JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 16, No. 2, March-April 1993

Gain-Scheduled Missile Autopilot Design Using Linear Parameter Varying Transformations

Jeff S. Shamma*
University of Texas at Austin, Austin, Texas 78712
and
James R. Cloutier†
U.S. Air Force Armament Directorate, Eglin Air Force Base, Florida 32542

This paper presents a gain-scheduled design for a missile longitudinal autopilot. The gain-scheduled design is novel in that it does not involve linearizations about trim conditions of the missile dynamics. Rather, the missile dynamics are brought to a quasilinear parameter varying (LPV) form via a state transformation. An LPV system is defined as a linear system whose dynamics depend on an exogenous variable whose values are unknown a priori but can be measured upon system operation. In this case, the variable is the angle of attack. This is actually an endogenous variable, hence the expression "quasi-LPV." Once in a quasi-LPV form, a robust controller using μ synthesis is designed to achieve angle-of-attack control via fin deflections. The final design is an inner/outer-loop structure, with angle-of-attack control being the inner loop and normal acceleration control being the outer loop.

I. Introduction

FUTURE tactical missiles will be required to operate over an expanded flight envelope to meet the challenge of highly maneuverable tactical aircraft. In such a scenario, an autopilot derived from linearization about a single flight condition will be unable to achieve suitable performance over all envisioned operating conditions. A particular challenge is that of the missile endgame. This involves the final few seconds before delivery of ordnance. During this phase, a missile autopilot can expect large and rapidly time-varying acceleration commands from the guidance law. In turn, the missile is operating at a high and rapidly changing angle of attack.

Traditionally, satisfactory performance across the flight envelope can be attained by gain scheduling local autopilot controllers to yield a global controller. Often the angle of attack is used as a scheduling variable. However, during the rapid transitions in the missile endgame, a fundamental guideline of gain scheduling to "schedule on a slow variable" is violated. Given the existing track record of gain scheduling, any improvement in the gain-scheduling design procedure—especially in the endgame—could have an important impact on future missile autopilot designs.

In this paper, we present a novel approach to gain-scheduled missile autopilot design. The missile control problem under consideration is normal acceleration control of the longitudinal dynamics during the missile endgame. In standard gain scheduling, the design plants consist of a collection of linearizations about equilibrium conditions indexed by the scheduling variable, in this case the angle of attack α (see Refs. 1 and 2). In the present approach, the design plants also consist of a family of linear plants indexed by the angle of attack. A key difference between the present approach and standard gain scheduling is that this family is not the result of linearizations. Rather, it is derived via a state transformation of the original missile dynamics (i.e., an alternate selection of state variables). Since no linearization is involved, the approach is not limited by the local nature of standard gain-scheduled designs.

Since gain scheduling generally encounters families of linear plants indexed by a scheduling variable, we shall refer to such a family as a linear parameter varying (LPV) plant. LPV plants *differ* from linear time-varying plants in that the time variations (i.e., the scheduling variable) is unknown a priori but may be measured/estimated upon operation of the feedback system. We shall call such a family quasi-LPV in case the scheduling variable is actually endogenous to the state dynamics (as in the missile problem). In Refs. 1 and 2, it was shown that LPV and quasi-LPV dynamics form the underlying structure of gain-scheduled designs.

The design for the resulting quasi-LPV system is performed via μ synthesis.[3] Briefly, μ synthesis exploits the structure of performance requirements and robustness considerations to achieve robust performance in a nonconservative manner. Thus, the present approach makes use of gain scheduling's ability to incorporate modern linear synthesis techniques into a nonlinear design.

Another feature in the present approach is its interpretation of an inner/outer-loop approach to nonlinear control design. In standard gain scheduling (as well as geometric nonlinear control[4]), one often applies an inner-loop feedback. In gain scheduling, this feedback is an update of the current trim condition. In geometric nonlinear control, this feedback serves to invert certain system dynamics to yield linear behavior in the modified plant. In either case, unless the inner-loop robustly performs its task, the outer-loop performance and even stability can be destroyed. In other words, any inner/outer-loop approach must be built from the inside out. Reference 1 presents a more detailed discussion of this possibility in the context of standard gain scheduling.

The present approach also takes an inner/outer-loop approach to the autopilot design. The inner loop consists of a robust angle-of-attack servo. The reason for the inner loop is that nonlinear gain-scheduling techniques prefer to directly control the scheduling variable. Such an inner/outer-loop decomposition was also employed in Ref. 5, where the reasoning was to avoid nonminimum phase dynamics from the fin deflection to normal acceleration.

The actual regulated variable of interest is the normal acceleration. Thus, the outer loop serves to generate angle-of-attack commands α_c to obtain the desired normal acceleration. A *consequence* of the inner-loop design is that the dynamics from the commanded angle of attack α_c to the angle of attack α exhibit a linear behavior *within the bandwidth* of the inner-

Received Sept. 30, 1991; revision received April 14, 1992; accepted for publication May 29, 1992. This paper is declared a work of the U.S. Government and is not subject to copyright protection in the United States.

*Assistant Professor, Department of Aerospace Engineering and Engineering Mechanics. Member AIAA.

†Senior Scientist, Guidance and Controls Branch, Wright Laboratory WL/MNAG. Associate Fellow AIAA.

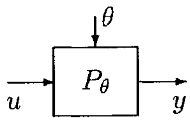

Fig. 1 Linear parameter varying (LPV) system.

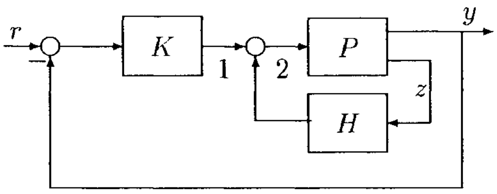

Fig. 2 Loop breaking points with different properties.

loop design. Thus, as in geometric nonlinear control, the inner loop linearizes certain dynamics. However, the approximate linearization stems from the natural *linearizing effect of feedback* (see Ref. 6) as opposed to an exact linear geometric condition on the plant state dynamics. Thus, the outer-loop design is essentially a linear design. However, in the design process, it is acknowledged that the linear behavior due to the inner loop is *approximate* and *band limited.*

The remainder of this paper is organized as follows. First we present the nonlinear missile dynamics under consideration. Then we review background material on LPV systems and μ synthesis. Next, the design and simulation results are presented for the missile autopilot. Finally, we state some concluding remarks.

II. Missile Dynamics

The missile dynamics considered here are taken from Ref. 7. These dynamics are representative of a missile traveling at Mach 3 at an altitude of 20,000 ft. However, they do not correspond to any particular missile airframe.

The nonlinear dynamics are as follows:

$$\dot{\alpha} = f \frac{g \cos(\alpha/f)}{WV} Z + q \tag{1}$$

$$\dot{q} = fm/I_{yy} \tag{2}$$

where

d = reference diameter, 0.75 ft
f = radians-to-degrees conversion, $180/\pi$
g = acceleration of gravity, 32.2 ft/s^2
I_{yy} = pitch moment of inertia, 182.5 slug-ft^2
m = $C_m QSd$ = pitch moment, ft-lb
Q = dynamic pressure, 6132.8 lb-ft^2
q = pitch rate, deg/s
S = reference area, 0.44 ft^2
V = speed, 3109.3 ft/s
W = weight, 450 lb
Z = $C_Z QS$ = normal force, lb
α = angle of attack, deg

The normal force and pitch moment aerodynamic coefficients are approximated by

$$C_Z = \phi_Z(\alpha) + b_Z \delta \tag{3}$$

$$C_m = \phi_m(\alpha) + b_m \delta \tag{4}$$

where

b_m = −0.206
b_Z = −0.034
δ = fin deflection, deg
$\phi_m(\alpha) = 0.000215\alpha^3 - 0.0195\alpha|\alpha| + 0.051\alpha$
$\phi_Z(\alpha) = 0.000103\alpha^3 - 0.00945\alpha|\alpha| - 0.170\alpha$

These approximations are accurate for α in the range of ± 20 deg.

Finally, the missile tailfin actuator is modeled as the second-order system with transfer function

$$\delta(s) = \frac{\omega_a^2}{s^2 + 1.4\omega_a s + \omega_a^2} \delta_c(s) \tag{5}$$

where

δ_c = commanded fin deflection, deg
ω_a = actuator bandwidth, 150 rad/s

The autopilot will be required to control the normal acceleration (expressed in g)

$$\eta_Z = Z/W \tag{6}$$

via commanded fin deflections δ_c. The general performance objective is to track acceleration step commands with a steady-state accuracy of less than 0.5% and a time constant of 0.2 s. Of course, the controller is band limited by flexible mode dynamics and actuator/sensor nonlinearities (e.g., rate saturations).[7]

III. Background Theory

A. LPV Systems

An LPV system[1,2,8] is defined as a linear system whose coefficients depend on an exogenous time-varying parameter. Let $y = P_\theta u$ be an LPV system as in Fig. 1. A possible realization for P_θ is

$$\dot{x} = A(\theta)x + B(\theta)u \tag{7}$$

$$y = C(\theta)x \tag{8}$$

The exogenous parameter θ is unknown a priori; however, it can be measured/estimated upon operation of the system. The reason for the special nomenclature is to distinguish LPV systems from linear time-varying systems for which the time variations are known beforehand (as in periodic systems). Typical a priori assumptions on θ are bounds on its magnitude and rate of change.

A gain-scheduled approach to controlling an LPV system is to design a collection of controllers based on frozen parameter values. This leads to an LPV controller K_θ. It was shown in Refs. 1 and 2 that this approach has guaranteed robustness and performance properties provided that the parameter time variations are "sufficiently slow." Quantitative statements qualifying sufficiently slow are provided in Refs. 1 and 2. Of course, sufficiently slow is with regards to the closed-loop system dynamics. Work on modifying gain scheduling to accommodate arbitrarily fast parameter time variations is in progress (see Refs. 9 and 10).

In Refs. 1, 2, and 8, it was shown that LPV systems provide the underlying framework for nonlinear gain-scheduled systems. To see this relationship, consider the nonlinear square plant

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} z \\ w \end{pmatrix} = f(z) + A(z)\begin{pmatrix} z \\ w \end{pmatrix} + B(z)u \tag{9}$$

where u is the control input and z is the controlled output. For this system, the nonlinearities depend only on the controlled output. Such systems are subsequently called "output-nonlinear" systems. Note that the missile dynamics are output nonlinear with the angle of attack α as the controlled output as follows:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \alpha \\ q \end{pmatrix} = \begin{bmatrix} \dfrac{fgQS\cos(\alpha/f)}{WV}\phi_Z(\alpha) \\ \dfrac{fQSd}{I_{yy}}\phi_m(\alpha) \end{bmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \alpha \\ q \end{pmatrix}$$

$$+ \begin{bmatrix} \dfrac{fgQSb_Z\cos(\alpha/f)}{WV} \\ \dfrac{fQSdb_m}{I_{yy}} \end{bmatrix}\delta \tag{10}$$

Section IV discusses how to accommodate normal acceleration η_Z as the controlled output even though the dynamics are output nonlinear in α.

We assume that there exist continuously differentiable functions $w_{eq}(z)$ and $u_{eq}(z)$ such that

$$0 = f(z) + A(z)\begin{bmatrix} z \\ w_{eq}(z) \end{bmatrix} + B(z)u_{eq}(z) \tag{11}$$

In other words, we have a family of equilibrium states parameterized by the controlled output z. For the missile problem, we have

$$\delta_{eq}(\alpha) = -\phi_m(\alpha)/b_m \tag{12}$$

$$q_{eq}(\alpha) = -\frac{fgQS\cos(\alpha/f)}{WV}\left[\phi_Z(\alpha) - \frac{b_Z}{b_m}\phi_m(\alpha)\right] \tag{13}$$

Let $A(z)$ and $B(z)$ be partitioned

$$A(z) = \begin{bmatrix} A_{11}(z) & A_{12}(z) \\ A_{21}(z) & A_{22}(z) \end{bmatrix}, \qquad B(z) = \begin{bmatrix} B_1(z) \\ B_2(z) \end{bmatrix} \tag{14}$$

to conform with $(z \quad w)^T$. Then it is easy to show that the state dynamics may be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z \\ w - w_{eq}(z) \end{bmatrix} = \begin{bmatrix} 0 & A_{12}(z) \\ 0 & A_{22}(z) - Dw_{eq}(z)A_{12}(z) \end{bmatrix}\begin{bmatrix} z \\ w - w_{eq}(z) \end{bmatrix} + \begin{bmatrix} B_1(z) \\ B_2(z) - Dw_{eq}(z)B_1(z) \end{bmatrix}\left[u - u_{eq}(z)\right] \tag{15}$$

Thus, we have *transformed* the original dynamics into a quasi-LPV form, with the variable z as the "exogenous" parameter. In case all nonlinearities are not contained in the output, the previous transformation will be approximate up to first-order terms in $w - w_{eq}(z)$.[1] It is interesting to note that this quasi-LPV family *is not* the same family we would obtain by performing linearizations about equilibrium conditions.

Now, although we may use the previous quasi-LPV plant as the design plant, a possible drawback is the inner-loop feedback term $u_{eq}(z)$. More precisely, if one were to design a controller for the previous quasi-LPV plant, the actual applied control signal would be

$$u = u_{eq}(z) + \tilde{u} \tag{16}$$

where $\tilde{u}$ is the controller output. Even though the outer loop may have guaranteed robustness properties, the inner-loop feedback $u_{eq}(z)$ can destroy these properties by adversely exciting flexible mode dynamics.[1,8] This is illustrated in Fig. 2. In this figure, the block P represents the plant dynamics, the block H represents an inner feedback to update the trim condition $u_{eq}(z)$, and the block K represents a controller designed using the previous quasi-LPV plant. In this figure, it is possible that unmodeled dynamics at breaking point 2 can destroy performance or even be destabilizing, whereas robust performance is obtained for the same unmodeled dynamics at breaking point 1. Note that actuator dynamics occur at the plant input 1 and not the controller output 2. In the missile problem, we have $\delta_{eq}(\alpha) = -\phi_m(\alpha)/b_m$. Thus fast angle-of-attack variations (as in the endgame) could excite neglected flexible mode dynamics.

This problem can be avoided by augmenting integrators at the plant input. Let

$$u = \int v \tag{17}$$

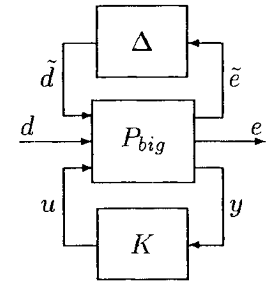

Fig. 3 μ synthesis interconnection structure.

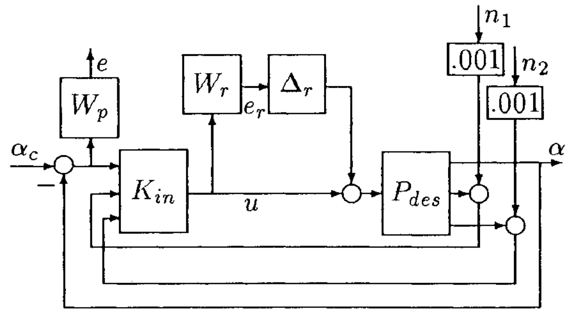

Fig. 4 Inner-loop angle-of-attack control.

Then the system dynamics take the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} z \\ w - w_{eq}(z) \\ u - u_{eq}(z) \end{bmatrix} = \begin{bmatrix} 0 & A_{12}(z) & B_1(z) \\ 0 & A_{22}(z) - Dw_{eq}(z)A_{12}(z) & B_2(z) - Dw_{eq}(z)B_1(z) \\ 0 & -Du_{eq}(z)A_{12}(z) & -Du_{eq}(z)B_1(z) \end{bmatrix} \times \begin{bmatrix} z \\ w - w_{eq}(z) \\ u - u_{eq}(z) \end{bmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v \tag{18}$$

Now if we design a controller without using the state $u - u_{eq}(z)$ for feedback, no inner-loop feedback of a trim condition is applied. Thus, any robustness properties of the quasi-LPV design remain intact.

The forthcoming missile design uses the previous representation of the missile dynamics for the autopilot design. This representation is the result of a *state transformation* only. That is, no approximation/linearization of the original dynamics has occurred. As mentioned earlier, in case the system dynamics are nonlinear in w as well, then the previous representation is accurate up to first order in $w - w_{eq}(z)$. Systems for which higher order terms in $w - w_{eq}(z)$ are large are not well suited for gain-scheduling in the first place. In other words, gain scheduling seeks to exploit predominantly output-nonlinear dynamics.

A gain-scheduled approach to control design for quasi-LPV systems resembles that for LPV systems. Namely, a series of designs are performed for frozen z values of the state-space matrices. This leads to a quasi-LPV controller with z as the external parameter.

B. μ Synthesis

In this section, we present a very brief overview of μ synthesis for linear plants. See Ref. 3 for a more detailed discussion.

First, we establish the following notation (see Ref. 11). For a time signal g, we define $\|g\|$ as

$$\|g\| \stackrel{\text{def}}{=} \left[\int_0^\infty g^T(t) g(t)\, dt \right]^{1/2} \tag{19}$$

For a stable dynamical system H, we define $\|H\|$ as

$$\|H\| \stackrel{\text{def}}{=} \sup_{\substack{g \neq 0 \\ \|g\| < \infty}} \frac{\|Hg\|}{\|g\|} \tag{20}$$

In case H is linear time invariant, then

$$\|H\| = \sup_\omega \sigma_{\max}[H(j\omega)] \tag{21}$$

Figure 3 shows the general structure for μ synthesis. In this figure, the block P_{big} denotes the "generalized plant," i.e., the plant to be controlled as well as various weightings/normalizations on time signals and modeling errors. The block K denotes the controller. The block Δ denotes a block-diagonal system of linear time-varying perturbations. This is a slight departure from the usual assumption of linear time-invariant perturbations. This assumption seems more appropriate since the quasi-LPV dynamics are really nonlinear. We assume Δ has been normalized (via weightings absorbed into P_{big}), so that $\|\Delta\| < 1$. Let $H(K,\Delta)$ denote the closed-loop dynamics from d to e. The objective is to design a controller K to minimize $\|H(K,\Delta)\|$ over all admissible perturbations Δ. Typically, the problem is normalized so that $\|H(K,\Delta)\| \leq 1$ for all $\|\Delta\| < 1$ implies robust performance, i.e., performance for all admissible perturbations.

The μ synthesis design procedure is described as follows. Let $\tilde{H}(K)$ denote the closed-loop dynamics

$$\begin{pmatrix} \tilde{d} \\ d \end{pmatrix} \mapsto \begin{pmatrix} \tilde{e} \\ e \end{pmatrix}$$

The objective is to find a controller K_* and a constant diagonal scaling D_* such that $\|D_* \tilde{H}(K_*) D_*^{-1}\| \leq 1$. This implies robust performance for all admissible perturbations. The diagonal structure of D_* is set to appropriately match the diagonal structure of Δ. See Ref. 3 for more details.

A design via μ synthesis seeks to achieve the aforementioned goal by minimizing $\|D\tilde{H}(K)D^{-1}\|$ over stabilizing K and appropriate diagonal D. This quantity is minimized by alternatively minimizing over K and D. First, design a controller K_0 such that $\|\tilde{H}(K_0)\|$ is minimized. Second, find a constant diagonal scaling D_0 such that $\|D_0\tilde{H}(K_0)D_0^{-1}\|$ is minimized. Next, design a controller K_1 such that $\|D_0\tilde{H}(K_1)D_0^{-1}\|$ is minimized. Then, find a constant diagonal scaling D_1 such that $\|D_1\tilde{H}(K_1)D_1^{-1}\|$ is minimized, etc. This process, known as D-K iteration, continues as long as each iteration provides a sufficient reduction in the cost function $\|D\tilde{H}D^{-1}\|$. Although D-K iteration need not find a global minimum, it can often lead to good results.

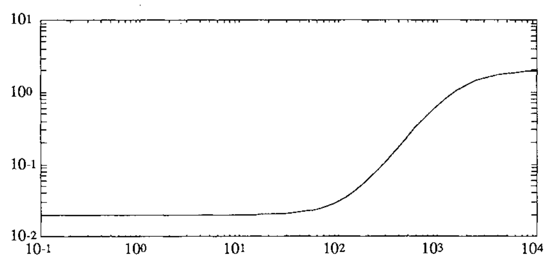

Fig. 5 Robustness weight $W_r(j\omega)$.

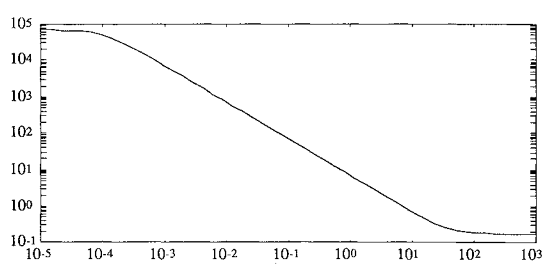

Fig. 6 Performance weight $W_p(j\omega)$ for angle-of-attack control.

IV. Missile Autopilot Design

In this section, we discuss the missile autopilot design. The objective is to control normal acceleration η via commanded fin deflections δ_c. We assume that the angle of attack α, the pitch rate q, and the normal acceleration η are available for feedback. The pitch rate and normal acceleration measurements are obtained via rate gyros and accelerometers, respectively.[7] However, in practice the angle of attack α must be estimated.

As mentioned in Sec. III.A, gain scheduling seeks to control the output variable of an output-nonlinear system. The longitudinal missile dynamics are output nonlinear with the angle of attack α as the output variable. Since normal acceleration η_Z is the regulated variable of interest, the gain-scheduled design is modified as follows. First, we design a controller for angle of attack. This constitutes the inner feedback loop. We then design an outer feedback loop to generate angle-of-attack commands to achieve desired normal accelerations.

A. Inner-Loop Angle-of-Attack Control

The inner feedback loop consists of angle-of-attack control. First, the missile dynamics are augmented with an integrator and transformed to a quasi-LPV form as in Sec. III.A. The actuator dynamics are then augmented onto the quasi-LPV missile dynamics. This process leads to a design plant P_{des} whose states are $[\alpha \quad q - q_{eq}(\alpha) \quad \delta - \delta_{eq}(\alpha) \quad x_a \quad \dot{x}_a]^T$, where x_a and $\dot{x}_a$ are the actuator state variables.

Figure 4 shows the block diagram used for the μ synthesis design. The block P_{des} denotes the quasi-LPV design plant. The input u to P_{des} is actually the time derivative of the commanded fin deflection. That is,

$$\delta_c = \int u \tag{22}$$

The measurements from P_{des} are the angle of attack α, the pitch rate trim deviation $q - q_{eq}(\alpha)$, and the normal acceleration trim deviation $\eta - \eta_{Z,eq}(\alpha)$. Note that

$$\eta_Z - \eta_{Z,eq}(\alpha) = \frac{QSb_Z}{W}[\delta - \delta_{eq}(\alpha)] \tag{23}$$

In the actual implementation, the values of $q - q_{eq}(\alpha)$ and $\eta - \eta_{eq}(\alpha)$ would be constructed from α, q, and η measurements.

The robustness and performance objectives are described as follows. The block Δ_r represents linear time-varying multiplicative perturbation weighted by W_r. This uncertainty reflects actuator phase/gain uncertainty and flexible mode dynamics. Figure 5 shows the frequency response of W_r, where

$$W_r(s) = 2\frac{(s+100)(s+200)}{(s+1000)(s+2000)} \tag{24}$$

The performance objective is to keep $\|\alpha_c \mapsto e\| \leq 1$. The performance weight

$$W_p(s) = \frac{7(s+40)}{40(s+0.0001)} \tag{25}$$

has a frequency response shown in Fig. 6. Finally, the signals n_1 and n_2 are small noises injected to satisfy certain rank con-

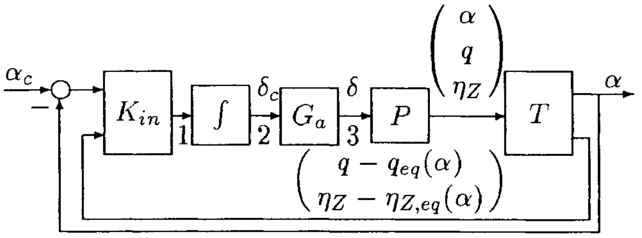

Fig. 7 Implementation of K_{inner}.

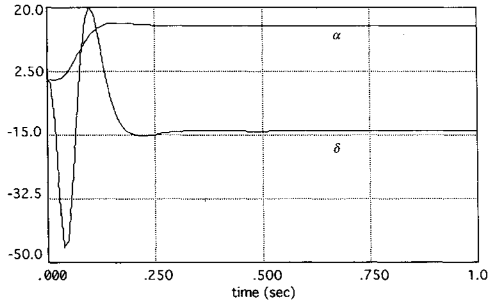

Fig. 8 Angle-of-attack step response.

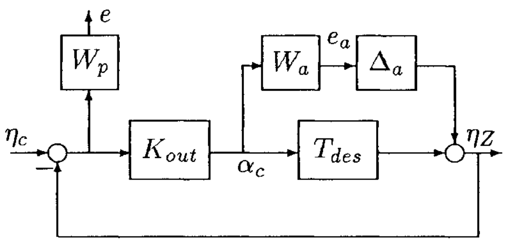

Fig. 9 Outer-loop acceleration control.

ditions in the μ synthesis design. In terms of Fig. 3, the interconnection structure is

$$d = \alpha_c \tag{26}$$

$$\tilde{d} = \begin{bmatrix} d_r \\ \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \end{bmatrix} \tag{27}$$

$$y = \begin{bmatrix} \alpha_c - \alpha \\ q - q_{eq}(\alpha) + n_1 \\ \eta - \eta_{eq}(\alpha) + n_2 \end{bmatrix} \tag{28}$$

$$\tilde{e} = \begin{pmatrix} e_r \\ 0.001e \end{pmatrix} \tag{29}$$

and u and e as shown in Fig. 4.

A μ synthesis design procedure was performed with this interconnection structure at the set point $\alpha = 0$. That is, the α-dependent coefficient matrices of the quasi-LPV plant P_{des} were evaluated at $\alpha = 0$ for the design. The first pass led to a frozen α robust performance level of 1.09. In terms of Sec. III.B, the cost function $\|D\tilde{H}(K)D^{-1}\| = 1.09$. After six iterations, this value was reduced to 0.5232.

Now a gain-scheduled design procedure would typically involve repeating the fixed-α designs for several α set points. However, it turns out the $\alpha = 0$ controller delivered robust performance for all α in the range ± 20 deg. Thus, no controller gain scheduling was required. For this particular airframe, the missile dynamics at $\alpha = 0$ are statically unstable and become stable at higher values of α. It is believed that the μ synthesis procedure—which is an optimization—is most constrained at $\alpha = 0$, thereby resulting in the $\alpha = 0$ design providing uniform robust performance. Note that, even though K_{inner} stems from only one linear design, it is still a nonlinear controller in that it uses $q - q_{eq}(\alpha)$ and $\eta - \eta_{Z,eq}(\alpha)$ as inputs.

Figure 7 shows the actual implementation of K_{inner}. The block P denotes the nonlinear missile dynamics. The block G_a denotes the actuator dynamics. The block T denotes a transformation of the actual measurements into their "deviation from trim" form. Note that the inner loop was designed to give guaranteed robustness properties at loop breaking point 1 (Fig. 7). That is, the design was to deliver robust performance for all admissible linear time-varying perturbations. However, we see from Fig. 7 that similar robustness properties for linear time-invariant perturbations are obtained at loop breaking points 2 or 3, where actual model deviations are likely to occur. As mentioned in Sec. III.A, this is one of the benefits of augmenting integrators and *not* having an inner loop that updates the trim control input.[1]

Regarding stability properites of the feedback system, it is shown in Refs. 1 and 8 that robust stability and robust performance is maintained provided there are sufficiently slow time variations in the scheduling variable. Although Refs. 1 and 8 provide quantitative statements regarding sufficiently slow, an application of these inequalities is likely to lead to conservative conclusions. Rather, the following qualitative interpretation is more suggestive. Namely, a sufficiently slow requirement is with regards to the closed-loop system dynamics. In this case, the closed-loop system has a bandwidth of about 120 rad/s. Thus, one may expect robust performance for angle-of-

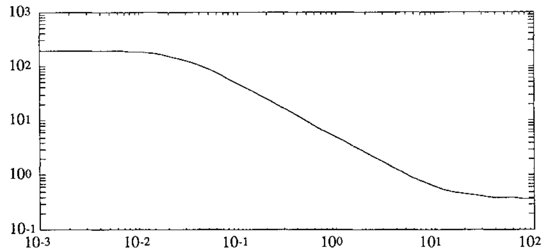

Fig. 10 Performance weight $W_p(j\omega)$ for acceleration control.

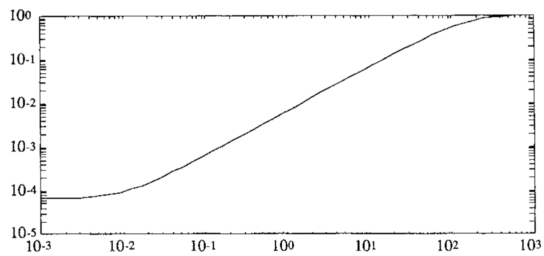

Fig. 11 Robustness weight $W_a(j\omega)$.

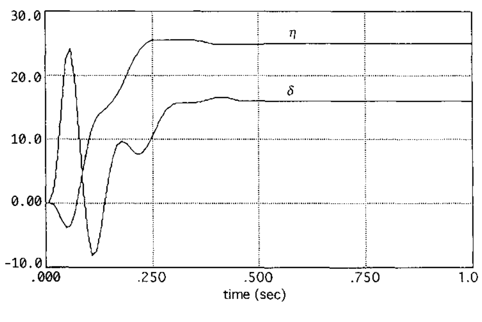

Fig. 12 Acceleration step response.

attack time variations of the same order. Of course, only extensive simulations, nonconservative stability criteria, or an alternate scheduling procedure entirely can really confirm/refute closed-loop robust performance.[2]

Finally, Fig. 8 shows the response to a 15-deg α step command. Note that this step command leads to excessively large fin deflections. This will not be the case for the overall controller, since the outer feedback loop will be designed to discourage drastic angle-of-attack commands, such as a significantly large step.

B. Outer-Loop Normal Acceleration Control

The outer-loop feedback is designed to generate angle-of-attack commands α_c to achieve desired normal accelerations. As mentioned in Sec. II, the performance requirements are to track acceleration step commands with a steady-state accuracy of less than 0.5% and a time constant of 0.2 s. However, the outer loop is band limited by the inner feedback loop. That is, the outer loop should only generate angle-of-attack commands α_c within the bandwidth of the inner loop. It is over this range that we have a reasonable model of the inner-loop behavior.

The first part of this design is to obtain the appropriate quasi-LPV system dynamics from α_c to η_Z. The inner loop leads to quasi-LPV dynamics from α_c to the states $[\alpha \quad q-q_{eq}(\alpha) \quad \delta-\delta_{eq}]^T$. However, the normal acceleration is given by

$$\eta_Z = \eta_{Z,eq} + \left[\eta_Z - \eta_{Z,eq}(\alpha)\right]$$
$$= \frac{QS}{W}\left[\phi_Z(\alpha) - \frac{b_Z}{b_m}\phi_m(\alpha)\right] + \frac{QSb_Z}{W}\left[\delta - \delta_{eq}(\alpha)\right] \tag{30}$$

Thus the normal acceleration η_Z is a nonlinear function of the quasi-LPV states. Performing a linear approximation of the term $[\phi_Z(\alpha) - (b_Z/b_m)\phi_m(\alpha)]$ leads to

$$\eta_Z \cong -1.54\alpha + \frac{QSb_Z}{W}\left[\delta - \delta_{eq}(\alpha)\right] \tag{31}$$

This leads to an output coefficient matrix that approximates the normal acceleration by a linear function of the quasi-LPV states.

Figure 9 shows the block diagram used for the μ synthesis design. The block T_{des} denotes the closed-loop dynamics from α_c to η_z. The measurement is $\eta_c - \eta_Z$, where η_c is the commanded acceleration. The performance objective is to keep $\|\eta_c \mapsto e\| \leq 1$, where

$$W_p(s) = \frac{15s + 200}{40s + 1} \tag{32}$$

Figure 10 shows the frequency response for W_p. The block Δ_a represents an additive perturbation on the closed inner-loop dynamics. The weighting (cf., Fig. 11)

$$W_a(s) = \frac{s + 0.01}{s + 150} \tag{33}$$

reflects that the inner-loop model is fairly accurate at low frequencies. However, the model is less accurate for high-frequency angle-of-attack commands. In terms of Fig. 3, the interconnection structure is

$$d = \eta_c \tag{34}$$

$$\tilde{d} = d_a \tag{35}$$

$$y = \alpha_c - \alpha \tag{36}$$

$$\tilde{d} = e_a \tag{37}$$

$$u = \alpha_c \tag{38}$$

and e as shown in Fig. 9.

A μ synthesis design procedure was performed with this interconnection structure at the set point $\alpha = 0$. The first pass led to a frozen α robust performance level of 3.125. After two iterations, this value was reduced to 0.732.

Once again, the controller for $\alpha = 0$ proved adequate for the entire α range of $\alpha \pm 20$ deg. Thus, no gain scheduling is required. Recall that this was the case in the inner-loop α control. For the inner loop, this was due to the airframe dynamics at $\alpha = 0$ being the most difficult to control. However, it is believed that the outer loop not requiring gain scheduling is to be expected in general. Recall that the inner loop was the result of a robust servo design. This feedback in itself has a linearizing effect on the missile dynamics (see Ref. 6). For example, one has that $\alpha \cong I\alpha_c$, for the class of α_c within the

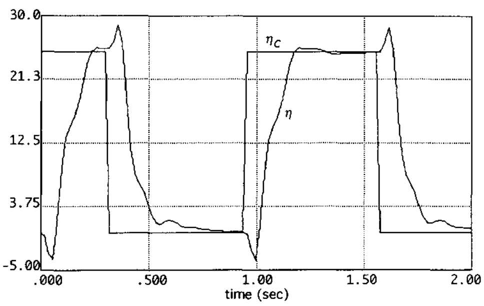

Fig. 13 Acceleration square wave response.

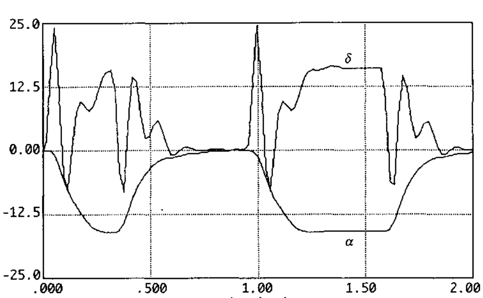

Fig. 14 Fin and angle-of-attack square wave response.

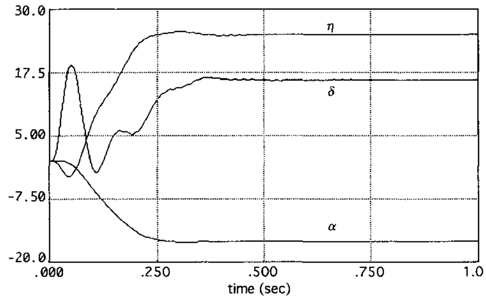

Fig. 15 Step response with $\gamma = 3.5$ gain margin.

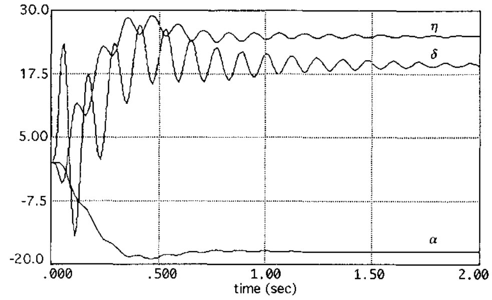

Fig. 16 Step response with −10% perturbation in $\phi_Z(\alpha)$.

inner-loop bandwidth. Outside of this bandwidth, the linear behavior will deteriorate—hence the additive uncertainty weighting W_a. Thus, although not needing gain scheduling in the inner loop is not typical, it seems reasonable in the outer loop. Note that the linearization through feedback *differs* from that of geometric feedback linearization (see Ref. 4). In this case, the linearization is due to a robust servo design. Hence it is approximate and band limited. Furthermore, the design of the outer loop takes this into account.

Figure 12 shows the response to a $25g$ step command. Note that large fin deflections do not occur as in the angle-of-attack step response (Fig. 8). As mentioned earlier, this is due to the outer loop acknowledging the band-limited performance of the inner loop. During the missile endgame, the guidance law typically generates large rapidly varying acceleration commands. To illustrate the performance in such a scenario, Fig 13 shows the response to a square-wave command oscillating between $25g$ and $0g$. Figure 14 shows the fin deflections and angle-of-attack response.

The stability and performance robustness was tested for model uncertainties not addressed explicitly in the μ synthesis design. The results are described as follows. The issued fin command δ_c was perturbed by a scalar gain γ. Good performance was achieved for γ up to 3.5; stability was maintained for γ down to 0.5, but with poor performance. Stability and performance were maintained for a $+50\%$ uncertainty in $\phi_m(\alpha)$; stability was maintained for a -30% uncertainty. Stability was maintained for a -10% perturbation and a $+5\%$ perturbation in $\phi_Z(\alpha)$. It is believed the increased sensitivity was due to the cubic terms in the polynomial. In fact, stability was maintained for $\pm 20\%$ perturbations in the linear coefficient $C_{Z,\alpha}$, which nominally equals -0.170. The design was sensitive (i.e., tolerating $\pm 5\%$) to combined uncertainties involving both control gains and plant models, such as perturbing both $\phi_z(\alpha)$ and b_z. Representative $25g$ step command time responses are shown in Fig. 15, exhibiting good performance and in Fig. 16, exhibiting stability only.

Of course, all of these perturbations imply that the plant dynamics and the design model are different. A further implication is that the true plant dynamics and the transformation to equilibrium values are mismatched (cf., the block T in Fig. 7). A potential improvement would be to incorporate this mismatch into the design process.

V. Conclusions

This paper has presented a novel approach to gain-scheduled missile autopilot design for longitudinal missile dynamics. Some key features of this approach are as follows. First, the missile dynamics are brought to an LPV form via a state transformation rather than the usual linearization. Second, an integrator is augmented so that no "update of trim control" feedback loop is present. Finally, an inner/outer-loop decomposition is applied. It is believed that an effect of the inner loop is to linearize the missile dynamics in an approximate and band limited manner, thereby leading to a simplified outer-loop design with guaranteed inner-loop robustness properties.

Appendix: Compensator State Equations

Let $\{A,B,C,D\}$ denote the state dynamics

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du \tag{A1}$$

The inner-loop compensator $K_{\text{inner}} \equiv \{A_{\text{inner}}, B_{\text{inner}}, C_{\text{inner}}, D_{\text{inner}}\}$ has eight states, three inputs, and one output (cf., Fig. 7), with

$$A_{\text{inner}} = \begin{bmatrix} -2.2498e-04 & -5.8240e-01 & -2.1941e-02 & 7.5435e-14 & 2.8314e-16 & 3.1178e-10 & 2.4908e-10 & 4.3565e-11 \\ -7.9862e-08 & -6.3392e+02 & -2.3883e+01 & -1.0295e-13 & -3.8644e-16 & -1.8726e-09 & -3.3994e-10 & -5.9458e-11 \\ 1.1309e-07 & 2.5996e+03 & -1.1688e+03 & 1.0000e+00 & 2.1864e-14 & 1.0594e-07 & 1.9234e-08 & 3.3641e-09 \\ 3.7058e-05 & 9.6835e+05 & -8.2186e+05 & 5.8075e-09 & 1.0000e+00 & 1.0563e-04 & 1.9176e-05 & 3.3539e-06 \\ 4.6000e+06 & 1.3621e+08 & -2.3542e+08 & -2.5886e+04 & -2.2271e+02 & -6.1576e+07 & 9.0707e+06 & 3.8545e+07 \\ 6.5405e-17 & 2.4379e-18 & -6.5340e-18 & -4.8140e-20 & -1.8069e-22 & -9.9997e-05 & -1.5895e-16 & -2.7801e-17 \\ 2.0444e+02 & 7.0983e+00 & -2.0437e+01 & -1.5048e-01 & -5.6481e-04 & -2.7367e+03 & -5.9686e+02 & -8.6903e+01 \\ 2.0444e+02 & 7.0983e+00 & -2.0437e+01 & -1.5048e-01 & -5.6481e-04 & -2.7367e+03 & 4.0314e+02 & -2.8690e+02 \end{bmatrix} \tag{A2}$$

$$B_{\text{inner}} = \begin{bmatrix} -3.9368e-06 & 2.7690e-02 & 8.4886e-03 \\ -1.3999e-09 & 1.1082e+01 & 9.2208e+00 \\ 2.1174e-09 & -4.5494e+01 & -1.0091e+02 \\ 7.8654e-07 & -1.6945e+04 & -7.0956e+04 \\ 1.1058e-04 & -2.3807e+06 & -2.0286e+07 \\ 1.7499e-02 & -2.9233e-21 & 3.3947e-22 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{A3}$$

$$C_{\text{inner}} = (1.1683e+04 \quad 4.0565e+02 \quad -1.1679e+03 \quad -8.5994e+00 \quad -3.2277e-02 \quad -1.5640e+05 \quad 2.3039e+04 \quad 9.7899e+04) \tag{A4}$$

$$D_{\text{inner}} = 0 \quad 0 \quad 0) \tag{A5}$$

The outer-loop compensator $K_{\text{outer}} \equiv \{A_{\text{outer}}, B_{\text{outer}}, C_{\text{outer}}, D_{\text{outer}}\}$ has three states, one input, and one output (cf., Fig. 7), with

$$A_{\text{outer}} = \begin{pmatrix} -2.5009e-02 & 1.3148e-01 & 4.6253e-02 \\ 0 & -1.8846e+01 & 5.9814e+01 \\ 0 & -7.2876e+01 & -3.2574e+00 \end{pmatrix} \tag{A6}$$

$$B_{\text{outer}} = \begin{pmatrix} 2.1391e+00 \\ -2.7653e+00 \\ -1.6766e+00 \end{pmatrix} \tag{A7}$$

$$C_{\text{outer}} = (-2.1376e+00 \quad 3.2141e+00 \quad 3.4579e-01) \tag{A8}$$

$$D_{\text{outer}} = -1.6948e-02 \tag{A9}$$

Acknowledgments

This research was supported by an Air Force Office of Scientific Research (AFOSR) Summer Faculty Fellowship at the U.S. Air Force Armament Directorate, Eglin Air Force Base, FL, AFOSR Research Initiation Grant 92-70 subcontracted to the Research and Development Laboratory, and National Science Foundation Grant ECS-9296074. The authors thank Johnny Evers and Bob Reichert for helpful discussions.

References

[1]Shamma, J. S., "Analysis and Design of Gain Scheduled Control Systems," Ph.D. Thesis, Dept. of Mechanical Enginering, Massachusetts Inst. of Technology, Cambridge, MA, 1988.

[2]Shamma, J. S., and Athans, M., "Analysis of Nonlinear Gain Scheduled Control Systems," *IEEE Transactions on Automatic Control,* Aug. 1990, pp. 898–907.

[3]Balas, G., Doyle, J. C., Glover, K., Packard, A., and Smith, R., "μ-Analysis and Synthesis Toolbox: μ-Tools," MUSYN, Inc., and The Mathworks, Inc., Dec. 1990.

[4]Isidori, A., *Nonlinear Control Systems: An Introduction,* Vol. 72, Lecture Notes in Control and Information Science, Springer-Verlag, Berlin, 1985.

[5]Tahk, M., Briggs, M. M., and Menon, P. K. A., "Applications of Plant Inversion via State Feedback to Missile Autopilot Design," *Proceedings of the 27th IEEE Conference on Decision and Control* (Austin, TX), Dec. 1988, pp. 730–735.

[6]Desoer, C. A., and Wang, Y.-T., "Foundations of Feedback Theory for Nonlinear Dynamical Systems," *IEEE Transactions on Circuits and Systems,* Vol. CAS-27, No. 2, Feb. 1980, pp. 104–123.

[7]Reichert, R., "Modern Robust Control for Missile Autopilot Design," *Proceedings of the American Control Conference* (San Diego, CA), June 1990, pp. 2368–2373.

[8]Shamma, J. S., and Athans, M., "Guaranteed Properties of Gain Scheduled Control of Linear Parameter-Varying Plants," *Automatica,* Vol. 27, No. 3, May 1991, pp. 559–565.

[9]Shahruz, S. M., and Behtash, S., "Design of Controllers for Linear Parameter Varying Systems by the Gain Scheduling Technique," Berkeley Engineering Research Inst., Memorandum BERI M90/1, Univ. of California, Berkeley, CA, Jan. 1990.

[10]Shamma, J. S., and Athans, M., "Gain Scheduling: Potential Hazards and Possible Remedies," *IEEE Control Systems Magazine,* June 1992.

[11]Desoer, C. A., and Vidyasagar, M., *Feedback Systems: Input-Output Properties,* Academic, New York, 1975.