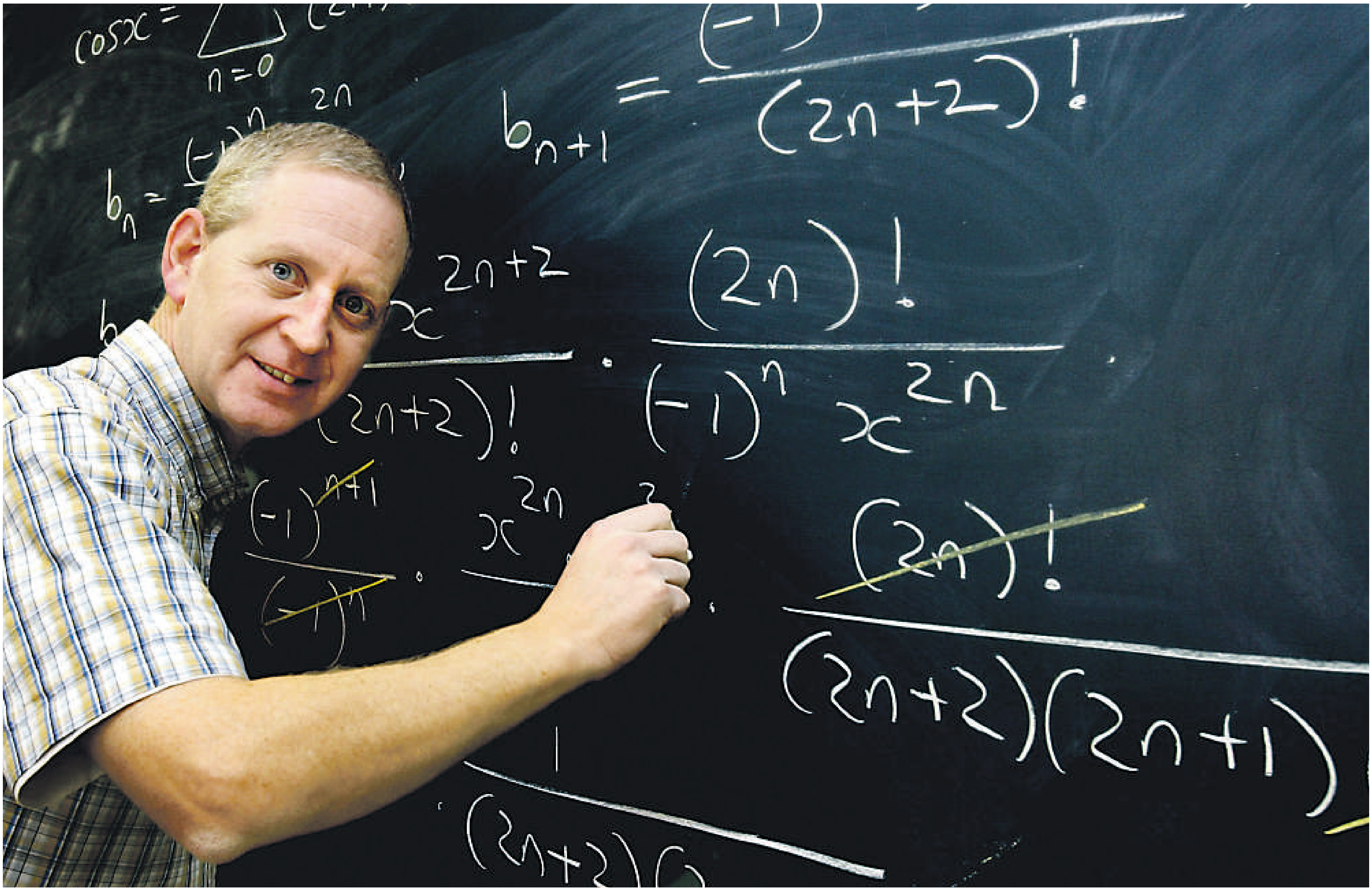
Irish Independent Institute of Education

exam brief

PART 1 • 06.02.2008

The winning numbers

28 pages of Leaving Cert Maths



The complete guide to Leaving Cert Maths this year, by **Aidan Roantree,** one of the country's top Maths teachers.

Our **28 page supplement** covers every aspect of the **Maths** courses at **Higher and Ordinary Levels** with sample Questions and Answers.

It's a no-brainer. If you want to maximise your grade, here's your guide to getting the winning numbers

The A1 Student • Dos and Don'ts • Tips from the top Teacher

Welcome to our guide to the Leaving Cert in 2008

WELCOME TO OUR GUIDE TO THE LEAVING CERT IN 2008.We begin the series of supplements today with the ultimate guide to the Maths exams, at both Higher and Ordinary levels.

This is the first of five Leaving Cert 2008 supplements which are being published by the Irish Independent in association with The Institute of Education.

The Institute of Education is Ireland's leading private tuition college, sending more students to university than any other school over the past few years. Part of its success is attributed to the outstanding teacher notes supplied to its students.

These notes, together with special additional advice from the teachers, form the basis of this series of supplements. They provide an overview of the entire course in each subject with invaluable practical advice on how to study and how to maximise exam performance.

Last year the Institute was the No 1 provider of students to UCD, Trinity, the Royal College of Surgeons, DCU and DIT. Now with our ExamBrief series, all students can benefit from the notes and advice that have been so successful at the Institute.

The Leaving Cert ExamBrief supplements begin today and will continue every Wednesday over the next four weeks. The supplements are available only with the Irish Independent and offer the only in-depth exam preparation guides available with an Irish newspaper. Unlike other supplements which have appeared, they cover the complete course in each subject featured.

This year our Leaving Cert ExamBrief series is even more extensive than in previous years. Subjects are being grouped thematically for the first time. Today's 28-page supplement is entirely devoted to Maths, both Higher and Ordinary, offering an unmissable guide to the complete course at both levels. It is written by Aidan Roantree, one of the country's top Maths teachers. It includes sample Questions and Answers, tips on tackling the subject over the remaining three months, and advice from an A1 student from last year.

Next week ExamBrief will be a Languages Supplement, covering English, Irish and French.

The series will include a Sciences Supplement, covering Physics, Chemistry and Biology.

There will also be a Money Supplement, covering Economics, Business and Accounting.

Other subjects will be included in a final supplement.

Supplements Editor: John Spain

Raymond Kearns, Director of the Institute of Education

You don't have to be Einstein

Mastering Maths - Concept and Execution

By Aidan Roantree, Maths Teacher, Institute of Education

To master Maths, you must master both concept and execution. A weakness in either area will stop you achieving a high grade.

To master concepts, you must try to understand the purpose of the methods and formulae you meet. If you merely learn off a technique, by doing repeated examples of the same type, and yet miss the point of what it is driving at, all it needs is one small, subtle twist in the exam question and you will be lost. And don't be deceived, many (b) parts and most (c) parts contain such subtle twists. This is how they distinguish the A1 students from the C students.

So you should try your utmost to understand what you are doing. Maths is not Biology, Business or Home Economics, where you will be rewarded for having learned many relevant facts. To each their own.

Ask questions, ask why, ask why not! Of yourself, your teacher, your friends. Don't settle for platitudes! Your aim should be that by the time the Leaving Cert. comes, there should be few, if any, concepts that you are not comfortable with. Be positive and aggressive.

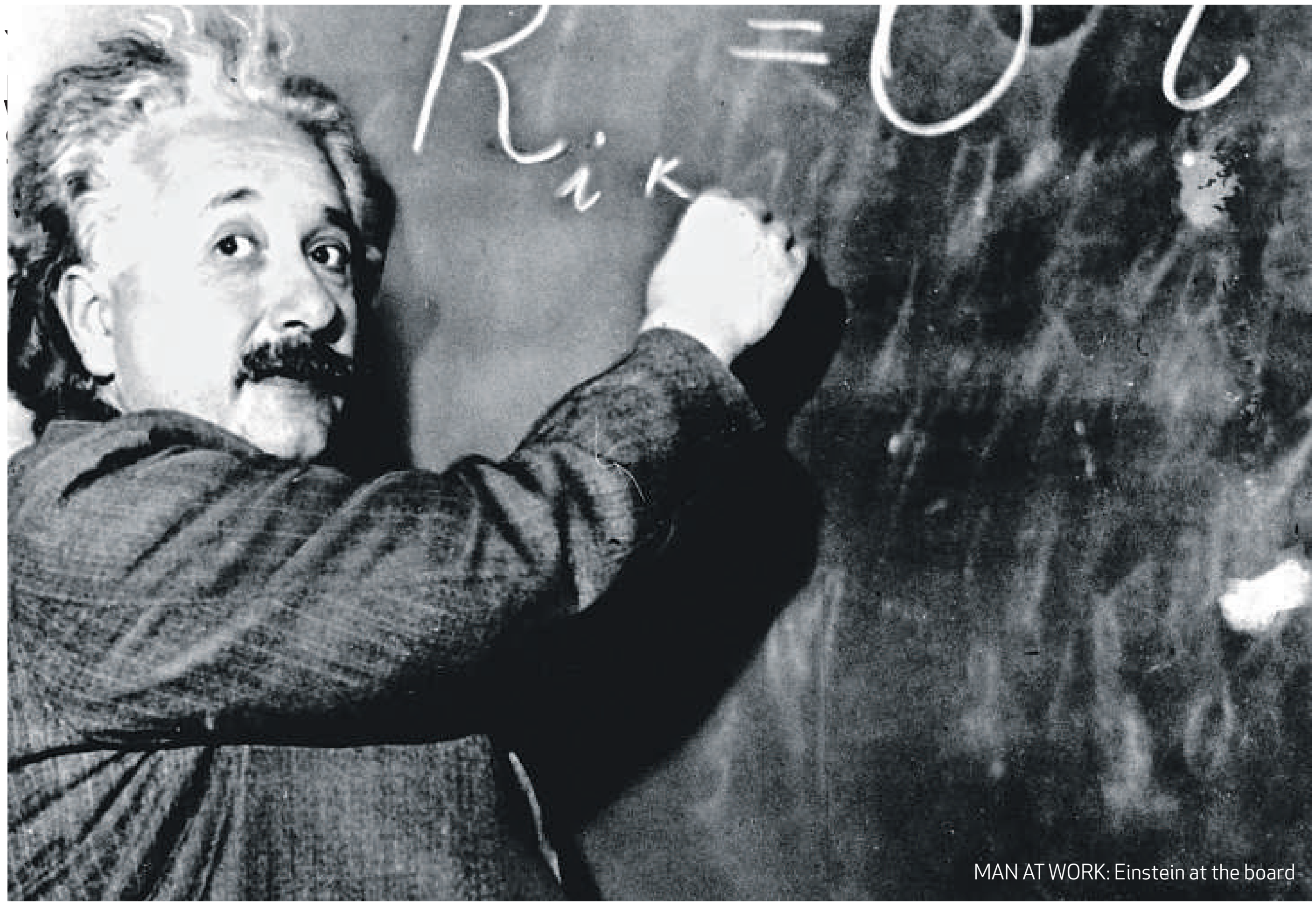
Some people are blessed with 'mathematical ability', just as others are blessed with abilities in sport, English or music. Being blessed with mathematical ability means they can grasp new mathematical ideas fully in an instant, e.g. Albert Einstein. Don't you hate it when the student who keeps getting As, if not 100%, says that he/she only spends 10 minutes a night doing maths, and you spend two hours? People have different abilities: it is a fact of life.

Even though you may not be an Albert Einstein (who is?), you give yourself the best chance of maximising your grade by trying to understand as much as possible. There will always be a few questions for which nobody seems to have an answer, but keep trying.

Unfortunately, mastering concept alone does not guarantee a high grade. How often do we meet students who say: 'I can understand fully what is going on in class, but I just can't do the questions myself'?

The diagnosis here is that the execution is weak, i.e. they know what they are supposed to do, they just can't do it. This is almost always because they have basic weaknesses in their Algebra, Trig or Differentiation, but most importantly Algebra.

Weakness with Algebra is the most common reason why a good, intelligent student doesn't perform to their ability in Maths. Should this describe you, the best advice is to go back through Algebra: expressions, equations (with an equal to sign), brackets, fractions, etc., with a fine tooth comb and practise, practise, until you understand all the techniques perfectly, and develop good habits in the way you write Maths. You might consider this demeaning, but, hey, do you want to improve your grade in Maths or not? The choice is yours.



MAN AT WORK: Einstein at the board

For 2008 - the advice is to leave nothing out

THE TEACHER'S VIEW

The aim of the current Higher Level syllabus, introduced in 1992, which was intended to be shorter and more straightforward than the one it replaced, was to make Higher maths appeal to more students.

For a while it seemed to be succeeding. In the mid nineties, nearly 11,000 students took the Higher level exam. Also, the percentage of As rose from about 5pc fifteen years ago to about 15.4pc last year. This is a little behind Physics, Chemistry and Biology, but ahead of all the other major subjects.

Even more impressive is the fact that 80.1% of students who sat the Higher Level papers obtained a C grade or higher. Only 3.9pc of students failed Higher Level maths in 2007. These figures have been roughly the same over the last few years.

So even though Higher Level maths seems to have re-acquired its reputation as a difficult, elite subject, which is reflected by the drop in numbers to about 8,400 in 2007, the students who take this level are doing extremely well in the Leaving Cert. exam.

One concern is that as the syllabus and exams grow old, which they are now doing, the search for new questions becomes harder. This has resulted in a number of questions on unlikely topics being asked in the last couple of years. Examples are the inverse tan graph in 2006 and concurrent lines in 2007. Because of this, you should leave nothing out when preparing each topic.

At Ordinary Level, the figures in 2007 were 13.9pc of students getting an A, and 67.9pc getting a grade C or higher. While not as good as the Higher Level stats, it is still reasonably encouraging for the majority of students.

But the big news, and the big concern, is that 11.6% of students failed the Ordinary Level exam. This means that just over 4,000 students did not get an important qualification in maths. This figure was roughly in line with previous years.

Prior to 2007, a lot of the blame for this situation was laid at the door of very difficult and challenging exam papers (for the level). However, the 2007 papers were generally well received, and some expected a reduction in the failure rate. That it didn't happen was perhaps because there is a minority of students for whom this course is simply not suitable, and who find themselves able to write little or nothing in the exams. So a small easing in standard is going to have little effect on the failure rate.

If you feel that you are at risk, you should practise writing down as much as possible for each question you attempt. Writing down something is always better than writing down nothing. Marks are given for any small effort in the right direction, and great flexibility is usually allowed. By making every effort with each question, even if you think you haven't a clue, you can dramatically improve your performance.- **AR**

Higher level
Paper 1

Paper One

Paper 1 contains eight questions, and students are required to answer any six of these, for 50 marks each. The time allowed is two and a half hours. This equates to twenty five minutes per question. However, this doesn't take into account that you will have to start by reading the paper to choose your questions, and that difficult parts may have to be revisited. In practice, you should aim to complete as many questions as possible within a twenty minute time limit.

Many students enter the exam hall for Paper 1 with the six questions that they intend to do already chosen. In a lot of cases, they have not even revised for the other two questions. This is a highly risky strategy. On occasion, some of the more popular questions have contained unusual and very difficult parts. Students with only six questions prepared are then forced to tackle these, often with catastrophic effects to their morale and eventual grade. The best advice is to enter the exam with at least one standby question.

Algebra and calculus dominate Paper 1. Of the eight questions, two full questions are dedicated to algebra and three to calculus. One of the other questions deals with complex numbers and matrices, which rely heavily on algebra. The remaining two questions, which cover sequences and series, binomial, induction and algebra, are difficult to predict. Nevertheless, if you prepare these topics, you may well be rewarded with relatively easy questions, as has happened many times in the past.

Algebra

The first two questions on Paper 1 always cover the topic of algebra. These are extremely popular questions with almost all students, but nevertheless you should carefully examine the questions you are presented with before rushing in to attempt them. If there are unusual or difficult parts, you should consider carefully whether or not you would be better off attempting other questions instead.

Within these two questions there can be as many as six to ten separate parts, which means that a wide range of question types can be asked in any given year.

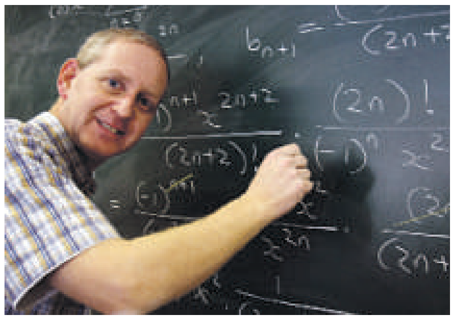
A number of question types occur with great frequency, others less often. Some of the most popular topics are:

* simultaneous equations (every year),
* abstract quadratic factors of cubics (most years),
* the Factor Theorem (most years),
* roots of quadratic equations (most years),
* inequalities, both solving and proving (very often).

Other topics should not be neglected, as there will almost certainly be one or more parts on the likes of surds, powers, logs, fractions, function notation, etc. In particular, it is unwise to assume that logs will not be examined in Questions 1 and 2. There is no such guarantee. You only have to look at Question 2(c) 2006.

Below is a list of the topics that you should study in algebra.

1. **Fractions**
   e.g. show that
   $$\frac{6}{x-2}+\frac{2+2x}{2-x}$$
   reduces to a constant, for all $x \in \mathbf{R}$, $x \neq 2$
2. **Identities**
   e.g. find real numbers a and b if
   $$a(5x-2)+b(3x+4)=21x-2,$$
   for all $x \in \mathbf{R}$
3. **Surds**
   e.g. simplify
   $$\left(1+\sqrt{a}-\sqrt{a+1}\right)\left(1+\sqrt{a}+\sqrt{a+1}\right)$$
   where $a \in \mathbf{R}$, $a > 0$
4. **Irrational equations**
   e.g. solve
   $$\sqrt{2x-1}+\sqrt{x-1}=5,$$
   for $x \in \mathbf{R}$
5. **Proof of the Factor Theorem**
6. **Use of the Factor Theorem to factorise cubics and solve cubic equations**
   e.g. if $(2x-1)$ is a factor of
   $$f(x)=2x^3+kx^2-11x-6,$$
   find the value of $k \in \mathbf{R}$, and find the other two factors of $f(x)$
7. **Quadratic factor of a cubic**
   e.g. if x^2+2b is a factor of
   x^3+bx^2+ax+c, show that $a^2=2c$
8. **Linear simultaneous equations**
   e.g. solve the simultaneous equations
   $$3x-5y-z=-3$$
   $$2x+y-3z=-9$$
   $$x+3y+2z=7$$
9. **Linear, non-linear simultaneous equations**
   e.g. solve the simultaneous equations
   $$2x+y=8$$
   $$x^2+y^2=52$$
10. **Modulus inequalities**
    e.g. solve $|5x-1|<9$
11. **Solving quadratic equations**
    e.g. solve $x^2-9x+18=0$ and hence solve
    $$(x^2+x)+\frac{18}{x^2+x}=9$$
12. **Nature of quadratic roots**
    e.g. show that for all $a \in \mathbf{Z}$, the roots of the equation
    $$x^2-4ax+(3a^2+4a-4)=0$$
    are integers
13. **Alpha and beta roots of quadratics**
    e.g. if α, β are the roots of
    $$x^2-px-q=0,$$
    express $\alpha^2+\beta^2$ in terms of p and q, and hence construct a quadratic equation with roots $\alpha+2\beta,\ 2\alpha+\beta$
14. **Function notation**
    e.g. if $f(x)=2x+1$ and $g(x)=-6x-4$, show that
    $$f(f(x)+g(x))=g(x)-f(x)$$
15. **Abstract inequalities**
    e.g. if $x, y \in \mathbf{R}$, prove that
    $$x^2+y^2 \geq \frac{1}{2}(x+y)^2$$
16. **Rational inequalities**
    e.g. solve
    $$\frac{x+3}{x-4}>-2, \quad x \in \mathbf{R},\ x \neq 4$$
17. **Sequence notation**
    e.g. if $u_n=5(2^n)$, show that
    $$u_{n+2}+u_{n+1}-6u_n=0$$
18. **Logs and log equations**
    e.g. solve the equation
    $$\log_3(5x+1)=2\log_3(x+1),$$
    for $x>-1$
19. **Equations with the unknown in the index**
    e.g. solve the equation
    $$2^{2x+1}-9(2^x)+4=0,$$
    for $x \in \mathbf{R}$.



AIDAN ROANTREE

Aidan Roantree is Senior Mathematics Teacher at the Institute of Education, where he has been teaching maths, at both higher and ordinary level, and applied maths since 1986. He is the author of over a dozen textbooks, including the two volume series 'Leaving Certificate Maths for Higher Level' and 'Maths in Focus' for ordinary level students. Over fifteen years he has given many talks and lectures, concerning aspects of the courses, to both students and teachers. He has written the Leaving Cert. maths articles for the Exam Brief supplement to the Irish Independent for the last fifteen years. He has been editor of 'Science Plus', the monthly science and maths journal for Leaving Cert. students for the last twenty years.

Scribble box

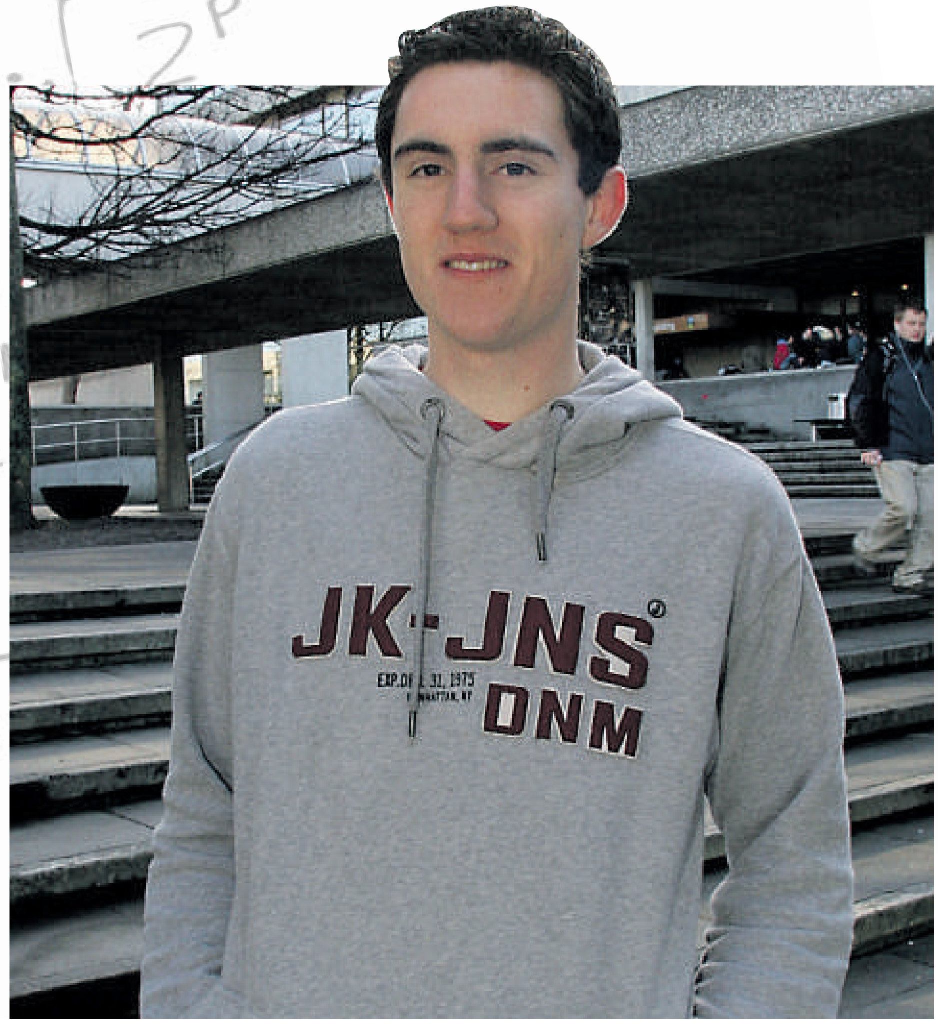
Complex Numbers and Matrices

Question 3, on complex numbers and matrices is another very popular question. In most years, two out of the three parts of the question deal with complex numbers, while the remaining part covers matrices. Matrices was only the (a) part in 2007, so don't be surprised to see a more substantial part on matrices this year.

Complex numbers can be divided into three rough areas:

* algebra of complex numbers; TO PAGE 6

Higher level
The A1 Student's view



How Richard got his sums right

Name: Richard Ryan
From: Ballyhendricken, Kilkenny.
Schools: Repeated the LC in The Institute of Education, Dublin in 2007, 7 A1s.
Results: Maths A1, Applied Maths A1, Physics A1, Chemistry A1, Biology A1, Agricultural Science A1, Spanish A1.
College: Now in UCD doing Veterinary Medicine.

LEAVING CERT Higher Level Maths is not an easy subject. However, it is a logical subject and with work it can be easy or at least less difficult. One of the advantages of Maths, as with all the sciences, is that it is a case of being right or wrong and what's right is set in stone, unlike some other subjects where the two examiners could give different marks to an answer based on their own opinions.

So now, how can maths become an easy subject for you? PRACTISE!!! That may seem obvious but it can be very hard to get around to as 6th year is very busy for most students. Try to make a routine, not necessarily a study timetable as they don't work for a lot of people, myself included. Get up at the same time every morning and be prepared to either study a lot in the evenings or, if you are a morning person, set the alarm clock a few hours earlier and study before school. This does require effort but if you go to sleep earlier too you'll be surprised how sharp your mind is in the morning.

Just saying practise isn't much help as you need to know what to practise. Leaving Cert past papers are crucial as they provide an invaluable insight into how the examiner tends to set the paper. Remember, the syllabus is a finite list so the style of questions has to repeat from time to time! The papers are best done question by question first, as in prepare Algebra for example and then do all the past algebra questions.

CONTEXT

Doing the questions "in context" like this, is a good way to iron out anything you don't know about a topic but remember that you must do them in a realistic time as you would in the LC exam and try to have them marked by a teacher. Also it might be an idea to leave one or two papers completely untried and you could attempt these in full in the final weeks before the exam itself.

Make sure you are prepared before you start trying Leaving Cert questions as there is no benefit from starting a question and then going to your notes to check something. Leaving Cert questions, especially (c) parts tend to look easy once you see them done out but the problem is that you won't have seen the ones coming up next June. So the key is that once you start a practise question don't go back to your notes. Try your best to get it out in the time limit and then realise that you weren't as prepared as you'd have liked to have been.

A lot of students around the country seem to be trying to cut out parts of the maths course in recent years. This is a terrible idea, as seen in 2006 with the Inverse Trig Graph which the majority of people didn't prepare. Also many students don't realise that topics can overlap in questions.

So do your best to be capable of doing any of the eight questions on either paper because on the day any of them could be very, very difficult and you may have to try a different question, which could be rather hard if you've only prepared six questions!

Algebra is a must know as it is involved in so many of the topics and while many students despise it, Sequences & Series can also pop up unexpectedly destroying otherwise nice questions on the students who left the topic out.

So do your best to be capable of doing any of the eight questions on either paper because on the day any of them could be very, very difficult and you may have to try a different question, which could be rather hard if you've only prepared six questions! The option in maths is a key question for students as it must be included in the marks for paper two, even if you scored 5marks on it and did six questions perfectly in section A! So whichever options you do - the vast majority of people only prepare one option due to time constraints - it must be prepared perfectly.

THEOREMS

Theorems in Maths are such easy marks for students who are prepared to put in the work to learn them. The best way to learn them is to write them out until you can do it without any help from the notes. This is best done by thinking about the steps involved logically and trying to understand what exactly you are doing because it is always easier to learn something you understand. Once they are learned off, don't forget to practise them from time to time as you don't want to draw a blank in June.

Breaks are very important when studying, especially in maths where you need to stay sharp mentally. Do not use energy drinks. Lots of sleep and drinking water are much better ideas. Take small breaks during study, like 10mins every hour or 30mins every two hours depending on what suits you personally. Most people find they have their own methods anyway and that other methods don't work for them. Try to get some exercise during breaks as this clears your mind.

When you go into the exam in June, don't just rush in blindly and do the first 6 questions, take time at the start, about five or ten minutes, to read through the paper and analyse the questions; this is where the past paper practise pays off.

CONTEXT

Choose your questions carefully and try your best to stick to your guns once the decision is made. The majority of people can't write fast enough to do more than six questions per paper so stopping halfway through and starting a different question could be disastrous. If you do finish early make sure to re-read the whole paper, make sure you answered enough questions and check for mistakes. You'd be surprised how many marks are lost on slips and blunders.

If you think you have time to do another question be careful, you might be better off going back and trying to redo a part of a question which you couldn't get out. If you have 6 questions done to a good standard, an extra question would have to be answered better than one of these for it to have been worthwhile, and if you are rushing at the end you have to consider whether this is possible.

Higher level
Paper 1

FROM PAGE 4

$-, -, \times, \div, \sqrt{\ }, =$, conjugate, Argand diagram and modulus,

* complex equations, including but by no means confined to the Conjugate Roots Theorem,
* polar form, De Moivre's Theorem and its applications to trig identities, powers and roots.

Last year most of these topics were covered by the Leaving Cert. question. The only topic here that has not been examined recently is trig identities.

In matrices, you must be able to:

* $-, -, \times$ matrices and find their inverses,
* solve matrix equations, including simultaneous equations,
* deal with the construction $P^{-1}AP$.

A. Complex Numbers

1. **Equality of complex numbers**
   e.g. find the complex number $z = x + yi$ if
   $$2z + (1+i)\bar{z} = 5 + 3i,$$
   where $\bar{z}$ is the conjugate of z
2. **Addition, subtraction and multiplication**
   e.g. if $z = 2 - 3i$ and $w = 5 + i$, express in the form $a + ib$:
   $$iz(2z - w)$$
3. **Conjugate and division**
   e.g. express $\dfrac{9+4i}{1+i}$ in the form $a + bi$
4. **Square roots**
   e.g. find the real numbers a and b if
   $$(a + bi)^2 = 21 + 20i$$
5. **Argand diagram and modulus**
   e.g. $z = 3 + 4i$ and $w = -2 + i$. Plot $z + w$ on an Argand diagram and investigate if
   $$|z + w| = |z| + |w|$$
6. **Complex equations**
   e.g. if $3 + i$ is a root of the equation
   $$z^2 - kz + (7 - i) = 0,$$
   find the value of $k \in \mathbf{R}$, and find the other root of this equation
7. **Conjugate Roots Theorem**
   e.g. if $2 - 3i$ is a root of the equation
   $$z^3 + az^2 + bz - 65 = 0, \quad a, b \in \mathbf{R},$$
   find the values of a and b and the other roots of the equation
8. **Polar Form**
   e.g. express $-\sqrt{2} - \sqrt{2}\,i$ in the form $r(\cos\alpha + i\sin\alpha)$
9. **Proof of De Moivre's Theorem**
10. **De Moivre: Trigonometric identities**
    e.g. use De Moivre's Theorem to express $\sin 3\theta$ as a polynomial in $\sin\theta$
11. **De Moivre: Large powers**
    e.g. express $(-1 + i)^{13}$ in the form $a + bi$
12. **De Moivre: Roots**
    e.g. express the solutions of the equation
    $$z^6 - 64$$
    in the form $a + bi$.

B. Matrices

1. **Adding, subtracting**
   e.g. if $A = \begin{pmatrix} 17 & -9 \\ 4 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -3 \\ -1 & 4 \end{pmatrix}$, find the matrix X if
   $$3X + 2B = A$$
2. **Multiplication**
   e.g. express
   $$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$
   in the form $ax^2 + bxy + cy^2$
3. **Inverse matrix**
   e.g. if $M = \begin{pmatrix} 5 & 3 \\ -4 & -2 \end{pmatrix}$, find M^{-1}
4. **Solving equations**
   e.g. if $A = \begin{pmatrix} 3 & -1 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}$,
   (i) find the matrix X if $AX = B$
   (ii) find the matrix Y if $YA = B$
5. **Simultaneous equations**
   e.g. use matrix methods to solve the simultaneous equations
   $$2x + y = 7$$
   $$5x - 2y = -5.$$

Sequences, Series, Binomial and Induction

Questions 4 and 5 have traditionally been the least popular questions on Paper 1. However, on occasion one or other of these have been among the easiest questions on the Paper. It can sometimes be very difficult to convince students that these questions should be read before making a decision as to what six questions to choose.

Why the negative attitude? Mainstream questions on sequences and series, while not very exciting, are at least manageable. However, when students go back through the past papers and see the once-off, completely unforeseen questions, which only appeal to the high-flying students, they can be turned off these questions. Lest this turns you off, please note that such questions do not always appear, and so the questions are well worth reading.

It is also becoming difficult to predict what topics are going to be examined in which question. The binomial theorem has been examined in both questions often enough so that it has no home. But proof by induction, which used to be a banker for Question 5, last year occurred in Question 4.

Don't write off these questions. In an ideal world, Question 5 would contain an easy part on the binomial, then some algebra, followed by a proof by induction.

A. Sequences and Series

1. **Sequence notation**
   e.g. if $u_n = n!(n+2)$, show that
   $$(n+1)u_n + (n+1)! = u_{n+1}$$
2. **The result $u_n = S_n - S_{n-1}$**
   e.g. $S_n = u_1 + u_2 + u_3 + \ldots + u_n$. For this series
   $$S_n = 5n^2 + 3^n.$$
   Find u_2 and $u_3 + u_4$.
3. **Arithmetic sequences and series**
   e.g. in an arithmetic series, the sum of the first eight terms is 164 and the sum of the next six terms is 333. Find the first term and the common difference.
4. **Geometric sequences and series**
   e.g. the first term of a geometric series is $\frac{3}{5}$ and the fourth term is $\frac{75}{8}$. Find the sixth term and the sum of the first six terms.
5. **Infinite geometric series**
   e.g. a geometric series with common ratio $0{\cdot}8$ has a sum to infinity of 250. Find the fourth term of the series.
6. **Telescoping series**
   e.g. express $\dfrac{1}{(2r-1)(2r-1)}$ in the form $\dfrac{A}{2r-1} + \dfrac{B}{2r+1}$,
   and hence evaluate $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)}$
7. **Powers of natural numbers**
   e.g. evaluate $\displaystyle\sum_{n=1}^{13}(n+1)(n-1)$.

B. Binomial Theorem

1. **Binomial coefficients**
   e.g. show that
   $$\binom{n+1}{2} + \binom{n}{2} = n^2$$
2. **Binomial expansions**
   e.g. expand and simplify
   $$(x + \sqrt{2})^4 - (x - \sqrt{2})^4$$
3. **Given terms in the expansion**
   e.g. if
   $$(1 + ax)^n = 1 + 20x + 150x^2 + \ldots$$
   find the values of a and n
4. **General terms**
   e.g. find the coefficient of the term x in the expansion of
   $$\left(2x^2 - \frac{1}{2x}\right)^8.$$

C. Proof by Induction

1. **Formula for the sum of a series**
   e.g. prove by induction that
   $$(2)(5) + (3)(6) + \ldots + (n+1)(n+4) = \frac{n(n+4)(n+5)}{3}$$
2. **Divisibility proofs**
   e.g. prove by induction that
   $$7^n + 2^{2n+1}$$
   is divisible by 3, for all $n \in \mathbf{N}$
3. **Inequality proofs**
   e.g. prove by induction that
   $$3^n > n^2,$$
   for all $n \in \mathbf{N}$, $n \geq 2$.

Differentiation

The two questions on differentiation cover the mechanics of differentiation and its applications. You can expect to see parts on the mechanics and on the applications in both Question 6 and Question 7.

These questions are extremely popular, but students do not always achieve the marks they expect from these questions. This is usually because students do not apply the rules and methods of differentiation accurately enough, leading to a seepage of marks. The moral of the story is to pay great attention to detail and to be very, very careful.

There are certain topics that occur regularly and should be afforded special attention:

* proofs: six first principles and four rules (very often)
* chain rule (every year)

Higher level
Paper 1

- parametric differentiation (almost every year)
- implicit differentiation (very often).

From the applications of differentiation, sketching rational curves and real roots of cubic equations are probably worth extra study this year.

You should also carefully go through the questions from the past five years where algebra or trigonometry are required to re-write derivatives in some specified form. This pattern is likely to continue.

1. **Differentiation from first principles**
   (There are only six functions that you can be asked to differentiate from first principles: $x^2, x^3, \frac{1}{x}, \sqrt{x}, \sin x, \cos x$)
   e.g. differentiate $\sqrt{x}$ with respect to x from first principles
2. **Differentiation proofs**
   e.g. prove, from first principles,
   $$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx},$$
   where $u = u(x)$ and $v = v(x)$
3. **Differentiation by rule**
   e.g. find $\frac{dy}{dx}$ if
   (i) $y = \sin^{-1}(3x-1)$
   (ii) $y = e^{2\cos x}\sin x$
   (iii) $y = \ln\frac{x^2-4}{\sqrt{x+2}}$
4. **Implicit differentiation**
   e.g. find $\frac{dy}{dx}$ if
   $$6x^2 + 7y^2 - 2x^2y^2 = 8x$$
5. **Parametric differentiation**
   e.g. find the value of $\frac{dy}{dx}$
   when $t = 0$ if
   $$x = t + t\cos t, \quad y = t - t\sin t,$$
6. **Max, min and points of inflection**
   e.g. find the co-ordinates of the local maximum and local minimum points of the curve
   $$y = xe^{-x^2}, \quad \text{for } x \in \mathbf{R}$$
7. **Cubic curves and equations**
   e.g. determine the values of $k \in \mathbf{R}$ for which the equation
   $$x^3 - 3x^2 - 24x + k = 0$$
   has three real roots
8. **Rational curves**
   e.g. $f(x) = \frac{2x+1}{x-2}, \quad x \neq 2$
   (i) Show that this curve has no turning points or points of inflection.
   (ii) Find the asymptotes of $y = f(x)$.
   (iii) Draw a rough sketch of $y = f(x)$.
9. **Newton-Raphson**
   e.g. taking $x_1 = 0$ as the first approximation to a real root of the equation
   $$x^3 - 10x + 3 = 0,$$
   use the Newton-Raphson method to find x_2 and x_3, the second and third approximations
10. **Rates of change**
    e.g. V and x are connected by the equation
    $$V = (3x-5)^3$$
    If $\frac{dx}{dt} = 4$, find $\frac{dV}{dt}$ when $x = 2$.

Scribble box

Integration

Integration is the topic covered in the last question, Question 8, on Paper 1. Like the other two calculus questions, this question is extremely popular with students.

It helps that this question is one of the more predictable on Paper 1. The (a) part will contain one or two straightforward indefinite integrals (no limits), not requiring any substitutions. The (b) and (c) parts will contain two or three more involved integrals, along with probably one part on areas or volumes.

One of the main problems you will meet in integration is being able to classify integrals given to you. You should practise with a wide range of integrals from different sources, making careful note of the correct approach to each type.

For questions on areas, it is important to analyse the given region properly, in particular, noting points of intersection of curves.

1. **(a) part indefinite integrals**
   e.g. find
   (i) $\int \frac{1}{x^2}dx$
   (ii) $\int \sin 5x\,dx$
   (iii) $\int x^2(x+1)\,dx$
2. **Substitution**
   e.g. evaluate
   (i) $\int_0^{\frac{\pi}{4}} \frac{\cos x\,dx}{\sin^2 x - 4}$
   (ii) $\int_0^2 4x(x^2-1)^4\,dx$
   (iii) $\int_0^3 \sqrt{9-x^2}\,dx$
3. **Trigonometric integrals**
   e.g. evaluate
   (i) $\int_0^{\frac{\pi}{4}} 2\cos^2 2x\,dx$
   (ii) $\int_0^{\frac{\pi}{6}} 2\cos 8x\cos 4x\,dx$
   (iii) $\int_0^{\frac{\pi}{2}} \sin x\cos^3 x\,dx$
4. **Rational integrals**
   e.g. evaluate
   (i) $\int_0^1 \frac{2x^2-3x+5}{x+2}dx$
   (ii) $\int_{-2}^{-1} \frac{dx}{x^2+4x+5}$
   (iii) $\int_0^2 \frac{3x-2}{3x^2-4x+7}dx$
5. **Area by integration**
   e.g. find the area of the region bounded by the curve $y = x^2+1$ and the line $y = 5$
6. **Volumes of rotation**
   e.g. find, by integration methods, the volume of the sphere generated by rotating the circle
   $$x^2 + y^2 = 9$$
   about the x-axis.

SAMPLE QUESTIONS

1. Algebra

Question

(a) Express
$$\frac{2}{(x-1)(x+1)} + \frac{1}{(x+1)(x+2)}$$
in the form $\frac{k}{x^2+px+q}$, where k, p and q are constants.

(b) (i) Solve the inequality
$$\frac{2x-1}{x+2} < 1,$$
for $x \in \mathbf{R}$, $x \neq -2$.

(ii) If $(x+k)^2 + 4x + 8 = (x+t)^2$, for all $x \in \mathbf{R}$, find the real numbers k and t.

(c) $f(x) = \frac{x}{x+a}$,
for $x \in \mathbf{R}$, $a \in \mathbf{R}$, $x \neq -a$.

(i) Show that $f(-2a-x) = 2 - f(x)$, for all $x \in \mathbf{R}$, $x \neq -a$.

(ii) If $-a < p < q$ and $a > 0$, simplify $f(q) - f(p)$ and show that $f(q) > f(p)$.

Higher level
Paper 1

SAMPLE QUESTIONS

Solution

(a) $\dfrac{2}{(x-1)(x+1)}+\dfrac{1}{(x+1)(x+2)}$

$=\dfrac{2(x+2)-1(x-1)}{(x-1)(x+1)(x+2)}$

$=\dfrac{3x+3}{(x-1)(x+1)(x+2)}$

$=\dfrac{3}{(x-1)(x+2)}$

$=\dfrac{3}{x^2+x-2}.$

(b) (i) $\dfrac{2x-1}{x+2}<1$

$\dfrac{(2x-1)(x+2)^2}{x+2}<1(x+2)^2$

$(2x-1)(x+2)<(x+2)^2$

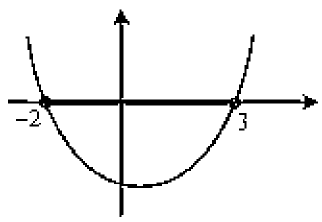
$2x^2+3x-2<x^2+4x+4$

$x^2-x-6<0$

If $x^2-x-6=0$

$(x+2)(x-3)=0$

$x=-2$ or $x=3$



Thus $-2<x<3$.

(ii) $(x+k)^2+4x+8=(x-t)^2$ for all x

$x^2+2kx-k^2+4x+8$

$=x^2+2tx+t^2$ for all x

$x^2+(2k+4)x+(k^2+8)$

$=x^2+2tx+t^2$ for all x

Putting like to like,

1: $2k+4=2t$

$k=t-2$

2: $k^2-8=t^2$

$(t-2)^2+8=t^2$

$t^2-4t+4+8=t^2$

$12=4t$

$t=3$ and $k=3-2=1.$

(c) (i) $f(x)=\dfrac{x}{x+a}$

$f(-2a-x)=\dfrac{(-2a-x)}{(-2a-x)-a}$

$=\dfrac{-2a-x}{-a-x}$

$=\dfrac{2a+x}{a+x}$

$2-f(x)=2-\dfrac{x}{x+a}$

$=\dfrac{2(x+a)-x}{x+a}$

$=\dfrac{x+2a}{x+a}=f(-2a-x)$

(ii) $f(q)=\dfrac{q}{q+a}$ and $f(p)=\dfrac{p}{p+a}.$

$f(q)-f(p)=\dfrac{q}{q-a}-\dfrac{p}{p+a}$

$=\dfrac{q(p-a)-p(q-a)}{(q+a)(p+a)}$

$=\dfrac{qp+aq-pq-pa}{(q-a)(p+a)}$

$=\dfrac{a(q-p)}{(q+a)(p+a)}$

As $-a<p<q$ and $a>0$,

$0<p+a<q+a$ and $q-p>0$.

Thus

$f(q)-f(p)=\dfrac{(+)(-)}{(+)(-)}$

$f(q)-f(p)>0$

$f(q)>f(p).$

2. Complex Numbers and Matrices

Question

(a) If $A=\begin{pmatrix}2&-1\\3&4\end{pmatrix}$ and $B=\begin{pmatrix}5&7\\3&4\end{pmatrix}$, find the matrix X such that

$XB=A.$

(b) (i) Show that $3-i$ is a root of the equation

$z^2-(4-i)z+5-5i=0,$

and find the other root of this equation.

(ii) $z_1=\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$ and

$z_2=\cos\dfrac{\pi}{3}-i\sin\dfrac{\pi}{3}.$

Evaluate z_1z_2, giving your answer in the form $x+iy$.

(c) Write $\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}$ in the form

$r(\cos\theta+i\sin\theta)$, and hence evaluate

$\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{15}.$

Solution

(a) $A=\begin{pmatrix}2&-1\\3&4\end{pmatrix}$ and $B=\begin{pmatrix}5&7\\3&4\end{pmatrix}.$

$\det(B)=|B|=20-21=-1.$

$B^{-1}=\dfrac{1}{-1}\begin{pmatrix}4&-7\\-3&5\end{pmatrix}=\begin{pmatrix}-4&7\\3&-5\end{pmatrix}$

$XB=A$

$XBB^{-1}=AB^{-1}$

$X=AB^{-1}=\begin{pmatrix}2&-1\\3&4\end{pmatrix}\begin{pmatrix}-4&7\\3&-5\end{pmatrix}$

$=\begin{pmatrix}-11&19\\0&1\end{pmatrix}.$

(b) (i) $z^2-(4-i)z+(5-5i)=0$

$z=3+i$:

$(3+i)^2-(4-i)(3+i)$

$+(5-5i)=0$

$9+6i+i^2-12-4i$

$+3i-i^2+5-5i=0$

$9+6i-1-12-i$

$-1+5-5i=0$

$(9-1-12-1+5)-(6i-i-5i)=0$

$0=0$

Thus $3+i$ is a root.

If the roots are $z=\alpha=3+i$ and $z=\beta$, then

$\alpha+\beta=\dfrac{-b}{a}=4-i$

$\beta=(4-i)-\alpha$

$=(4-i)-(3+i)$

$=1-2i.$

(ii) $z_1=\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$

$z_2=\cos\dfrac{\pi}{3}-i\sin\dfrac{\pi}{3}$

$=\cos\left(-\dfrac{\pi}{3}\right)+i\sin\left(-\dfrac{\pi}{3}\right)$

$z_1z_2=\left[\cos\left(\dfrac{2\pi}{3}\right)+i\sin\left(\dfrac{2\pi}{3}\right)\right]$

$\times\left[\cos\left(-\dfrac{\pi}{3}\right)-i\sin\left(-\dfrac{\pi}{3}\right)\right]$

$=\cos\left(\dfrac{2\pi}{3}-\dfrac{\pi}{3}\right)$

$+i\sin\left(\dfrac{2\pi}{3}-\dfrac{\pi}{3}\right)$

$=\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}$

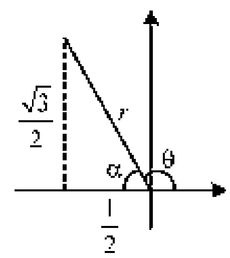
$=\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}.$

(c) (i) $\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}=\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\times\dfrac{1+\sqrt{3}i}{1+\sqrt{3}i}$

$=\dfrac{1+2\sqrt{3}i-3}{1+3}$

$=\dfrac{-2+2\sqrt{3}i}{4}$

$=-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$



$r=\sqrt{\left(\dfrac{1}{2}\right)^2+\left(\dfrac{\sqrt{3}}{2}\right)^2}$

$=\sqrt{\dfrac{1}{4}+\dfrac{3}{4}}=\sqrt{1}=1$

$\tan\alpha=\dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$

$\alpha=\dfrac{\pi}{3}$ (or 60°)

$\theta=\dfrac{2\pi}{3}$ (or 120°)

Thus

$\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}=1\left(\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right)$

or $1(\cos 120°+i\sin 120°)$

(ii) $\left[\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right]^{15}=\left[\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}\right]^{15}$

[or 120° instead of $\dfrac{2\pi}{3}$]

$=\left[\cos\dfrac{30\pi}{3}+i\sin\dfrac{30\pi}{3}\right]$

$=\cos 10\pi+i\sin 10\pi$

$=\cos 0+i\sin 0$

$=1+0i$

$=1.$

3. Differentiation

Question

(a) Find the slope of the tangent to the curve

$x^2-xy=y+13$

at the point $(3,2)$.

(b) (i) The parametric equations of a curve are:

$x=e^{2t}+1,\quad y=1-e^t.$

Find the value of $\dfrac{dy}{dx}$ when $t=0$.

Higher level
Paper 1

SAMPLE QUESTIONS

(ii) Taking $x_1 = 1$ as the first approximation to a root of the equation
$$x^3 + x^2 + k = 0,$$
where k is a constant, the Newton-Raphson method gives $x_2 = \frac{6}{5}$, where x_2 is the second approximation. Determine the value of k.

(c) If $y = \sqrt{\frac{\cos x}{1-\sin x}}$, for $0 < x < \frac{\pi}{2}$, show that
$$\frac{dy}{dx} = \frac{1}{2\sqrt{\cos x}\sqrt{1-\sin x}}.$$

Solution

(a)
$$x^2 + xy = y + 13$$
$$2x + \left[x\frac{dy}{dx} + y(1)\right] = \frac{dy}{dx}$$
$$x\frac{dy}{dx} - \frac{dy}{dx} = -2x - y$$
$$\frac{dy}{dx}(1-x) = 2x + y$$
$$\frac{dy}{dx} = \frac{2x+y}{1-x}$$
At $(3,2)$, the slope of the tangent is
$$\frac{6+2}{1-3} = -4.$$

(b) (i)
$$x = e^{2t} + 1 \qquad y = 1 - e^t$$
$$\frac{dx}{dt} = 2e^{2t} \qquad \frac{dy}{dt} = -e^t$$
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-e^t}{2e^{2t}} = \frac{-1}{2e^t}$$
When $t = 0$, $\frac{dy}{dx} = \frac{-1}{2e^0} = \frac{-1}{2}$.

(ii)
$$f(x) = x^3 + x^2 + k$$
$$f'(x) = 3x^2 + 2x$$
Newton-Raphson:
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
$$x_1 = 1,\ x_2 = \frac{6}{5}:$$
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$\frac{6}{5} = 1 - \frac{1+1+k}{3+2}$$
$$\frac{6}{5} = 1 - \frac{2+k}{5}$$
$$6 = 5 - (2+k)$$
$$6 = 3 - k$$
$$k = -3.$$

(c)
$$y = \sqrt{\frac{\cos x}{1-\sin x}} = \left(\frac{\cos x}{1-\sin x}\right)^{\frac{1}{2}}$$
$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{\cos x}{1-\sin x}\right)^{-\frac{1}{2}} \times \frac{(1-\sin x)(-\sin x) - (\cos x)(-\cos x)}{(1-\sin x)^2}$$
$$\frac{dy}{dx} = \frac{1}{2}\frac{(\cos x)^{-\frac{1}{2}}}{(1-\sin x)^{-\frac{1}{2}}} \times \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$$
$$\frac{dy}{dx} = \frac{1-\sin x}{2(\cos x)^{\frac{1}{2}}(1-\sin x)^{\frac{3}{2}}}$$
$$\frac{dy}{dx} = \frac{1}{2(\cos x)^{\frac{1}{2}}(1-\sin x)^{\frac{1}{2}}}$$
$$\frac{dy}{dx} = \frac{1}{2\sqrt{\cos x}\sqrt{1-\sin x}}.$$

4. Integration

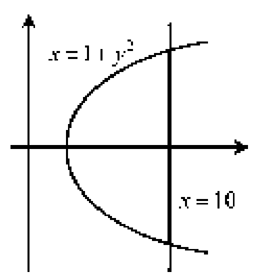
Question

(a) Find (i) $\int(\sqrt{x} + x)dx$

(ii) $\int \cos 4x\, dx$.

(b) (i) Evaluate $\int_1^5 \sqrt{2x-1}\, dx$.

(ii) The shaded region shown is bounded by the curve $x = 1 + y^2$ and the line $x = 10$. Find the area of this region.



(c) Evaluate $\int_2^4 \sqrt{16 - x^2}\, dx$.

Solution

(a) (i)
$$\int(\sqrt{x} + x)dx = \int\left(x^{\frac{1}{2}} + x\right)dx$$
$$= \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2 + c$$

(ii)
$$\int \cos 4x\, dx = \frac{1}{4}\sin 4x + c$$

(b) (i)
$$I = \int_1^5 \sqrt{2x-1}\, dx$$

$$\theta = \sin^{-1}\frac{x}{4}$$
when $x = 2$, $\theta = \frac{\pi}{6}$

when $x = 4$, $\theta = \frac{\pi}{2}$
$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4\cos\theta . 4\cos\theta\, d\theta$$
$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 16\cos^2\theta\, d\theta$$
$$= \frac{16}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \cos 2\theta)\, d\theta$$
$$= 8\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$
$$= 8\left[\left(\frac{\pi}{2} + 0\right) - \left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right)\right]$$
$$= 4\pi - \frac{4\pi}{3} - 2\sqrt{3}$$
$$= \frac{8\pi}{3} - 2\sqrt{3}.$$

Scribble box

Higher level
Paper 2

The one thing that Paper 1 and Paper 2 have in common is that students are required to answer six questions in two and a half hours. However, on Paper 1, there is a straight-forward choice of six questions out of eight. But, on Paper 2, there are two separate sections. Section A, on the core course, contains seven questions, of which students must answer five. Section B contains four questions from which students have to choose just one.

While Paper 1 is heavily based on algebra and calculus, Paper 2 leans more towards geometry, trigonometry and probability. As such, the algebra content is much reduced, being mostly confined to solving linear, quadratic and simultaneous equations. Even though many students see paper 2 as a welcome relief after Paper 1, there is nevertheless an increase in the number of formulae and methods to be learned. Also, a completely different mindset is required to tackle probability questions from that required for Paper 1.

One word of warning is not to approach Section A with only five questions prepared. This can be highly dangerous. If one (or more!) of the questions you intend doing turns out to be unusually difficult or strange, then you have no fall-back question. A classic example of this was Question 5, on trigonometry in 2006. If, for example, you are not a big fan of probability, you should work at enough so that at least you have an option if the worst comes to the worst.

In Section B, by far the most popular question is Question 8, on Further Calculus and Series. This question is chosen by well over 90% of students annually. It goes without saying that your differentiation and integration will have to be up to scratch to tackle this topic.

The Circle

The first question on Paper 1 examines co-ordinate geometry of the circle. This is really the second co-ordinate geometry topic, after the line. It will be necessary to use many of the formula from the line, e.g. the distance, slope, equation of a line and perpendicular distance formulae.

The key topics to prepare for the later parts of the question are:

* finding the equation of an awkward circle, either by geometry (as in 2007) or by using the g, f, c method (as in 2004)
* finding the equations of tangents and chords, particularly by using the perpendicular distance formulae.

If the information given about a circle, or a line, can be represented geometrically, then it is a good idea to draw a rough diagram. This can serve a dual purpose. It can serve to show the approach that should be taken, and it can provide one way of checking the answer. If the information given about a circle is not easily drawn, this probably means that the better way of finding its equation is to use the g, f, c method.

Unfortunately, many students start Paper 2 by trying Question 1, often without reading the other questions. This is not wise, as there is no guarantee that this question is the easiest on the paper. Indeed it has sometimes been among the longer and more involved. You should always read all the questions, and start with the question which you consider to be the easiest for you.

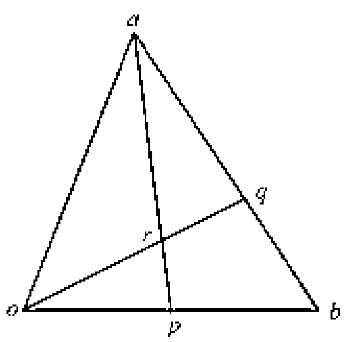
1. **Equation:** $x^2+y^2=r^2$
   e.g. find the equation of the circle, with centre $(0,0)$ and which has the line $2x+3y=26$ as a tangent
2. **Equation:** $(x-h)^2+(y-k)^2=r^2$
   e.g. find the equation of the circle that has centre $(6,3)$ and has the line $x=1$ as a tangent
3. **Equation:** $x^2+y^2+2gx+2fy+c=0$
   e.g. find the co-ordinates of the centre of the circle
   $$x^2+y^2+6x-2y+k=0$$
   and the value of k if the length of its radius is 5
4. **g, f, c method**
   e.g. find the equation of the circle which contains the points $(0,2)$ and $(1,5)$ and which has its centre on the line $x+5y-15=0$
5. **Parametric equations**
   e.g. find the Cartesian equation, the centre and the radius of the circle given by
   $$x=7-4\cos\theta,\quad y=-1+4\sin\theta$$
6. **Touching circles**
   e.g. investigate if the circles
   $$x^2+y^2-2x-4y-4=0 \text{ and}$$
   $$x^2+y^2-8x-12y+48=0$$
   intersect at a single point
7. **Intersection of a line and a circle**
   e.g. find the points of intersection of the line $x+2y=12$ and the circle
   $$x^2+y^2-2x-6y=0$$
8. **Proof of tangent formula**
9. **Tangent at a point**
   e.g. find the equation of the tangent to the circle $x^2-y^2-2x+4y=0$ at the point $(3,-3)$
10. **Tangents and chords**
    e.g. find the equations of the tangents that can be drawn from the point $(-3,-4)$ to the circle
    $$x^2+y^2-4x-2y-5=0.$$

Vectors

Vectors is the topic covered by Question 2 each year. Although some students do not study vectors and so cannot attempt this question, it is nevertheless usually one of the more accessible questions on Paper 2. Again, there have been exceptions, most notably the (c) part in 1999 and the (c) part in 2003. So again, it is important to be able to switch questions, if necessary.

Students tend to prefer questions involving $\vec{i}$ and $\vec{j}$ vectors and the scalar product, and are likely to be rewarded in most parts of the question. However, general plane vectors usually appear at some stage, and so should not be ignored. Indeed a number of questions mix ideas from both sources.

1. **General vectors**
   e.g. Let oab be a triangle. Let p be the midpoint of $[ob]$ and $q\in[ab]$ such that $|aq|:|qb|=3:2$. Let ap and oq intersect at the point r.
   (i) Express $\vec{q}$ in terms of $\vec{a}$ and $\vec{b}$.
   (ii) Express $\vec{r}$ in terms of $\vec{a}$ and $\vec{b}$.



2. **$\vec{i}$ and $\vec{j}$ vectors**
   e.g. $\vec{p}=5\vec{i}-2\vec{j}$, $\vec{q}=-3\vec{i}+4\vec{j}$ and $\vec{r}=2\vec{i}+\vec{j}$.
   (i) Write $\overline{qp}$ in terms of $\vec{i}$ and $\vec{j}$.
   (ii) Write $\vec{t}$ in terms of $\vec{i}$ and $\vec{j}$ if $\overline{pt}=\overline{rq}-\vec{p}$.
3. **Modulus**
   e.g. if $\vec{p}=-3\vec{i}+k\vec{j}$, for $k\in\mathbf{R}$, find the value of k if $|\vec{p}|=\sqrt{58}$
4. **Scalar product**
   e.g. if $\vec{a}=3\vec{i}-4\vec{j}$, $\vec{b}=-5\vec{i}+\vec{j}$ and $\vec{c}=3\vec{i}-7\vec{j}$, calculate
   (i) $\vec{c}.\vec{b}$.
   (ii) $\overline{ab}.\overline{bc}$
5. **Geometric properties of scalar product**
   e.g. $\vec{a}=4\vec{i}-3\vec{j}$, $\vec{b}=\vec{i}+2\vec{j}$ and $\vec{c}=10\vec{i}-k\vec{j}$.
   (i) Find the measure of the angle between $\vec{a}$ and $\vec{b}$, correct to the nearest degree.
   (ii) Determine the value of $k\in\mathbf{R}$ if $\overline{ab}\perp\vec{c}$.
6. **Related (perpendicular) vector**
   e.g. if $\vec{a}=2\vec{i}-3\vec{j}$ and $\vec{b}=-5\vec{i}+2\vec{j}$,
   (i) Find $(\vec{a}+\vec{b})^{\perp}$.
   (ii) Investigate if $\vec{a}.\vec{b}^{\perp}=\vec{a}^{\perp}.\vec{b}$.

The Line and Transformations

Co-ordinate geometry of the line and linear transformations will be examined in Question 3. Possibly because of familiarity with co-ordinate geometry since second year or third year, most students are well disposed towards this question. However, that does not mean that it without its difficulties.

Many question parts from previous years required no more than Junior Cert material to fully answer the question. Yet some of these were difficult (b) and (c) parts!

The new formulae for Leaving Cert. are those for the angles between two lines, the perpendicular distance and for concurrent lines. The last of these was asked as a major part last year, and so may or may not re-appear this year.

In transformations, the main types of questions are finding the images of points, lines and line segments. You should also watch questions where an image line is given and we have to work back to the original line.

A. The Line

1. **Basic concepts**
   e.g. if $b=(k,4)$, $c=(1,1)$ and $|bc|=3\sqrt{2}$,
   find the two possible values of $k\in\mathbf{R}$

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2. **Divisors of a line segment**
e.g. $a=(-5,1)$ and $b=(9,-6)$; find the co-ordinates of the point c which divides $|ab|$ internally in the ratio $5:2$
3. **Proof of angle between two lines formula**
4. **Angle between two lines**
e.g. find, correct to the nearest degree, the larger angle between the lines $3x+5y=0$ and $2x-3y+1=0$
5. **Concurrent lines**
e.g. find the equation of the line which contains the point of intersection of the lines $x+y-5=0$ and $x-2y-4=0$ and which has slope $\frac{4}{3}$
6. **Proof of perpendicular distance formula**
(This is well worth paying special attention to, as it has not been examined since 1998.)
7. **Perpendicular distance formula**
e.g. find the perpendicular distance from the point $(-3,7)$ to the line containing the points $(2,-4)$ and $(-6,2)$.

B. Transformations

1. **Images of points**
e.g. f is the transformation
$(x,y)\to(x',y')$
where $x'=-x+3y$, $y'=2x+y$.
If $p=(-3,1)$, $q=(0,7)$ and $r=(2,4)$, find $f(p)$, $f(q)$ and $f(r)$.
Investigate if
area $\Delta\, pqr$ = area $\Delta\, f(p)f(q)f(r)$.
2. **Image of a line**
e.g. f is the transformation
$(x,y)\to(x',y')$
where $x'=-3x+y$, $y'=2x+3y$.
(i) L is the line $3x-4y=8$. Find the equation of $f(L)$.
(ii) M is the line $3x-4y=k$. Find the equation of $f(M)$, and verify that $f(M)\parallel f(L)$.
3. **Image of a line segment**
e.g. f is the transformation
$(x,y)\to(x',y')$
where $x'=x+2y$, $y'=-x+3y$.
(i) If $a=(4,-1)$ and $b=(-1,2)$, show that
$x=4-5t$, $y=-1+3t$,
for $0\le t\le 1$, are parametric equations of $[ab]$.
(ii) Prove that $f([ab])$ is a line segment.

Scribble box

Trigonometry

After the huge scare in 2006, the two trigonometry questions, Questions 4 and 5, returned to normal in 2007. Although many students say that they don't like trig, most end up doing at least one of these questions in the Leaving Cert. Most students also recognise that they need to be quite proficient with trig for many other areas, especially differentiation, integration and complex numbers.

Certain patterns have emerged over the years (2006 excluded) in the two trig questions. Although there is no guarantee of a continuation, it is still worth looking closely at the patterns.

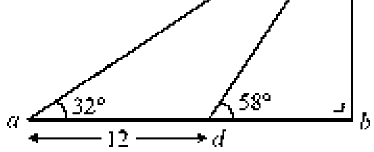
For starters, for the last six years, Question 4(b) has dealt with trig equations, along with an associated trig identity. Also Question 4(c) has been a practical problem connected with circles. You could do far worse than go through these questions in great detail.

For a similar time-frame, Question 5(b) and 5(c) have dealt with proving and using trig identities, and 3-dimensional practical problems, with the latter usually occupying the (c) part. Again this is worth noting.

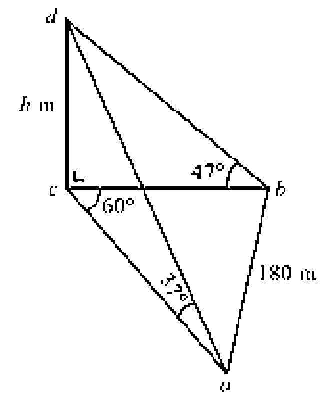
Besides practical trig, i.e. the sine and cosine rules, arc and sectors, the next most important area to study is trig identities. The first thing to do is to learn the twelve set proofs on the course: you are more than likely to find one of them on the paper in June. Proving other trig identities has not been that difficult for a number of years now.

Finally, inverse trig graphs and trig limits might appear, but are less likely.

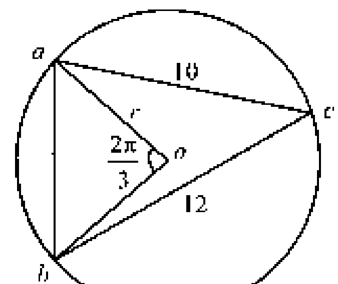
1. **Basic definitions**
e.g. if $\sin A=\frac{t}{t+1}$, for $0°\le A\le 90°$, express $\tan A$ in terms of $t\in\mathbb{N}$
2. **Right-angled triangles**
e.g. In the triangle shown below, $ab\perp bc$. If $|ad|=12$, $|\angle cab|=32°$ and $|\angle cdb|=58°$, find $|cb|$, correct to two decimal places.



3. **General triangles**



e.g. a, b and c are three points on horizontal ground, with $|ab|=180$ m and $|\angle acb|=60°$.
A vertical mast, $[cd]$, of height h m, is placed at c. The angle of elevation of d from a is $37°$, while the angle of elevation of d from b is $47°$.
(i) Express $|ac|$ and $|bc|$ in terms of h.
(ii) Hence, or otherwise, find h, correct to the nearest metre.
4. **Arcs and sectors**



e.g. a, b and c are three points on a circle with centre o and radius r.
$|ac|=10$, $|bc|=12$ and $|\angle oab|=\frac{2\pi}{3}$.
(i) Calculate $|ab|$, correct to two decimal places.
(ii) Find the value of r, correct to two decimal places.
(iii) Calculate the area of the shaded region shown.
5. **Twelve standard proofs**
e.g. prove that
$\sin 2A=\frac{2\tan A}{1+\tan^2 A}$,
and use this identity to express $\tan 75°$ in surd form
6. **Basic trigonometric identities**
e.g. prove that
$(\sin A+\cos A)^2+(\sin A-\cos A)^2=2$

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7. **Other identities**
e.g. prove that
$$\frac{\sin(A+B)}{\sin(A-B)}=\frac{\tan A+\tan B}{\tan A-\tan B}$$

8. **Simple trigonometric equations**
e.g. solve the equation
$$\sin\theta=-\frac{\sqrt{3}}{2},$$
for $0^\circ\le\theta\le 360^\circ$

9. **Harder trigonometric equations**
e.g. solve the equation
$$\sin^2\theta-\cos\theta+1=0,$$
for $0^\circ\le\theta\le 360^\circ$

10. **Trigonometric limits**
e.g. evaluate
$$\lim_{x\to 0}\frac{\sin 3x}{x\cos x}.$$

Probability

Section A finishes with two questions which are commonly known as the probability questions. However, a better name would be 'discrete maths' as the questions cover probability, statistics and difference equations.

Certain patterns are well established with respect to the format of Questions 6 and 7. Question 6(b) has for many years dealt with difference equations. On one or two occasions, if it is particularly difficult, it has moved to the (c) part. Question 7(c) has always dealt with abstract statistics questions.

The remaining four parts of the two questions then deal with arrangements, combinations and probability. An odd time, one of these four parts has dealt with numerical statistics, e.g. the weighted mean.

Because of this, students who are not at all well disposed towards probability are often surprised to find that some Leaving Cert. questions contain only one part on probability itself.

For safety reasons, you should have at least one of these questions ready to go if one of your more popular questions is unusually difficult. This is of course, assuming that you are not a probability buff, in which case you will have these questions near the top of your to-do list.

1. **Fundamental principle of counting**
e.g. how many different four digit numbers greater than 5000 can be formed from the digits 2, 4, 5, 8, 9 if each digit can be used only once in any given number?
How many of these numbers are odd?

2. **Arrangements (permutations)**
e.g. how many arrangements are possible of all of the letters of the word COURTED?
In how many of these arrangements are the three vowels side by side?
In how many of these arrangements do the three vowels occupy the last three positions?

3. **Combinations (choices)**
e.g. a woman has eleven close friends.
(i) In how many ways can she invite five of them to dinner?
(ii) In how many ways can she invite five of them if two are married and will only attend if both are invited?
(iii) In how many ways can she invite five of them if two of her friends are not on speaking terms and will not attend together?

4. **Probability**
e.g. three discs are chosen at random, without replacement, from a bag containing 3 red, 8 blue and 7 white discs. Find the probability that the chosen discs will be
(i) all blue.
(ii) one of each colour.
(iii) two of one colour and one of a different colour.

5. **The mean and weighted mean**
e.g. if the mean of 8, $x+1$, 2, $2x+1$ is $x+2$, find the value of x

6. **Standard deviation**
e.g. if σ is the standard deviation of
$$a,\ b,\ c,$$
show that the standard deviation of
$$4a-1,\ 4b-1,\ 4c-1$$
is 4σ

7. **Proof of difference equations formula**

8. **Difference equations**
e.g. solve the difference equation
$$2u_{n+2}-3u_{n+1}+u_n=0,$$
if $u_0=5$ and $u_1=\frac{7}{2}$.

Further Calculus and Series

Question 8, the first question in Section B, is by far the most popular option chosen by students each year. In the vast majority of cases, this is because it is the only option topic they studied in school. Studying one option topic is more or less unavoidable, due to the length of the course and the time constraints.

Further calculus and series is composed of four major topics: integration by parts, the Ratio Test, Maclaurin series and maximum and minimum problems. These are quite distinct topics, with the exception of the use of the Ratio Test with Maclaurin series.

Of the four topics, the only one which has occurred every year is maximum and minimum problems. Last year this was just the (a) part of the question. It is likely to be a more substantial part this year.

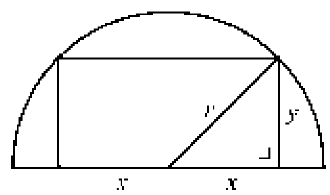
One other key idea to watch is writing down an expression for the general term in a series obtained from the Maclaurin series formula. This is traditionally a hugely problematic area for students. Failure to obtain a general term expression can mean that we may not be able to complete the question, if we are subsequently asked to use the Ratio Test to test the series for convergence.

1. **Integration by parts**
e.g. use integration by parts to find
$$\int_1^e x\ln x\,dx.$$

2. **Ratio Test**
e.g. show that the series $\sum_{n=0}^{\infty}\frac{(-1)^n x^{n-1}}{(2n+1)!}$ is convergent for all $x\in\mathbf{R}$

3. **Maclaurin series**
e.g. the Maclaurin series for $f(x)$ is
$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\ldots$$
(i) Derive the Maclaurin series for $f(x)=\cos x$, up to and including the term containing x^5.
(ii) Write down the general term, and use the Ratio Test to show that the series converges for all $x\in\mathbf{R}$.

4. **Maximum and minimum problems**
e.g.



A rectangle of dimensions $2x$ and y is drawn inside a semi-circle of radius r as shown.
Express the area of the rectangle in terms of x and r.
Find the value of x for which the area of the rectangle is a maximum, and find this maximum area.

Scribble box

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SAMPLE QUESTIONS

1. The Circle

Question

(a) The circle C has the lines $x=1$, $x=7$, $y=-2$ and $y=4$ as tangents. Find the equation of C.

(b) The circle S_1 has equation
$$x^2+y^2+2x-4y-15=0$$
and the circle S_2 has equation
$$x^2+y^2-10x-10y+k=0,$$
where k is a constant.

(i) Find the value of k if S_1 and S_2 touch externally.

(ii) With this value of k, S_1 and S_2 touch at the point a. Find the equation of the common tangent to S_1 and S_2 at a.

(c) The line $L: x+5y-27=0$ intersects the circle S at the points a and b where $|ab|=\sqrt{26}$. The centre of S is $(-1,3)$.

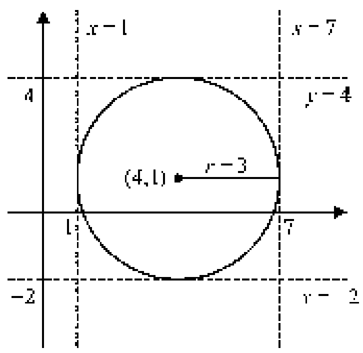
(i) Find the perpendicular distance from the centre of S to L.

(ii) Find the equation of the circle S.

(iii) Find the equations of the two tangents to S which have slope $\frac{2}{3}$.

Solution

a) (i)



From the diagram, centre $=(4,1)$ and radius length $=3$.

Thus the equation of the circle C is
$$(x-4)^2+(y-1)^2=9.$$

(b) (i) $S_1: x^2+y^2+2x-4y-15=0$

centre $=(-1,2)$

radius $=r_1$

$=\sqrt{1+4+15}=\sqrt{20}$

$S_2: x^2+y^2-10x-10y+k=0$

centre $=(5,5)$

radius $=r_2$

$=\sqrt{25+25-k}$

$=\sqrt{50-k}$

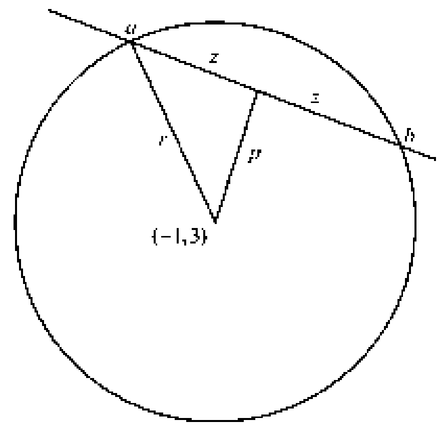
Also, distance between the centres is

$$d=\sqrt{(5+1)^2+(5-2)^2}=\sqrt{45}$$

If the circles touch externally, then
$$r_1+r_2=d$$
$$\sqrt{20}+\sqrt{50-k}=\sqrt{45}$$
$$2\sqrt{5}+\sqrt{50-k}=3\sqrt{5}$$
$$\sqrt{50-k}=\sqrt{5}$$
$$50-k=5$$
$$k=45.$$

(ii) The equation of the common tangent to both circles at their point of intersection is
$$S_1-S_2=0$$
$$(x^2+y^2+2x-4y-15)$$
$$-(x^2+y^2-10x-10y+45)=0$$
$$12x+6y-60=0$$
$$2x+y-10=0.$$

(c) (i)



Let p be the perpendicular distance from the centre to the line $L: x+5y-27=0$. Then
$$p=\frac{|(-1)+5(3)-27|}{\sqrt{1^2+5^2}}$$
$$=\frac{|-13|}{\sqrt{26}}=\frac{13}{\sqrt{13}\sqrt{2}}=\frac{\sqrt{13}}{\sqrt{2}}$$

(ii) Let the length of the chord, $|ab|$, be $2z$. Then
$$2z=\sqrt{26}$$
$$z=\frac{\sqrt{26}}{2}=\sqrt{\frac{26}{4}}=\sqrt{\frac{13}{2}}$$

If r is the length of the radius, then
$$r^2=p^2+z^2$$
$$r^2=\frac{13}{2}+\frac{13}{2}=13$$

The equation of the circle S is
$$(x+1)^2+(y-3)^2=13$$
or $\quad x^2+y^2+2x-6y-3=0.$

(iii) Let the equation of a tangent to the circle C with slope $\frac{2}{3}$ be
$$2x-3y+k=0.$$

Then the perpendicular distance from the centre $(-1,3)$ to this tangent is $r=\sqrt{13}$.
$$\frac{|2(-1)-3(3)+k|}{\sqrt{2^2+(-3)^2}}=\sqrt{13}$$
$$|k-11|=13$$
$$k-11=13 \quad \text{or} \quad k-11=-13$$
$$k=24 \quad \text{or} \quad k=-2$$

The required tangents are $2x-3y+24=0$ and $2x-3y-2=0$.

2. The Line and Transformations

Question

(a) $p(2t-3, 5t+1)$ is on a line L, for all values of $t\in\mathbb{R}$. Write the equation of L in the form $ax+by+c=0$.

(b) (i) $M: y=m_1x+c_1$ and $N: y=m_2x+c_2$ are two intersecting lines. If θ is an angle between M and N, prove that
$$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}.$$

(ii) If the angle between the line $y=mx+c$ and the line $x+y=7$ is $\tan^{-1}\frac{2}{3}$, find the two possible values of m.

(c) f is the transformation $(x,y)\to(x',y')$ where $x'=x+2y$ and $y'=3x-y$.

(i) Express x and y in terms of x' and y'.

(ii) L is the line $4x+7y+3=0$. Find the equation of $f(L)$.

(iii) K is a line such that $f(K)$ contains the point $(1,2)$ and is perpendicular to $f(L)$. Find the equation of $f(K)$ and the equation of K.

Solution

(a) $p(2t-3, 5t+1)$ is on L. Thus

$x=2t-3 \qquad y=5t+1$

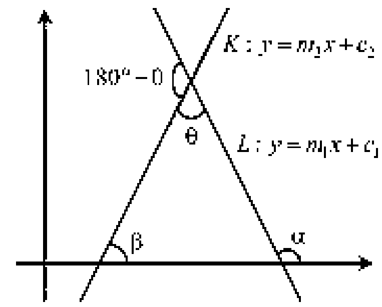
$x+3=2t \qquad y-1=5t$

$\frac{x+3}{2}=t \qquad \frac{y-1}{5}=t$

Then $\frac{x+3}{2}=\frac{y-1}{5}$
$$5x+15=2y-2$$
$$5x-2y+17=0$$
is the equation of L.

(b) (i) Let α be the angle between L and the positive x-axis. Let β be the angle between K and the positive x-axis, and let θ be an angle between L and K.

$\tan\alpha=m_1$ and $\tan\beta=m_2$.



$$\tan\theta=\tan(\alpha-\beta)$$
$$\tan\theta=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$
$$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}$$

(ii) Let $m_1=m$ be the slope of $y=mx+c$ and $m_2=-1$ be the slope of the line $x+y=7$.

$\theta=\tan^{-1}\frac{2}{3}$ is the angle between these lines. Thus $\tan\theta=\frac{2}{3}$.

Then
$$\tan\theta=\pm\frac{m_1-m_2}{1+m_1m_2}$$
$$\frac{2}{3}=\pm\frac{m+1}{1-m}$$

Case 1:
$$\frac{2}{3}=\frac{m+1}{1-m}$$
$$2-2m=3m+3$$
$$-1=5m$$
$$m=-\frac{1}{5}$$

Case 2:
$$\frac{2}{3}=-\frac{m+1}{1-m}$$
$$2-2m=-3m-3$$
$$m=-5$$

(c) (i) $f:(x,y)\to(x',y')$ where $x'=x+2y$ and $y'=3x-y$

Then
$$x+2y=x'$$
$$6x-2y=2y'$$
$$7x\quad=x'+2y'$$
$$x=\frac{x'+2y'}{7}$$

Also

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Paper 2

SAMPLE QUESTIONS

$$3x + 6y = 3x'$$
$$-3x + y = -y'$$
$$7y = 3x' - y'$$
$$y = \frac{3x' - y'}{7}$$

(ii) $L:\ 4x + 7y + 3 = 0$

$f(L)$:

$$4\left(\frac{x' + 2y'}{7}\right) + 7\left(\frac{3x' - y'}{7}\right) + 3 = 0$$
$$4x' + 8y' + 21x' - 7y' + 21 = 0$$
$$25x' + y' + 21 = 0$$

(iii) Slope of $f(L) = -25$

As $f(K) \perp f(L)$, slope of $f(K) = \frac{1}{25}$.

$f(K)$ contains the point $(1, 2)$. Thus the equation of $f(K)$ is

$$y' - 2 = \frac{1}{25}(x' - 1)$$
$$25y' - 50 = x' - 1$$
$$x' - 25y' + 49 = 0$$

Then the equation of K:

$$(x + 2y) - 25(3x - y) + 49 = 0$$
$$x + 2y - 75x + 25y + 49 = 0$$
$$-74x + 27y + 49 = 0$$

or $\quad 74x - 27y - 49 = 0$.

3. Trigonometry

Question

(a) If $\tan A = \frac{3}{4}$, for $0° < A < 90°$, express

(i) $\cos A$, (ii) $\cos 2A$,

in the form $\frac{p}{q}$, where $p, q \in \mathbb{N}$.

(b) (i) Express $\cos 3x - \cos x$ as a product of sines.

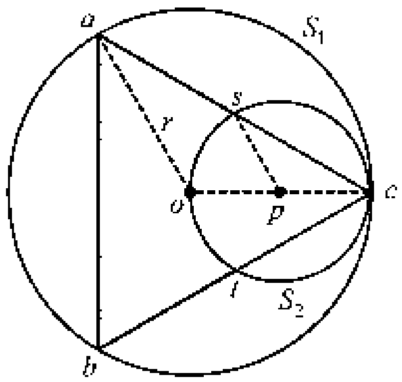
(ii) Hence, or otherwise, find all solutions of the equation

$\sin x + \cos 3x - \cos x = 0$,

for $0° \leq x \leq 360°$.

(c) S_1 is a circle with centre o and radius length r containing the points a, b and c such that abc is an equilateral triangle.

S_2 is a circle with centre p and $[oc]$ as a diameter. S_2 intersects $[ac]$ and $[bc]$ at s and t respectively.



(i) Express $|ab|$ and the area of the triangle abc in terms of r.

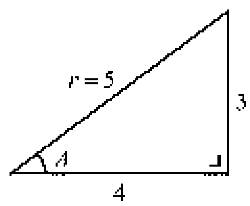
(ii) Express the area of the triangle cps in terms of r.

(iii) Express the area of the shaded region in terms of r and π.

Solution

(a) (i) From the triangle,

$$r^2 = 4^2 + 3^2$$
$$r^2 = 25$$
$$r = 5$$



Thus $\quad \cos A = \frac{4}{5}$

(ii) $\cos 2A = \cos^2 A - \sin^2 A$

$$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$$
$$= \frac{16}{25} - \frac{9}{25}$$
$$= \frac{7}{25}.$$

(b) (i) $\cos 3x - \cos x$

$$= -2\sin\left(\frac{3x + x}{2}\right)\sin\left(\frac{3x - x}{2}\right)$$
$$= -2\sin 2x \sin x$$

(ii) $\sin x + [\cos 3x - \cos x] = 0$

$$\sin x - 2\sin 2x \sin x = 0$$
$$\sin x(1 - 2\sin 2x) = 0$$
$$\sin x = 0 \quad \text{or} \quad 1 - 2\sin 2x = 0$$
$$\sin x = 0 \quad \text{or} \quad \sin 2x = \frac{1}{2}$$

$\sin x = 0$:

$x = 0°, 180°, 360°$

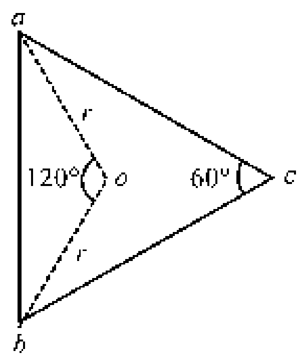
$\sin 2x = \frac{1}{2}$:

$2x = 30°, 150°, 390°, 510°$

$x = 15°, 75°, 195°, 255°$

(c) (i) Using the Cosine Rule on the triangle oab.

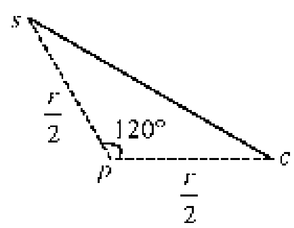
$$|ab|^2 = r^2 + r^2 - 2r.r.\cos 120°$$
$$|ab|^2 = 2r^2 - 2r^2\left(-\frac{1}{2}\right)$$
$$|ab|^2 = 3r^2$$
$$|ab| = \sqrt{3}\, r$$



Also

$$\text{area } \Delta abc = \frac{1}{2}|ab||bc|\sin 60°$$
$$= \frac{1}{2}\left(r\sqrt{3}\right)\left(r\sqrt{3}\right)\frac{\sqrt{3}}{2}$$
$$= \frac{3\sqrt{3}r^2}{4}$$

(ii)



$$\text{Area } \Delta cps = \frac{1}{2}\left(\frac{r}{2}\right)\left(\frac{r}{2}\right)\sin 120°$$
$$= \frac{r^2}{8}\left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{\sqrt{3}r^2}{16}$$

(iii) Shaded area

= area $\Delta abc - 2$ area Δspc

− area sector $spto$

$$= \frac{3\sqrt{3}r^2}{4} - \frac{\sqrt{3}r^2}{8} - \frac{1}{2}\left(\frac{r}{2}\right)^2\left(\frac{2\pi}{3}\right)$$
$$= \frac{\sqrt{3}r^2}{8}(6 - 1) - \frac{\pi r^2}{12}$$
$$= \frac{5\sqrt{3}}{8}r^2 - \frac{\pi r^2}{12}.$$

4. Probability

Question

(a) A student sits five exams to obtain a qualification. The weights for these exams are 5, 2, 3, 1, 4 respectively. The student scores 73%, 60%, 45% and 48% in the first four exams. What percentage score is required in the final exam to obtain a weighted mean percentage of 60%?

(b) $u_n = k\left(\frac{1}{2}\right)^n + m\left(\frac{2}{3}\right)^n$ is the general solution of the difference equation

$$6u_{n-2} + pu_{n+1} + qu_n = 0,$$

where p and q are constants.

Determine the value of p and the value of q.

If $u_0 = 5$ and $u_1 = 3$, determine the values of the constants k and m, and find u_4 in the form $\frac{a}{b}$, where $a, b \in \mathbb{N}$.

(c) A shelf contains six different books in English, five different books in French and four different books in German.

(i) If two books are picked at random, what is the probability that they are in the same language?

(ii) If two books are picked at random, what is the probability that they are in different languages?

(iii) If three books are picked at random, what is the probability that they are not all in the same language?

(iv) If six books are picked at random, what is the probability that there are exactly two in each language?

Solution

(a) Let x% be the required mark in the last test to get a weighted mean of 60%.

Then

$$60 = \{(73 \times 5) - (60 \times 2) + (46 \times 3) - (48 \times 1) + (x \times 4)\} / (5 + 2 - 3 + 1 + 4)$$
$$60 = \frac{668 + 4x}{15}$$
$$900 = 668 + 4x$$
$$232 = 4x$$
$$x = 58$$

(b) From $u_n = k\left(\frac{1}{2}\right)^n + m\left(\frac{2}{3}\right)^n$, $\frac{1}{2}$ and $\frac{2}{3}$ are the roots of the characteristic equation.

$$\text{sum of roots} = \frac{1}{2} - \frac{2}{3} = \frac{7}{6}$$
$$\text{product of roots} = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$$

Higher level
Paper 2

SAMPLE QUESTIONS

Characteristic equation:

$$x^2 - \frac{7}{6}x + \frac{1}{3} = 0$$

or $6x^2 - 7x + 2 = 0$

Comparing this with

$$6u_{n+2} - pu_{n+1} + qu_n = 0,$$

we get $p = -7$ and $q = 2$.

If $u_n = k\left(\frac{1}{2}\right)^n + m\left(\frac{2}{3}\right)^n$.

$u_0 = 5$: $k\left(\frac{1}{2}\right)^0 + m\left(\frac{2}{3}\right)^0 = 5$

$k + m = 5$ ...1

$u_1 = 3$: $k\left(\frac{1}{2}\right)^1 + m\left(\frac{2}{3}\right)^1 = 3$

$3k + 4m = 18$ ...2

Thus

1×-3: $-3k - 3m = -15$

2: $3k + 4m = 18$

$m = 3$

and $k = 2$.

Then

$$u_4 = 2\left(\frac{1}{2}\right)^4 + 3\left(\frac{2}{3}\right)^4 = \frac{1}{8} + \frac{16}{27} = \frac{155}{216}.$$

(c) 6 English, 5 French and 4 German books.

(i) Experiment:
choose 2 books from 15

$$n = \#S = \binom{15}{2} = 105$$

E: choose 2 books in same language
= (2 from 6 Eng)
or (2 from 5 Fr)
or (2 from 4 Ger)

$$r = \#E = \binom{6}{2} + \binom{5}{2} + \binom{4}{2} = 31$$

$$P(E) = P(\text{same language}) = \frac{31}{105}$$

(ii) Success = 2 books in different languages
Failure = 2 books in the same language

By (i),

$$P(\text{failure}) = P(\text{same lang.}) = \frac{31}{105}$$

$$P(\text{success}) = 1 - P(\text{failure})$$

$$P(\text{diff lang.}) = 1 - \frac{31}{105} = \frac{74}{105}$$

(iii) Experiment:
choose 3 books from 15

$$n = \#S = \binom{15}{3} = 455$$

Success: not all in the same language
Failure: all in the same language

$$r = \#E = \binom{6}{3} + \binom{5}{3} + \binom{4}{3} = 34$$

$$P(\text{failure}) = \frac{34}{455}$$

$$P(\text{success}) = 1 - \frac{34}{455} = \frac{421}{455}$$

(iv) Experiment:
choose 6 books from 15

$$n = \#S = \binom{15}{6} = 5005$$

E: choose (2 from 6 Eng)
and (2 from 5 Fre)
and (2 from 4 Ger)

$$r = \#S = \binom{6}{2} \times \binom{5}{2} \times \binom{4}{2}$$

$r = 900$

$$P(E) = P(\text{two in each language})$$

$$= \frac{900}{5005}$$

$$= \frac{180}{1001}.$$

5. Further Calculus

(a) Use integration by parts to evaluate

$$\int_0^1 (2x+1)e^x \, dx.$$

(b) A rectangular block has a square base and a total surface area of 54 cm^2.
Find the maximum possible volume of the block.

(c) **(i)** Find the value of $k \in \mathbf{R}$ if

$$\tan^{-1}\frac{1}{2} + \tan^{-1}k = \frac{\pi}{4}.$$

(ii) The Maclaurin series for $\tan^{-1}x$ is

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

Use the first four terms of this series to approximate $\tan^{-1}\frac{1}{2}$ and hence find an approximation for π, correct to three decimal places.

Solution

(a) $I = \int_0^1 (2x+1)e^x \, dx$

Let $u = 2x+1$ $\quad dv = e^x dx$

$\frac{du}{dx} = 2$ $\quad v = \int e^x dx$

$du = 2dx$ $\quad v = e^x$

$$I = \int_0^1 u\,dv = uv\Big|_0^1 - \int_0^1 v\,du$$

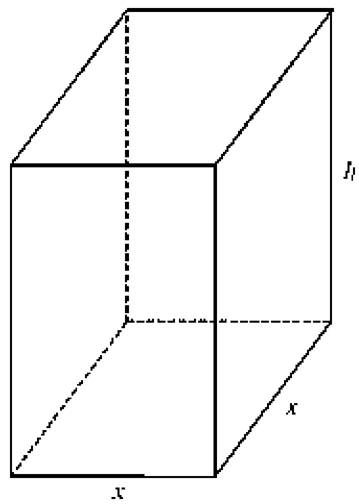
$$= \Big[(2x+1)e^x\Big]_0^1 - \int_0^1 e^x .2dx$$

$$= (3e-1) - 2\Big[e^x\Big]_0^1$$

$$= (3e-1) - 2(e-1)$$

$$= e - 1.$$

(b) Let x be the length of a side of the square base, and let h be the height of the block.



To be a maximum:

$V = x^2h$

Given:

Total surface area = 54

$2(x^2) + 4(xh) = 54$

$2xh = 27 - x^2$

$$h = \frac{27 - x^2}{2x}$$

Then

$$V = x^2\left(\frac{27 - x^2}{2x}\right)$$

$$V = \frac{1}{2}(27x - x^3)$$

Thus

$$\frac{dV}{dx} = \frac{1}{2}(27 - 3x^2)$$

Put $\frac{dV}{dx} = 0$:

$27 - 3x^2 = 0$

$x^2 = 9$

$x = 3$

The volume of the block is a maximum when $x = 3$.

When $x = 3$, the maximum volume of the block is

$$\frac{1}{2}\left[27(3) - (3)^3\right] = \frac{54}{2} = 27 \text{ cm}^3.$$

(c) **(i)** $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}k = \frac{\pi}{4}$

$$\tan^{-1}\left(\frac{\frac{1}{2} + k}{1 - \frac{1}{2}k}\right) = \frac{\pi}{4}$$

$$\frac{1+2k}{2-k} = \tan\frac{\pi}{4}$$

$$\frac{1+2k}{2-k} = 1$$

$1 + 2k = 2 - k$

$3k = 1$

$$k = \frac{1}{3}$$

(ii) $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$

$$\tan^{-1}\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right) - \frac{1}{3}\left(\frac{1}{2}\right)^3 + \frac{1}{5}\left(\frac{1}{2}\right)^5 - \frac{1}{7}\left(\frac{1}{2}\right)^7$$

$= 0 \cdot 463467261$

$$\tan^{-1}\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right) - \frac{1}{3}\left(\frac{1}{3}\right)^3 + \frac{1}{5}\left(\frac{1}{3}\right)^5 - \frac{1}{7}\left(\frac{1}{3}\right)^7$$

$= 0 \cdot 321745378$

Then

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$$

$= 0 \cdot 463467261$
$+ 0 \cdot 321745378$

$$\frac{\pi}{4} = 0 \cdot 785212639$$

$\pi = 3 \cdot 140850558$

$\pi = 3 \cdot 141$, correct to three decimal places.

Scribble box

The 2007 A1 club at the Institute of Education

Institute students who obtained six or more A1 grades (600 points) in Leaving Certificate 2007.
From left: Grainne Rooney - Medicine, NUI Galway; Emer Gilhooley - Medicine, Trinity; Rosie Plunkett - Medicine, Trinity; Niall Nelligan - Medicine, UCD; Gillian Judge - Medicine, Trinity; Amy Kelly - Teaching St Pats; Olivia Murphy - Medicine, UCD

Dos and Don'ts: How to avoid the common mistakes in Maths exams

By Aidan Roantree

Dos:

■ Arrive with plenty of time to spare. If you are rushing into the exam, you will not be in the right frame of mind to do yourself justice, at least for the first while.

■ Start immediately, by reading the entire paper, or at least any question that you may possibly do.

■ Answer the questions on the basis of 'easiest first'. Start with the question you consider the easiest, then do the next easiest, and so on.

■ Read each question you are attempting extremely carefully, making note of all that is required of you.

■ Show your calculations. You sometimes (not always) get full marks for a correct answer without work, but if you get the wrong answer, you get nothing.

■ Plan your timing carefully. In theory you have 25 minutes per question. In practice, you should aim to complete each question in 20 minutes. This will leave a little time for reading the paper at the beginning and checking answers at the end.

■ Watch the key words, or even underline them. Some examples are prove, verify, show, find, solve, evaluate, graph, plot. If a question says 'hence', as distinct from 'hence, or otherwise', you must use what went before to complete what follows. Using any other method will not get you the marks.

■ Before leaving a question, check that you have answered everything required. If there is some part you cannot do, leave space to return later.

■ Make some attempt at every part of the questions you are doing. Any right step will get you at least the attempt mark for that part.

■ Realise the importance of algebra, and the need for care and accuracy when using algebra.

Don'ts

■ Don't, if at all possible, rely on exactly six questions on each paper. Try to have a standby question, should one of your preferred questions not be as easy as you hoped.

■ Don't write out the question, or any part of it in the exam. This wastes valuable time and is completely unnecessary.

■ Don't forget to bring your calculator, pens, maths instruments, and perhaps some sugary sweets or chocolate into the exam. By the end, you could probably do with the energy boost.

■ Don't do any rough work on your exam paper. Everything should be done in your answer-book. Rough work often merits marks.

■ Don't perform any difficult calculations in your head. Use your calculator.

■ Don't forget that you must attempt one question from Section B on Paper 2.

■ Don't bring in a new calculator bought on the morning of the exam. You need to be fully familiar with all the common operations on your calculator.

■ Don't spend time trying to guess answers. Even if you get lucky every now and then, the risk is too great that you are just wasting time.

■ Don't daydream, or become worried about how you are doing. There will be time for both after the exam. Stay focussed.

■ Don't spend too much time on a difficult final part: there are probably only going to be a few marks allocated to it.

Ordinary level
Paper 1

Your Leaving Cert. maths exams will begin on June 6th with Paper 1. This is a Friday, and the exam is in the morning. By this stage, English, Home Economics and Chemistry will be over. Even if you do neither of the latter two subjects, you should be well into the mindset for doing exams.

But preparing for a maths exam requires you to focus differently from other subjects. Maths is a very precise subject, and we don't wax eloquent about quadratic equations in the same way that we can about a Yeats poem.

Paper 1 is very heavily biased towards algebra. Of the eight questions on this paper, only Question 1, on arithmetic and money, rarely has any algebra. Questions 2 and 3 are entirely dedicated to algebra itself. Question 4, on complex numbers, involves much algebraic manipulation. Question 5, on sequences and series, has a little less algebra.

Questions 6 and 8, on functions, graphs and differentiation, and Question 7, on differentiation alone, involve their fair share of solving equations and rewriting expressions.

All in all, if you are weak at algebra, you will find Paper 1 troublesome. With this in mind, you should focus now on making sure that your grasp of the main algebraic techniques is adequate.

Even though it might appear to be very basic, and almost a waste of time, you should thoroughly revise the following algebraic techniques:

* substituting values into expressions,
* simplifying expressions,
* removing brackets by multiplication,
* inserting brackets by taking out a common factor,
* rearranging formulae,
* simplifying equations,
* solving linear equations,
* solving quadratic equations
* using *f(x)* notation.

These techniques will be frequently required throughout both papers, but especially on Paper 1. If one piece of advice were to be singled out, it would be to treat the equal to sign, '=', with respect. Don't leave it out: it is the verb in a mathematical sentence. Try to avoid mathematical islands, i.e. expressions written on their own, not linked to anything.

Although we are aware of what topics will be examined in which questions, it is still not a good idea to enter the exam with just six questions prepared. The content and question style can change quite a bit in any given year. This is notably true of Question 1, Question 6 and Question 8. You should enter the exam with an open mind, read the entire paper and then choose the questions which suit you best this year.

Arithmetic and Money

The topic covered by Question 1 is arithmetic and money. There are a few formulae you need to learn here:

* compound interest rule: for each year

$$A = P\left(1+\frac{R}{100}\right)$$

* compound interest formula: after n years

$$A = P\left(1+\frac{R}{100}\right)^n$$

* percentage error

$$=\frac{|\text{true value} - \text{estimate}|}{\text{true value}}\times 100\%$$

There are also a number of methods which must be known:

* dealing with fractions, percentages, ratios and proportions,
* calculating income tax,
* using scientific notation,
* handling time and speed calculations.

This however, is not the full story. Each year, a major part of Question 1 concerns a real-life, practical situation, where all that is required is basic common sense, and good numerical skills. You should look carefully at the questions of this type that have been asked for the last ten or so years. However, expect a new scenario to appear this year. Unless you are comfortable dealing with such practical problems, you should probably consider leaving out this question.

1. **Fractions**
   e.g. $\frac{3}{7}$ of a sum of money is €264. Find the sum of money.
2. **Ratios**
   e.g. express the ratio
   $$\frac{3}{2}:\frac{5}{3}:\frac{7}{4}$$
   in the form $p:q:r$, where $p, q, r \in \mathbb{N}$
3. **Proportional parts**
   e.g. a sum of money is divided between Sean, Therese and Mary in the ratio $5:2:4$. If Sean gets €200,
   (i) what is the sum of money,
   (ii) how much do Therese and Mary get?
4. **Direct proportion**
   e.g. 7 diaries cost €39·55. How many of these diaries can be bought for €50·85?
5. **Currency conversions**
   e.g. a woman wants to change 412500 Japanese Yen into euro. The exchange rate is €1 = 165 Yen. The bank charges €20 for this conversion. How much, in euro, does the woman get?
6. **Percentages**
   e.g. express 624 grams as a percentage of 2·6 kilograms
7. **VAT**
   e.g. a computer is for sale at €1149·50, inclusive of VAT at 21%. What is the price of the computer exclusive of VAT?
8. **Profit and loss**
   e.g. a car dealer sells a car for €17400, making a profit of 20%. How much had she paid for the car?
9. **Income tax**
   e.g. Mark earns €5860 in a month and has tax credits of €240 for that month. The standard rate cut off point is €2950 and the standard and higher rates of tax are 20% and 41% respectively. Calculate Mark's take home pay for the month.
10. **Compound interest rule**
    e.g. a man invests €8500. He gets 3.5% per annum compound interest in the first year and 4.2% in the second year. Calculate the value of his investment at the end of the second year.
11. **Compound interest formula**
    e.g. what sum of money, invested now at 3% per annum compound interest, will amount to €4919·50 in seven years time?
12. **Time and speed**
    e.g. a woman drives from Wexford to Sligo. She leaves Wexford at 9:35 and arrives in Sligo at 16:23. If the distance from Wexford to Sligo is 307 km, what was her average speed, correct to the nearest km/h?
13. **Scientific notation**
    e.g. write as a decimal
    $$(7\cdot 9\times 10^7)-(8\cdot 2\times 10^6)$$
14. **Approximation and error**
    e.g. if $60 - 35$ is taken as an approximation for $58\cdot 7 + 34\cdot 8$, calculate, correct to two decimal places, the percentage error.

Algebra

Algebra is the topic covered in Questions 2 and 3. Most students end up trying at least one of these questions. It is not possible to reliably predict what will appear in each question, so it will be necessary to study all of algebra to be sure of even one question.

The only formulae that you need to learn in algebra are the quadratic formula

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

and the laws of indices.

But there are many, many methods that have to be revised. The most important of these are:

* re-arranging a formula,
* solving linear equations,
* solving quadratic equations,
* solving equations with fractions,
* solving simultaneous equations,
* using the Factor Theorem,
* solving cubic equations,
* rewriting powers, roots and surds,
* solving equations using the laws of indices,
* solving inequalities.

As you revise these topics, you should concentrate on paying attention to detail. This is not just a request to be as neat as your personality allows, it is a warning that sloppy work usually results from sloppy thoughts, and generally leads to more mistakes than those made by an organised student. Even if you are not an organised person, pretend for one day that you are!

1. **Evaluating expressions**
   e.g. find the value of $a^2 - 2ab$ when $a = 2$ and $b = 3$
2. **Linear equations**
   e.g. solve for x:
   $$2(5x-11)-4(5-x)$$
3. **Linear equations with fractions**
   e.g. solve for x:
   $$\frac{4-x}{3}=\frac{3x+2}{5}$$
4. **Manipulating formulae**
   e.g. write a in terms of b and c if
   $$\frac{a+2b}{4}=3a+c$$
5. **Linear simultaneous equations**
   e.g. solve the simultaneous equations
   $$x+3y=8$$
   $$2x-y=-5$$
6. **Quadratic equations**
   e.g. solve for $x \in \mathbf{R}$,
   $$2x^2-7x-4=0$$
7. **Linear/non-linear simultaneous equations**
   e.g. solve the simultaneous equations
   $$x+2y=5,\quad x^2-y^2=8$$
8. **Factor theorem**
   e.g. if $(x+2)$ is a factor of $x^3+kx^2-8x-12$, find the value of k
9. **Cubic equations**
   e.g. solve the equation

Ordinary level

Paper 1

$$x^3 + 2x^2 - 13x + 10 = 0$$

10. **Forming an equation**
    e.g. form a cubic equation with roots $4, -1$ and -2
11. **Equations with fractions**
    e.g. solve the equation
    $$\frac{2}{x+1} + \frac{3}{x+2} = 2, \quad x \neq -1, \ x \neq -2$$
12. **Powers**
    e.g. write as a power of 5:
    $$\frac{25}{5^{1-x}}$$
13. **Equations with the unknown in the index**
    e.g. solve for x:
    $$4^{x-1} = \left(\frac{8}{\sqrt{2}}\right)^3$$
14. **Inequalities**
    e.g. solve the inequality for $x \in \mathbf{R}$:
    $$\frac{x+2}{3} < \frac{2x-1}{5}.$$

Complex Numbers

Complex numbers, which is examined in Question 4 each year, has been remarkably consistent over the years. Most formulae and methods occur in any given year. Because of this predictably, this question enjoys great popularity with students. Put simply, if you tackle Question 4 for the last six or seven years, you will probably do very well with this question.

The only formulae to be learned are:

* quadratic formula:
  $$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
* conjugate:
  if $z = a + bi$, then $\bar{z} = a - bi$
* modulus:
  $$|a + bi| = \sqrt{a^2 + b^2}$$

In addition to this, the following methods should be mastered:

* adding, subtracting, multiplying complex numbers (remember $i^2 = -1$)
* division, by using the conjugate,
* representing complex numbers on an Argand diagram,
* equality:
  if $a + bi = c + di$
  then $a = c$
  and $b = d$.

1. **Addition and subtraction**
   e.g. if $z = 8 + 3i$ and $w = -4 - 3i$,
   express in the form $a + bi$:
   (i) $z + w$, (ii) $2z - w$
2. **Multiplication**
   e.g. if $z = -1 - 5i$ and $w = 2 + 7i$,
   express in the form $a + bi$:
   (i) zw
   (ii) z^2
3. **Conjugate and division**
   e.g. express $\frac{7+4i}{1+2i}$ in the form $a + bi$,
   where $i^2 = -1$
4. **Argand diagram**
   e.g. represent each of the following complex numbers on an Argand diagram:
   (i) $3 - 5i$, (ii) $-2 + 4i$,
   (iii) -2, (iv) $-i$
5. **Modulus**
   e.g. if $z = -1 + 3i$ and $w = 2 + 5i$,
   calculate (i) $|z|$, (ii) $|w|$,
   (iii) $|z - w|$
6. **Complex equations**
   e.g. if $z = 4 - 3i$, write $z^2 + 17$ in the form $a + bi$, $a, b \in \mathbf{R}$. Hence solve for real k:
   $$k(z^2 + 17) = |z|(1 - i).$$
7. **Quadratic equations**
   e.g. solve the following equation for $z \in \mathbf{C}$:
   $$z^2 - 2z + 65 = 0$$
8. **More quadratic equations**
   e.g. find the value of the real number a if $8 - 2i$ is a root of $z^2 + az + 68 = 0$,
   and find the other root.

Sequences and Series

The sequences and series question, Question 5, used to be one of the easiest on Paper 1. However, its standard has been raised over the last five or six years, and it is now on a par with the other questions.

This topic relies more on formulae than the previous topics. Nevertheless, substituting into a formula alone will not answer the question. We will also have to understand the symbols and concepts, and use a good bit of algebra.

The formulae you must know are:

* for all series:
  $$S_n = T_1 + T_2 + T_3 + \ldots + T_n,$$
* arithmetic general term:
  $$T_n = a + (n-1)d,$$
  where $a = T_1$ and $d = T_2 - T_1$,
* arithmetic sum to n terms:
  $$S_n = \frac{n}{2}\left[2a + (n-1)d\right],$$
* geometric general term:
  $$T_n = ar^{n-1},$$
  where $a = T_1$ and $r = \frac{T_2}{T_1}$,
* geometric sum to n terms:
  $$S_n = \frac{a\left(1 - r^n\right)}{1 - r}.$$

You should also be able to prove that a given sequence is arithmetic or geometric.

1. **Sequence notation**
   e.g. the nth term of a sequence is given by $T_n = 2n - 1$.
   (i) Find the first two terms of the sequence.
   (ii) Show that $T_2 + T_5 = T_3 + T_4$.
2. **Series**
   e.g. for the series $a + b + 7 + \ldots$,
   $S_1 = 2$ and $S_2 = 5$.
   (i) Find the value of a and the value of b.
   (ii) Find S_3.
3. **Arithmetic sequences**
   e.g. the first two terms of an arithmetic sequence are 7, 10. Find
   (i) a, the first term,
   (ii) d, the common difference,
   (iii) T_n, in terms of n,
   (iv) the value of n if $T_n = 55$.
4. **Arithmetic series**
   e.g. for the arithmetic series
   $$4 + 9 + 14 + \ldots$$
   (i) Find a and d.
   (ii) Express S_n in terms of n.
   (iii) Hence find S_{10}.
5. **Arithmetic problems**
   e.g. for an arithmetic series, $T_8 = 25$ and $S_9 = 81$.
   (i) Find a and d.
   (ii) Find S_n in terms of n.
6. **Proving a sequence is arithmetic**
   e.g. $T_n = 3n + 2$ is a sequence.
   (i) Express T_{n-1} in terms of n.
   (ii) Hence prove that T_n is an arithmetic sequence.
7. **Geometric sequences**
   e.g. The nth term of a geometric sequence is
   $$T_n = 3^{n-1}.$$
   (i) Find a and r.
   (ii) Show that $T_3 > T_1 + T_2$.
8. **Geometric problems**
   e.g. the first three terms of a geometric sequence are
   $$x - 3,\ 1 - x,\ x + 3.$$
   Find the value of x and the fourth term of the sequence.
9. **Geometric series**
   e.g. the first two terms of a geometric series are
   $$2 + 1 + \ldots$$
   (i) Find a and r.
   (ii) Find S_6 in the form $\frac{a}{b}$, $a, b \in \mathbf{N}$.
   (iii) Write S_n in the form $k\left(1 - \frac{1}{c^n}\right)$,
   where k and c are constants.
10. **Proving a sequence is geometric**
    e.g. $T_n = 4(5^n)$ is a sequence.
    (i) Express T_{n+1} in terms of n.
    (ii) Hence prove that T_n is a geometric sequence.

Differentiation

Differentiation is such an important topic in maths at this level and beyond. To reflect this, differentiation is examined in each of the last three questions on Paper 1. Because you can only leave out two questions, you will therefore have to perform differentiation somewhere on Paper 1. (For exam historians, there was only one exception to this: in 2005, only two questions contained differentiation.)

The main differentiation question, Question 7, is one of the most reliable on Paper 1. The (a) part is usually one or two easy derivatives. The (b) part typically asks two of the three rules. The (c) part examines rates of change, including velocity and acceleration. This last part can be problematic if not properly prepared.

Up until recently, the use of differentiation in Questions 6 and 8 was limited to finding slopes and equations of tangents and maximum and minimum points. Now, differentiation from first principles and even direct questions on the rules of differentiation can, and have been, asked in these questions.

Most of the formulae required for differentiation are contained in the maths tables. These include how to differentiate powers, the Product Rule and the Quotient Rule. You should make sure you know where these are in the tables.

The main formulae that you have to learn are:

* differentiation from first principles:
  $$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad \text{or}$$
  $$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x},$$
* the Chain Rule:
  $$\frac{d}{dx}(x^2 + 2x)^3 = 3(x^2 + 2x)^2(2x + 2)$$
* velocity and acceleration:

Ordinary level
Paper 1

if x is distance, then

velocity, $v = \frac{dx}{dt}$, and

acceleration, $a = \frac{dv}{dt}$.

1. **Differentiation from first principles**
   e.g. differentiate $x - 2x^2$ with respect to x from first principles
2. **Differentiation by rule**
   e.g. differentiate with respect to x
   (i) $7x^6$
   (ii) $3 - 2x + 5x^2 - 2x^3$
   (iii) $\frac{3}{x^2}$
3. **Product rule**
   e.g. use the Product Rule to find $\frac{dy}{dx}$ if
   $$y = (5x+7)(2x^2 - 3x + 5)$$
4. **Quotient rule**
   e.g. find $\frac{dy}{dx}$ if
   $$y = \frac{x^2+3}{2x-1}$$
5. **Chain Rule**
   e.g. find $\frac{dy}{dx}$ if
   $$y = (3x^2 - 5x + 8)^4$$
6. **Evaluating derivatives**
   e.g. if $y = \frac{x^2-1}{x+2}$, find the value of $\frac{dy}{dx}$ at $x = 1$
7. **Rates of change**
   e.g. the external surface area, A cm^2, is given by
   $$A = 2x^2 + 30x,$$
   where x is one of the dimensions of the base. Find the rate of change of the surface area as x changes, when $x = 4$.
8. **Velocity and acceleration**
   e.g. a car passes a point p as it slows down to rest. The distance, x metres, travelled by the car t seconds after passing p is given by
   $$x = 6t - \frac{1}{2}t^2.$$
   (i) Find the speed of the car as it passes p.
   (ii) Find the time taken by the car to come to rest.
   (iii) Find the distance travelled by the car in coming to rest.

Functions and Graphs

Many students are wary of Questions 6 and 8, but most are forced to attempt at least one of them. This reluctance is understandable, as the questions have varied greatly over the years.

This topic relies more on methods than formulae. The main methods are:

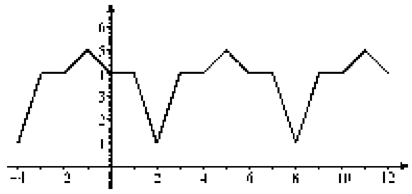
* using function notation,
* solving simple equations,
* plotting linear, quadratic and cubic curves,
* answering questions on curves we have drawn,
* finding slopes and equations of tangents, using differentiation,
* finding local max and local min points, using $\frac{dy}{dx} = 0$,
* finding the period and the range of a supplied periodic graph,
* drawing reciprocal graphs, e.g. $y = \frac{1}{x-2}$
* differentiating such functions, and finding points where the tangents have a given slope.

What is not clear is what is going to appear where. For the last four years, reciprocal graphs have occurred as a major part of Question 8. But these questions have been far more wide-ranging than just sketching the curve. They have ranged from function notation, to graphs, to differentiation, and back again. These should be studied intently, if you harbour any intention of tackling Question 8.

1. **Function notation**
   e.g. $f(x) = x^2 - 2x + 5$, for all $x \in \mathbb{R}$.
   (i) Find $f(1)$ and $f(4)$.
   (ii) Find $f(x+1)$, and write your answer in the form $ax^2 + bx + c$.
   (iii) For what value of x is $f(x) = 3$?
2. **Linear graphs**
   e.g. the distance, D km, between two ships t hours after one ship leaves a harbour is given by
   $$D = 8 + 2 \cdot 5t.$$
   Draw the graph of D against t, placing t on the horizontal axis, for $0 \le t \le 4$.
   Use your graph to estimate the length of time for which the distance between the ships is between 10 km and 15 km.
3. **Quadratic graphs**
   e.g. sketch a graph of
   $$f(x) = x^2 - 4x - 3, \text{ for } -1 \le x \le 5$$

Scribble box

4. **Cubic graphs**
   e.g. sketch a graph of
   $$f(x) = x^3 + 2x^2 - 10x + 5,$$
   for $-5 \le x \le 3$
5. **Questions on graphs**
   e.g. draw a graph of the function
   $$f(x) = x^3 - 2x^2 - 7x + 4,$$
   in the domain $-3 \le x \le 4$.
   Use your graph to estimate the values of x for which
   (i) $x^3 - 2x^2 - 7x = 0$,
   (ii) $f(x)$ is increasing,
   (iv) $x > 0$ and $f(x)$ is decreasing
6. **Turning points**
   e.g. let $f(x) = x^3 + 3x^2 - 9x - 5$.
   (i) Find $f'(x)$, the derivative of $f(x)$.
   (ii) Find the co-ordinates of the local maximum and local minimum points of the curve $y = f(x)$.
7. **Reciprocal graphs**
   e.g. draw a graph of the function
   $$f(x) = \frac{1}{x}$$
   for $-4 \le x \le 4$, $x \in \mathbb{R}$, $x \ne 0$.
   Find $f'(x)$, the derivative of $f(x)$.
   Find the co-ordinates of the points on the curve $y = f(x)$ at which the tangents have a slope of -1.
   Show that the curve has no turning points.
8. **Periodic graphs**



   e.g. a section of the graph of the periodic function, $y = f(x)$, is shown above.
   Find
   (i) the period and the range of the function,
   (ii) $f(19)$.

SAMPLE QUESTIONS

1. Arithmetic and Money

Question

(a) Express 720 cubic centimetres as a fraction of $1 \cdot 2$ litres. Give your answer in its simplest form.
Note: 1 litre = 1000 cubic centimetres.

Ordinary level
Paper 1

SAMPLE QUESTIONS

(b) (i) Calculate the value of

$$\frac{(2\cdot 04\times 10^8)+(3\cdot 05\times 10^7)}{3\cdot 5\times 10^4},$$

and write your answer in decimal form.

(ii) Declan spends €23·87 in a grocery shop and €11·29 in a hardware shop. He approximates his total expenditure to be €(24 + 12). Calculate, correct to one decimal place, his percentage error.

(c) Hugh has a net income of €4577 for a particular month. His tax credits are €270 for the month and the standard rate cut off point is €2950. The standard and higher rates of tax are 20% and 41% respectively. Calculate his gross income for the month.

Solution

(a) $1\cdot 2$ litres $= 1200\text{ cm}^3$

Fraction $= \frac{720}{1200} = \frac{3}{5}$

(b) (i) By calculator,

$$\frac{(2\cdot 04\times 10^8)+(3\cdot 05\times 10^7)}{3\cdot 5\times 10^4}$$
$$= 6700$$

(ii) True value $= €23\cdot 87 + €11\cdot 29$
$= €35\cdot 16$

Estimate $= €24 + €12 = €36.$

$$\%\text{ error} = \frac{|35\cdot 16 - 36|}{35\cdot 16}\times 100\%$$
$$= \frac{0\cdot 84}{35\cdot 16}\times 100\%$$
$$= 2\cdot 4\%.$$

(c) Let $€x$ be the gross income.

Standard rate: €2950@20%
$= €590$

Higher rate: $€(x - 2950)$ @41%
$= €(0\cdot 41x - 1290\cdot 50)$

Gross tax:
$= €590 + €(0\cdot 41x - 1290\cdot 50)$
$= €(0\cdot 41x - 619\cdot 50)$

Tax credits:
$= €270$

Net tax:
$= €(0\cdot 41x - 619\cdot 50) - €270$
$= €(0\cdot 41x - 889\cdot 50)$

Then

Gross income:
$= €x$

Net tax:
$= €(0\cdot 41x - 889\cdot 50)$

Net income:
$= €x - €(0\cdot 41x - 889\cdot 50)$
$= €(0\cdot 59x + 889\cdot 50)$

Given:

$$0\cdot 59x + 889\cdot 50 = 4577$$
$$0\cdot 59x = 3687\cdot 50$$
$$x = \frac{3687\cdot 50}{0\cdot 59}$$
$$x = 6250$$

His gross income for the month is €6250.

2. Algebra

Question

(a) Solve for $x \in \mathbf{R}$:

$$3(2-x)+5 = 2(4-x)$$

(b) (i) Express c in terms of a and b if

$$b = \frac{4a-3c}{5}.$$

Find the value of c when $a = 12$ and $b = 5$.

(ii) $(x-2)$ is a factor of $2x^3 - 11x^2 + kx - 6$. Find the value of $k \in \mathbf{R}$.

(c) (i) Solve for $x \in \mathbf{R}$:

$$3^{3x+1} = \frac{9^x}{\sqrt{3}}$$

(ii) Solve the equation

$$(\sqrt{2x}+\sqrt{x+1})(\sqrt{2x}-\sqrt{x+1}) = 5.$$

for $x \in \mathbf{R}$, $x > 0$.

Solution

(a)
$$3(2-x)+5 = 2(4-x)$$
$$6-3x+5 = 8-2x$$
$$-3x+11 = 8-2x$$
$$-3x = -3-2x$$
$$-x = -3$$
$$x = 3$$

(b) (i)
$$b = \frac{4a-3c}{5}$$
$$5b = 4a-3c$$
$$5b+3c = 4a$$
$$3c = 4a-5b$$
$$c = \frac{4a-5b}{3}$$

$a = 12$, $b = 5$:

$$c = \frac{4(12)-5(5)}{3} = \frac{23}{3}$$

(ii) Let $f(x) = 2x^3 - 11x^2 + kx - 6$.

Put $x - 2 = 0$. Thus $x = 2$. As $x - 2$ is a factor,

$$f(2) = 0$$
$$2(2)^3 - 11(2)^2 + k(2) - 6 = 0$$
$$16 - 44 + 2k - 6 = 0$$
$$2k = 34$$
$$k = 17$$

(c) (i)
$$3^{3x+1} = \frac{9^x}{\sqrt{3}}$$
$$3^{3x+1} = \frac{(3^2)^x}{3^{\frac{1}{2}}}$$
$$3^{3x+1} = 3^{2x-\frac{1}{2}}$$
$$3x+1 = 2x - \frac{1}{2}$$
$$6x+2 = 4x-1$$
$$2x = -3$$
$$x = -\frac{3}{2}$$

(ii)
$$(\sqrt{2x}+\sqrt{x+1})(\sqrt{2x}-\sqrt{x+1}) = 5$$
$$\sqrt{2x}(\sqrt{2x}-\sqrt{x+1}) + \sqrt{x+1}(\sqrt{2x}-\sqrt{x+1}) = 5$$
$$2x - \sqrt{2x}\sqrt{x+1} + \sqrt{2x}\sqrt{x+1} - (x+1) = 5$$
$$2x-(x+1) = 5$$
$$x-1 = 5$$
$$x = 6.$$

3. Complex Numbers

Question

(a) Express in the form $a+bi$, where $a, b \in \mathbf{R}$ and $i^2 = -1$:

$$(-1+4i)^2.$$

(b) (i) If $z = 2-3i$ and $w = 4+i$, investigate if

$$|z+w| = |z|+|w|.$$

(ii) Solve for $z \in \mathbf{Z}$:

$$z^2 - 2z + 5 = 0$$

(c) Let $z = 1+i$.

(i) Express $z - \frac{1}{z}$ in the form $x+yi$, $x, y \in \mathbf{R}$.

(ii) Find the values of the real numbers t and k if

$$k\left(z-\frac{1}{z}\right)+ti = 2+i.$$

Solution

(a)
$$(-1+4i)^2 = (-1+4i)(-1+4i)$$
$$= -1(-1+4i)+4i(-1+4i)$$
$$= 1-4i-4i+16i^2$$
$$= 1-8i+16(-1)$$
$$= -15-8i$$

(b) (i)
$$|z| = |2-3i|$$
$$= \sqrt{2^2+(-3)^2}$$
$$= \sqrt{13} \approx 3\cdot 61$$
$$|w| = |4+i|$$
$$= \sqrt{4^2+1^2}$$
$$= \sqrt{17} \approx 4\cdot 12$$
$$|z+w| = |(2-3i)+(4+i)|$$
$$= |6-2i|$$
$$= \sqrt{6^2+(-2)^2}$$
$$= \sqrt{40} \approx 6\cdot 32$$

As $|z|+|w| = 3\cdot 61+4\cdot 12$
$= 7\cdot 73 \neq 6\cdot 32$

$$|z|+|w| \neq |z+w|.$$

(ii)
$$z^2-2z+5 = 0$$
$$[a = 1,\ b = -2,\ c = 5]$$
$$z = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)}$$
$$= \frac{2\pm\sqrt{4-20}}{2}$$
$$= \frac{2\pm\sqrt{-16}}{2}$$
$$= \frac{2\pm 4i}{2}$$
$$= 1\pm 2i$$

The roots are $1+2i$ and $1-2i$.

(c) (i)
$$\frac{1}{z} = \frac{1}{1+i}\times\frac{1-i}{1-i}$$
$$= \frac{1-i}{1-i+i-i^2}$$
$$= \frac{1-i}{1+1}$$
$$= \frac{1-i}{2}$$
$$= \frac{1}{2}-\frac{1}{2}i$$
$$z-\frac{1}{z} = (1+i)-\left(\frac{1}{2}-\frac{1}{2}i\right)$$
$$= \left(1-\frac{1}{2}\right)+\left(1+\frac{1}{2}\right)i$$
$$= \frac{1}{2}+\frac{3}{2}i$$

(ii)
$$k\left(z-\frac{1}{z}\right)+ti = 2+i$$
$$k\left(\frac{1}{2}+\frac{3}{2}i\right)+ti = 2+i$$
$$k(1+3i)+2ti = 4+2i$$
$$k+3ki+2ti = 4+2i$$
$$k+(3k+2t)i = 4+2i$$

Re = Re:
$$k = 4 \quad \ldots 1$$

Im = Im:
$$3k+2t = 2 \quad \ldots 2$$

2:
$$3(4)+2t = 2$$
$$2t = -10$$
$$t = -5.$$

Ordinary level
Paper 1

SAMPLE QUESTIONS

4. Functions, Graphs & Differentiation

Question

(a) $g(x) = 7 + 3x$, for all $x \in \mathbb{R}$.

(i) Find $g(2)$ and $g(15)$.

(ii) Find the constant k if $g(15) = k\,g(2)$.

(b) (i) Find the value of $\frac{dy}{dx}$ at $x = 1$ if $y = \frac{x^2 - 2}{2x - 3}$.

(ii) Find the value of $\frac{dy}{dx}$ when $x = -2$ if $y = (5x^2 + 8x - 5)^8$.

(c) Let $f(x) = \frac{1}{x - 2}$, $x \in \mathbb{R}$, $x \neq 2$.

(i) Complete the following table:

x	$f(x)$
−2	
−1	
0	
1	
1·5	
2·5	
3	
4	
5	
6	

(ii) Draw the graph of $y = f(x)$ in the domain $-2 \leq x \leq 6$.

(iii) Find $f'(x)$, the derivative of $f(x)$.

(iv) Find the co-ordinates of the points on the curve $y = f(x)$ at which the slope of the tangent is $-\frac{1}{4}$.

Solution

(a) $g(x) = 7 + 3x$

(i) $g(2) = 7 + 3(2) = 13$

$g(15) = 7 + 3(15) = 52$

(ii) $g(15) = k\,g(2)$

$52 = k(13)$

$52 = 13k$

$k = 4$

(b) (i) $y = \frac{x^2 - 2}{2x + 3}$

$u = x^2 - 2$ $\qquad$ $v = 2x + 3$

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(2x + 3)(2x) - (x^2 - 2)(2)}{(2x + 3)^2}$$

At $x = 1$,

$$\frac{dy}{dx} = \frac{(5)(2) - (-1)(2)}{(5)^2}$$

$$= \frac{12}{25}$$

(ii) $y = (5x^2 + 8x - 5)^8$

$$\frac{dy}{dx} = 8(5x^2 - 8x - 5)^7(10x - 8)$$

At $x = -2$,

$$\frac{dy}{dx} = 8(-1)^7(-12)$$

$$= 8(-1)(-12)$$

$$= 96$$

(c) (i) $y = f(x) = \frac{1}{x - 2}$

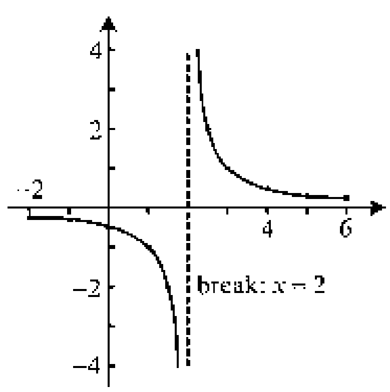
Asymptote (break): $x - 2 = 0$

$x = 2$

Table:

x	$f(x)$
−2	$-\frac{1}{4}$
−1	$-\frac{1}{3}$
0	$-\frac{1}{2}$
1	−1
1·5	−2
2·5	2
3	1
4	$\frac{1}{2}$
5	$\frac{1}{3}$
6	$\frac{1}{4}$

(ii) Graph:



(iii) $f(x) = \frac{1}{x - 2} = (x - 2)^{-1}$

$$f'(x) = -(x - 2)^{-2} = \frac{-1}{(x - 2)^2}$$

(iv) Slope $= -\frac{1}{4}$

$$\frac{-1}{(x - 2)^2} = \frac{-1}{4}$$

$4 = (x - 2)^2$

$x - 2 = 2$ or $x - 2 = -2$

$x = 4$ or $x = 0$

Points are $\left(4, \frac{1}{2}\right)$ and $\left(0, -\frac{1}{2}\right)$.

Getting the right results: Two Institute of Education students Jennifer Kenny-Boyd (left) and Caroline Marron celebrating after getting their Leaving Cert results last year. Both got the points they needed to do the courses they wanted.

Ordinary level
Paper 2

Paper Two

As has been the case for the last couple of years, you will have the week-end to recover from Maths Paper 1, and get ready for Maths Paper 2, which will be held on the morning of Monday, June 9th. It goes without saying that, no matter what happens on Paper 1, you should not indulge in post-mortems, but rather focus on the task ahead. You should also be aware that the majority of students perform better on Paper 2. This should offer some comfort, but doesn't leave any room for complacency.

Because Maths Paper 2 is to be followed in rapid succession over the next few days by Irish, Business, French and History, you will not have as much time as you may think over the week-end to prepare for Paper 2. The weekend should be left for rapid revision, going through the key formulae and methods for each of the topics you may attempt.

All your serious revision must be done in the weeks and months prior to the start of the Leaving Cert.

Unlike Paper 1, Paper 2 contains two sections. Section A contains seven questions, of which you must attempt five. Section B contains four questions, one on each of the four option topics, of which you only have to attempt one.

Because of time constraints, most students have only studied one option topic. For this reason, they will have no choice in Section B. It is also common for students not to cover certain topics in Section A. For example, many students do not cover geometry and enlargements, Question 4, due to perceived difficulty, dislike and/or disinterest. Also, no few students find it hard to relate to probability, Question 6, and so have no intention of trying this question.

All of this leaves many students entering Paper 2 with their six questions pre-determined, i.e. with absolutely no choice. On Paper 1, this would be highly dangerous, but on Paper 2 it is not so bad.

In general, Paper 2 is more benign than Paper 1, having fewer variations from year to year. It also has less algebra content than Paper 1, which is seen as a good thing by most people.

Nevertheless, each topic you intend to tackle must be treated with respect and revised in depth. This is especially true if you enter the exam with little or no choice.

One thing to be wary of with Paper 2 is the timing. Many questions involve drawing graphs, e.g. the co-ordinate geometry questions, statistics and linear programming. You should practice drawing these accurately, efficiently, but most importantly, quickly.

Areas and Volumes

Question 1, dealing with areas and volumes, is a very popular question with most students. There are a number of good reasons for this. First of all, much of the material is familiar since Junior Cert., and so is seen as not being too difficult.

Secondly, the three parts of the question have conformed to a regular pattern over the last good number of years. There is nothing to suggest that this pattern is to be disrupted this year.

The (a) part traditionally deals with a plane figure, e.g. a triangle, a circle or a sector, and asks students to calculate a length or an area. The basic formulae are either well known or in the maths tables, except

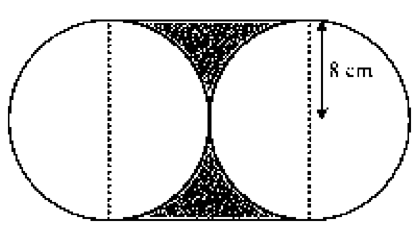
$$\text{length of arc} = 2\pi r \times \frac{A}{360} \quad \text{and}$$

$$\text{area of sector} = \pi r^2 \times \frac{A}{360}.$$

Since 1996, the (b) part has always dealt with the use of Simpson's Rule to approximate an area. These have been very similar to each other, and can be well prepared. There is a form of Simpson's Rule on page 42 of the Maths tables. However, this is very abstract and most find it inaccessible. You are well advised to learn your own form of the Rule, or learn how to lay out the calculation in the form of a table.

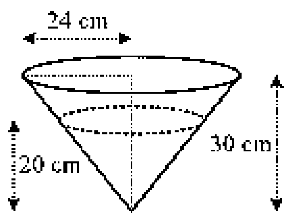
The (c) part always deals with the volume of a solid, which may be a compound solid. There is usually an easy introduction, but before the end you will probably have to solve an equation to calculate a dimension. The required formulae are on page 7 of the Maths tables.

1. **Areas**
   e.g. two circles of radius length 8 cm touch at a single point, as shown.



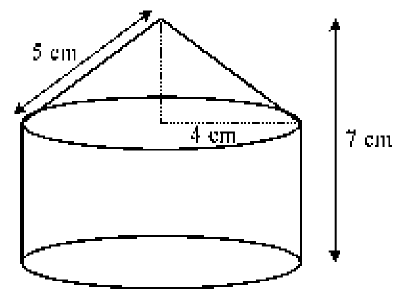
Taking $\pi = 3{\cdot}14$, calculate the shaded area in the diagram.

2. **Prisms**
   e.g. the triangular base of a prism has area 16 cm^2 and the height of the prism is 22 cm. Calculate the volume of the prism.
3. **Spheres and hemispheres**
   e.g. a sphere has diameter 16 cm. Express its volume in terms of π.
4. **Cylinders**
   e.g. calculate, in terms of π, the volume of a cylinder which has a base of radius 8 cm and a height of 16 cm
5. **Cones**



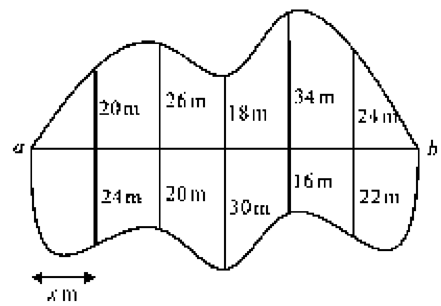
e.g. a hollow cone has its axis vertical and its apex below as shown. Its height is 30 cm and the radius of its base is 24 cm. Water is poured in to a height of 20 cm. Express the volume of the water as a percentage of the volume of the cone.

6. **Compound volumes**



e.g. a solid figure consists of a cylinder topped by a cone. The radius of both the cylinder and the cone is 4 cm. The slant height of the cone is 5 cm and the total height of the figure is 7 cm.
(i) Find the height of the cone.
(ii) Find the height of the cylinder.
(iii) Find the volume of the figure in terms of π.

7. **Forming equations**
   e.g. a sphere has a volume of 1 litre, i.e. 1000 cm^3. Taking $\pi = 3{\cdot}14$, find the length of the radius of the sphere, correct to one decimal place.
8. **Equal volumes**
   e.g. a solid metal sphere has radius 6 cm. Express the volume of the sphere in terms of π.
   The sphere is melted down and recast as a solid cylinder of height $4{\cdot}5$ cm. Find the length of the radius of the cylinder.
9. **Simpson's Rule**
   e.g. the sketch below shows a small plot of land.



At equal intervals of x metres along $[ab]$, perpendicular measurements are made to the edges of the plot. The measurements to the top edge are 20 m, 26 m, 18 m, 34 m and 24 m.
The measurements to the bottom edge are 24 m, 20 m,30 m, 16 m and 22 m.
At a and b, the measurements are 0 m.
Using Simpson's Rule, the area of the plot is estimated to be 3968 m^2.
Calculate x.

The Line

Question 2 deals with co-ordinate geometry of the line, and is attempted by almost every student. Again, most of the basic material is familiar from Junior Cert., and familiarity breeds contentment.

However, this question is taken for granted at your peril. There are many formulae to be learned, and quite a few methods. For each formula, you must learn the formula *exactly*, know precisely when it is used and know how to substitute values into the formula.

A list of the key formula is:
* distance formula,
* midpoint formula,
* slope formula,
* equation of a line formula,
* area of a triangle formula.

The following methods are also important:
* slopes of perpendicular lines,
* determining if a point is on a line,
* plotting lines,
* finding the points of intersection of two lines,
* translations.

Crucially, you should also practise multi-part questions, which often form the (b) or (c) part of Leaving Cert. questions. This is where a number of successive, linked parts are asked. A diagram is usually essential to keep track of what is going on.

Ordinary level
Paper 2

1. **Distance and midpoint**
   e.g. find the co-ordinates of m, the midpoint of $[ab]$, if $a = (-1,6)$ and $b = (7,2)$. Verify that $|am| = |mb|$.
2. **Slope**
   e.g. $p = (-1,3)$ and $q = (5,-1)$. Find the slope of the line pq and investigate if pq is perpendicular to the line with equation $3x - 2y = 7$.
3. **Plotting lines**
   e.g. plot the line $3x - y = 9$ and verify that the line contains the point $(2,-3)$
4. **Equation of a line**
   e.g. find the equation of the line ab if $a = (2,-3)$ and $b = (4,5)$
5. **Connected lines**
   e.g. find the equation of the line through $a = (-1,6)$ which is perpendicular to the line $x + 3y = 8$
6. **Translations**
   e.g. $a = (3,-4)$, $b = (-1,-2)$, $c = (5,1)$. Find the co-ordinates of d if $abcd$ is a parallelogram.
7. **Area of a triangle**
   e.g. find the area of the triangle with vertices $(5,-1)$, $(2,7)$, $(-3,2)$.

The Circle

Question 3 also deals with co-ordinate geometry, but this time of the circle. It is nearly as popular as the line question, although it is probably the question with the greatest algebra content on Paper 2.

First of all, it is important to be aware that every formula and method from the line may be required here in Question 3. Because of this, there are only two new formulae here, along with a few new methods.

The required formulae are both for the equation of a circle, and are:

* circle, centre $(0,0)$, radius r:
  $x^2 + y^2 = r^2$
* circle, centre (h,k), radius r:
  $(x-h)^2 + (y-k)^2 = r^2$.

The methods include:

* determining if a point is on, inside or outside a given circle,
* finding the centre and the radius, given, for example, the endpoints of a diameter,
* finding the equation of a tangent at a point on a circle,
* finding the equation of a parallel tangent,
* finding the points of intersection of a line and a circle with centre $(0,0)$.

The last method here involves solving a linear, non-linear system of simultaneous equations, which is first studied in algebra, but should be revised for the circle.

1. **The equation $x^2 + y^2 = r^2$**
   e.g. find the equation of the circle with centre $(0,0)$ and which contains the point $(-5,2)$
2. **The equation $(x-h)^2 + (y-k)^2 = r^2$**
   e.g. find the equation of the circle which has $[pq]$ as a diameter, where $p = (7,-2)$ and $q = (-1,6)$
3. **Point on a circle**
   e.g. the point $(k,7)$ belongs to the circle
   $(x+2)^2 + (y-4)^2 = 34$.
   Find the value of the real number k.
4. **Finding the centre and the radius**
   e.g. find the co-ordinates of the centre and the length of the radius of the circles
   (i) $x^2 + y^2 = 53$
   (ii) $(x+5)^2 + (y-1)^2 = 17$
5. **Position of a point relative to a circle**
   e.g. investigate if the point $(6,-4)$ is outside the circle
   $(x+4)^2 + (y-1)^2 = 45$
6. **Equation of a tangent**
   e.g. $p(3,1)$ is a point on the circle
   $(x-2)^2 + (y+3)^2 = 17$.
   (i) Find the slope of the tangent to the circle at p.
   (ii) Find the equation of this tangent.
7. **Parallel tangents**
   e.g. find the equation of T, the tangent to the circle $(x+4)^2 + (y-3)^2 = 74$ at the point $p(3,-2)$.
   Find the equation of T_1, the other tangent to the circle which is parallel to T.
8. **Intersection of a line and a circle**
   e.g. find the co-ordinates of the points of intersection of the line $3x + y = 10$ and the circle $x^2 + y^2 = 20$.

Geometry and Enlargements

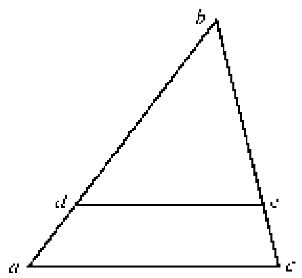
Very few students study geometry theorems and enlargements, which is examined in Question 4. And among those who do, fewer intend to tackle the question in the Leaving Cert. The perception is that geometry proofs are boring, fussy and pointless. Naturally, many teachers disagree, but still find it difficult to instil any enthusiasm among students.

However, if you can overlook such negative attitudes and focus on picking questions that will generate good marks, this may be one to consider. The (a) part always contains a very easy application of one of the theorems.

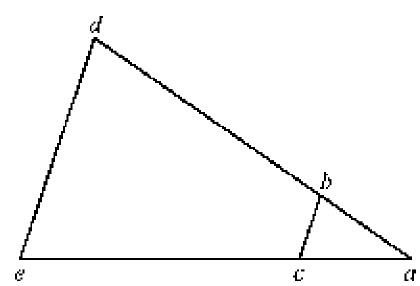
Then the (b) part always asks one of ten standard proofs. Although tedious, if you go to the trouble of learning them, it is a guaranteed 20 marks.

The (c) part then covers enlargements. In practice, questions on enlargements are just an application of Theorem 5 from geometry. However, you have to learn some terminology, e.g. centre of enlargement and scale factor. There is also one result to be remembered, i.e. that if the scale factor is k, then the area of a region is multiplied by k^2 to find the area of its image.

1. **Proofs of theorems**
   e.g. prove that if three parallel lines make intercepts of equal length on a transversal, then they will also make intercepts of equal length on any other transversal.
2. **Questions on theorems**
   e.g. in the diagram, $de \parallel ac$, $|bd| = 9$, $|ad| = 3$ and $|be| = 7$; find $|ec|$



3. **Enlargements**
   e.g. the triangle ade is the image of the triangle abc under an enlargement, centre a. If $|ad| = 7$, $|ae| = 10 \cdot 5$ and $|bd| = 5$, find
   (i) the scale factor of the enlargement,
   (ii) $|ac|$,
   (iii) the area of Δade if the area of $\Delta abc = 2$.



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Maths

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Trigonometry

Question 5 examines trigonometry, and is one of the more mechanical questions on Paper 2. It is popular for both negative and positive reasons. As a negative, it is seen by some as 'not as bad as some of the other questions'.

As a positive, many recognise that nearly all the required formulae are in the maths tables, the questions tend to be of a practical nature. For the most part, and usually involve using a calculator.

The only formulae that need to be learned are basic, or used elsewhere:

* Pythagoras' theorem:

  $a^2 + b^2 = c^2$
* trig ratios:

  $\sin = \frac{\text{opp}}{\text{hyp}}$, $\cos = \frac{\text{adj}}{\text{hyp}}$, $\tan = \frac{\text{opp}}{\text{adj}}$
* length of arc and area of sector (see Areas and Volumes).

Of course, you must completely master using your calculator to work out trig functions, e.g. $\sin 57°$, and the inverse trig functions, e.g. to find the angle A such that $\cos A = 0 \cdot 45$.

For general triangles, you should know where to find the area of a triangle formula, the Sine Rule and the Cosine Rule in the maths tables. You must also know how to decide which rule to use, and how to use it correctly.

More complicated questions, usually (c) parts, involve more than one triangle. Here you must know how to decide which triangle to start with and how to proceed.

Finally, know how to use the compound angle formulae, e.g.

$\sin(A + B) = \sin A \cos B + \cos A \sin B$,

which are given on page 9 of the tables.

1. **Using a calculator**

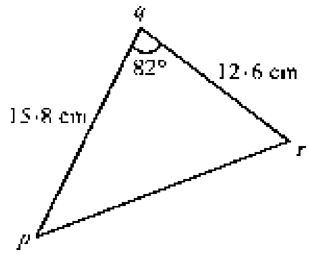
   e.g. use your calculator to find

   (i) $\cos 39°$

   (ii) the angle $0° < A < 90°$, to the nearest degree, if $\sin A = 0 \cdot 6157$
2. **Connected ratios**

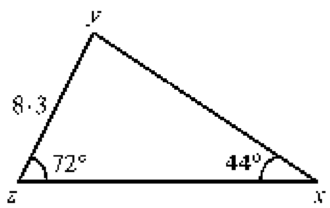
   e.g. if $\sin A = \frac{9}{41}$, for $0° \le A \le 90°$, express $\tan A$ in the form $\frac{p}{q}$, where $p, q \in Z$
3. **Area of a triangle**

   e.g. in the triangle pqr, $|pq| = 15 \cdot 8$, $|qr| = 12 \cdot 6$ and $|\angle pqr| = 82°$.



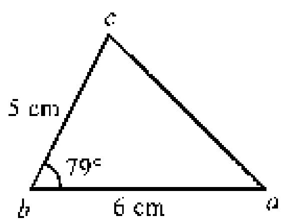
   Find the area of the triangle pqr, correct to one decimal place.
4. **Sine Rule**

   e.g. in the triangle xyz, $|yz| = 8 \cdot 3$, $|\angle yxz| = 44°$ and $|\angle yzx| = 72°$.



   Find $|xy|$, correct to one decimal place.
5. **Cosine Rule**

   e.g. in the triangle abc, $|ab| = 6$ cm, $|bc| = 5$ cm and $|\angle abc| = 79°$. Find $|ac|$, correct to one decimal place.



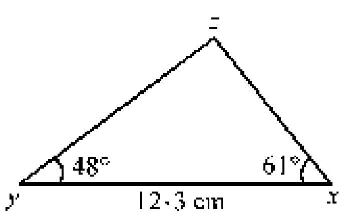
6. **Solving triangles**

   e.g. in the triangle xyz, $|xy| = 12 \cdot 3$ cm, $|\angle xyz| = 48°$ and $|\angle yxz| = 61°$. Find:

   (i) $|\angle xzy|$.

   (ii) $|xz|$, correct to one decimal place,

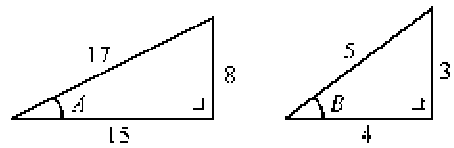
   (iii) $|yz|$, correct to one decimal place.



7. **Arcs and sectors**

   e.g. a circle has radius of length 21 cm and centre o. A sector of this circle has an angle of 240° at o. Taking $\pi = 3 \cdot 14$, calculate the area of this sector.
8. **Compound angles**

   e.g. the angles A and B are shown in the right-angled triangles below.



   Use the triangles to find as fractions

   (i) $\cos(A + B)$,

   (ii) $\sin(A + B)$.

Probability

Probability, which is examined in Question 6, is a completely different brand of maths than the rest of our course. There is no algebra, there are few formulae, and most of the calculations involve counting, usually by calculator.

The formula we use are:

* one thing **and** another:

  multiply numbers of outcomes
* one way **or** another way:

  add numbers of outcomes
* arrange n different objects:

  $n!$ button on a calculator
* choose r objects from n different objects:

  $n\,C\,r$ button on a calculator
* probability of event, E:

  $P(E) = \frac{r}{n}$

  $= \frac{\text{no. of favourable outcomes}}{\text{total no. of outcomes}}$

Although this may sound easy, the real problem with some probability questions is to read a chunk of text, decide whether we ought to arrange or choose, or work out a probability, and decide how many objects are involved. All of this must be done before using the calculator to find the answer. This task is not everyone's cup of tea.

If we are asked to find a probability, it is important to realise that the answer must lie between 0 and 1.

1. **Fundamental principle of counting**

   e.g. Sean forms a password by taking one of the letters of his name along with two of the digits from 0 to 9, e.g. E26, N44.

   (i) How many passwords are possible?

   (ii) How many passwords start with a vowel?

   (iii) How many passwords contain two even numbers?
2. **Arrangements (permutations)**

   e.g. when all the letters of the word FRIENDLY are arranged,

   (i) how many arrangements are possible,

   (ii) how many of these arrangements start with the letter F,

   (iii) how many of these arrangements start with a vowel and end with Y?
3. **Combinations (choices)**

   e.g. from a group of twelve students and five teachers, a committee of four is to be chosen.

   (i) How many committees are possible if there are no restrictions?

   (ii) How many committees are possible if there must be two students and two teachers?
4. **Probability**

   e.g. a club has 12 men and 18 women as members. 8 of the men are full members and 4 are associate members. 6 of the women are full members and 12 are associate members.

   A single club member is chosen at random. What is the probability that

   (i) the club member chosen is an associate member?

   (ii) the club member chosen is a full member who is a woman?
5. **Two-stage probability**

   e.g. a bag contains seven blue beads, four yellow beads and five white beads. Two beads are chosen at random from the bag.

   (i) What is the probability that the two beads chosen are blue?

   (ii) What is the probability that neither bead is white?

   (iii) What is the probability that one bead is yellow and the other is white?

Statistics

The last question in Section A, which deals with statistics, is one of the most popular on Paper 2. This is perhaps because it contains graphs and a number of easy to apply formulae.

There are two types of graph that you can be asked to construct: a histogram and a cumulative frequency curve. These are very different, and great care must be taken with their construction. For example, you must use graph paper, draw the axes with a ruler and put scales on the axes. For histograms, the heights of the rectangles must be correct, as must the locations of their bases. For a cumulative frequency curve, the correct points must be plotted and joined up. We can also be asked to deduce values, e.g. the median, interquartile range, from a cumulative frequency curve.

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In addition to graphs, we can be asked to perform a number of calculations:

* the median of a list:
  this is simply the middle number when arranged in order,
* the mean of a list:
  $$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n},$$
* the mean of a table:
  use a formula or your own constructed table,
* standard deviation:
  use a formula or table to find the standard deviation of a list or table.

A word of warning: do not do one of these calculations entirely on a calculator, show no working, and just write down the answer. If you get the answer right: great. But if it is wrong, you stand to lose all marks for that part.

1. **Histograms**
   e.g. a group of students was asked how many hours they spent watching television on a particular night and the results are given in the table below.

hours	0 – 1	1 – 2	2 – 3	3 – 5
students	12	16	8	10

   (i) How many students were in the class?
   (ii) Represent this data on a histogram.
   (iii) What is the greatest possible number of students who could have watched no television at all?

2. **Mean**
   e.g. if the mean of the numbers 4, x, 2, 8, 6 is x, find the value of x.
3. **Weighted mean**
   e.g. calculate the weighted mean of the results 6, 10, 15, 3, 7 whose respective weights are 2, 1, 1, 4, 2
4. **Standard deviation**
   e.g. calculate the mean and the standard deviation of the following distribution:

Result	1	2	3	4
Frequency	20	14	10	5

5. **Cumulative frequency curve**
   e.g. the weights, in kg, of a number of parcels passing through a sorting office was recorded as follows.

kg	0 – 2	2 – 4	4 – 6	6 – 8
Number	12	24	10	6

   Complete the following cumulative frequency table.

kg	< 2	< 4	< 6	≤ 8
Number				

   Draw a cumulative frequency curve and use it to estimate the number of parcels which weighed
   (i) less than 5 kg,
   (ii) more than 3 kg,
   (iii) between 3 kg and 5 kg.

6. **Median and interquartile range**
   e.g. the ages (in years) of a number of children in a playground were recorded and are shown in the table below.

Age	0 – 3	3 – 6	6 – 9	9 – 12
No.	9	17	10	6

   Draw a cumulative frequency curve and use it to estimate
   (i) the median age of the children,
   (ii) the interquartile range.

OPTIONS

Section B of Paper 2 contains one question on each of the four option topics:

Question 8: Further geometry
Question 9: Vectors
Question 10: Further sequences and series
Question 11: Linear programming

Students are only required to answer one question from this section, and for this reason most students only study one of these topics.

Of the four questions, Question 11 is the most popular, probably because it is seen as being an extension of co-ordinate geometry. The (a) part of this question involves drawing or naming one or more half planes. The (b) part contains a practical situation that has to be converted into inequalities which are then plotted.

The other options may not be as popular as linear programming, but each has its own following. It is sufficient to say that whichever option topic you study, make sure you are very familiar with the types of questions asked on that topic.

SAMPLE QUESTIONS

1. The Line

Question

(a) Find the slope of the line containing the points $(-1, 4)$ and $(2, 3)$.

(b) Plot the line $7x - 2y = 14$.

(c) L is the line $2x - 3y = 4$.
(i) Verify that $a(-1, -2)$ belongs to L.
(ii) K is the line which contains the point $b(9, -4)$ and which is perpendicular to L. Find the equation of K.
(iii) L and K intersect at the point c. Find the co-ordinates of c.
(iv) Find the area of the triangle abc.

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Solution

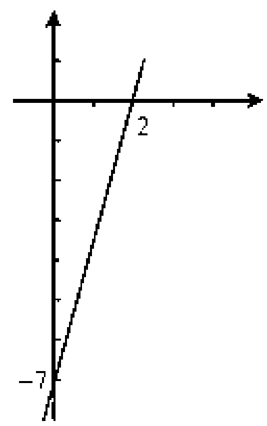
(a)
(x_1, y_1) (x_2, y_2)
$(-1, 4)$ $(2, 3)$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4}{2 + 1} = \frac{-1}{3}$$

(b) $7x - 2y = 14$

x-axis: Let $y = 0$.
$4x - 2(0) = 14$
$x = 2$
$(2, 0)$

y-axis: Let $x = 0$.
$7(0) - 2y = 14$
$y = -7$
$(0, -7)$.



(c) (i) L: $2x - 3y = 4$
$a(-1, -2)$:
$2(-1) - 3(-2) = 4$
$-2 + 6 = 4$
$4 = 4$ True.
Thus a belongs to L.

(ii) L: $2x - 3y = 4$
$[a = 2,\ b = -3]$

$$m = -\frac{a}{b} = -\frac{2}{-3} = \frac{2}{3}$$

$K \perp L$.

Slope of $K = -\frac{3}{2}$.

(x_1, y_1) m
$b(9, -4)$ $-\frac{3}{2}$

Equation of K:

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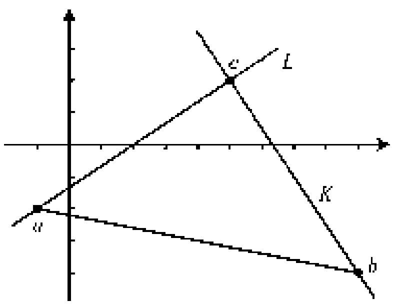
$$(y+4) = -\frac{3}{2}(x-9)$$



(iii)
$$2(y+4) = -3(x-9)$$
$$2y+8 = -3x-27$$
$$2y = -3x+19$$
$$3x+2y = 19$$
is the equation of K.

L: $2x-3y=4$
K: $3x+2y=19$

Then
$L\times 2$: $4x-6y=8$
$K\times 3$: $9x+6y=57$
$13x \quad = 65$
$x=5$

K: $3(5)+2y=19$
$15+2y=19$
$2y=4$
$y=2$

Thus $c=(5,2)$.

(iv) Translating,

$a(-1,-2)$	$b(9,-4)$	$c(5,2)$
1,2	1,2	1,2
$(0,0)$	$(10,-2)$	$(6,4)$
	(x_1,y_1)	(x_2,y_2)

$$\text{Area } \Delta = \frac{1}{2}\left|(10)(4)-(6)(-2)\right|$$
$$= \frac{1}{2}\left|40+12\right|$$
$$= \frac{1}{2}\left|52\right|$$
$$= \frac{1}{2}(52)$$
$$= 26 \text{ square units.}$$

2. The Circle

Question

(a) Determine, by calculation, if the point $(3,-3)$ is inside the circle $x^2+y^2=20$.

(b) Find the points of intersection of the line $x+y=3$ and the circle $x^2+y^2=29$.

(c) $a(0,3)$, $b(4,-3)$ and $c(-2,-7)$ are the vertices of a triangle.

(i) Verify that ab is perpendicular to bc.

(ii) Find the equation of the circle C which contains a, b and c.

Solution

(a)
$$x^2+y^2=20$$
$(3,-3)$: $(3)^2+(-3)^2 \quad 20$
$$9+9 \quad 20$$
$$18<20$$
Thus $(3,-3)$ is inside the circle.

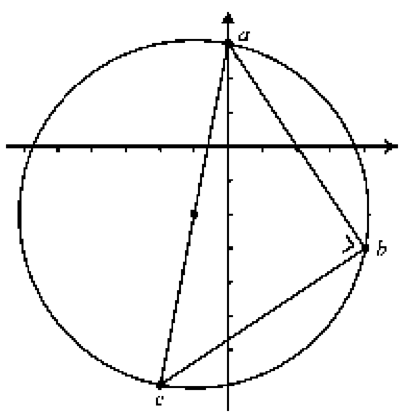
(b) L: $x+y=3$
$y=3-x$

C: $x^2+y^2=29$
$$x^2+(3-x)^2=29$$
$$x^2+(3-x)(3-x)=29$$
$$x^2+9-3x-3x+x^2=29$$
$$2x^2-6x-20=0$$
$$x^2-3x-10=0$$
$$(x-5)(x+2)=0$$
$$x-5=0 \quad \text{or} \quad x+2=0$$
$$x=5 \quad \text{or} \quad x=-2$$

L: $y=3-x$
$x=5$: $y=3-(5)=-2$
One point of intersection is $(5,-2)$.
$x=-2$: $y=3-(-2)=5$
The other point of intersection is $(-2,5)$.

(c) (i) Let m_1 be the slope of ab, where $a=(0,3)$ and $b=(4,-3)$.
$$m_1 = \frac{-3-3}{4-0} = \frac{-6}{4} = \frac{-3}{2}$$
Let m_2 be the slope of bc, where $b=(4,-3)$ and $c=(-2,-7)$.
$$m_2 = \frac{-7+3}{-2-4} = \frac{-4}{-6} = \frac{2}{3}$$
$$\text{As } m_1.m_2 = \left(\frac{-3}{2}\right)\left(\frac{2}{3}\right) = -1.$$
$ab \perp bc$.

(ii) Centre of C is the midpoint of $[ac]$.
$$(h,k) = \left(\frac{0-2}{2},\frac{3-7}{2}\right)$$
$$=(-1,-2)$$



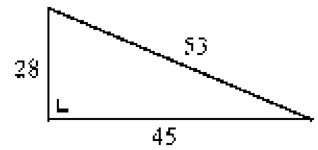
Radius of C is the distance from $(-1,-2)$ to $a=(0,3)$.
$$r=\sqrt{(0+1)^2+(3+2)^2}$$
$$=\sqrt{1^2+5^2}$$
$$=\sqrt{26}$$

Thus the equation of C is
$$(x-h)^2+(y-k)^2=r^2$$
$$(x+1)^2+(y-2)^2=26.$$

3. Trigonometry

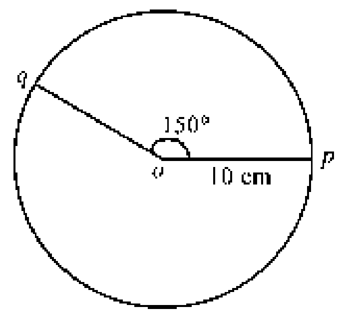
Question

(a) Copy the triangle shown below and indicate on it the angle A if $\tan A = \frac{45}{28}$.



Show that $\sin A + \cos A < \tan A$.

(b) A circle has centre o, and radius length 10 cm. opq is a sector of this circle and $|\angle poq| = 150°$.

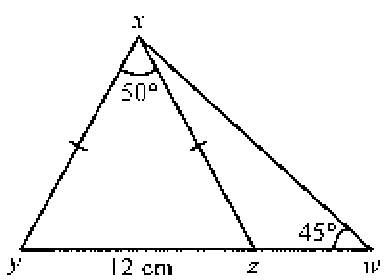


Find, in terms of π,

(i) the length of the minor arc pq,

(ii) the area of the sector opq.

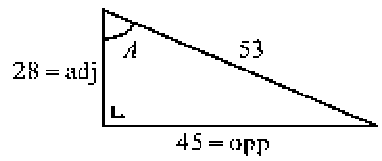
(c) In the diagram,
$|yz| = 12$ cm,
$|xy| = |xz|$,
$|\angle yxz| = 50°$ and
$|\angle zwx| = 45°$.



Calculate
(i) $|xz|$, correct to one decimal place,
(ii) $|xw|$, correct to one decimal place.

Solution

(a) The angle A is shown below.
$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{45}{28}$$

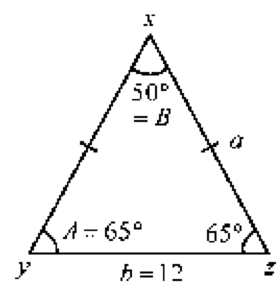


Then
$$\sin A = \frac{\text{opp}}{\text{hyp}} = \frac{45}{53}$$
$$\cos A = \frac{\text{adj}}{\text{hyp}} = \frac{28}{53}$$
$$\sin A + \cos A = \frac{45}{53} + \frac{28}{53} = \frac{73}{53} = 1\cdot 377$$
$$\tan A = \frac{45}{28} = 1\cdot 607$$
Thus $\sin A + \cos A < \tan A$.

(b) (i) Length of arc $= 2\pi r \times \frac{A}{360}$
$$= 2\pi(10)\times\frac{150}{360}$$
$$= 20\pi\times\frac{5}{12}$$
$$= \frac{25\pi}{3} \text{ cm}$$

(ii) Area sector $= \pi r^2 \times \frac{A}{360}$
$$= \pi(10)^2\times\frac{150}{360}$$
$$= 100\pi\times\frac{5}{12}$$
$$= \frac{125\pi}{3} \text{ cm}^2.$$

(c) (i) Δxyz.
As the triangle is isosceles.
$|\angle xyz| = |\angle xzy|$.



Thus

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$$2|\angle xyz| + 50° = 180°$$
$$2|\angle xyz| = 130°$$
$$|\angle xyz| = 65°$$
$$= |\angle xzy|.$$

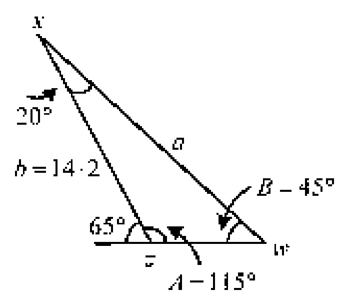
Let $a = |xz|$.

By the Sine Rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 65°} = \frac{12}{\sin 50°}$$
$$a = \frac{12 \sin 65°}{\sin 50°}$$
$$a = 14 \cdot 2 \text{ cm}.$$

(ii) Δxwz.

$$|\angle xzw| = 180° - 65°$$
$$= 115°$$
$$|\angle zxw| + 115° + 45° = 180°$$
$$|\angle zxw| = 20°.$$



Let $a = |xw|$.

By the Sine Rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 115°} = \frac{14 \cdot 2}{\sin 45°}$$
$$a = \frac{14 \cdot 2 \sin 115°}{\sin 45°}$$
$$a = 18 \cdot 2 \text{ cm}.$$

Scribble box

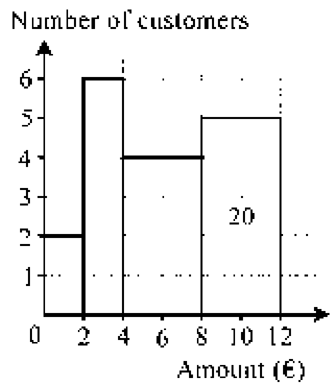
4. Statistics

Question

(a) The mean of the numbers 3, 4, 5, 6, 7 is 5. Calculate the standard deviation correct to one decimal place.

(b) The numbers 6, 11, 13, 3, 7 have weights 2, 5, 1, x, 1 respectively. If the weighted mean is 8, find the value of x.

(c) The amounts of money spent by a number of customers in a convenience store in a one hour period are shown in the histogram below.



(i) Copy and complete the following grouped frequency table.

Amount (in €)	0 - 2	2 - 4	4 - 8	8 - 12
No. of customers				20

(ii) By taking the data at the mid-interval values, estimate the mean amount spent by customers. Give your answer correct to the nearest cent.

(iii) What is the greatest number of customers who could have spent less than €6 in the convenience store?

Solution

(a) 3, 4, 5, 6, 7.

$\bar{x} = 5$

Table:

x	$x - \bar{x}$	$(x - \bar{x})^2$
3	−2	4
4	−1	1
5	0	0
6	1	1
7	2	4
	Total	10

Then

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$
$$= \sqrt{\frac{10}{5}} = \sqrt{2} = 1 \cdot 4.$$

(b) Table:

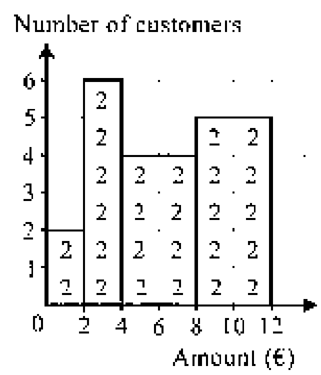
x	w	xw
6	2	12
11	5	55
13	1	13
3	x	$3x$
7	1	7
Totals	$x + 9$	$87 + 3x$

Given:

weighted mean = 8

$$\frac{\Sigma xw}{\Sigma w} = 8$$
$$\frac{87 + 3x}{x + 9} = 8$$
$$87 + 3x = 8x + 72$$
$$15 = 5x$$
$$x = 3.$$

(c) (i)



(As the frequency of '8 − 12' is 20, and there are 10 blocks in this rectangle, each block represents 2 customers.)

Filling in the table:

Amount (in €)	0 - 2	2 - 4	4 - 8	8 - 12
No. of customers	4	12	16	20

(ii) Table:

Interval	x	f	xf
0 − 2	1	4	4
2 − 4	3	12	36
4 − 8	6	16	96
8 − 12	10	20	200
	Totals	52	336

Then

$$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{336}{52} = 6 \cdot 46.$$

Thus the mean amount spent is €6·46.

(iii) The greatest number of customers who could have spent less than €6 is $4 + 12 + 16 = 32$.

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