All horses are the same color

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Problem: Prove that all horses in the world are of the same color (the color itself is not important; could be white, purple, green with orange polka dots, etc.). That is: if we have any set of *n* horses (*n* any integer greater than or equal to 1), all horses in this set should have the same color. This way, our proof would be independent of the actual number of live horses present on earth at a given time. (We are ignoring problems with information transfer arising from relativity.)

Solution: By mathematical induction. The smallest case in which we can prove the statement is $n = 1$. Clearly, any single horse is the same color as itself. (Hint: Yep, this is perfectly legitimate. Move on.) Say we happen to know that the statement is true for some $n = k \geq 1$ (induction hypothesis), or, that any set of *k* horses would always have a consistent color within the set. Now we want to show that any set of $k+1$ horses would have the same color.

So beg, borrow, or steal $k+1$ horses, and line them up. For example, here's a picture:

Horse 1 Horse 2 … … Horse 16 Horse 17

(then $k = 16$ and $k+1 = 17$ in this example). The leftmost *k* horses, as a subset, would share the same color according to our induction hypothesis:

Horse 1 Horse 2 …. … Horse 16: same color	Horse 17 ?

Wait… we could instead leave out Horse 1 and look at the remaining *k* horses to the right. This set of *k* horses should also be of the same color (possibly different from the first):

Horse 1 ?	Horse 2 Horse 3 … … Horse 17: same color

The $k-1$ horses in the intersection (all except the first and the last) would have the color of the first set AND of the second… and since we don't expect horses to change colors spontaneously (that's why the statement is not about chameleons), these two colors must be the same! But then, Horse 1 and Horse $k+1$ are also included in these two groups respectively, so they must share the common color as well.

Horse 1 Horse 2 … … Horse 16 Horse 17: same color (as the middle ones)

Exercise: Something must be wrong with this induction proof, though. What is it? Explain clearly, making references to the principle of mathematical induction. If you can't, we may just have to live in a world of pink horses.