Physics Graduate School Qualifying Examination

Fall 2007 Part I

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. **<u>Start each problem on a new pack of correspondingly numbered paper and use only one side of each sheet.</u> Place your 3-digit identification number in the upper right hand corner of each and every answer sheet. All sheets, which you receive, should be handed in, even if blank.** Your 3-digit ID number is located on your envelope. All problems carry the same weight.

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- Use correct vector notation when appropriate.
- Your work must be legible and clear.

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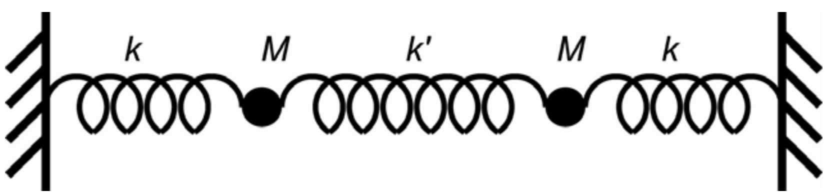
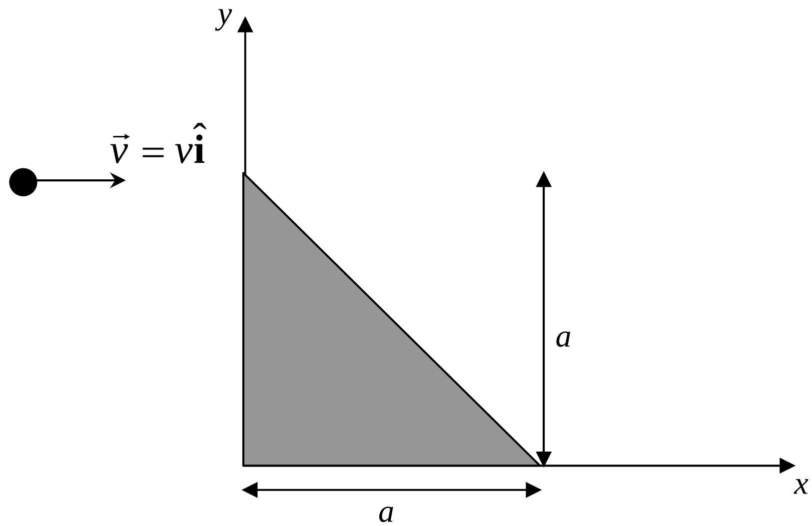
1) Consider two identical with masses M on springs of force constant k that are attached to fixed walls. The two masses are connected by a spring of force constant k'.



Calculate the fundamental frequencies of the coupled system and interpret.

2) A thin triangular lamina of mass M with dimensions as shown below, is hit by a small ball of putty of mass $m << M$ which moves with velocity $\vec{v} = v\hat{\mathbf{i}}$



Before the collision, the position of the center-of-mass of the lamina is located at

$$x_{cm} = a/3$$
$$y_{cm} = a/3$$

and the moment of inertia about its center of mass (rotation axis parallel to z-axis) is

$$I_{cm} = Ma^2/9$$

(a) If the putty hits the corner of the lamina at the point $y = a$ and sticks there, calculate the velocity of the center-of-mass of the lamina after the collisions.

(b) Calculate the angular velocity of the lamina about its center of mass after the collision.

Assume that the mass of the putty is small enough that its contribution to moment of inertia of the lamina + putty system is negligible.

3) A proton moves in a circle of radius r in a constant magnetic field that is perpendicular to the plane of the circle. The magnitude of the magnetic field is B.

a) Calculate the angular frequency at which the proton moves around the circular path.

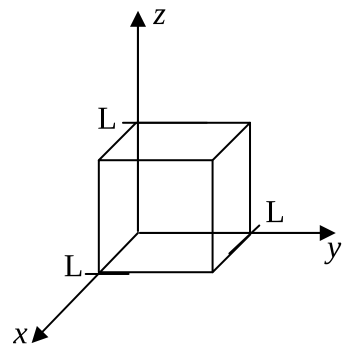
b) What is the kinetic energy of the proton?

Hints: $F_c = mv^2 / r$; $\vec{F}_B = q(\vec{v} \times \vec{B})$

4. Suppose we have a time-independent current density given by

$$\vec{J}(\vec{r}) = az\hat{z}$$

where a is a constant. Let Q be the total charge contained in the cube bounded by $0<x<L$, $0<y<L$, $0<z<L$.



Compute $\dfrac{dQ}{dt}$.

5) At time t=0, a particle is in a state which is a linear superposition of two energy eigenstates, i.e. the normalized wavefunction of the particle is given by,

$$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_1(x) + \psi_2(x)\right)$$

where $\psi_i(x)$ are the normalized wavefunctions, i= 1 or 2, of the energy eigenstates. These energy eigenstates have energy eigenvalues of E_i , i= 1 or 2.

Suppose a measurable physical quantity of the particle is described by the time independent quantum operator $\hat{A}$. Assume that the expectation value of $\hat{A}$ is given by *A(t)*.

a) **What is the wavefunction of the particle at time *t*=*T*?**

b) **In this superposition state, the expectation of $\hat{A}$, *A(t)*, is expected to be a periodic function of *t*. What is the period?**

c) **Derive a condition so that *A(t)* is time independent.**

6) The spin functions for a free electron in a basis where the spin operator $\hat{s}_z$ is diagonal can be written as

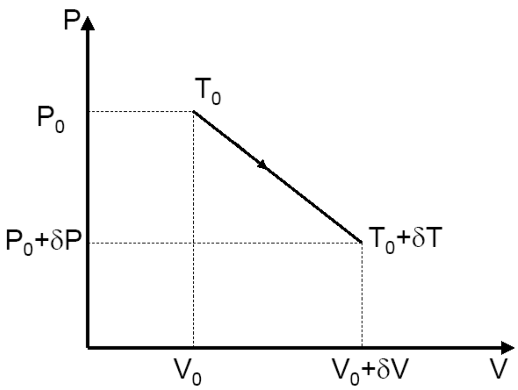
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

with the eigenvalues of $\hat{s}_z$ (taking $\hbar$ =1) of these states being +1/2 and -1/2 , respectively.

Using this basis find the normalized eigenfunction of $\hat{s}_y$ with eigenvalue -1/2. Also, find the probability that s_z = +1 in that state.

$$\hat{s}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \;;\; \hat{s}_y = \frac{i}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \;;\; \hat{s}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

7. An ideal gas is in equilibrium at a given initial temperature $T_{0,}$ initial pressure $P_{0,}$ and volume V_0. The volume of the gas is **increased** by the infinitesimal amount $\delta V=V-V_0$ such that its pressure changes by $\delta P = b\times\delta V$. Note that b is a constant which can be either positive, negative, or zero (in the diagram below it is negative).



For what values of *b* is δT positive?

8. Classically, the fusion of 2 protons (ionized H) in a hot gas is hindered by the electrostatic repulsion between the 2 charged particles.

(a) Calculate the gas temperature required for 2 protons to overcome the electrostatic repulsion between them and fuse at a separation distance of *r* .

From a quantum mechanical point of view the fusion of 2 protons can occur when the distance between them is comparable to their DeBroglie wavelength (i.e. they can tunnel through the electrostatic barrier at that distance).

(b) Find an expression for the DeBroglie wavelength (λ) for which the kinetic energy of a proton is equal to the electrostatic potential energy between two protons when they are separated by this same λ. Then use this separation distance λ to find the temperature of a gas that allows fusion to occur.

Physics Graduate School Qualifying Examination

Fall 2007 Part II

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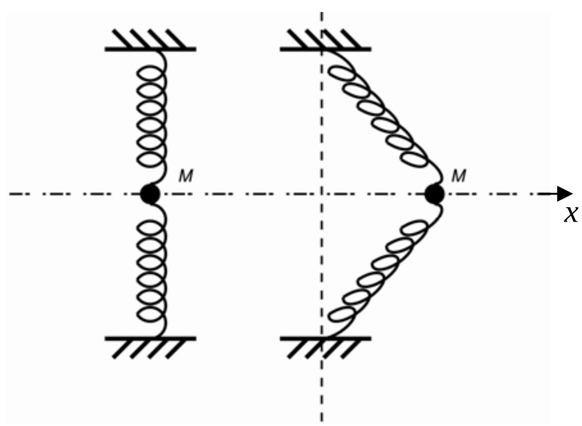
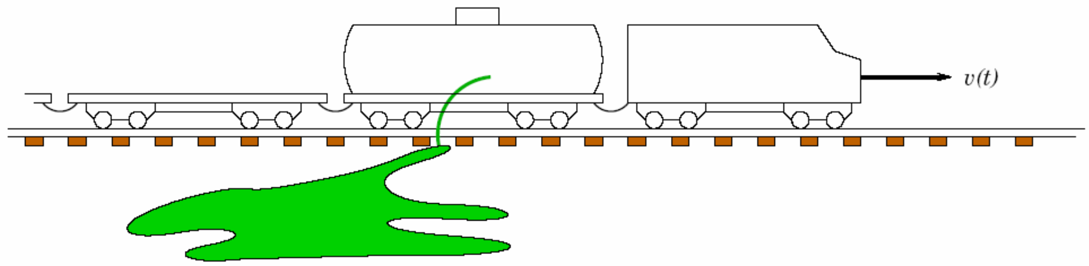
1. Consider a mass M suspended between two identical springs as shown below (left).



a) Calculate the force on the mass exactly for displacements along the mid-plane (above right), if the springs are initially free from tension. (neglect gravity)

b) What is the form of the force for small displacements? In this limit, is the system linear?

2. An engine pulls a train in which a train car carrying toxic goo has sprung a leak as shown. If the engine applies a constant force *F* to the train which has an initial mass, *M*, calculate the velocity of the train as a function of time, assuming that the toxic goo leaks out at a constant rate.



3. In a perfect conductor, the conductivity is infinite, so $\vec{E} = 0$ $(\vec{j} = \sigma\vec{E})$, and any net charge resides on the surface (just as it does for an imperfect conductor in electrostatics).

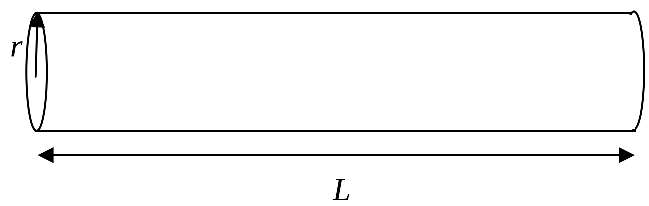
(a) Show that the magnetic field is constant $\left(\frac{\partial\vec{B}}{\partial t} = 0\right)$ **inside a perfect conductor.**

(b) Show that the magnetic flux through a perfectly conducting loop is constant.

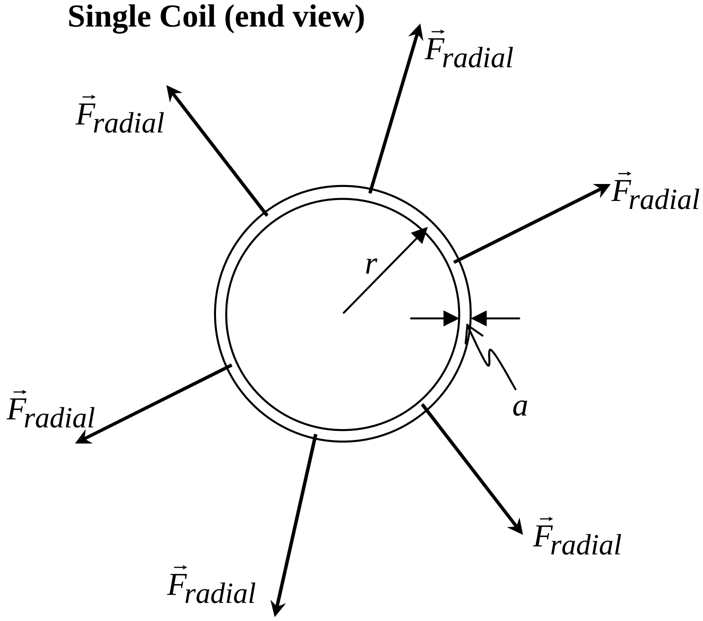
A superconductor is a perfect conductor with the additional property that the (constant) magnetic field inside is zero. (This flux exclusion is known as the Meissner effect.)

(c) Show that the current in a superconductor is confined to the surface.

4. The length of a cylindrical solenoid (L) is much larger than its radius r. The coil is made of a wire of square cross-section with a size $a<r$, the coil is tightly wound. When current is passed through the coil the solenoid feels a radially outward pressure due to a magnetic force acting radially outward on the wire. **Calculate this pressure.**



Single Coil (end view)



5. Two quantum mechanical particles have the same mass, m, and are in a one dimensional space. An attractive force between them grows linearly as the separation distance between them, therefore, the total system is described by the following Hamiltonian

$$H = \frac{p_1^2 + p_2^2}{2m} + \frac{1}{2} k (x_1 - x_2)^2$$

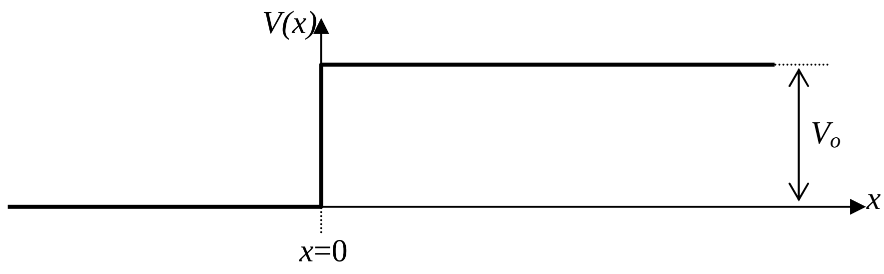
Where p_1, p_2, x_1, and x_2 are the momenta and positions of the two particles, respectively. k is a constant.

(a) **What is the ground state energy of the system if the two particles are identical bosons?**

(b) **What is the ground state energy of the system if the two particles are identical spin 1/2 fermions in a triplet state?**

6. A particle of mass m is incident from the right on the step potential given below:

$$V(x) = \begin{cases} 0 \ ; & x < 0 \\ V_o \ ; & x \geq 0 \end{cases}$$



The particle has an energy of $E>V_0$. As the particle comes in from the right what is:

a) The reflection coefficient from this step potential? That is the ratio of the reflected current to the incident current.

b) **The transmission coefficient from this step potential? That is the ratio of the transmitted current to the incident current.**

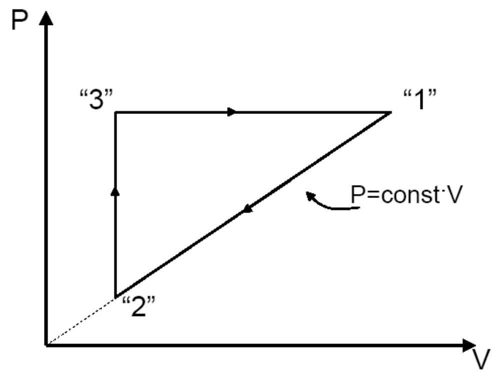
7. Stars are radiating gas spheres in a state of hydrostatic equilibrium (i.e. the pressure forces are in balance with the gravitational forces at every point within the star). As such, stars are governed by the virial theorem which states that the sum of twice the total thermal energy and the total gravitational potential energy vanishes. That is

$$2U + E_{grav} = 0.$$

(a) Find an expression for the total gravitational potential energy of the Sun. Assume the density of the Sun is constant. Write your answer in terms G, M_{sun} , and R_{sun} (the Universal Gravitational Constant, the mass of the Sun, and the radius of the Sun, respectively).

(b) Then using that expression and the virial theorem find the average temperature of the Sun. In addition to assuming a constant density for the Sun, also assume that it is an ideal gas composed entirely of hydrogen (H) which has a mass m_{H}.

8. A monatomic ideal gas works according to the cycle shown in the figure.



a. Calculate the total work done over the entire cycle.

b. Calculate in each of the segments the:

   i. Net heat absorbed in 1 to 2

   ii. Net heat absorbed in 2 to 3

   iii. Net heat absorbed in 3 to 1