# Plato's Forms, Pythagorean Mathematics, and Stichometry 

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This essay advances a series of strong theses about Plato's philosophy and aims to corroborate the view of Aristotle and other members of the early Academy that it was fundamentally Pythagorean. The argument reconceptualises the literary strategies of the dialogues and thus requires some historical stage-setting. The argument is inductive in form. This means that it gains in strength by accumulating a variety of independent, yet mutually reinforcing kinds of evidence.

Stichometric analyses find unexpected evidence for Pythagoreanism in the dialogues themselves, and thereby develop a pregnant argument made by Sayre. He observed, in short, that the structure of the text of the Statesman seemed to reflect its metaphysics. There is at the mathematical centre of the dialogue 'reference to the middle between Excess and Deficiency,' and thus 'the dialogue begins with a reference to a beginning, ends with reference to an ending, and includes reference at its midpoint to a mean or middle. ${ }^{11}$ This correspondence between textual structure and metaphysics is substantially amplified here. In fact, the

* This line of research originated when I was teaching one course devoted to Plato's Republic, and so rehearsing the literature on its puzzling structure, and another course on the history of mathematics, including Pythagorean music theory. Thank you to Michael Worboys and my friends and colleagues in CHSTM at Manchester University, where this paper was first read. My work on Plato began in the philosophy departments at Notre Dame and Stanford, and I thank my friends, colleagues, and teachers from those institutions. For help with early drafts of this paper, thanks to Anthony Lesser, John Pickstone, Thomas Uebel, Michael Rush, John Shand, and Louise Crascall.

1 Sayre [58], 183. Cf. Statesman, 283c3-5c2. Sayre proceeds to make a similar argument about the structure of the Philebus.

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musical structure Sayre takes as paradigmatic of Plato's mathematical ontology can, it will be argued, be found embedded in the dialogues. ${ }^{2}$

Recent work on the Derveni Papyrus has helped to spark a widespread reappraisal of the interest in symbolical and allegorical writing in antiquity. There is now a consensus that such literary techniques were an important theme of discussions in the circles around Socrates. ${ }^{3}$ Although it is common for symbolical texts to avoid describing their underlying structure in any open way, the many passages about symbolism and allegory in Plato's dialogues attest to his interest in these debates. ${ }^{4}$ Plato's motivations for adopting this form of symbolic writing are best assessed after the evidence has been presented. There is some evidence, reviewed below, that certain ancient readers of the dialogues did in fact recognise their stichometric structures.

Manual stichometric studies of Plato's dialogues have been carried out in various ways by Birt, Schanz, Harris, Dodds, Berti, and others, but were primarily aimed at inferring the lengths of the lines and columns on the papyrus sources of surviving manuscripts. ${ }^{5}$ This is apparently the first report of computer-based, stichometric investigations of Plato's dialogues. This lacuna is surprising in an era when computerised, stylometric studies have been undertaken by a number of scholars. ${ }^{6}$ Although the data described below reveals some unexpected features of the dialogues, it is in retrospect natural that Plato would have given his works mathematical form. The dialogues reflect the revolution in mathematics that affected several of the arts and sciences during the

[^1]fifth century, and mathematics is thought to have been important in the early Academy. Plato's dialogues, of course, generally champion the importance of mathematics for philosophy and education. Embedding mathematical forms in their surface narratives also accords with the dialogue's core philosophical conception of 'forms beneath appearances.'

A surprisingly large number of authors in later times, especially those associated with some strand of Platonism, gave their longer works a mathematical organisation. ${ }^{7}$ The regular, mathematical divisions of the Divine Comedy are visible on the poem's face. A generation ago, scholars in English literature and Renaissance Platonism were surprised when it was discovered that some of Spenser's well-known poems contained elaborate schemes of astronomical and calendrical symbolism. This shifted the direction of Spenser studies and is now a well-developed part of ongoing research and undergraduate curricula. ${ }^{8}$ There was some debate about how these structures escaped notice during years of intensive study of Spenser's works. ${ }^{9}$ The eventual diagnosis was perhaps two-fold. First, since the history of science and of philosophy then gave short shrift to the pseudo-sciences of earlier times, scholars may not have been sensitive to the subtle strategies of the so-called 'Pythagorean aesthetic,' according to which a literary work should mirror the supposed, underlying structure of the cosmos. Second, unravelling Spenser's symbolism required familiarity with three subjects, Renaissance astronomy, Spenser's Platonistic philosophy, and literary allegory, which are often separated into distinct academic disciplines today. Although features of these later works should not be read back into Plato, their scholarly study provides methodological precedents and recalls a genre which has reappeared in different periods and different cultures. ${ }^{10}$

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## 1 Ancient Stichometry

The practice of counting the number of syllables in a line or the number of lines in a stanza was already routine in archaic poetry. Vitruvius, without giving his source, reports a tradition that 'Pythagoreans' and some comic playwrights mathematically organised longer works. ${ }^{11}$ Some set speeches in Euripides' plays have similar lengths and he has been suspected of counting the number of lines even in these larger textual units. The German scholars Ludwig and Biehl have found various sorts of evidence for underlying stichometric structures in his plays, but no consensus about their claims has emerged. ${ }^{12}$

Papyrologists, aiming to reassemble the scraps and fragments of ancient papyri, have been particularly interested in the at times widespread, ancient scribal practice of counting the number of lines in a scroll. These counts are perhaps analogous to and fulfil some of the same functions as our page numbers. The recording of total counts on the scrolls themselves was common; some also had partial line counts at regular intervals in the margins (as with our Stephanus numbers). The standard work on this 'total stichometry' or 'partial stichometry', Ohly's Stichometrische Untersuchungen, finds evidence (too extensive to be reviewed here) that the practice was already common during Plato's lifetime. ${ }^{13}$ Callimachus' catalogue, compiled about a century after Plato's death, recorded the stichometric totals for each of the scrolls in the library of Alexandria. ${ }^{14}$ Diogenes Laertius' report that Aristotle's writings amounted to 445,270 lines may have derived from the Alexandrian catalogues. ${ }^{15}$ In this context, any authors with Pythagorean inclinations could avail themselves of stichometric counts to organise their works.

There were several motivations for stichometry. First, scribes were often paid by the line; their rates were sometimes set by (still extant) legal decrees. Second, the cost of a scroll may in part have depended

## 11 De Arch V, Prol 5

12 Ludwig [49] and Biehl [11]. There is a positive appraisal of Ludwig's work in Michelini [50].

13 Ohly [54]. Lang [46] is a more recent, less reliable review of the subject.
14 See Blum [13, 157-8] and Ohly [54]. Shironi [63] argues that a critical edition of Plato was made at Alexandria.

15 V 27. Totals are given for Speusippus (43,475, IV.5), Theophrastus (232,800, V 50), Xenocrates (224,239, IV 14), and others. See Blum [13], 201.
upon its length. Like our page numbers, the stichometric totals were thus important information for customers. Third, the stichometric counts helped to identify works (in a period when texts were not always given titles) and were a guarantee that copies were complete. ${ }^{16}$ Fourth, some believe that classical literary scrolls were manufactured in expensive, standardised lengths, and that authors would thus have had an incentive to plan their compositions so that they would fit into a standard scroll and not waste large blank spaces. ${ }^{17}$ The columns and lines in classical literary papyri were typically fairly uniform, ${ }^{18}$ and this would have made it easier for authors (and their readers) to maintain accurate counts. Birt went so far as to conclude that this was the norm, 'dass kein classischer Shriftsteller zu componieren vermocht hat ohne zu disponieren. ${ }^{19}$

The hypothesis, to be tested below, that Plato was a stichometrist and counting his lines, columns, syllables, or letters suggests some method is needed to measure and identify locations within his texts. The traditional Stephanus page numbers supply only a rough and approximate measure. Henri Estienne's 1578 edition of the dialogues made no attempt to ensure that his columns, our Stephanus pages, were uniform. Their length varied significantly to accommodate his notes and Latin translation. ${ }^{20}$

Construction of more precise measures faces several complications. Punctuation, indentation, and inter-word spaces were probably not used in any significant way in Plato's original compositions. ${ }^{21}$ Measures were constructed here by stripping out everything but the letters of the Greek alphabet, counting them, and then marking regular intervals

16 Cf. the Anonymous Commentary on the Theaetetus. Bastianini, Sedley, et al. [61], col. III.28-37, 268, cf. 486 and Ohly [54], 103.

17 Johnson [43] finds no evidence for such standardisation in the later papyri from Oxyrhyinchus.
See the illustration in Johnson [43], $x$, and also Birt [12], etc.
19 Birt [12], 342.
20 Estienne [55]. There are many anomalies in the Stephanus numbers. The paragraphs $a$ through $e$ are also of variable length and are occasionally omitted altogether. In the Republic and Laws there are gaps in the Stephanus page numbers between books.

21 Thompson [72], etc. There may also have been no indications of change in speaker.

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within the original text. The computer programs were verified in several ways. ${ }^{22}$ The algorithms employed were complicated and will be described separately. ${ }^{23}$

Although textual critics debate the point, some hold that Plato's texts survived repeated copying in relatively good shape, ${ }^{24}$ and the investigations reported below accord with that view. However, the many small emendations, corruptions, possible interpolations, and suspected losses of text listed in the Oxford Classical Texts (used here) and other critical editions of Plato's dialogues suggest there will be a limit to the accuracy of any measure of distance within the dialogues. Fortunately, a certain benign averaging works to improve the accuracy of measurements of relative lengths and ratios within the dialogues. The effects of a fairly uniform distribution of smaller scribal errors, omissions, and interpolations still uncorrected by textual critics would on average compensate for each other and would not therefore significantly disturb measurements of relative positions. A surprising technique for gauging the accuracy of these measurements, introduced below, indicates that they are generally accurate to within half of a percent. This is a testament to the quality of the scribal tradition, to the extraordinary efforts of generations of modern textual critics, and to the power of averaging.

## 2 Evidence within Individual Dialogues

Once the Stephanus numbers are replaced with accurate measures of relative location, the placement of key concepts and episodes at mathematically significant locations in the dialogues is readily apparent. It is diffcult to convey in short compass the regularity of these patterns,

22 Short blocks of text easily counted by hand were duplicated many times and combined; the computer correctly counted the total. The texts of some dialogues were also separated into several parts to check that the computer's counts of the parts exactly added to its counts for the whole (linearity preserves ratios).

23 They excluded the large number of 'invisible characters' in the Unicode files from the counts by examining each character's hexadecimal representation.
24 The question is reviewed in Böter [14]. Brockmann [18], 5-16 and Irigoin [41] contain historical surveys of modern textual criticism of Plato's dialogues. Slings' work on the text of the Republic is collected in [65]. The older edition of the Republic by Adam [1] contains a convenient overview of the categories and locations of its textual problems (see Index III under 'Manuscripts').
which extend through the length of a dialogue. Here, the strategy is to focus on a few easily measured features. Although each may appear trivial in itself, they provide a preliminary form of evidence for the stichometric structure of the dialogues. Later sections build up to more elaborate kinds of evidence, and finally turn toward some philosophical implications.

Some dialogues, like the Menexenus, the Symposium, and the Phaedrus, contain set speeches clearly demarcated from the surrounding text. The lengths of some of these speeches provide evidence that the composition of each dialogue was stichometrically organised.

In the Menexenus, for example, Socrates' long speech lasts ten-twelfths of the length of the entire dialogue to within a fraction of a percent. ${ }^{25}$

In the Symposium, Pausanias' speech, Eryximachus' speech (including the repartee over Aristophanes' hiccups), and Aristophanes' speech are each about one-twelfth of the dialogue. Socrates' long speech, including his conversations with Agathon and Diotima, occupies three-twelfths or one quarter of the entire dialogue. Alcibiades' speech lasts about twotwelfths of the dialogue. ${ }^{26}$ These length measurements suggest that an interval of one-twelfth of the dialogue plays a fundamental role.

The relative location of the speeches within the Symposium provides another form of evidence for the importance of this unit. The beginning of Pausanias' speech is aligned with the point two-twelfths of the way through the dialogue, the beginning of Eryximachus' speech (with hiccups) with the three-twelfths point, and the beginning of Aristo-

25 Menex: Ten-twelfths of the total dialogue is 20,610 letters, and the speech is 20,601 letters or 99.95 percent of ten-twelfths. The speech lasts from 236 d 4 to 249 c 8 .
26 Symp: Pausanias' speech begins at 180c4 shortly before the two-twelfths point at 180e5, and ends at 185c3, three OCT lines after the three-twelfths point at 185 b 7.

Eryximachus' speech (with the exchanges about hiccups) begins at 185c5 shortly after three-twelfths, and ends with some repartee with Aristophanes at 189 c 1 shortly before the four-twelfths point at 189 d 6 .

Aristophanes' speech begins at 189c2 shortly before the four-twelfths point, and ends at 193e2, three OCT lines after the five-twelfths point at 189 d 7 .

Socrates begins responding to Agathon at 198a4, four lines before the central six-twelfths point at 198a8. After some banter, Socrates agrees to give a proper speech at 199b2-7 which is at the six and a quarter twelfths point (199b4). After quizzing Agathon and reporting his conversation with Diotima, Socrates stops at 212 c 3 which is one OCT line after the nine and a quarter twelfths point (212c2).

Alcibiades' agrees to give a speech at 214 c 6 shortly after the nine and threequarters point ( 214 c 1 ) and stops at 222 b 7 shortly before the eleven and three-quarters point at 222 e 7 .

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phanes' speech with the four-twelfths point. ${ }^{27}$ The climactic, rhetorical fireworks in praise of Eros that conclude Agathon's speech occur at sixtwelfths, the centre of the dialogue. ${ }^{28}$ This scale of one to twelve plays a role even within the longer speeches. For example, the highlights of Diotima's speech, her talk of intimate contact with Beauty and her description, at the top of her 'ladder', of transcendent Beauty as the form of the One, are aligned with points eight- and nine-twelfths of the way through the dialogue, and thus are also separated by an interval of one-twelfth. ${ }^{29}$

In the Phaedrus, Socrates' second speech is three times as long as his first speech to within a fraction of a percent. The first speech is somewhat longer than one-twelfth of the dialogue and the second is somewhat longer than three-twelfths. The beginning of the second speech occurs shortly before the four-twelfth point and the end is aligned with the seven-twelfth point. ${ }^{30}$

The structure of arguments within individual dialogues is often organised around this scale of twelfths. Many examples could be given. In the Phaedo, Socrates concludes his argument for immortality from cyclic generation at the third twelfth; immediately thereafter he begins the argument from recollection which concludes at four-twelfths. ${ }^{31}$ In

27 See the previous note.
The conclusion of Agathon's speech is at 197e8; the centre of the dialogue is four OCT lines later at the six-twelfths point, 198a4.

29 Diotima's Beauty is at 206d2-7a4, which includes the eight-twelfths point at 206e34. Socrates' speech concludes with Diotima's description of Beauty as the form of the One, which begins at 210 e 2 . This passage includes the calculated nine-twelfths point at 211b4, which is three lines after the phrase 'eternally existing one-form, in and of itself' at 211b1-2.

30 Socrates' shorter first speech is 99.9 percent of one-third of the second speech. Socrates' second speech (243e9-57b6) has 21,508 characters, and one-third of this is 7,169 characters. The first speech has 7,163 characters (237a7-41d1). The speeches do contain prologues, interludes, and quotations. If the prologue of the first speech (237a7-b1) is not counted, then the first speech has 6,743 characters, which is close to one-twelfth of the dialogue $(6,744)$. Lysias' speech early in the Phaedrus is criticised for its lack of organisation; its length is 5,750 characters, which is not close to one twelfth of the entire dialogue.
31 The three-twelfths point of the Phaedo is at 72 c 8 ; the conclusion of the cyclic generation arguments is at 72c5-e2, i.e., at the three-twelfths point.

The Phaedo's argument for immortality from recollection begins at 72e3, immediately after the three-twelfths point, and concludes at 77c6-d5, i.e., at the fourtwelfths point ( 77 c 8 ).
the Euthyphro, the first definition of holiness is at three-twelfths and the second definition is at fourtwelfths. ${ }^{32}$ In the Apology, Socrates begins his investigation of the oracle's claim that he is 'wisest' at the two-twelfths point and concludes it at threetwelfths. ${ }^{33}$

Measurements of the absolute lengths of the dialogues also suggest that the number twelve has some architectural importance. There is substantial evidence, reviewed at length in Birt [12] and Ohly [54], that the lengths of classical prose compositions were typically measured by the number of standard 'lines,' which were each as long as one of the hexameter lines of epic poetry. Perhaps verse compositions like Homer's poems were the first longer texts written down in Greek and thereby established his line-length as a conventional unit even for prose. Ohly finds the earliest reference to measuring texts in these heroic lines in Plato's Laws, and concludes from a variety of pieces of evidence that "... in Platos alter wurde also der Hexameter bereits als Maßeinheit verwandt ..." ${ }^{\prime 34}$ Some surviving fragments of earlier Greek papyri with historical, philosophical, or literary prose compositions are written in such lines. Using a figure of thirty-five letters per hexameter line, ${ }^{35}$ calculations of the total number of lines in the dialogues produce, with about

32 In the Euthyphro, the first definition of holiness is at 5d8-e2, i.e., at the three-twelfths point (d9). The second definition of holiness is at 6e10-7a1; the four-twelfths point is at the word 'paradigm' at e4-5.

33 In the Apology, the investigation begins at 21b8-9, i.e., at the two-twelfths point, which is at the word sophos in line b9. Socrates concludes his discussion of his investigations at 23 c 1 and turns to his own teachings at the next line (pros de toitois ...); the four-twelfths point is at this line, 23 c 2 .
34 Laws 958e9-9a1 is discussed in Ohly [54], 93.
35 The number of letters per standard line is discussed by Graux [29], Birt [12], and Ohly [54]. Birt [12], 202 concluded: 'This standard line (Normalzeile) of circa 35 letters therefore ... dominated book production unchanged through at least five hundred years from Dionysius' copy of Thucydides until the time of Justinian.' Schanz examined the ninth-century Clarke codex of Plato's dialogues at Oxford and argued that the partial stichometric notations in its margins descended from earlier copies. His analysis found that scribes had employed average line lengths of 35.56 letters in the Cratylus and 34.32 letters in the Symposium. Although there is thus some uncertainty in the line length used in the present calculations, the round numbers which emerge here are additional evidence for a figure close to 35 for Plato's texts. (It was common for the actual lines employed in papyri to have lengths different from the 35-letter line which served as a conventional unit of measurement, as Birt [12], 210 ff . and Blum [13] discuss.) My work in progress will discuss the principles determining the absolute lengths of the dialogues.
one or two percent accuracy, impressively round numbers involving multiples of the number twelve:

- The Apology is 1200 lines, or 100 per twelfth.
- The Protagoras, Cratylus, Philebus, and the Symposium are each 2400 lines, or 200 per twelfth.
- The Gorgias is 3600 lines, or 300 per twelfth.
- The Republic is 12,000 lines, or 1000 per twelfth.
- the Laws is 14,400 lines, or 1200 lines per twelfth.

In sum, the lengths of speeches, the position of speeches within the dialogues, the location of significant turns in the arguments, and the absolute lengths of the dialogues all provide evidence for an underlying stichometric organisation and, in particular, for the importance of a twelve-part structure.

## 3 Parallels Between Dialogues

This evidence for a common twelve-part stichometric structure within individual dialogues suggests that they be read side-by-side, in order to compare their structures. Despite the different subjects of the dialogues, such comparisons reveal a surprising number of parallel passages, i.e., passages with similar content at the same relative locations in different dialogues. Many examples could be given. Here one, clear example is considered in a range of dialogues, both early and late. These passages suggest that Plato is employing the rhetorical figure of 'variation,' a technique common in symbolic and allegorical writing, in which the expression of a single idea is repeatedly altered to suit various contexts.

The Republic's discussion of philosopher-kings and the form of the ideal just man occurs at the centre of the dialogue. Comparisons between the dialogues shows that passages describing the divine wisdom and justice of the ideal philosopher often recur near the centre. These terms also, of course, occur elsewhere in the dialogues, and that raises the chance that the following parallels are a coincidence. The immediate argument here against this possibility is simply the specificity, similarity, and precise locations of the passages:

Republic (50.0-50.5p): Socrates seeks justice and the just man who 'participates' in it, invokes Zeus, and first mentions the philoso-pher-kings who will lead (hegemoneuō) the city. ${ }^{36}$

Phaedrus (49.5-50.3p): the followers of Zeus, the god of justice, seek a beloved with a 'philosophical' nature, who is a leader (hegemonikos), and 'participates' in the nature of god. The followers of Hera, on the other hand, seek a beloved with a 'kingly' nature. ${ }^{37}$

Symposium (49.4-50.0p): Agathon praises Eros for being 'the best and most beautiful leader' (hegemon) and for being a 'spectacle to the wise and admirable to the gods' (including Zeus), and Socrates, perhaps for Plato an ideal philosopher and embodiment of Eros, jocularly claims to be a prophet (generally, a kind of divine knowledge). ${ }^{38}$

Apart from the explicit repetition of forms of 'hegemon,' these three passages share a number of elements: Zeus and justice, the philosopher's relation to divinity, and the notion of ruling or leading.

The Cratylus is useful for investigating parallels between the dialogues. Its series of etymologies is not organised in detail by any over-arching argument or narrative; the locations of the various terms analysed there, which typically appear only once, is generally determined by the underlying network of parallels between the dialogues. Here, for example, our leitmotiv occurs at the centre and nowhere else:

Cratylus (47.7-51.3p): the etymologies of wisdom, knowledge, the good, justice, Zeus, and of nous which rules itself and orders all things. ${ }^{39}$

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Some dialogues show this ideal philosopher in action at their centre, and repeat the cluster 'philosophy, justice, and god':

Apology (49.1-50.7p): Socrates claims to be wiser because he knows nothing - except that injustice is wrong for man and god, he will not give up philosophy, and he will obey the god. ${ }^{4}$

Euthydemus (48.6-49.9p): one must philosophise, knowledge is more valuable than gold, and knowledge makes one immortal. ${ }^{41}$

Euthyphro (48-50p): the gods dispute about justice, Socrates seeks to become wiser by being taught what the gods believe is correct (i.e., just), and will sing the praises of wisdom. ${ }^{42}$

Gorgias (49.1-50.1p): Socrates asks about the nature of wisdom, behaves like an ideal philosopher by admitting his ignorance and seeking correction, and doubts whether justice is the stronger ruling over the weaker. ${ }^{43}$

Finally, the Timaeus interrupts a long passage on natural philosophy at the centre of the dialogue with a paragraph of Pythagorean theology. Since justice is sometimes for Plato a kind of harmony, ${ }^{44}$ this passage would itself constitute an example of just and divine rule:

Timaeus (49.4-49.5p): Necessity willingly or unwillingly obeys God, who harmonises everything in the universe according to precise proportions. ${ }^{45}$

[^4]Other examples could be given, but these passages are evidence that it is no coincidence that the Republic's philosopher-kings are mentioned at the centre of the dialogue: the dialogues typically allude to the philosopher's divine wisdom and just rule at that point.

Despite the humorous and sceptical tone of the Cratylus, Sedley has argued at length that Plato took the practice of philosophical etymology and punning seriously. ${ }^{46}$ The centre of that dialogue hems and haws evasively about the etymology of 'justice' (412e2 ff.), but Aristotle offers one etymology as if it were a well-known matter of fact: 'dikaios is so-named because it is [cutting] in half (dicha), as if someone would say dichaios [instead of dikaios], and call a judge a dichastēs [instead of dikastēs]. ${ }^{47}$ This may be one reason why references to justice are lodged consistently at the halfway point of the dialogues. Although passages with puns alluding to their mathematical location within the text, like the Statesman's reference to 'middle' at the midpoint, are common in the dialogues, they are not pursued here since the brevity of puns makes them hard to interpret rigourously. ${ }^{48}$

## 4 Ranges of Positive and Negative Concepts

Careful study of the parallels between the dialogues leads to another feature of their shared stichometric structure. Side-by-side comparisons of passages at the same relative locations shows that concepts with negative valuations within the dialogues, like disease, dishonesty, Hades, the body, difference, and negation, tend to cluster in definite ranges and at a definite locations, such as around and between the points ten and eleven twelfths of the way through the dialogues. Similarly, positive concepts, like the forms, virtue, the gods, goodness, justice, and the soul, tend to occur in distinct and equally definite ranges. These tendencies are never absolute, but the mixture of concepts in these ranges is clearly dominated either by more negative or by more positive concepts, as can

[^5]perhaps best be made clear by the examples below. The evidence for these negative and positive ranges within the dialogues is introduced here; the next section advances an interpretation of this pattern.

The range of negative concepts from the tenth to the eleventh twelfth contain some of the more vivid passages in Plato's dialogues:

Apology (10-11): Socrates is found guilty and then sentenced to death. ${ }^{49}$

Phaedo (10-11): wicked souls are condemned to Hades at ten-twelfths, our world is a mere muddy hollow, geography of the underworld, and, at eleven twelfths, abysmal Tartarus and the filthy River Styx. ${ }^{50}$

Phaedrus (10-11): rhetoric is like the groping of the blind at ten-twelfths, is merely persuasive, and needs not truth but probability; at eleven-twelfths, writing produces forgetfulness and the mere appearance of wisdom. ${ }^{51}$

Republic (10-11): at ten twelfths, the woes of the tyrant, murder, anarchy, and lawlessness are described, the critique and banishment of merely imitative poets follows. ${ }^{52}$

Symposium (10-11): the notorious Alcibiades begins his drunken speech just before ten-twelfths, Socrates is compared to a Satyr at ten-twelfths, Alcibiades' scandalous attempt to seduce Socrates, and, at eleven-twelfths, Alcibiades' pain, rejection, and shame. ${ }^{53}$

[^6]Timaeus (10-11): old age and death, diseases and corruption of the body, and, at eleven twelfths, madness and diseases of the soul. ${ }^{54}$
In contrast, positive concepts dominate around and between the points eight and nine twelfths of the way through the dialogues.

Apology (8-9): the courageous conclusion to Socrates' first defence, and, at nine-twelfths, affirmation of his belief in God. ${ }^{55}$

Phaedo (8-9): the theory of forms is introduced at length and used to prove again that the soul is immortal. ${ }^{56}$

Phaedrus (8-9): Socrates advocates a higher art of speaking (dialectics) which knows reality, employs the Method of Division to clearly define things, is well-organised like a living body, and finds a single idea among particulars. ${ }^{57}$

Republic (8-9): ascension to reality by the true philosopher, at eighttwelfths the study of the One turns the soul to the vision of reality, mathematics and the sciences lead the soul upward, dialectics, the nuptial number, and the beginning of the transformation from aristocracy, at nine-twelfths timocracy as an intermediate mixture of aristocracy and oligarchy. ${ }^{58}$

Symposium (8-9): intellectual reproduction stimulated by Beauty, the higher nature of love, and, at ninth twelfths, the conclusion of Diotima's Ladder with an ascent to the form of the One. ${ }^{59}$

[^7]Timaeus (8-9): the demiurge constructs all things, including the soul, according to the Good and harmonious proportion, the organ of divination, and the ensouled body. ${ }^{60}$

Although all these ranges contain a mixture of good and bad, the predominance of one or the other is clear.

## 5 Musical Interpretation of the Stichometric Structures

As the many passages about music in the Republic show, music theory and 'harmonics' were already a relatively advanced and much debated science by Plato's time. Two aspects are relevant here. The Pythagoreans famously associated the intervals between notes in musical scales with ratios between small whole numbers. An octave, for example, corresponded to a one-totwo ratio. It was convenient and common in some quarters to associate important notes in musical scales with the integers six, eight, nine, and twelve. Since twelve has many factors, musically important intervals within the octave could then be represented by simple ratios between integers. ${ }^{61}$ This six-to-twelve representation of the octave was ascribed to Pythagoras, was known before Plato to the Pythagorean Philolaus, was embedded in the multi-octave scale discussed in the Timaeus, was mentioned in the Platonic Epinomis, and was well-known to Aristotle and Aristoxenus. ${ }^{62}$ The twelve-part structure of the dialogues detected above together with the prominence of the number twelve in Greek music theory suggests that the stichometric

[^8]structure of the dialogues is a musical scale. ${ }^{63}$ Plato used this musical scale as an outline, pegging key concepts and turns in the argument to steps in the scale.

A second aspect of Greek musical theory helps confirm this interpretation. Greek theorists like Plato's correspondent Archytas constructed measures of relative musical harmony. That is, they ranked pairs of notes according to whether the pairs were more or less harmonious. The evidence for these theories is discussed by Huffman, West, and Barker. ${ }^{64}$ In short, pairs of notes which correspond to ratios whose lowest terms involve only small whole numbers were considered more harmonious, and the smaller the numbers the greater the harmony. Ratios involving larger integers were relatively dissonant or disharmonious.

The distribution of positively and negatively valued concepts examined above is correlated with and therefore explained by these measures of relative harmony. According to Greek theory, the third (1:4), fourth (1:3), sixth (1:2), eighth (2:3) and ninth (3:4) notes on the twelve-note scale will best harmonise with the twelfth. Passages near these relatively harmonious notes are dominated by positively valued concepts, while passages near dissonant notes (the fifth, seventh, tenth, and eleventh) are dominated by negative ones. Passages near other notes (the first and second) tend to be more neutral. ${ }^{65}$

Examination of the dialogues suggests that the arrangement and treatment of their themes were influenced by the harmonic structure of the underlying musical scale. In the Symposium, for example, even a brief survey suggests a correspondence with harmonic theory:

63 I discuss the similar, regular scales of the 'Harmonists' (Barker [7]) in my work in progress.
64 For Archytas, see A17 and the following survey of theories of relative harmony in Huffmann [40], 428 ff . West [73], Barker [5], Barker [6], 71-75, and Barker [7]. In English, the theory of relative harmony is sometimes called a theory of 'consonance' or 'concord.'

65 In some theories, a function of the sum of the numbers in the ratios (after reduction to lowest terms) was used to measure consonance. The sum for the first note in the scale (one-to-twelve ratio) is thirteen, but the the sum for the fifth note (five-totwelve ratio) is seventeen. Thus the first note is less dissonant than the fifth (when each is sounded together with the twelfth).

0-2+, Neutral: introduction and middling speeches of Phaedrus and Pausanias. ${ }^{66}$

3+, Harmonic: Eryximachus' explicit discussion of erotic and musical harmony.
4+, Harmonic: Aristophanes: Apollo, the god of music, heals and fits together the severed creatures ('harmonise' is literally 'fit together'); Hephaestus offers to fuse two lovers.

5+, Disharmonic: Agathon's arty rhetoric, faulty logic, and beauty without truth.

6+, Harmonic: Praise of Eros, Socrates (ideal philosopher) wants truth not the appearance of beauty.

7+, Disharmonic: the refutations of Agathon and the young Socrates (elenchus is a kind of disagreement or discord), Diotima's story of debauchery among the gods, Eros is not a god.
8-9+, Harmonic: Diotima on Beauty and ascent to the One (as above).
10-11+, Disharmonic: Alcibiades and his shame (as above).
A separate essay will review several further kinds of evidence for this musical interpretation of the stichometric structure. For example, the musical and music-related passages which recur through the Republic are lodged at musically significant locations on a twelve-note scale. At a third of the way through the Republic, Socrates condemns innovations in music, mentions the musical expert Damon, and says the guardians must build their guardhouse in music; at two-thirds, there is a long discussion of music, harmony, and mathematics. ${ }^{67}$ That essay will also provide evidence for a fine-grained musical structure between the twelve notes. ${ }^{68}$ In sum, both the twelve-part structure and the distribution of positive and negative concepts found in the dialogues are explained by interpreting the underlying stichometric structure as a musical scale.

[^9]About fifteen dialogues which have been ascribed to Plato in antiquity are of doubtful authorship. ${ }^{69}$ No claim about the authenticity or spuriousness of any of these dialogues is made here. The presence of stichometric structure may only evidence an imitator's discerning acquaintance with the genuine dialogues. The immediate importance of these dialogues for the present study is that they serve as a kind of check or control on the methods employed here. They show that, among texts written with a subject and style similar to Plato's, the methods can distinguish between those which have and do not have stichometric structure. That is, the tests for stichometric structure are stringent enough to be falsified by some data.

Four of the works which were investigated are generally agreed to be spurious: On Justice, the Minos, On Virtue, and the Eryxias. These were found to have no detectable stichometric structure. On the other hand, the authenticity of the First Alcibiades, the Cleitophon, and the Epinomis has been, however cautiously, defended by some modern scholars. ${ }^{70}$ There is strong evidence in each of these last three dialogues for the presence of the twelve-part musical structure. ${ }^{71}$

## 7 Ancient Interpretations of Plato as a Pythagorean

Plato's philosophy has been associated with or even identified with Pythagoreanism by Aristotle, ${ }^{72}$ by members of the early Academy, by

[^10]'Neo-Pythagoreans' around the first century BCE, and by ancient and Renaissance Neo-Platonists, but modern scholars have often strongly denied this, in part because Pythagoras and the Pythagoreans are hardly mentioned in the dialogues. ${ }^{73}$ Historians of early Pythagoreanism like Burkert and Huffmann have regarded the views of the early Academy as some sort of mythologising propaganda which contaminated the later tradition, and emphasised the elements in Plato's philosophy which ill accord with earlier Pythagorean tradition. ${ }^{74}$ Even the Timaeus is regarded as a Platonic rather than an orthodox Pythagorean dialogue. ${ }^{75}$ There have been dissenters, however, who emphasised the scattered evidence in the dialogues for some Pythagorean influence on Plato. ${ }^{76}$

In the centuries after Plato, there was an apparent resurgence of interest in the connection between Plato and Pythagoras among the so-called 'NeoPythagoreans. ${ }^{77}$ Perhaps in the first century BCE, Numenius, for example, entitled one of his lost works On the Secrets or Reserved Doctrines in Plato. ${ }^{78}$ Although sceptical, Tarrant summarises the Neo-Pythagoreans' approach to interpreting Plato as follows (italics original):

All this suggests [their] belief that Pythagorean doctrines are hidden in Plato, who for one reason or another is reluctant to reveal them, and that true Pythagoreanism can be teased out of Platonic texts by in-depth interpretation. Like Thrasyllus, [other Neo-Pythagoreans such as] Moderatus, Numenius, and Numenius' friend Cronius were all supposed to have written on the first principles of Plato and Pythagoras in such a way that they had somehow anticipated Plotinus... So it would seem safe to say that something quite esoteric is regularly being detected beneath Plato's text, concealing details of the allegedly Pythagorean

[^11]metaphysic that Pythagoreans, almost as a matter of faith, supposed to exist there. ${ }^{79}$

Thrasyllus, the first-century editor of Plato's dialogues and a philosopher in the court of Tiberius, is given as the source for some long passages in the work by Theon of Smyrna, On the Mathematics Needed for Understanding Plato (perhaps early second century CE). Although important for the history of mathematics, Theon's book has puzzled Plato scholars because much of it seems irrelevant to interpreting the dialogues. Theon seems to have been a mere compiler of earlier handbooks and gives little guidance on why this or that topic is included. However, Theon does give an extended exposition of the same musical scale with twelve-regularly spaced notes which is found here in the dialogues. ${ }^{80}$ This never appears openly in Plato's dialogues. Theon says explicitly that he is here following Thrasyllus. Theon also discusses the theory of relative harmony, attributing the theory to Pythagoras, Archytas, and Eudoxus. ${ }^{81}$ Thus some Neo-Pythagoreans (i) asserted there were reserved or concealed metaphysical doctrines in Plato's dialogues, (ii) associated these with Pythagoras, (iii) discussed - in the context of interpreting Plato - the scale of twelve, regularly spaced notes, and (iv) reviewed the theory of relative harmony. All this suggests the need to re-evaluate the scant remains of these early Neo-Pythagoreans. There is at least a prima facie case that they recognised the musical structure of the dialogues. ${ }^{82}$

## $8 \quad$ Verifying the Stichometric Measurements

This section mounts an elaborate argument with important dividends. It aims in the first place to show that the computer's measurements of relative location within the dialogues are accurate, and thereby to validate both the conceptual framework and the particular counting algorithms adopted. In the second place, the argument deepens the connection between the mathematical structure of the dialogues and Plato's Pythagoreanism.

[^12]There is a literature, carefully reviewed by Balashov and briefly by Herz-Fischler, on the question of whether the Divided Line in the Republic was meant to be divided at the Golden Mean. ${ }^{83}$ Although the Golden Mean is not explicitly mentioned in the simile nor anywhere else in Plato, ${ }^{84}$ the question is whether the passage in the Republic is an allusion to a theme which was at times prominent in later Pythagorean and numerological lore. The Golden Mean occurs in the 'pentagram', allegedly an esoteric symbol of the Pythagoreans constructed by extending the sides of a pentagon to form a five-pointed star. ${ }^{85}$ Euclid's Elements contains a number of theorems involving the pentagon and the Golden Mean and, as discussed in HerzFischler's monograph $A$ Mathematical History of the Golden Number, these theorems are generally held to have been developed before or during Plato's lifetime. ${ }^{86}$

Stichometric measurements bring a new datum to this debate over the Divided Line. The numerical value of the Golden Mean is, to three places, 0.618, and thus a unit length divided at the Golden Mean will be divided at 61.8 percent. ${ }^{87}$ Surprisingly, the Republic's discussion of the Divided Line begins at 61.7 percent of the way through the text. ${ }^{88}$ By itself, this could be a coincidence, but the other dialogues typically contain allusions to the Golden Mean near 61.8p.

Euclid's definition of the Golden Mean is phrased in terms of its technical name:

83 As reviewed by Balashov [4] and Herz-Fischler [36], 84-5, Brumbaugh, Gibson, Des Jardins, and Dreher find that the Republic does allude to the Golden Mean; Cherniss opposes; Balashov holds that it is possible but unproven.

84 Burkert [19], 453n28 sees tenuous evidence in a vague passage in the Platonic Hippias Major but see Herz-Fischler [36], 85.

85 Burkert [19], 176, 452, Herz-Fishler [36], III.
86 Herz-Fischler [36]; see also Heath's stronger view about Plato's role in [25], v II, 99.

87 Notation and terminology vary slightly. Sometimes the Golden Mean is said to be the reciprocal of 0.618 , that is, 1.618 . Which number is dubbed the Golden Mean is arbitrary. The value of the Golden Mean is found by solving a quadratic equation. If 1: $x:: x:(1-x)$ then $x^{2}+x-1=0$, whose positive root is $(\sqrt{5}-1) / 2$.
88 The good is in proportion to itself at 508 b 13 ( 61.7 p ) The Golden Mean (61.8p) is nine lines later at 508c9. Glaucon there asks for an explanation, and this leads to the long passage about the divided line, which is first mentioned at 509d6 (62.2p).

A straight line is said to have been cut in 'extreme and mean ratio' (akron kai meson logon) when as the whole line is to the greater section, so is the greater to the less. ${ }^{89}$

Thus the Golden mean is defined by relations between a line and its own parts: in various ways they are greater than, less than, and equal to each other. A passage in the Parmenides at the location of the Golden Mean recalls Euclid's language:

Parmenides (61.7-61.8p): The One is equal and greater and less than itself ... And if greater and less and equal, it would be of equal measures and more and less than itself ... and in number less and more ... ${ }^{9}$

The similarity of the content of this passage to Euclid's definition, together with its location, suggests that this is also an allusion to the Golden Mean. References to the mean or middle occur in other dialogues near 61.8p. ${ }^{91}$

This evidence corroborates the literature which associated the Republic's Divided Line with the Golden Mean, and connects the stichometric structure of the dialogues in yet another way to Pythagorean themes. Moreover, since the value of the Golden Mean is known precisely, this evidence provides an important check on the accuracy of the computer's calculations. The discrepancy between the relative location of a passage in the modern text as calculated by the computer and its location within Plato's autograph ${ }^{92}$ can be measured. This discrepancy is generally less than half a percent. This means it is possible to reach

[^13]behind the scribal tradition, and evaluate the integrity of texts descended from the classical period.

## 9 Motivation

It is difficult today, in our culture of open publication, to appreciate the many motivations in ancient cultures for 'reserving' doctrines. Burkert's monograph on sects, cults, and clubs in antiquity made the point that secrecy was then 'normal. ${ }^{93}$ Regardless of their content, secrets played an important sociological role by fostering a sense of inclusiveness and elite privilege that helped hold groups together. Plato's dialogues often refer to reserving doctrines, and his characters seem to treat this as unremarkable. ${ }^{94}$

In antiquity, the Pythagoreans were reputed to be a persecuted sect which resorted to various means to keep their identities and doctrines secret. ${ }^{95}$ Historians of ancient literary criticism sometimes attribute to them a role in the invention of 'symbolism' and allegory. ${ }^{96}$ The Derveni Papyrus has recently provided a glimpse of an insider's approach to the ancient amalgam of philosophy, reserved doctrines, and allegoresis. ${ }^{97}$

The question of why an author of Plato's magnitude resorts to a style of writing with secondary or symbolic levels can have no simple answer. Since intentions are, strictly speaking, inaccessible, we can at best enumerate candidate motivations. The musico-mathematical structures in the dialogues may serve several purposes.

First, they make the literary text a concrete instance of the metaphysics. According to the Pythagorean 'harmony of the spheres,' whose early attestations include the Republic and De Caelo, ${ }^{98}$ the whole cosmos has a musico-mathematical structure which, later sources tell us, was

[^14]perceptible only to the superior reason of philosophers like Pythagoras himself. ${ }^{99}$ There is an analogy between, on the one hand, an imperceptible music reflecting proportions in the constitution of the cosmos and, on the other, a musical scale which organises a dialogue and yet is submerged beneath its surface - an unheard melody finally accessible to reason and measurement. Just as the individual in the Republic mirrors the order of the polis, and the individual in the Timaeus mirrors the order of the cosmos, each dialogue is another microcosm mirroring macrocosmic principles of order: 'the dialogue is a cosmos. ${ }^{100}$ Although the difference in context will make us cautious, the literature on Renaissance Platonism and Pythagoreanism has examined the relevant literary genre at great length. Neuse's succinct summary is suggestive here:
... a cardinal feature of the Pythagorean aesthetic [was] the hidden or implicit harmony which the artist was supposed to impose upon his work. Thus the numerical-symbolic structure [in a poem by the Platonist Spenser] serves, in Pythagorean fashion, to express its secret affinity with the mathematical order of the universe ... For combined with its demand for an abstract structure or pattern, Humanist Pythagorism had a conception of artistic production [in which the ...] artist's imagination must enter into, become identified with Nature's course, and produce images as by her agency ... ${ }^{101}$

Thus a philosophical or literary work would not reveal nature's order but participate in it. The aim was not representation and imitation, but reproduction of form. Just as nature's astronomical or physical laws lie beneath the observable phenomena, a literary work would have a corresponding and similarly latent structure. Without pursuing questions of influence, there is at least an analogy here to the structure of the dialogues which may shed light on Plato's motivations.

Second, the structures unify the texts. Although unity is in various ways a major theme of the dialogues and they themselves say that texts

[^15]and speeches should be unified, ${ }^{102}$ an extensive literature criticises the dialogues' sometimes meandering conversations for lacking literary unity. The Republic in particular, which roams from political science and poetry criticism to arguments about immortality, and seems to shift from Plato's early style in the first book to the metaphysical concerns of his middle period, has been thought an arbitrary composite of separate tracts written at different times. ${ }^{103}$ The literature on this issue will need to be re-examined for, beneath the surface narratives, the dialogues have a strong formal unity.

Third, the underlying structures in the dialogues serve to convey, with the allegorist's peculiar balance between communication and concealment, further philosophical doctrine. Plato's dialogues have sometimes been interpreted as deliberately aporetic exercises aimed at galvanising inquiry or even as empty exercises in scepticism. However, the same philosophical approach the dialogues urge toward the world - the search for underlying forms, the attempt to resolve puzzles about appearances, and the employment of reason and measurement - will, when applied to the dialogues themselves, lead to symbolic structures which carry additional doctrinal content. As subsequent work will endeavour to show, Plato's positive philosophical programme is in the underlying forms.

## Conclusion

There are now several kinds of evidence that Plato's dialogues have a stichometric structure: the lengths of speeches, the alignment of some speeches and key concepts with the twelfths, the parallel passages, and the parallel negative and positive ranges. The musical interpretation of these features is natural and coherent: a twelve-note scale with harmonic and dissonant ranges underlies the surface narrative of the dialogues. The evidence and its interpretation fit the historical context: stichometry was a common practice and applied to Plato's dialogues, allegory was widely debated, the new mathematics was promoted by

102 Halliwell [30], 333, etc. lists various pronouncements in the dialogues about literary unity.

103 See, for example, Rutherford [56] on 'separtists' vs. 'unitarians', Lear [47], Lesser [48], Annas [2], and Krohn [44] on the Republic.

Plato and the Academy, the numeric representation of musical scales and harmonic theory were well-known, Plato's correspondents, colleagues, and followers associated him with Pythagoreanism, and the Neo-Pythagoreans made the scale of twelve, regularly spaced notes part of their studies of the metaphysics allegedly hidden in the dialogues.

Methodologically, the inductive arguments for these claims have been secured by presenting extensive, mutually reinforcing lines of evidence. There were two kinds of controls or checks. The absence of stichometric structure in Platonic dialogues which are by general consent spurious shows that the techniques employed here rigourously distinguish structured from similar but unstructured texts. The specific measurements of relative location were confirmed to within a half a percent by the passages alluding to the Golden Mean, which generally validates the algorithms and programs employed.

Claims like these raise more questions than can be addressed within the confines of an essay. Though the evidence reported here will need to be verified and debated, it does clarify, in a surprising way, Aristotle's once puzzling view that Plato was a Pythagorean.

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[^1]:    2 Sayre [58], 149-70, 197 ff. and Sayre [57], 128-9, cf. 114-5, 116. See also Miller [51], 266n29.

    3 See Boys-Stones [15], Brisson [17], Ford [26], Sedley [62], and Struck [66]. Lamberton [45] gives examples of cases in late antiquity where critical discussions of symbolism and allegory spawned literary works which employed the techniques discussed.

    4 There are many references and some longer passages on 'symbols' and 'allegory' (hyponoia, ainigma, etymology, etc.) in the dialogues (see Struck [66] and Ford [26] for reviews). Ford [26], 86-7 offers a carefully modulated judgement of Plato's much-debated views on allegory: 'Allegoresis is viewed by Plato as an uncertain method and dangerous where children are concerned, but he never denies outright the possibility of its being used in a more philosophical way ... Plato's disquiet is focused on popularisers of subtle interpretation, not on the method itself ...'
    5 Birt [12], 440, etc., Schanz [60], Harris [32], Dodds [23], 46, Berti [9]
    6 Reviewed in Brandwood [16].

[^2]:    7 Fowler [27] surveys the genre. See also Heninger's works [34] and [35].
    8 Hieatt [37]. There is a consensus that Spenser's Epithalamion has a clear, mathematical structure; debates over whether the Fairie Queene has a similar structure have been inconclusive. See the review articles on number symbolism, etc. in the Spenser Encyclopaedia, edited by Hamilton [31].
    9 Fowler [27], Hamilton [31], etc.
    10 Mann's Doktor Faustus is a major example of this tradition from the last century. Literature on its Pythagorean, symbolically marked, musical structure is reviewed in Bergsten [8].

[^3]:    36 Resp: calculations place the center of the dialogue at $472 \mathrm{~b} 8=50.0 \mathrm{p}$, which is the passage about the ideal just man; Zeus at 472e8; philosopher-kings at $473 \mathrm{c} 11=$ 50.5p; 'hegemon' at 474c1-3.

    37 Phdr: 252e1-3b2; the calculated center of the Phaedrus is within this passage at the phrase humans 'participate in god,' 253a4-5.
    38 Symp: Zeus at 197b3, 'hegemon' at 197e2-3, cf. d3, 'spectacle' at 197d5-6, prophet at 198a4-a10; the centre of the Symposium is in this passage at a8.

    39 Crat: centre at 412e3, the passage from 411 d 4 to 413 d 2 considers the listed terms. Nous is autokratōr at 413c5-7.

[^4]:    40 Apol: wiser at 29b4, injustice at b6, philosophy at c8, the god at d3-4, the centre at c4.

    41 Euthd: philosophy at 288d6 ff., knowledge at d8 ff., immortality at 289b1, the centre at 289b2.

    42 Euthph: justice at 8e7, wiser at 9a2, what the gods believe at 9a8-b1, and praising wisdom at 9b3; the centre at 9a8.

    43 Grg: wisdom at 487c5 and e4; justice at 488b2-6; 50.0p is between these passages at 488a7, where Socrates behaves like a philosopher.

    44 For example, Resp 443 c 4 ff .
    45 Ti: 56c3-7; centre at 56e3.

[^5]:    46 Sedley [62]
    47 Eth Nic, 1132a30 ff
    48 Many examples could be given: Sayre's cutting in half at the halfway point of the Statesman ( $284 \mathrm{e} 2=6 / 12$ exactly), 'try to cut this into three' at the two-thirds point of the Philebus ( $48 \mathrm{~d} 4=8 / 12$ exactly), three kinds at the one-third point of the Symposium ( $189 \mathrm{~d} 7,4 / 12=\mathrm{d} 5$ ), etc.

[^6]:    $49 \mathrm{Ap}: 10 / 12=38 \mathrm{a} 1,11 / 12=40 \mathrm{a} 5$.
    50 Phd: $10 / 12=108 b 2,11 / 12=113 \mathrm{c} 1$; wicked souls in Hades at 108a7 ff., Earth a hollow 109a9 ff., underworld 111c5 ff., Tartarus at 113 b 4 etc., River Styx at 113c1-2.
    51 Phdr: $10 / 12=270 \mathrm{~d} 7,11 / 12=275 \mathrm{a} 7$; blind rhetoric at 270 e 2 , persuasive rhetoric 271c10 ff., no truth 272d2 ff., writing 274c5 ff.
    52 Resp: $10 / 12=573 \mathrm{e} 4 ; 11 / 12=599 \mathrm{c} 7$; tyrant 566 a 10 ff ., poets 595 a 1 ff . This range does include the three proofs that the just life is the best or happiest (580b1 ff.), which are evaluated by judges as in a theatrical contest.
    53 Symp: $10 / 12=215 c 1,11 / 12=219 \mathrm{~d} 5$; Alcibiades begins at 215 a 4 , Satyr simile at 215 b 4 ff ., seduction 217a2 ff., rejection 219 c 6 ff .

[^7]:    54 Ti: $10 / 12=80 \mathrm{~d} 7,11 / 12=86 \mathrm{~d} 3$; old age and death, 81 c 6 ff ., diseases of body, 81e686b1, of soul, 81e6 ff.

    55 Ap: 8/12 $=33 \mathrm{~d} 3 ; 9 / 12=35 \mathrm{~d} 3$; theism at d5 ff
    56 Phd: $8 / 12=97 e 4 ; 9 / 12=103 \mathrm{a} 6$; the theory of forms exposited from 100b1; the proof of immortality finishes between the nine-and ten-twelfths points at 106 e 5 ff .
    57 Phdr: $8 / 12=261 \mathrm{c} 6 ; 9 / 12=266 \mathrm{~b} 1$; the four aspects of dialectics are at 262b5-9, 263b6 ff., 264b2 ff., and 265d3 ff.

    58 Resp: $8 / 12=524 \mathrm{~d} 3 ; 9 / 12=549 \mathrm{e} 6$; ascension at 521 d , study of One at 525 a 1 , sciences from 526c7, dialectics from 532d7, nuptial number from 546b4, transformation begins at 547c5 where the discussion of timocracy also begins, timocracy as a mixture at 548c3-5 and 550b5, timocracy concludes at 550c1.

    59 Symp: $8 / 12=206 \mathrm{~d} 8,9 / 12=211 \mathrm{~b} 2$; contact with Beauty at 206d2 ff., form of the One at 211 b 1 .

[^8]:    60 Ti: $8 / 12=68 \mathrm{e} 2 ; 9 / 12=74 \mathrm{e} 5$; demiurge: 68 e 1 ff., divination: 71 d 5 ff., body constructed at 74d6 ff

    61 A fourth is eight-to-twelve; a fifth is nine-to-twelve; a whole tone is nine-to-eight.
    62 For Pythagoras and Philolaus, see Huffman [38], 54, 167 ff.; Ti 34b10 ff.; Epin 991a6b4; Aristotle. Metaph 1093a28-b5; Gibson [28], 684n16 concludes thatthis last passage shows Aristotle knew the 6-8-9-12 schema.

    This essay does not claim there is evidence in the dialogues for the existence of the monochord or its use. The long-standing controversy over the early history of this instrument is avoided here (see Burkert [19], 375n, West [73], Barker [5], and Barker [7] for contrary views and a brief survey of the debate). The embedded scale may be merely a mathematical or abstract construct or it may have been played on instruments with two or more strings. However natural, nothing here depends upon a monochord interpretation of the scale.

[^9]:    66 Symp: $1 / 12=176 \mathrm{c} 5,2 / 12=181 \mathrm{e} 3,3 / 12=185 \mathrm{~b} 6,4 / 12=189 \mathrm{~d} 5,5 / 12=193 \mathrm{~d} 8,6 / 12$ $=198 \mathrm{a} 8,7 / 12=202 \mathrm{c} 7,8 / 12=206 \mathrm{e} 1,9 / 12=211 \mathrm{~b} 4,10 / 12=215 \mathrm{c} 2,11 / 12=219 \mathrm{~d} 6$, $12 / 12=223 \mathrm{~d} 12$.

    67 Resp: one-third at 424b5 ff., two-thirds at 522a4-5c7.
    68 The so-called 'quartertones' mentioned in the Republic at 531a4.

[^10]:    69 There are older surveys of the Pseudo-Platonica in Heidel [33] and Schaarschmidt [59]. For recent debates see Döring, Erler, and Schorn [24].

    70 See Slings [64] on the Cleitophon and Denyer [21] on the First Alcibiades. There is a stronger inclination to doubt the authenticity of the Epinomis. Taran thought that neither ancient testimony nor the style of the Epinomis proved it inauthentic, but argued that doctrinal differences between it and the canonical dialogues suggested its spuriousness [67]. A.E. Taylor [69], $14 \mathrm{ff} ., 497 \mathrm{ff}$. and others have defended its authenticity.

    71 The evidence will be detailed in my work in progress.
    72 It may be inferred from Ross's edition of his fragments [3] and other passages that Aristotle, however knowledgeable, was a critic of and so an outsider to Pythagoreanism; there is no apparent evidence that Aristotle knew of the musical structure of the dialogues.

[^11]:    73 The only explicit references are in the Republic at 530d8 and 600b1-2. Here, 'early' and 'late' Pythagoreanism means before and after Plato.

    74 Burkert [19], Huffman [38]. See the strong statement in Jaeger [42], 97 ff .
    75 Huffman [39], 84 and Herz-Fischler [36] 83-84.
    76 Some passages seem clearly to point toward ideas associated with the Pythagoreans: the celestial harmony in the Republic's concluding myth, the prominence of mathematics in the Timaeus and elsewhere, doctrines like metempsychosis, the supposed correspondence with Archytas, etc.

    77 See Burkert [19], Dillon [22], Thesleff [70] and [71]
    78 Peri tōn para Platōni aporrētōn, Numenius (E. des Places) [53]

[^12]:    79 Tarrant [68], 84-85].
    80 Hiller 47.18-49.5 = Tarrant T13, Hiller 85.8-93.11 = Tarrant T14a.
    81 Hiller 56.9 ff., Archytas and Eudoxus at 61.12.
    82 The case is examined at greater length in my work in progress.

[^13]:    89 Bk VI, D 3, cf. Heath [25], v II, 188.
    $90151 \mathrm{~b} 5-\mathrm{c} 7,61.8 \mathrm{p}=\mathrm{b} 7$; the discussion of great and small lasts from 149 d 8 to 151 e 2 .
    91 For three examples: (i) Symp: Eros is en mesō at 203e5 $=61.0$ p, but philosophers and Eros are hoi metaksu' at 204b1 and b5, 61.8p $=204 \mathrm{~b} 6$. (ii) Phlb: 61.8p at 45e7; mēden agan at 45 e 1 . The Delphic Oracle was a temple to Apollo, a deity important to the Pythagoreans. (iii) Phdr: mesēmbria at 259a2, a6 and d8; 61.8p $=258 \mathrm{e} 7$.

    A wider survey of the dialogues in my work in progress suggests Plato associated the Golden mean with a doctrine similar to Aristotle's 'virtue is a mean,' which Aristotle himself associated with Pythagoreanism (Cf. Eth Nic 1106b28-a9, 1107b4-7. Aristotle refers to the Philebus at 1172 b 29 ff.).

    92 That is, before any corruptions introduced during its recopying and transmission disturbed the proportions of the original composition.

[^14]:    93 Burkert [20]
    Phd 62b3, Cra 413a3, etc
    Burkert [19], $114 \mathrm{ff} ., 166 \mathrm{ff}$. Iamblichus, $V P$, ch. 35. Thesleff [71] and [70] collects Hellenistic traditions about Pythagoras and his supposed followers.
    Struck [66], 96 ff.
    Betegh [10]
    Resp, Book X; Cael, 290b; Burkert [19], 350 ff. Huffman, following Burkert, argues that the doctrine probably formed part of Philolaus' system [38], 279 ff.

[^15]:    99 For example, Iamblichus, $V P, 66 \mathrm{ff}$.
    100 Anonymous Prolegomena to Platonic Philosophy (Westerink)
    101 Neuse [52], 166

