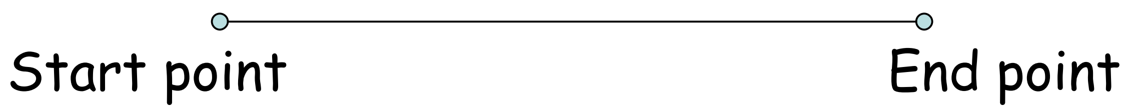
Stabbing Problem

Outline

- Stabbing problem
- Method 1
- Method 2
- Related problems

Stabbing Problem

- Given a set of n line segments S
- Input: query point q
- Output: the intervals that contain q



Stabbing Problem

- Brute force algorithm:
  - For query point q and every interval s_i in S, check if q overlaps with s_i
  - Time complexity: $O(n)$

Stabbing Problem

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  - For query point q and every interval s_i in S, check if q overlaps with s_i
  - Time complexity: $O(n)$
- Can we do faster?

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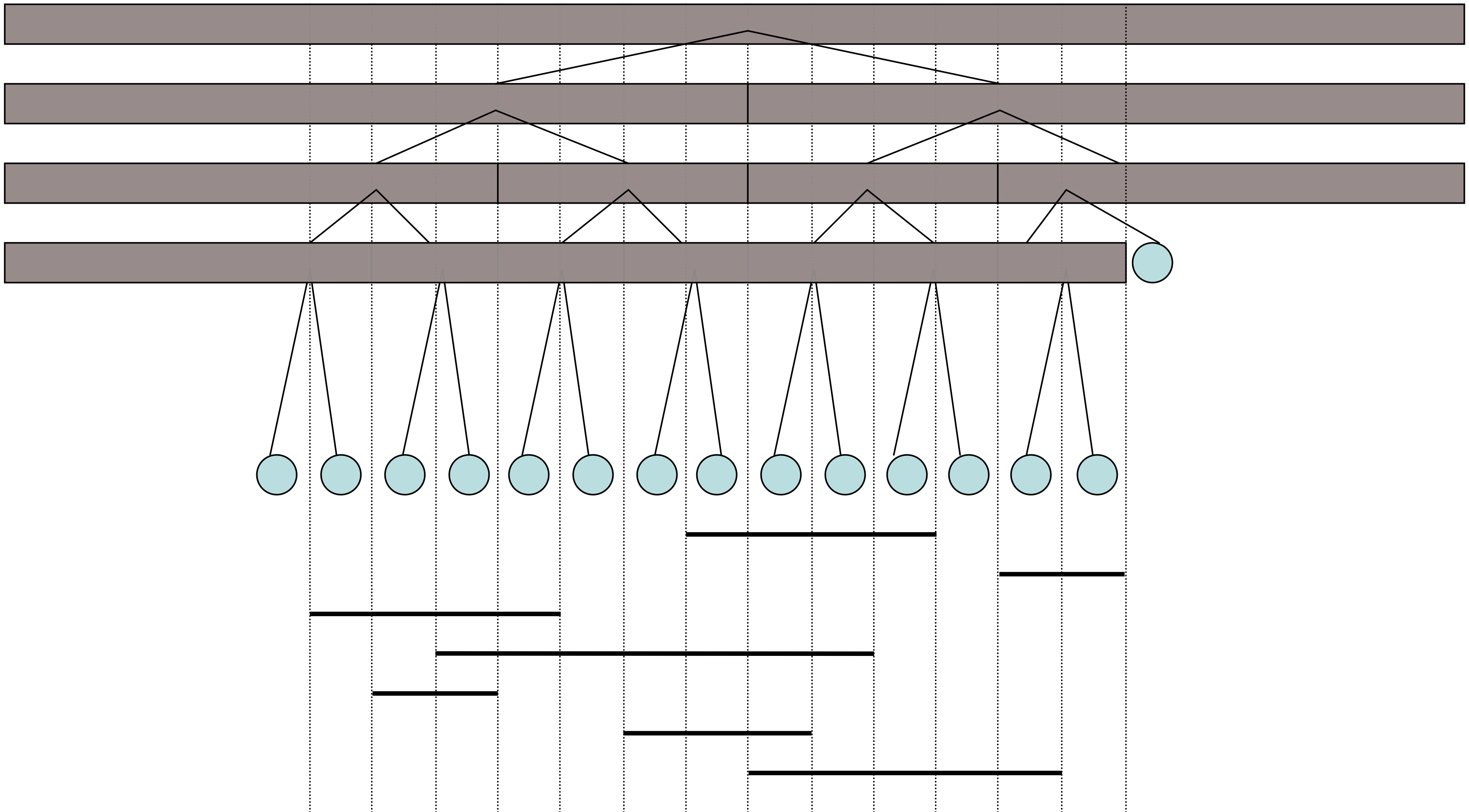
- Method 1 - segment tree

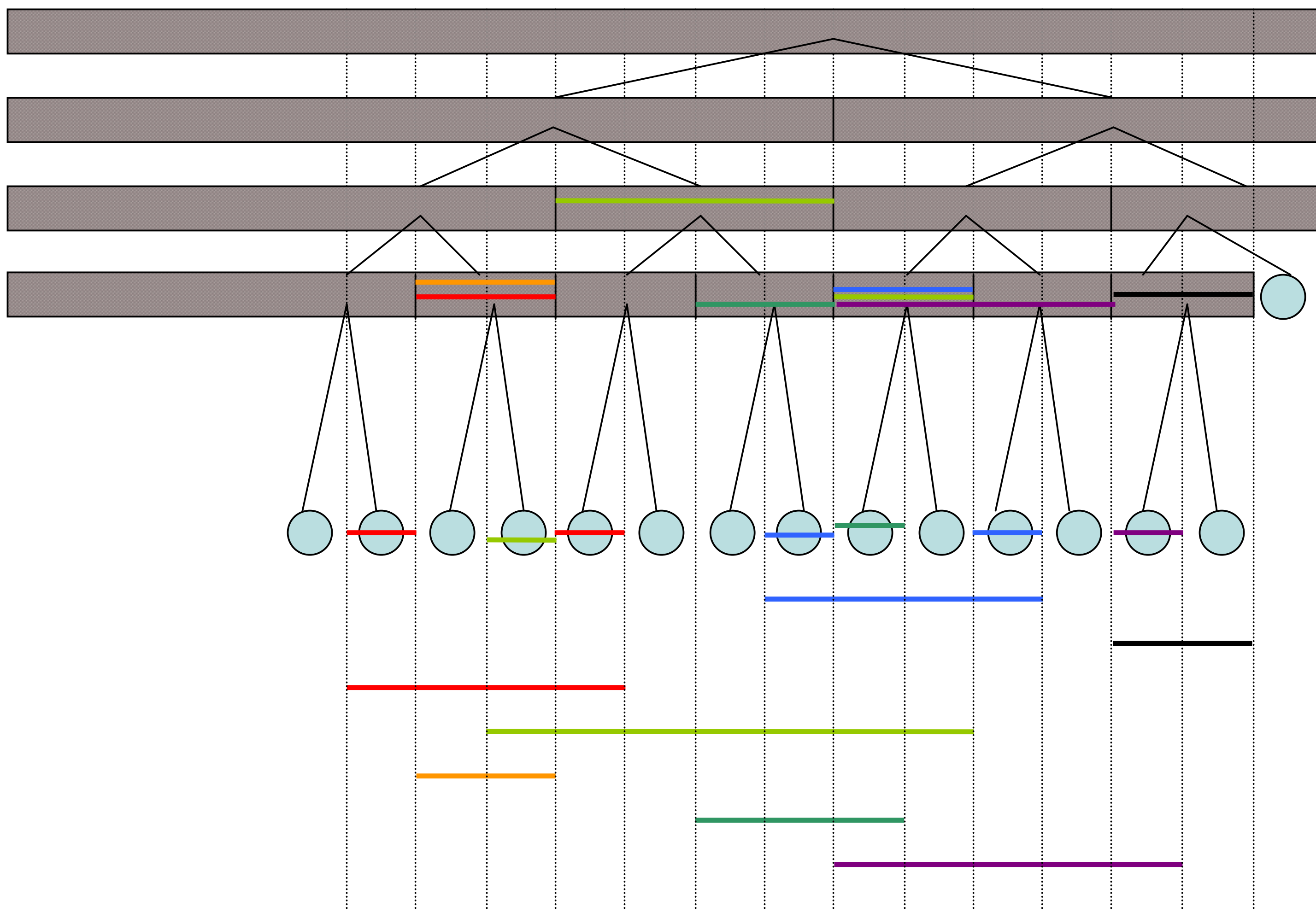
Segment Tree

- Preprocess:
  - Step 1: Sort by all the start points and end points
  - Step 2: By the 2n points, build a balanced binary search tree T
    - Height of T = O(log n)
  - Step 3: Insert the line segments into T
    - Insert a line segment needs O(log n) time

Example



Example

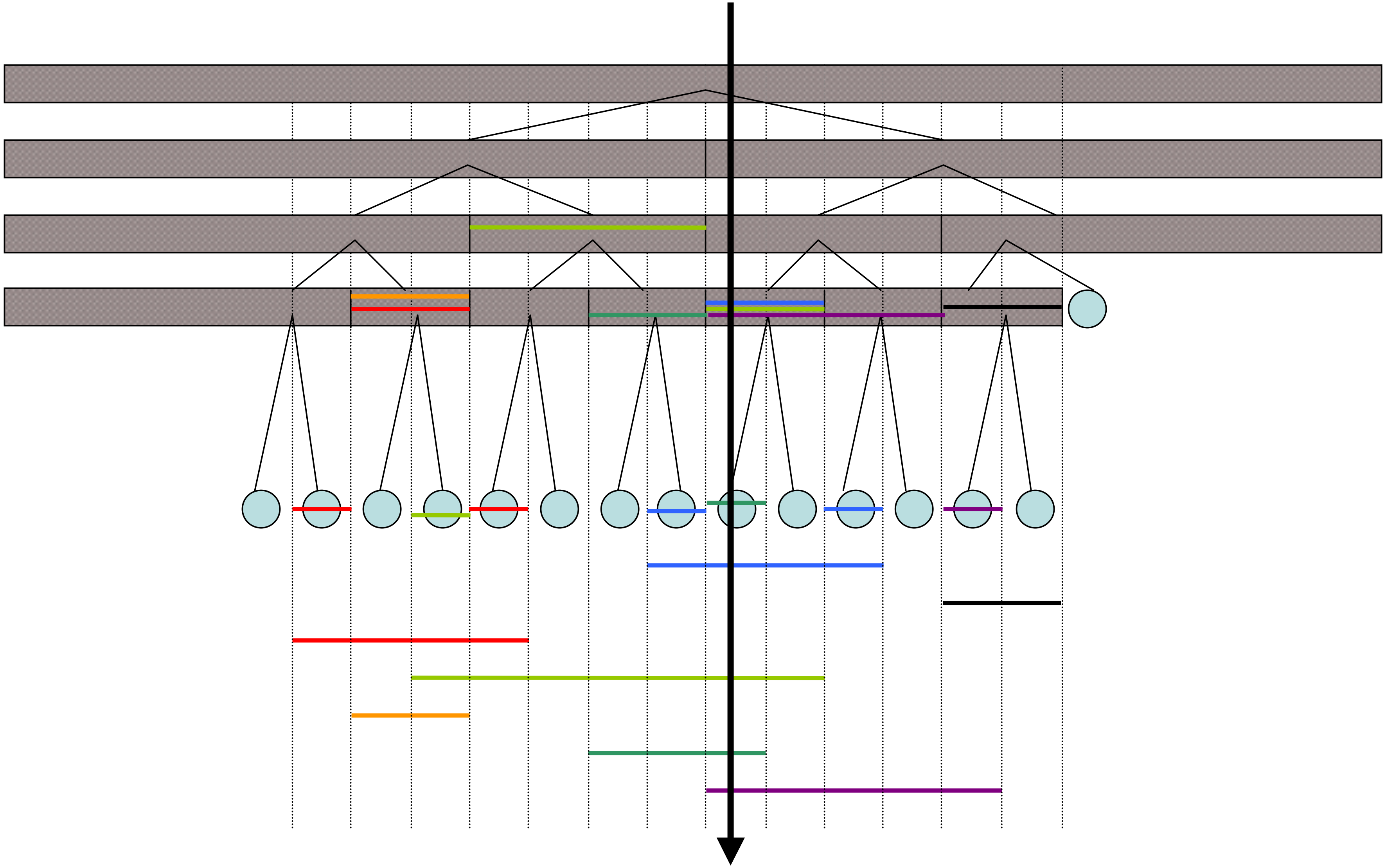


Segment Tree

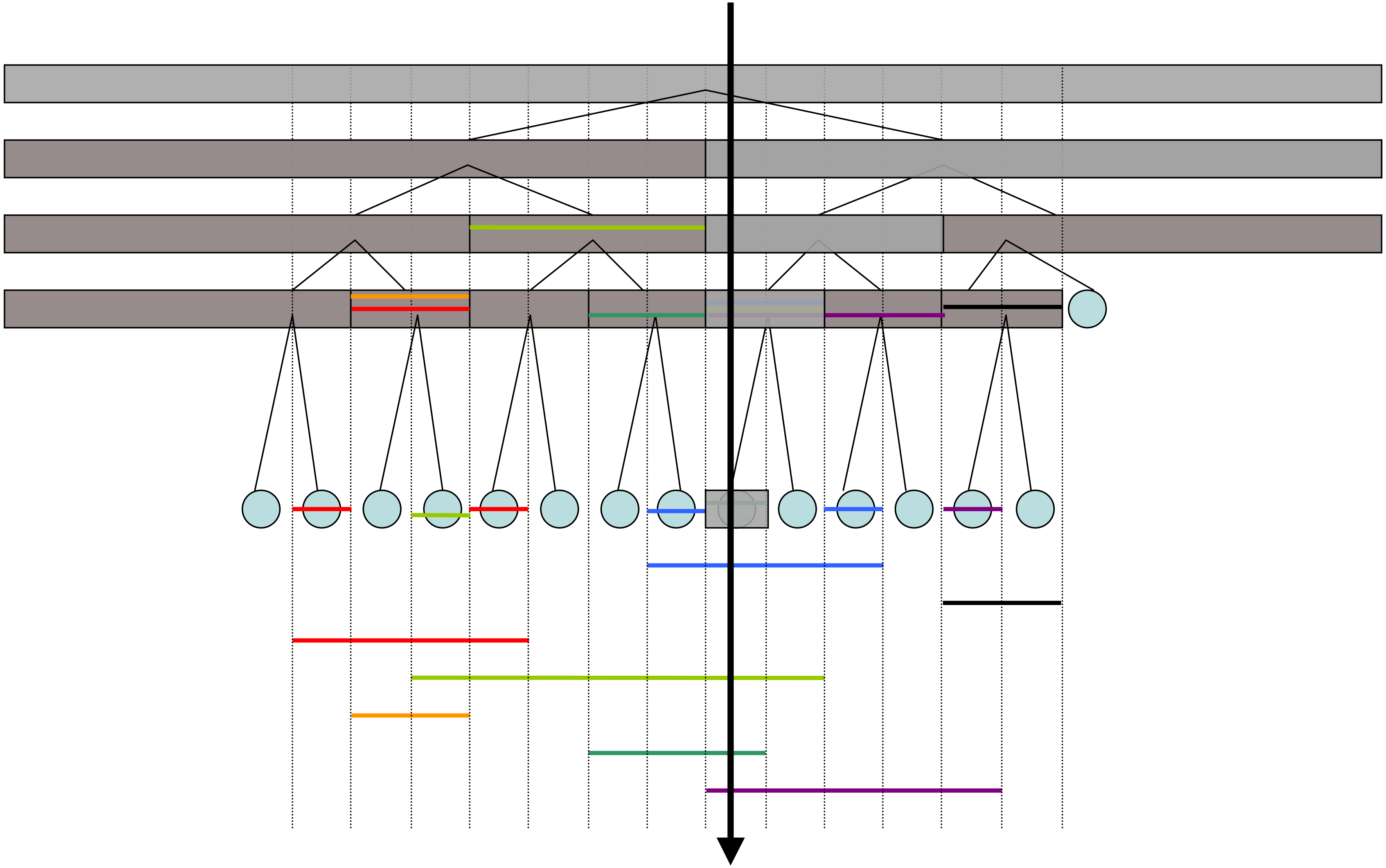
- Property: any segment is stored at most twice at each level of T
- Space complexity: $O(n \log n)$
- Preprocessing time: $O(n \log n)$

- Note: every node represents a segment

Segment Tree Query



Segment Tree Query



Segment Tree

- Query time:
  - $O(\log n + k_1 + k_2 + k_3 + \ldots + k_{\log n})$
    $= O(\log n + k)$
  - k_L: number of nodes reported on level L
- Output-sensitive
  - algorithms whose running time depends not only on the size of the input but also on the size of the output

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- Stabbing problem
- Method 1 - segment tree
- Method 2 - interval tree

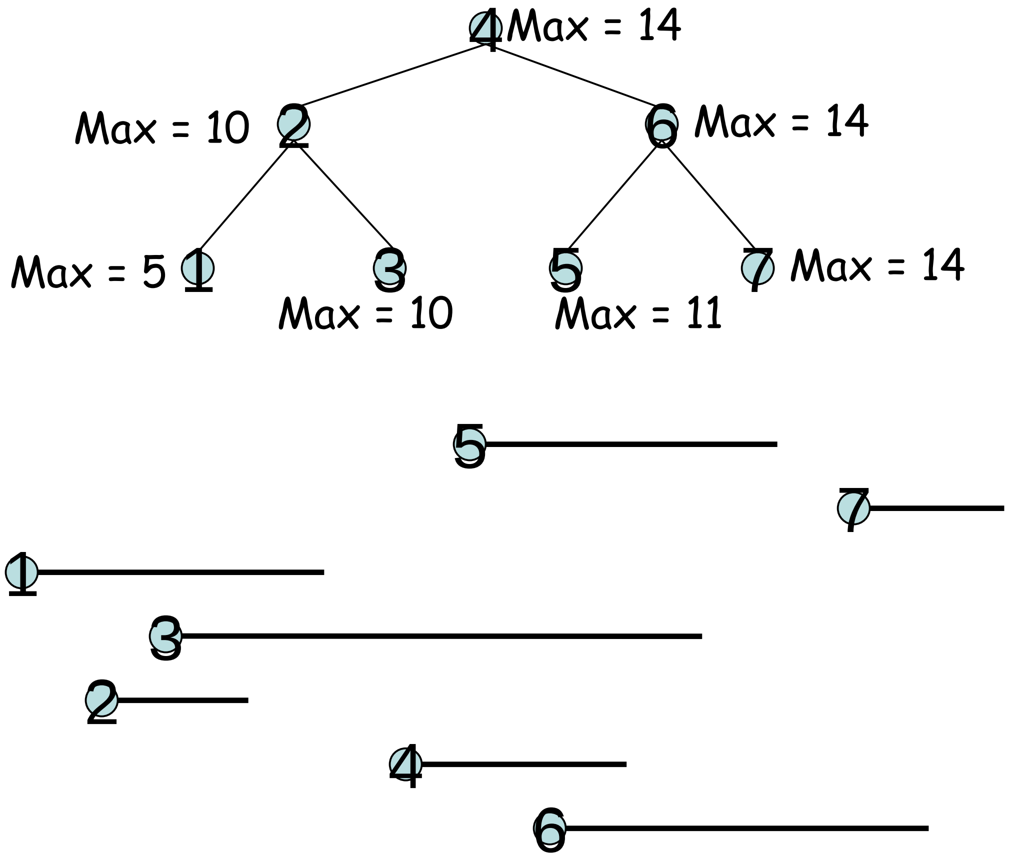
Interval Tree

- Preprocess:
  - Build a balanced binary search tree T for the n line segments by the start points
    - Each node v of T has information of the line segment and Max
    - Max: position of the righmost end points in subtree of root v

Interval Tree

- Preprocess time:
  - Build BBST: O(n log n)
    - Insert a line segment into T: O(log n)
  - Maintain Max: O(1)

Example



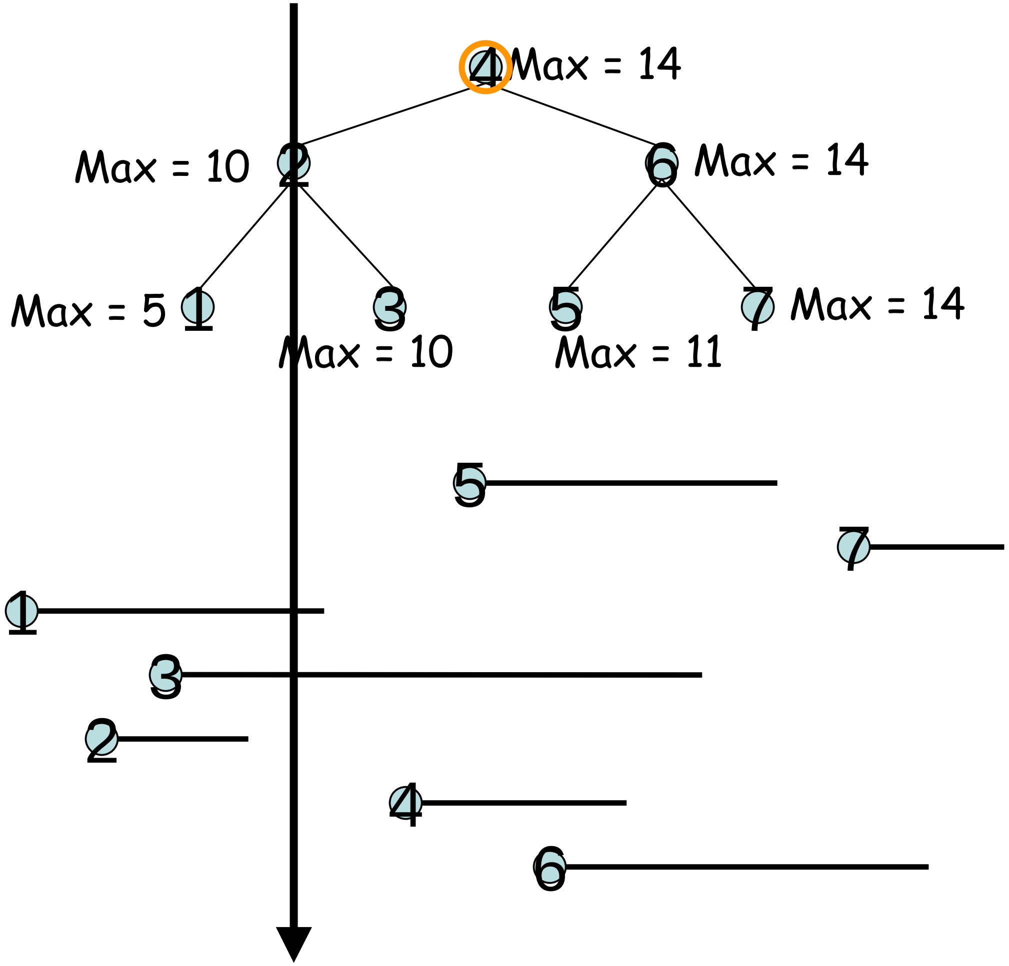
Interval Tree

- Space: O(n)
  - Each node represents a line segment
- Query time: O(k log n)

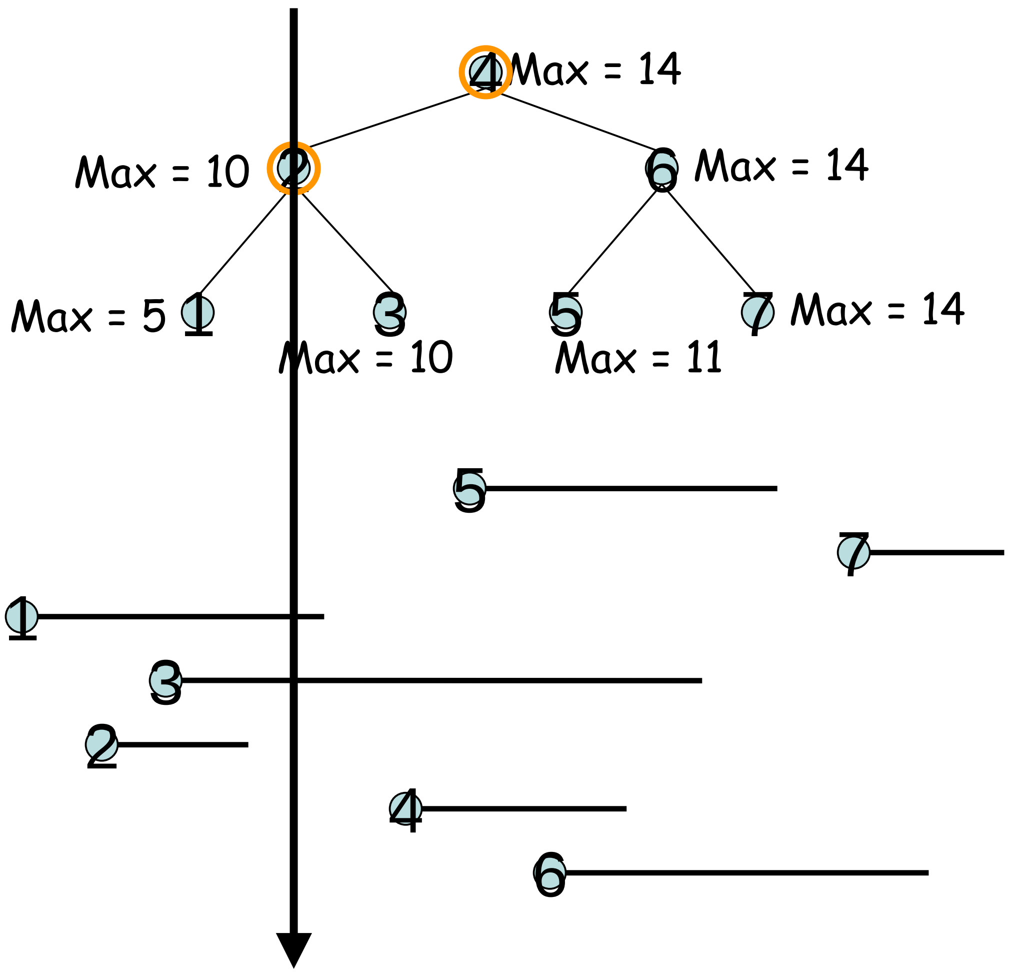
Interval Tree

- Query:
  - Step 1: check if query point q intersects with the line segment in node x
    - Yes -> report
  - Step 2: check if q > x.max
    - Yes -> complete
  - Step 3: check if q > x.startpoint
    - Yes -> recursively run on x.leftchild and x.rightchild
    - No -> recursively run on x.leftchild

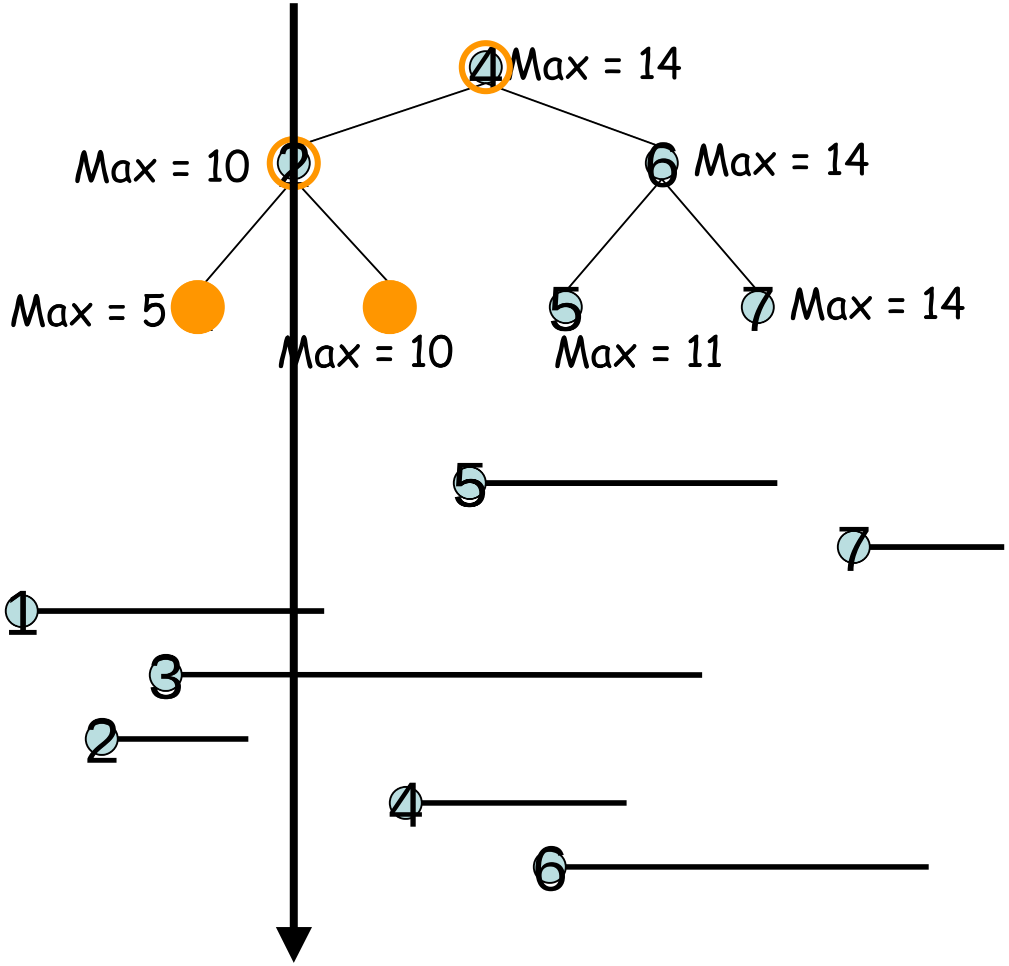
Example



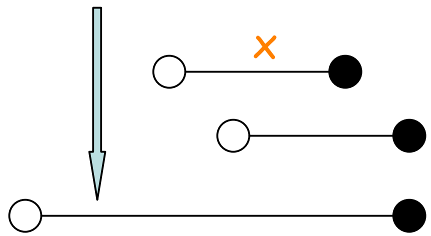
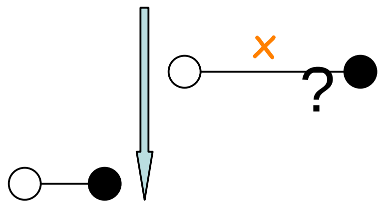
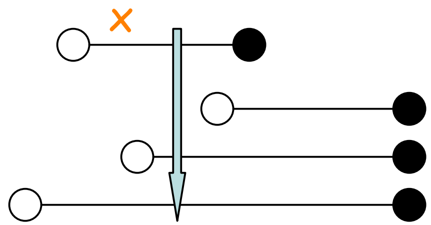
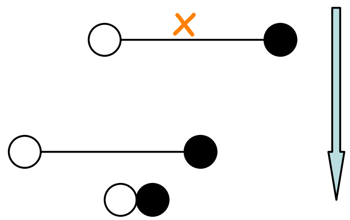
Example



Example



Interval Tree

	q < x.max	q > x.max
q < x.start		
q > x.start		

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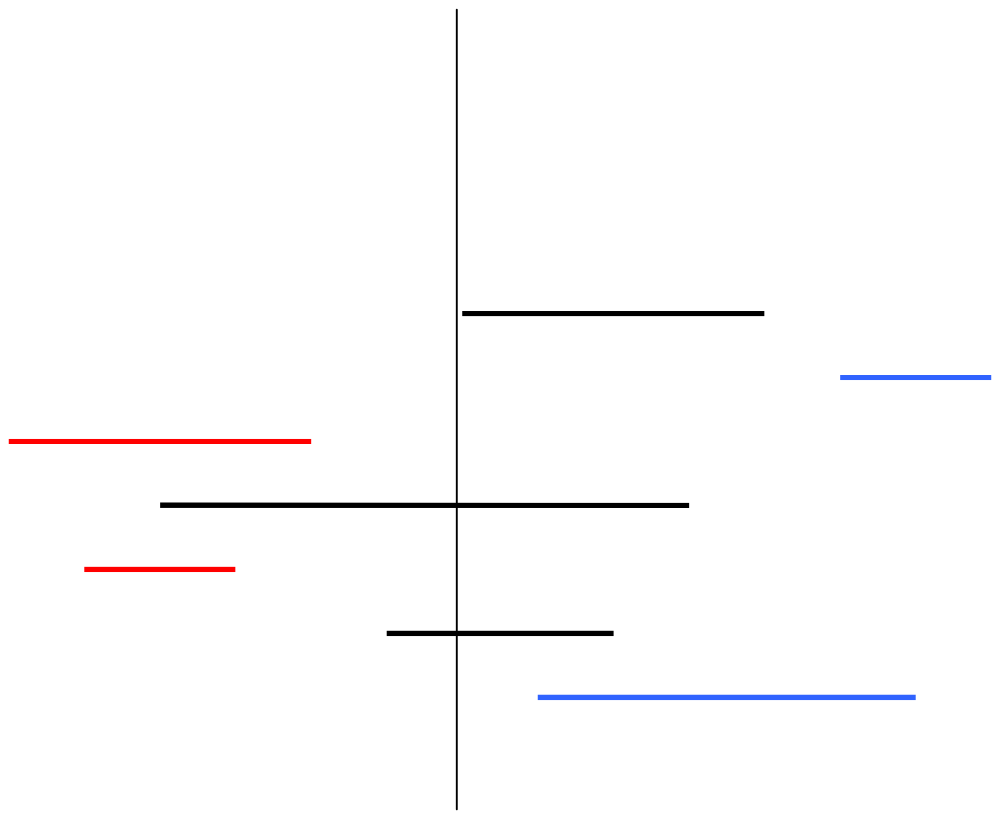
  (a completely different version)

Interval Tree 2



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Related Problem

- Higher-dimension Stabbing Problem
  - Solved by multi-level of segment trees
  - Space improvement if we use interval tree at deepest level

- Given a set of points, query rectangle
  - called Range Query Problem