1\rangle \sin(\Theta/2) + |0\rangle \cos(\Theta/2)$ . Thus, the state takes two revolutions of the Rabi oscillation to return to the original state.

Rabi oscillation of a qubit is an essential forerunner to arbitrary single qubit operations. Rabi oscillations of excitons confined in single quantum dots were observed two years ago by several



**Figure 1.** The population of the excited state goes through oscillations under a strong, resonant, coherent driving field. (Inset) Illustration of Rabi oscillations of a two-level system; the arrows represent Bloch vectors.



**Figure 2.** Single particle picture and excitation diagram of a two-bit system confined in a single quantum dot.  $|00\rangle$ ,  $|10\rangle$  ( $|01\rangle$ ) and  $|11\rangle$  denote, respectively, the crystal ground state, the exciton and the biexciton.

groups.<sup>1-4</sup> The population of a single exciton was driven coherently between the crystal ground state and the exciton state, as shown in Fig. 1. These experiments represent a major breakthrough on the road to building logic devices based on optically driven quantum dots because they demonstrate that it is possible to coherently control single solid state qubits by use of short laser pulses.

### Two-qubit gates

We now consider creation of a two-qubit system in a single quantum dot by use of optical excitation (see Fig. 2). Here, the qubit is the optical Bloch vector associated with the exciton transition. In the

electron and hole representation, it is clear that the valence band is fully occupied and the conduction band—in the absence of any optical excitation—is empty. The ground state is then mapped into the  $|00\rangle$  state. Two orthogonally polarized excitons can be excited by laser pulses of appropriate wavelength and polarization. These excitation states correspond to states  $|01\rangle$  and  $|10\rangle$ , respectively. One can also choose to create a bound state of the two excitons; such a biexciton state is mapped with state  $|11\rangle$ .

In the context of a discussion of logic gates, it is appropriate to transfer the relevant optical transitions from the electron-hole representation to the excitation



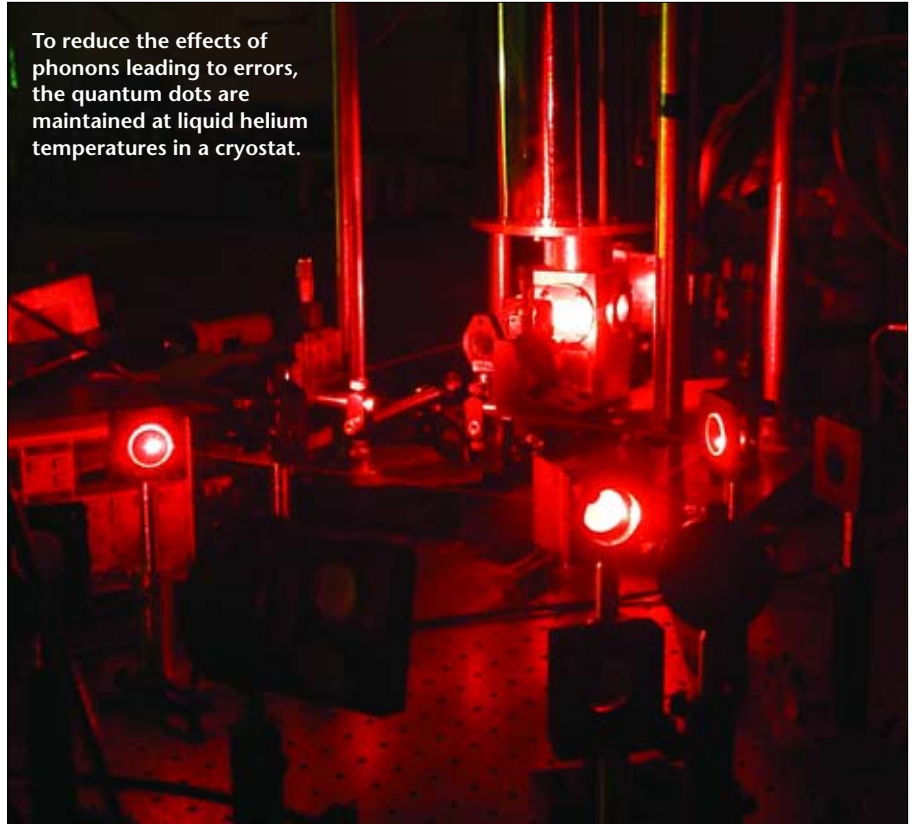
representation (see Fig. 2). An important feature evident in this four-level diagram is that the biexciton state has a large binding energy as a result of the enhanced Coulomb interaction caused by the three-dimensional quantum confinement in all quantum dots. The binding energy, which varies with the size of the quantum dots, is typically in the range of 3 to 4 meV. Each dipole-allowed transition can be driven independently by use of pulses of a few picoseconds—of properly selected polarization—because the bandwidth of such pulses is smaller than the biexciton binding energy.

Here is another way of understanding the implications of a large binding energy: the excitation of one exciton leads to a different excitation energy of the other exciton, shifting it down by an amount equal to the binding energy. This feature gives rise to the characteristic conditional dynamics needed to build a CNOT gate. If an optical pulse is designed to effect a  $\pi$ -rotation between the two states  $|10\rangle$  and  $|11\rangle$  without inducing other transitions, such as those that occur between  $|00\rangle$  and  $|01\rangle$ , then conditional on the presence of the first qubit only, the second qubit would be flipped, i.e.,

$$|00\rangle \rightarrow |00\rangle, |01\rangle \rightarrow |01\rangle, |10\rangle \rightarrow |11\rangle, |11\rangle \rightarrow |10\rangle.$$

Biexcitons confined in single quantum dots have been identified optically, and Rabi oscillations of the biexciton transition have also been recently demonstrated.<sup>5</sup> In Fig. 3(a), a single complete oscillation is observed, with the peak of the oscillation corresponding to a pulse area of  $\sim\pi$  and the trough corresponding to  $\sim 2\pi$ . Biexciton Rabi oscillations can also be observed as a function of the time delay between the excitation pulse and the measurement pulse at fixed excitation power [Figs. 3(b)–3(e)], where the target quantum states that follow the excitation pulse (indicated by the shaded area) are illustrated. The post-excitation-pulse evolution over time is caused by the population relaxation. When the pulse area exceeds  $\pi$ , stimulated emission begins to occur. This has been most clearly demonstrated in the case of a  $2\pi$  pulse, where the maximum population was excited and then immediately switched down.

To reduce the effects of phonons leading to errors, the quantum dots are maintained at liquid helium temperatures in a cryostat.



An important finding is the confirmation that the  $\pi$  pulse can serve as the operational pulse of a CNOT gate.

The performance of this exciton-based CNOT gate can be examined by comparing the physical truth table shown in Fig. 4 with the ideal one. As in the case of a classical gate, the truth table provides the population of each state at the output corresponding to a particular input. As an example, if the input state of the system is 10, after gate operation, the populations in states 00, 01, 10 and 11 are 0.14, 0.06, 0.17 and 0.63, respectively. The deviation of the truth table from the ideal values is due mainly to the population relaxation of the dipole-allowed transitions during the long operational pulse required for this sample.

Quantum coherence and quantum entanglement are critical determinants of the superior performance of quantum logic devices compared to that of classical devices. The complete wave function that immediately follows a pulse which simultaneously excites both excitonic states can be written as  $|\Psi\rangle = C_0|00\rangle + C_+|01\rangle + C_-|10\rangle + C_{++}|11\rangle$ . In an experiment aimed at creating the Bell state  $|01\rangle + |10\rangle$  by use of

pulses coupled to both orthogonal excitons, coefficients were estimated to be  $C_0=0.48$ ,  $C_+=C_-=0.62$ ,  $C_{++}=0$ . This led in our experiments to entanglement entropy of as high as 0.7—compared to the entropy of 1 that would characterize an ideal entangled state. Since entanglement entropy is a measure of the amount of quantum information that an entangled pair of qubits can carry, the ability to create an entangled pair of qubits with large entropy is critical for quantum information processing based on semiconductor quantum dots.

### The electron spin qubit: on the road to a quantum computer

To build a quantum computer capable of performing extensive number gate operations, interaction between qubits that reside in different dots is required. While researchers have suggested methods for controlled interaction between excitons in different dots (an example is cavity quantum electrodynamics), much experimental investigation will be required to further develop such schemes. While improvement of exciton-based qubit systems is desirable, our own

plan is to employ electron spin dynamics using the knowledge we have acquired in the area of exciton control.

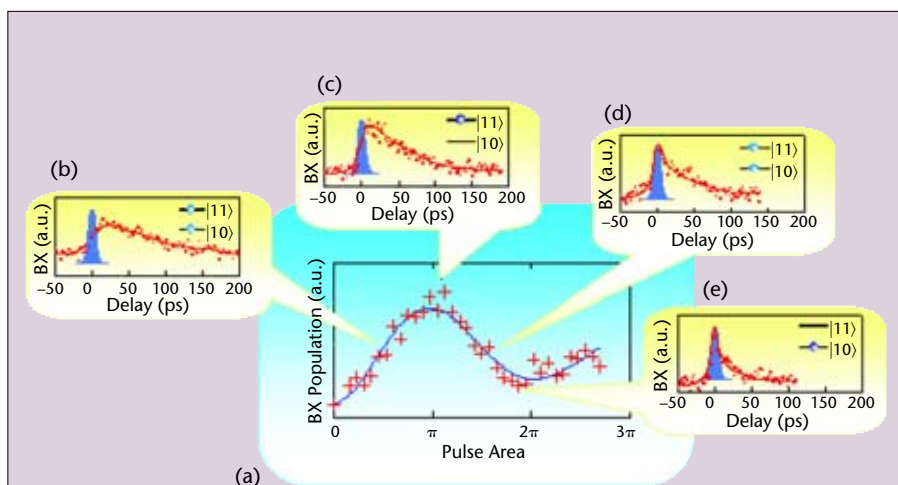
Electron spin is an ideal representation of a qubit. One can imagine the spin of an electron as a small magnet; associated with that magnet is a unit of angular momentum  $1/2\hbar$ . Electron spin can either point up, in representation of the binary state 1, or point down, in representation of the binary state 0. In a simple scheme, one can “dope” each quantum dot with a single electron. Each localized electron then serves as a qubit. The spin qubit has a much longer decoherence time than the exciton qubit. To manipulate the electron spin with short laser pulses, an exciton is usually created during the optical excitation. A trion (i.e., a charged exciton) is then formed by the exciton and the original extra electron in the quantum dot. Optical decoherence is avoided by virtual excitation of the exciton. We have also designed a Raman process to induce exchange interaction between spins in two dots; this process will give us a basic two-qubit gate for entanglement.<sup>6</sup> When this basic two-qubit gate for entanglement is combined with single qubit rotations, we have the basic building blocks for a quantum computer.

Many critical questions remain and require further investigation. Full characterization of the density matrix will be essential to development of a complete understanding of the quantum dynamics of the system, and a means for reliable readout and initialization will have to be experimentally explored. Progress in many different areas will be required; among them are improved control of materials fabrication and the development of more sophisticated schemes for optical excitation and coherent control.

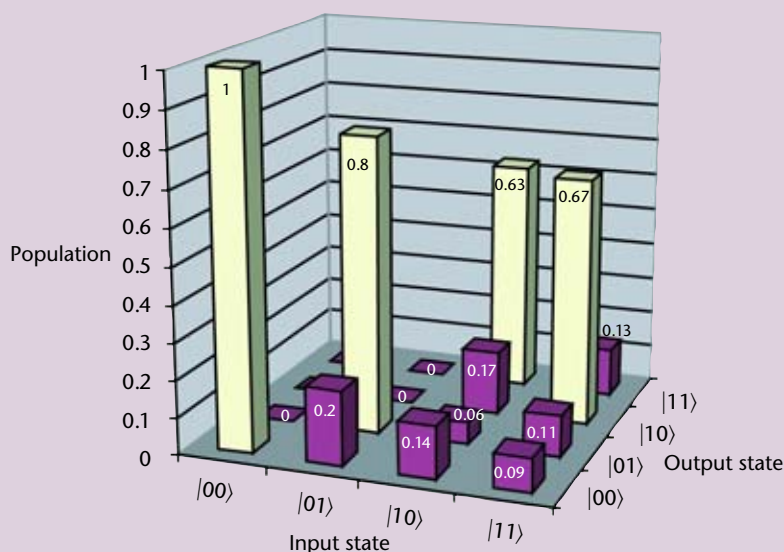
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**Figure 3.** Biexciton Rabi oscillations. (a) Measured biexciton population vs. excitation field amplitude. (b)-(e): Measured biexciton population vs. time delay between the “excitation pulse” and the “measurement pulse,” at fixed excitation power with target quantum states, following delivery of the pump pulse.



**Figure 4.** Truth table of the numerically simulated CNOT gate. In an ideal gate, the four highest bars are 1 and the others are 0.