Robust Formulations of the Cauchy Method Suitable for Microwave Duplexers Modeling

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***Abstract*—It is well known that a Vandermonde matrix generates an ill-conditioned system matrix when applied with finite numerical precision. This deficiency affects the Cauchy method by restricting its application to only lower order systems. This paper presents innovative, accurate, and robust formulations of the Cauchy method to rectify this limitation and make the Cauchy method suitable for the extraction of a high-order microwave duplexer polynomial model. The techniques employed are: the change of polynomial basis into the Krylov subspace and the precondition technique, both acting on the system matrix of the classic Cauchy method formulation. A novel formulation using the QR algorithm on the two characteristic functions of the duplexer and a suitable combination of the QR method and the precondition technique are then presented. Each of these procedures has been successfully verified by numerical application examples.**

***Index Terms*—Cauchy method, condition number, microwave duplexer, numerical analysis, robust modeling.**

I. Introduction

THE generation of reduced-order polynomial models from frequency sampled data is becoming the subject of more and more investigations and studies in the microwave community. The two most important applications concern: 1) the extrapolation/interpolation of electromagnetic (EM) simulated responses obtained though computationally expensive numerical methods and 2) the generation of circuit models from the measured (lossy) response of microwave networks (typically filters), suitable for automated tuning procedures.

The Cauchy method is a well-known and effective technique for generating reduced-order rational polynomial models from the response of a passive device [1]–[3]. Most of the past works have concerned group 1) of the above recalled applications; recently, efforts have been directed toward introducing a formulation consistent with the synthesis of equivalent lossless circuits, starting from lossy measured data, which is a fundamental requirement for computer-aided tuning [4], [5].

It is known that the classic formulation of the Cauchy method suffers from the limitation of an ill-conditioned Vandermonde matrix. In order to improve the robustness of the system matrix, techniques applied to microwave filters have been introduced [6], [7]. However, to date, these methods have not been applied to the extraction of models for more complex multiport systems. A possible example of such systems is the microwave duplexer, which is a combining network composed by two passband filters with the input ports connected through a suitable junction [8]. Since the overall order of a typical duplexer is at least the sum of the orders of the composing filters, this challenging application requires a more robust formulation for the Cauchy method.

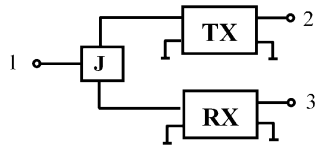

Fig. 1. General structure of a duplexer.

In this paper, a suitable formulation of the Cauchy method for generating a rational polynomial model of typical microwave duplexers is presented; moreover, additional accurate and robust techniques for the model extraction are introduced. Input frequency-sampled data can be either lossless (as those coming from EM simulations) or include losses (in case of computer-aided tuning applications); in this latter case, a suitable technique is presented in order to derive an approximate lossless polynomial model from the lossy data, which can be employed for the synthesis of the required equivalent circuits [8].

This paper is organized as follows. In Section II, the complete generation of the rational polynomial model from frequency-sampled data of a duplexer, using a modified formulation of the Cauchy method, is described. In Section III, two robust approaches are presented, involving the system matrix of the classic formulation of the Cauchy method: the change of polynomial basis (Krylov subspace) and the preconditioning technique. In Section IV, novel formulations of the Cauchy method based on the two duplexer characteristic functions are presented. The **QR** algorithm and a suitable combination of the **QR** algorithm and the preconditioning technique have been employed. Numerical application examples of these formulations are presented in Section V.

II. Cauchy Method: Model and Problem Formulation

A. Polynomial Modeling of the Duplexer

The typical configuration of a microwave duplexer is shown in Fig. 1 [8]. It is constituted by two bandpass filters [transmitter (TX) and receiver (RX)], connected through the three-port junction J, whose topology depends on the specific technology employed for the duplexer implementation. It is assumed that the RX passband $(f_{\text{low}}^{\text{RX}} - f_{\text{up}}^{\text{RX}})$ is below the TX passband $(f_{\text{low}}^{\text{TX}} -$

Manuscript received October 30, 2006; revised January 29, 2007.

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Digital Object Identifier 10.1109/TMTT.2007.895394

$f_{\rm up}^{\rm TX}$). In order to simplify the mathematical approach, it will be assumed in the following that the junction is a simple shunt connection of the two filters input ports.

The overall duplexer constitutes a particular kind of three-port network, which can be characterized, in the low-pass normalized frequency domain s, through a polynomial model; in the domain $s = j\Omega$, it is assumed that the outer passbands frequency limits ($f_{\rm low}^{\rm RX}$ and $f_{\rm up}^{\rm TX}$) are mapped to $-j$ and $+j$, respectively (various kinds of analytical frequency mapping can be used to this purpose; the most simple is the classic low-pass to bandpass transformation used for lumped resonators). Let observe that the polynomial modeling of a distributed network is intrinsically approximated (the accuracy decreases with the increase of the frequency span considered); in the case concerned here, the polynomial modeling gives good performances with narrow or moderate bandwidth networks.

It has been shown in [8] that, starting from four suitably computed polynomials, it is possible to synthesize a lossless duplexer, with the topology in Fig. 1, presenting the following scattering parameters:

$$S_{11}(s) = \frac{N(s)}{D(s)}$$
$$S_{21}(s) = \frac{P_{\rm TX}(s)}{D(s)}$$
$$S_{31}(s) = \frac{P_{\rm RX}(s)}{D(s)}. \tag{1}$$

Let $(n_{\rm TX}, m_{\rm TX})$ and $(n_{\rm RX}, m_{\rm RX})$ be the number of poles and the number of transmission zeros of the TX and RX filters, respectively. The following considerations can then be made concerning the above polynomials.

- The overall number of poles of the duplexer is $n_{\rm TX} + n_{\rm RX}$ [order of $D(s)$]. $N(s)$ also has the same order (i.e., the number of reflection zeros is $n_{\rm TX} + n_{\rm RX}$).
- The number of transmission zeros of S_{21} is $m_{\rm TX} + n_{\rm RX}$ [order of $P_{\rm TX}(s)$]; the additional $n_{\rm RX}$ zeros are produced by the loading of the RX filter at the TX filter input.
- Similarly, the number of transmission zeros of S_{31} is $m_{\rm RX} + n_{\rm TX}$ [order of $P_{\rm RX}(s)$].

The problem concerned with this study consists of the derivation of the above polynomials, given the scattering parameters in a specified frequency range (from EM simulations or measurements). In the case of computer-aided tuning, the derivation of the coupling matrices for the TX and RX filters is also required; this can be accomplished through the procedures presented in [8], which allows the evaluation of the characteristic polynomials associated to TX and RX filters. Once the topology of the two filters is specified, the synthesis methods available in the literature [9], [10] allow the evaluation of the required coupling matrices.

However, it must be observed that synthesis methods require the lossless condition to be satisfied, while the measured data from a real device actually include losses. It is then necessary to extrapolate from the lossy measured scattering parameters a lossless polynomial model. To this purpose, we have extended the method presented in [4] for two-port filters to the three-port duplexer structure. It can, in fact, be observed that, also in case of duplexers, the location of S_{11} zeros (reflection zeros), S_{21}, and S_{31} zeros (transmission zeros) are influenced very little by losses (if losses are not too large); thus, the polynomials $N(s), P_{\rm TX}(s)$ and $P_{\rm RX}(s)$ evaluated from the lossy measured data are practically coincident with those associated to the same network without losses. For evaluating these polynomials, the Cauchy method will be applied to the following characteristic functions (obtained from the measured data):

$$K_{\rm TX}(s) = \frac{S_{11}(s)}{S_{21}(s)} = \frac{N(s)}{P_{\rm TX}(s)}$$
$$K_{\rm RX}(s) = \frac{S_{11}(s)}{S_{31}(s)} = \frac{N(s)}{P_{\rm RX}(s)}. \tag{2}$$

Once $N(s), P_{\rm TX}(s)$, and $P_{\rm RX}(s)$ have been computed, $D(s)$ is obtained by imposing the unitary condition on the considered scattering parameters (S_{11}, S_{21}, and S_{31}), which can be expressed as follows (Feldtkeller equation):

$$N(s)N^*(-s^*) + P_{\rm RX}(s)P_{\rm RX}^*(-s^*) + P_{\rm TX}(s)P_{\rm TX}^*(-s^*) = D(s)D^*(-s^*). \tag{3}$$

The complex roots of the left-hand side of (3) are in pairs with an opposite signed real part. Selecting those with a negative real part, the poles of the lossless model for the considered duplexer are then obtained, as well as the polynomial $D(s)$. A lossless polynomial model can then be extrapolated from lossy measured data, which is suitable for the synthesis of lossless equivalent circuits.

It worthwhile to remark that, following the above procedure, the set of computed polynomials is fully compatible with the lossless condition; obviously this condition poses also constraints on the other scattering parameters (S_{22}, S_{33}, S_{23}), which can be univocally determined only after the synthesis of the overall duplexer network.

Another aspect to be considered is how the choice of the reference section at the duplexer input port affects the derived polynomial model; to this purpose, let observe that this choice has no influence on the position of reflection and transmission zeros so the estimation of the characteristic functions $K_{\rm TX}(s)$ and $K_{\rm RX}(s)$ does not change practically when the phase reference of the measured data is varied. The performance of the extracted polynomial model to reproduce the magnitude of the considered scattering parameters is then practically independent on the choice of the reference sections. A good match of the measured phases is also possible by adding a frequency linearly varying term (with a suitable slope) to the phase of S_{11}, S_{21}, and S_{31} computed from the polynomial model.

B. Formulation of the Cauchy Method

As above observed, the polynomials $K_{\rm TX}(s)$ and $K_{\rm RX}(s)$ can be derived using the Cauchy method, which will be briefly recalled in the following.

Let us define $K_{\rm TX}(s)$ and $K_{\rm RX}(s)$ as follows:

$$K_{\rm TX}(s) = \frac{S_{11}(s)}{S_{21}(s)} = \frac{\sum_{k=0}^{n_{\rm TX}+n_{\rm RX}} a_k s^k}{\sum_{k=0}^{m_{\rm TX}+n_{\rm RX}} c_k^{(1)} s^k}$$
$$K_{\rm RX}(s) = \frac{S_{11}(s)}{S_{31}(s)} = \frac{\sum_{k=0}^{n_{\rm TX}+n_{\rm RX}} a_k s^k}{\sum_{k=0}^{m_{\rm RX}+n_{\rm TX}} c_k^{(2)} s^k}. \tag{4}$$

Imposing that the equations in (4) are satisfied for a set of $N \geq 2\cdot(n_{\mathrm{TX}}+n_{\mathrm{RX}})+m_{\mathrm{TX}}+m_{\mathrm{RX}}+2$ not necessarily equally spaced frequency points, a system of N linear equations can be obtained, as shown in (5), at the bottom of this page, where $\mathbf{V}_m$ is a decreasing-power mth-order Vandermonde matrix defined as

$$\mathbf{V}_m = \begin{bmatrix} s_1^m & s_1^{m-1} & \dots & s_1^2 & s_1 & 1 \\ s_2^m & s_2^{m-1} & \dots & s_2^2 & s_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ s_N^m & s_N^{m-1} & \dots & s_N^2 & s_N & 1 \end{bmatrix} \in \mathbb{C}^{N\times(m+1)} \tag{6}$$

and $\mathbf{M}$ is the system matrix.

System (5) can be solved using the total least squares (TLS) method [11]. The coefficients a_k, $c_k^{(1)}$, and $c_k^{(2)}$, as well as the polynomials $N(s)$, $P_{\mathrm{TX}}(s)$, and $P_{\mathrm{RX}}(s)$ are obtained. Note that all the coefficients must be further normalized to satisfy the low-pass prototype feasibility [8].

Imposing (3) and taking the roots with a negative real part, the duplexer poles are then computed and the complete polynomial model of the overall duplexer is finally defined.

III. Modification of the System Matrix: Enhancing the Performance

A. Change of Polynomial Basis

The previous method presents serious limitations when finite numerical precision is used. This drawback arises from the fact that the Vandermonde matrix $\mathbf{V}$, as in (5), is extremely ill conditioned [12], [13], especially when its dimensions (i.e., the number of data points N or the model order) are moderately high. As a consequence, $\mathbf{M}$ is also very ill conditioned.

To address the problem of the ill conditioning due to the Vandermonde matrix, some studies has been introduced [6], [7]. Here, these studies are reviewed and modified.

To deal with the numerical rank deficiency problem of the Vandermonde matrix, a new orthonormal polynomial basis is generated. This basis spans the same space as the columns (*colspan*) of the Vandermonde matrix.

From the definition of a Vandermonde matrix (6), it can be observed that the kth column (with $k = 1, \dots, m+1$) of $\mathbf{V}_m$ is $[\mathbf{S}^{m-k+1}\cdot I]$, where $\mathbf{S} = \mathrm{diag}([s_1, s_2, \dots, s_N]) \in \mathbb{C}^{N\times N}$ and $I = [1, \dots, 1]^T \in \mathbb{R}^N$. Therefore, the columns of $\mathbf{V}_m$ generate a Krylov subspace, i.e., $\mathcal{K}_{m+1}(\mathbf{S}, I) = \mathrm{colspan}\{\mathbf{V}_m\}$ [14].

The Arnoldi process builds an orthonormal basis for a Krylov subspace for an assigned matrix. The implicit restarted Arnoldi (IRA) process with correction proposed by Daniel, Gragg, Kaufman, and Stewart (DGKS) [15] has been chosen (see the Appendix). The Arnoldi process produces the following factorization with a computational cost of $O(Nk)$ flops and $2Nk + O(k^2)$ storage (where k is the number of iterations, i.e., the number of obtained eigenvalues):

$$\mathbf{SQ} = \mathbf{QH} \tag{7}$$

where $\mathbf{Q} \in \mathbb{C}^{N\times k}$ has orthonormal columns (i.e., $\mathbf{QQ}^* = \mathbf{I}$) that spans $\mathcal{K}_{m+1}(\mathbf{S}, I)$ and $\mathbf{H} \in \mathbb{C}^{k\times k}$ is an upper Hessenberg matrix with strictly positive subdiagonal elements.

Since $\mathbf{S}$ is a skew-symmetric matrix ($\mathbf{S}^* = -\mathbf{S}$), then $\mathbf{H}$ must also be. Therefore, $\mathbf{H}$ has to be a tridiagonal matrix, hereinafter called $\mathbf{T}$.

With this new robust basis, a matrix equation analogous to (5) can be written, namely, (8), shown at the bottom of this page. This can be solved using the TLS method in the same manner as the classic formulation of the Cauchy method.

The resulting model is defined by

$$K_{\mathrm{TX}}(s) = \frac{S_{11}(s)}{S_{21}(s)} = \frac{\sum_{k=0}^{n_{\mathrm{TX}}+n_{\mathrm{RX}}} \tilde{a}_k p_k(s)}{\sum_{k=0}^{m_{\mathrm{TX}}+n_{\mathrm{RX}}} \tilde{c}_k^{(1)} p_k(s)}$$
$$K_{\mathrm{RX}}(s) = \frac{S_{11}(s)}{S_{31}(s)} = \frac{\sum_{k=0}^{n_{\mathrm{TX}}+n_{\mathrm{RX}}} \tilde{a}_k p_k(s)}{\sum_{k=0}^{m_{\mathrm{RX}}+n_{\mathrm{TX}}} \tilde{c}_k^{(2)} p_k(s)}. \tag{9}$$

where $p_k(s)$ are polynomials of order k, which satisfy the discrete orthogonality relation

$$\sum_{k=1}^{N} p_i(s_k) p_j(s_k) = \delta_{ij}. \tag{10}$$

Each family of orthogonal monic polynomials is defined by a three-term recursive expression that outlines the method for their building [16]. In this case, it corresponds to the following expression:

$$s\cdot p_{k-1} = \sum_{j=k-1}^{k+1} [\mathbf{T}]_{j,k}\cdot p_{j-1}. \tag{11}$$

Theoretically, the numerical stability of the process allows a more accurate solution of the system, a task that was not previously possible with double precision using the monomial base and the Vandermonde matrices due to the loss of numerical rank.

$$\begin{bmatrix} \mathbf{S}_{21}\mathbf{V}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}} & -\mathbf{S}_{11}\mathbf{V}_{m_{\mathrm{TX}}+n_{\mathrm{RX}}} & \mathbf{0} \\ \mathbf{S}_{31}\mathbf{V}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}} & \mathbf{0} & -\mathbf{S}_{11}\mathbf{V}_{m_{\mathrm{RX}}+n_{\mathrm{TX}}} \end{bmatrix} \begin{bmatrix} a \\ c^{(1)} \\ c^{(2)} \end{bmatrix} = [\mathbf{M}] \begin{bmatrix} a \\ c^{(1)} \\ c^{(2)} \end{bmatrix} = \mathbf{0} \tag{5}$$

$$\begin{bmatrix} \mathbf{S}_{21}\mathbf{Q}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}} & -\mathbf{S}_{11}\mathbf{Q}_{m_{\mathrm{TX}}+n_{\mathrm{RX}}} & \mathbf{0} \\ \mathbf{S}_{31}\mathbf{Q}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}} & \mathbf{0} & -\mathbf{S}_{11}\mathbf{Q}_{m_{\mathrm{RX}}+n_{\mathrm{TX}}} \end{bmatrix} \begin{bmatrix} \tilde{a} \\ \tilde{c}^{(1)} \\ \tilde{c}^{(2)} \end{bmatrix} = [\tilde{\mathbf{M}}] \begin{bmatrix} \tilde{a} \\ \tilde{c}^{(1)} \\ \tilde{c}^{(2)} \end{bmatrix} = 0 \tag{8}$$

B. Preconditioning Matrix

Using the orthonormal basis vectors, the condition number of the system matrix $\mathbf{M}$ decreases and the numerical solution is more reliable. However, this is not the best solution since increasing the number of poles of the filters composing the duplexer, i.e., by increasing the whole number of unknowns, it does not necessarily provides a more reliable solution.

In order to improve the condition number of the system matrix, the whole matrix has to be better conditioned. A new solution can be formed by using a preconditioning matrix [14]. The basic idea is to transform the system matrix into a better conditioned one through the following equation:

$$[\mathbf{P}^{-1}][\mathbf{M}]x = [\breve{\mathbf{M}}]x = 0 \tag{12}$$

where $\mathbf{P}$ is the (left) preconditioning matrix and x is the unknowns vector. $\mathbf{P}$ is any simple matrix that approximates $\mathbf{M}$.

$\mathbf{P}^{-1}$ is needed, and since $\mathbf{M}$ is not square, the inverse matrix can be obtained by the Moore–Penrose pseudoinverse [14], which is the best solution in the least squares sense and is noted as $\mathbf{P}^{-1} \approx \mathbf{M}^{\dagger}$ according to Watkins [14]. In this way, a simple preconditioning matrix, which approximates the system matrix, is found.

Note that, since the system (12) is homogeneous, it is not necessary to multiply the term vector by $\mathbf{P}^{-1}$, as in the classic method. This decreases the number of multiplications required to solve the system.

The TLS method can be used to solve the system (12).

IV. $\mathbf{QR}$ Algorithm for the Two Duplexer Characteristic Functions: A Fast and Accurate Solution

A. $\mathbf{QR}$ Algorithm

The $\mathbf{QR}$ algorithm is a widely used technique in numerical analysis. The $\mathbf{QR}$ algorithm decomposes a matrix as a product of an orthogonal matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$. In particular, the $\mathbf{QR}$ factorization with column pivoting [13], which is backward stable, is employed in this formulation. Using the $\mathbf{QR}$ algorithm, a new formulation of the Cauchy method for the duplexer model extraction can be achieved. This procedure avoids the unpleasant presence of blocks of zeros in the system matrix.

Starting from the matrix equations of the two characteristic functions,

$$[-\mathbf{S}_{11}\mathbf{V}_{m_{\mathrm{TX}}+n_{\mathrm{RX}}} \quad \mathbf{S}_{21}\mathbf{V}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}}] \begin{bmatrix} c^{(1)} \\ a \end{bmatrix} = [\mathbf{M}'] \begin{bmatrix} c^{(1)} \\ a \end{bmatrix} = 0 \tag{13}$$

and

$$[-\mathbf{S}_{11}\mathbf{V}_{m_{\mathrm{RX}}+n_{\mathrm{TX}}} \quad \mathbf{S}_{31}\mathbf{V}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}}] \begin{bmatrix} c^{(2)} \\ a \end{bmatrix} = [\mathbf{M}''] \begin{bmatrix} c^{(2)} \\ a \end{bmatrix} = 0 \tag{14}$$

a suitable $\mathbf{QR}$ decomposition can be realized over the two system matrices, as in the following equations:

$$[\mathbf{M}'] = [\mathbf{Q}'][\mathbf{R}'] = [\mathbf{Q}'] \begin{bmatrix} \mathbf{R}'_{11} & \mathbf{R}'_{12} \\ \mathbf{0} & \mathbf{R}'_{22} \end{bmatrix} \tag{15}$$

$$[\mathbf{M}''] = [\mathbf{Q}''][\mathbf{R}''] = [\mathbf{Q}''] \begin{bmatrix} \mathbf{R}''_{11} & \mathbf{R}''_{12} \\ \mathbf{0} & \mathbf{R}''_{22} \end{bmatrix}. \tag{16}$$

Now, both characteristic functions must have common reflection zeros; therefore, the following system is obtained:

$$\begin{bmatrix} \mathbf{R}'_{22} \\ \mathbf{R}''_{22} \end{bmatrix} a = 0. \tag{17}$$

A further $\mathbf{QR}$ decomposition of the previous system equation can be executed as follows:

$$\begin{bmatrix} \mathbf{R}'_{22} \\ \mathbf{R}''_{22} \end{bmatrix} = [\mathbf{Q}][\mathbf{R}] \tag{18}$$

resulting in the reduced system (17) expressed as

$$[\mathbf{R}]a = 0. \tag{19}$$

This again reduces the size of the system matrix, but preserves the eigenvalues of the system, and thus, a computed solution can be quickly achieved.

Once the coefficients a_k have been obtained by the TLS method, the coefficients $c_k^{(1)}$ and $c_k^{(2)}$ can then be found using the relationships (15) and (16), namely,

$$\begin{aligned} c^{(1)} &= -\mathbf{R}'_{11} \backslash \mathbf{R}'_{12} a \\ c^{(2)} &= -\mathbf{R}''_{11} \backslash \mathbf{R}''_{12} a \end{aligned} \tag{20}$$

where the operator $\backslash$ is the left matrix divide.

Imposing the unitary condition (3), the coefficients b_k can then be found and, therefore, the overall duplexer model can be realized.

B. Preconditioning and the $\mathbf{QR}$ Technique

In Section IV-A, it has been observed that the preconditioning technique is able to reduce the condition number of the system matrix and, therefore, increase the accuracy of the solution. Moreover, the powerful $\mathbf{QR}$ algorithm is able to decrease the computational time because it reduces the system matrix size while preserving the singular values, and thus, the same condition number. Here, a formulation that suitably combines these two techniques is presented.

Starting from the general system, as in (5), see (21), shown at the bottom of this page, the preconditioning technique is first

$$\begin{bmatrix} \mathbf{S}_{21}\mathbf{V}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}} & -\mathbf{S}_{11}\mathbf{V}_{m_{\mathrm{TX}}+n_{\mathrm{RX}}} & \mathbf{0} \\ \mathbf{S}_{31}\mathbf{V}_{n_{\mathrm{TX}}+n_{\mathrm{RX}}} & \mathbf{0} & -\mathbf{S}_{11}\mathbf{V}_{m_{\mathrm{RX}}+n_{\mathrm{TX}}} \end{bmatrix} \begin{bmatrix} a \\ c^{(1)} \\ c^{(2)} \end{bmatrix} = [\mathbf{M}] \begin{bmatrix} a \\ c^{(1)} \\ c^{(2)} \end{bmatrix} = 0 \tag{21}$$

applied, scaling the system matrix into a better conditioned one $\breve{\mathbf{M}}$, which is expressed by

$$[\mathbf{P}^{-1}][\mathbf{M}]\begin{bmatrix} a \\ c^{(1)} \\ c^{(2)} \end{bmatrix} = [\breve{\mathbf{M}}]\begin{bmatrix} a \\ c^{(1)} \\ c^{(2)} \end{bmatrix} = 0 \tag{22}$$

where $\mathbf{P}^{-1} \approx \mathbf{M}^{\dagger}$, i.e., the Moore–Penrose pseudoinverse [14]. The $\mathbf{QR}$ algorithm is then applied to the new system matrix $\breve{\mathbf{M}}$, resulting in

$$[\breve{\mathbf{M}}] = [\mathbf{Q}][\mathbf{R}]. \tag{23}$$

Even if the condition number is obviously the same because the matrix $\mathbf{R}$ has the same singular values of the original matrix $\breve{\mathbf{M}}$, the system can be solved faster because $\mathbf{R}$ is an upper triangular matrix. Hence, the following system:

$$[\mathbf{R}]\begin{bmatrix} a \\ c^{(1)} \\ c^{(2)} \end{bmatrix} = 0 \tag{24}$$

can be solved using the TLS method and its coefficients $[a\ c^{(1)}\ c^{(2)}]^T$ can be computed both in an efficient and accurate fashion.

This formulation is more efficient because it preserves the condition number obtained by the preconditioning techniques, but using the $\mathbf{QR}$ decomposition reduces the computational time.

V. Numerical Examples

A. Synthesized Test Duplexer

To test the performance of the new formulations of the Cauchy method, the scattering parameters of a suitably synthesized duplexer have been used. The four characteristic polynomials of this duplexer have been determined following the techniques described in [8]; from these polynomials, the response of the synthesized duplexer is evaluated and used in the various formulations of the Cauchy method investigated here for reconstructing the same polynomials. The performance of the novel proposed formulations can then be assessed by comparing the synthesized and the reconstructed polynomials.

In the example, each of the filters in the duplexer has ten poles, three assigned zeros, and a return loss of 23 dB. The synthesis of the duplexer is carried out in a normalized frequency domain, as in [8]; the RX filter passband spans from -1 to -0.14 and transmission zeros are placed at $0.15j,\ 0.3j,\ 0.35j$; the TX filter passband spans from $+0.14$ to $+1$ and the transmission zeros are placed at $-0.3j,\ -0.15j,\ -0.1j$. The number of complex unknowns of the system is 49, and the number of frequency points is 50.

As said above, what we wish to stress with this example is the ability of the proposed algorithms to accurately extract the duplexer polynomials; moreover, the ability in reproducing the original duplexer response is also considered.

Fig. 2 presents the duplexer response together with the one obtained from the polynomial model extracted by the classic formulation of the Cauchy method. It can be noted that the classic

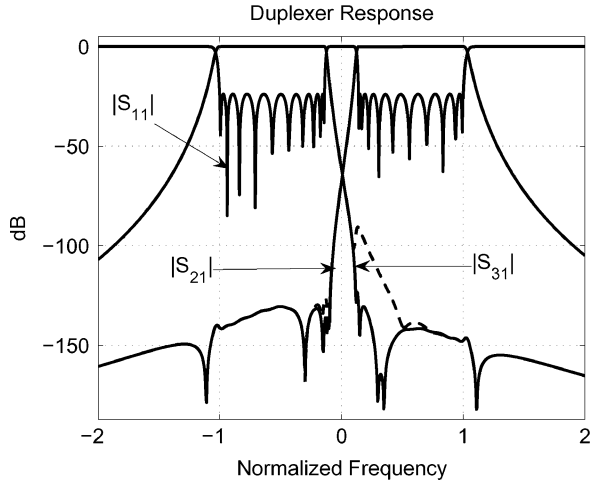

Fig. 2. Attenuation and return loss of the duplexer RX and TX filter's responses (solid line: synthesized polynomials; dashed lines: polynomial model obtained by the classic formulation of the Cauchy method).

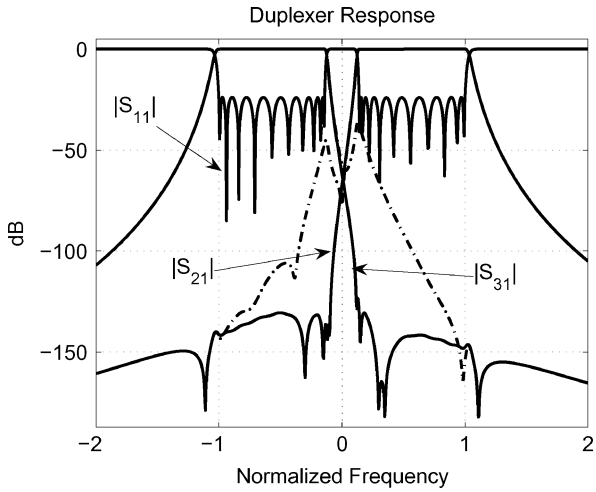

Fig. 3. Attenuation and return loss of the duplexer RX and TX filter's responses (solid line: synthesized polynomials; dashed–dotted lines: polynomial model obtained using the orthonormal polynomials).

formulation (dashed lines) fails to accurately reproduce the duplexer response in the out-of-band regions; this is due to the ill-conditioned Vandermonde matrix, which determines an inaccurate solution even with moderately high-order systems (such as the duplexer considered here).

Fig. 3 presents the duplexer response compared with the one obtained with the first new method investigated (IRA). It can be observed that the accuracy offered by this technique is even worse than that obtained with the classic formulation of the Cauchy method. However, it must be said that the change of the polynomial basis into the Krylov subspace has given good results in case of low-order systems (as pointed out in the literature [7]); in case of higher order systems (as the duplexer structure considered here), it offers performances that are not so good.

The other methods investigated here present much better performances in reproducing the synthesized duplexer response; in fact, the responses obtained from the polynomials computed with all the considered methods are practically indistinguishable from the original one, as depicted in Fig. 2. As a consequence,

TABLE I
PERFORMANCE COMPARISON AMONG THE PRESENTED TECHNIQUES

METHOD	$K(M)$	$E_{rel}(a)$	$E_{rel}(b)$	$E_{rel}(c^{(1)})$	$E_{rel}(c^{(2)})$	ACCURATE?	TIME (ms)
Vandermonde	10^{20}	$2.15{\cdot}10^{-8}$	$5.34{\cdot}10^{-7}$	$3.91{\cdot}10^{-9}$	$5.24{\cdot}10^{-8}$	NO	46
IRA	10^{16}	$2.65{\cdot}10^{-4}$	$9.71{\cdot}10^{-4}$	$1.58\cdot10^{-4}$	$3.47{\cdot}10^{-4}$	NO	391
Precond	10^{10}	$3.76{\cdot}10^{-9}$	$2.60{\cdot}10^{-7}$	$3.25\cdot10^{-9}$	$7.48{\cdot}10^{-9}$	YES	78
QR	10^{16}	$1.07\cdot10^{-8}$	$3.65{\cdot}10^{-7}$	$1.10{\cdot}10^{-8}$	$1.25{\cdot}10^{-8}$	YES	47
Precond + QR	10^{10}	$3.76{\cdot}10^{-9}$	$2.60{\cdot}10^{-7}$	$3.25{\cdot}10^{-9}$	$7.48{\cdot}10^{-9}$	YES	62

these formulations appear to be more robust than the classic formulation of the Cauchy method.

Although the responses appear to overlap, it is important to quantify the accuracy of the results. In order to evaluate the performance of the presented algorithms, let us introduce two parameters: the condition number and the relative error. The condition number is a useful measure of the sensitivity of the linear system (for its definition, see [13] and [14]). It gives information about how accurately one can solve the systems of equations. The condition number depends on the choice of the norm. Hereinafter we refer to a condition number computed with the Euclidean 2-norm or spectral norm of a matrix, i.e., the ratio between the two extreme singular values (it is worthwhile to note that the condition number is practically independent of the number of points N used to solve the system equations). With an ill-conditioned matrix, one expected to lose $\log_{10}(K(\mathbf{A}))$ digits in computing the solution [13], where $K(\mathbf{A})$ is the condition number of the matrix $\mathbf{A}$ (except under very special circumstances). According to Higham [17], the notation "$E_{\rm rel}(x)$" refers to the 2-norm relative error between the exact polynomial coefficients and the extracted solution for the parameter x. Relative error is connected with the notion of correct significant digits. It gives information about the accuracy of an algorithm.

Table I reports the summary of the results. The first column is relative to the condition number of the system matrix for all the presented formulations. The following columns refers to the relative errors of the extracted polynomial coefficients: $a, b, c^{(1)}$, and $c^{(2)}$. The last column indicates the computational time of the algorithm on a 1.73-GHz Intel Pentium Centrino processor.

It can be observed that the classic formulation of the Cauchy method suffers for the ill-conditioned Vandermonde matrix (its condition number is very high) and, therefore, the solution is not very accurate, as seen before (Fig. 2).

The change of the polynomial basis into the Krylov subspace does not furnish the expected improvement: the solution is again not accurate (even if its condition number is lower than the classic formulation) and, moreover, it is a time-consuming algorithm. Therefore, this method is not suited for the duplexer application concerned here.

The preconditioning technique has, in the considered example, the lowest condition number and the lowest relative errors, which reveal its very good ability to deal with high-order systems with a high accuracy.

The $\mathbf{QR}$ formulation is able to extract high-order systems a little less accurately than the preconditioning, but it is much faster. The preconditioning technique is expected to give a better accuracy since it yields a lower condition number.

The last algorithm combines the accuracy of the preconditioning technique with an increase in speed due to the $\mathbf{QR}$ algorithm, giving the best compromise between accuracy and computation time.

Finally, it can be expected that the new formulations can furnish even better performance as the system order increases (as in the case, for instance, of microwave multiplexers).

B. Data From a Network Model of the Duplexer

The proposed algorithms have also been verified with data coming from the circuit simulation of a designed and optimized duplexer presenting the following specifications:

- *RX Filter*: passband 880–918.5 MHz, return loss 16 dB, ten poles, two transmission zeros;
- *TX Filter*: passband 924–960 MHz, return loss 20 dB, ten poles, two transmission zeros.

The two filters have the same inline topology, with two additional couplings between resonators 2–4 and 6–8 for realizing the transmission zeros.

The three-port junction (Fig. 1) is constituted by two transmission line sections (of suitable length and characteristic impedance), which connect the input ports of the two filters with port 1 of the duplexer.

The circuit model employed for the duplexer representation includes distributed resonators constituted by short-circuited $\lambda_0/8$ transmission line sections resonating with lumped capacitors; the couplings are implemented through ideal admittance inverters. Losses are included in the simulation by assigning a finite unloaded $\mathbf{Q}$ (2000) to the resonators. The circuit parameters have been determined through a synthesis approach [8], followed by circuit optimization.

The scattering parameters S_{11}, S_{12}, and S_{13} have been computed at 150 equally spaced frequency points using a commercial circuit simulator; the polynomial model of the duplexer has been derived by using the $\mathbf{QR}$ method with the above introduced preconditioning. The roots of the computed polynomials are reported in Table II.

Fig. 4 shows the comparison between the circuit and polynomial responses of the considered duplexer.

Note the very good agreement between the two responses, even if the input data include losses; this confirms the assumption that, in case of low losses, reflection and transmission zeros derived from lossy data allow a sufficiently accurate lossless polynomial model to be computed.

TABLE II
ROOTS OF THE DUPLEXER CHARACTERISTIC POLYNOMIALS. THE HIGHEST DEGREE COEFFICIENT FOR N AND D IS 1, FOR P_{RX} IS $7.3283 \cdot 10^{-5}$ AND FOR P_{TX} IS $3.5522 \cdot 10^{-5}$

$N(s)$ roots	$D(s)$ roots	$P_{RX}(s)$ roots	$P_{TX}(s)$ roots
-0.0050 - 0.0173i	-0.0396 - 1.0564i	-0.0071 + 0.1128i	0.0130 - 0.0070i
0.0404 + 0.0037i	-0.1099 - 0.9971i	-0.0228 + 0.1335i	-0.0048 + 0.0181i
-0.0221 -0.0921i	-0.1452 - 0.8985i	0.0063 + 0.1481i	-0.0309 - 0.0016i
-0.0009 + 0.1261i	-0.0349 + 1.0418i	-0.0461 + 0.1752i	-0.0098 - 0.0443i
-0.0151 + 0.1542i	-0.0990 + 0.9864i	-0.0068 + 0.2229i	-0.0501 - 0.0915i
-0.0217 - 0.2031i	-0.1481 - 0.7199i	-0.0740 + 0.2899i	-0.0647 - 0.2039i
0.0231 + 0.2331i	-0.1404 + 0.8811i	-0.0735 + 0.4291i	-0.0994 - 0.3527i
-0.0223 + 0.2925i	-0.1637 + 0.7315i	-0.0954 + 0.5850i	-0.0890 - 0.5222i
-0.0621 - 0.3582i	-0.1497 - 0.5206i	-0.0809 + 0.7525i	-0.0858 - 0.7178i
0.0310 + 0.4292i	-0.1668 + 0.5697i	-0.0765 + 0.8988i	-0.0650 - 0.9007i
-0.0142 - 0.5192i	-0.1388 - 0.3422i	-0.0430 + 1.0096i	-0.0527 - 1.0230i
-0.0309 + 0.5772i	-0.1484 + 0.4140i	-0.0053 + 1.1038i	-0.0072 - 1.1030i
-0.0249 - 0.7174i	-0.1170 + 0.2846i		
0.0352 + 0.7185i	-0.0999 - 0.1895i		
-0.0281 + 0.8746i	-0.0767 + 0.1975i		
0.0049 - 0.9350i	-0.0386 + 0.1414i		
0.0957 - 0.9407i	-0.0111 + 0.1166i		
0.0497 + 0.9520i	-0.0636 - 0.0772i		
-0.0163 + 1.0071i	-0.0427 + 0.0014i		
-0.0299 - 1.0232	-0.0200 - 0.0024i		

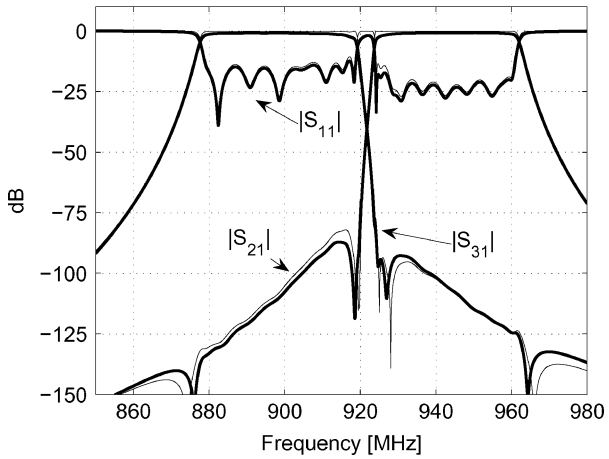

Fig. 4. Comparison between the circuit response (thick line) and the polynomial response (thin line) of the test duplexer.

Note also that the accuracy of the polynomial model does not depend on the actual topology of the three-port junction connecting the input ports of the two filters; this is, in general, true as far as the junction does not introduce additional poles (i.e., it has a nonresonant nature).

From the computed polynomials of the overall duplexer, with further suitable manipulations, the characteristic polynomials of the composing TX and RX filters have been derived [8]; the synthesis of the two filters has then been performed, allowing the evaluation of the coupling coefficients and the resonant frequencies. The values of these parameters are compared in Tables III and IV with the original ones employed in the simulated test duplexer.

A very satisfactory agreement between the original and extracted parameters can be observed, even if the extracted parameters have been obtained by assuming an ideal shunt connection of the input ports of TX and RX filters.

VI. CONCLUSION

Innovative formulations of the Cauchy method have been applied to the modeling of microwave duplexers, even in the presence of lossy measured data.

TABLE III
COMPARISON OF ORIGINAL AND EXTRACTED COUPLING COEFFICIENTS

	RX Filter		TX Filter	
	Original	Extracted	Original	Extracted
k_{12}	0.0355	0.0356	0.0300	0.0306
k_{23}	0.0232	0.0233	0.0193	0.0186
k_{34}	0.0211	0.0211	0.0182	0.0176
k_{45}	0.0238	0.0237	0.0207	0.0207
k_{56}	0.0235	0.0233	0.0206	0.0206
k_{67}	0.0202	0.0206	0.0177	0.0176
k_{78}	0.0207	0.0211	0.0184	0.0184
k_{89}	0.0259	0.0265	0.0230	0.0233
k_{910}	0.0347	0.0367	0.0327	0.0334
k_{24}	0.0117	0.0112	-0.0116	-0.0123
k_{68}	0.0125	0.0122	-0.0106	-0.0108

TABLE IV
COMPARISON OF ORIGINAL AND EXTRACTED RESONANT FREQUENCIES (IN MEGAHERTZ)

	RX Filter		TX Filter	
	Original	Extracted	Original	Extracted
f_1	898.3	889.0	945.7	952.0
f_2	897.4	898.2	944.3	944.3
f_3	907.8	908.0	932.7	932.2
f_4	897.7	898.1	942.9	942.9
f_5	898.6	897.8	942.4	942.3
f_6	897.8	898.5	943.4	943.5
f_7	909.3	908.8	932.9	932.7
f_8	898.6	898.2	942.6	942.6
f_9	898.4	897.5	942.7	942.7
f_{10}	898.0	898.4	942.8	942.8

The Cauchy method can usually only be applied to low-order models. In fact, this method is limited by the presence of the Vandermonde matrix, which generates an ill-conditioned system matrix. This paper has described novel accurate and robust techniques for high-order duplexer model extraction using modified versions of the Cauchy method.

The first method considered (basis change into the Krylov subspace using orthonormal polynomial basis) has proven to not be sufficiently accurate for the duplexer structures concerned here.

The other methods have instead offered noticeable improvements with respect to the basic formulation of the Cauchy method. In particular, the numerical stability of the preconditioning technique and the $\mathbf{QR}$ algorithm allow the solution of high-order systems, a task that was previously impossible to solve with the same accuracy using the ill-conditioned monomial basis and the Vandermonde matrix. At the moment, the preconditioning technique gives the best conditioned system matrix, while the $\mathbf{QR}$ process furnishes a very fast and accurate solution. A combination of these two methods gives an accurate and quite fast solution.

Each procedure has been successfully verified by a numerical application example. A polynomial model has been also extracted using the scattering parameters from a simulated test duplexer employing distributed (lossy) resonators; a very good agreement has been obtained between the original parameters (coupling coefficients and resonant frequencies) used in the simulated network and those associated to the synthesized filters obtained from the extracted polynomial model.

APPENDIX
ARNOLDI PROCESS

An orthonormal basis for *colspan* $\{\mathbf{V}_m\}$ can be generated by the well-known Arnoldi process since matrix $\mathbf{S} \in \mathbb{C}^{N \times N}$

is symmetric, but not Hermitian ($\mathbf{S}^* \neq \mathbf{S}$) [12], [13]. The Lanczos iteration cannot be used because it can be applied only to a Hermitian matrix [13], [18]. Moreover, it suffers, in practice, due to loss of orthogonality of the column basis vectors, a fact that is closely connected with the convergence of Ritz values to eigenvalues of $\mathbf{S}$. Due to these complexities, there is no known theorem stating that the Lanczos iteration is backward stable.

In particular, the k-*step Arnoldi* process [19] gives the following factorization:

$$\mathbf{S}\mathbf{Q}_k = \mathbf{Q}_k\mathbf{H}_k + f_k e_k^T \tag{25}$$

where $\mathbf{Q}_k \in \mathbb{C}^{N\times k}$ has orthonormal columns, i.e., $\mathbf{Q}_k^*\mathbf{Q}_k = \mathbf{I}_k$, and $\mathbf{I}_k$ is the identity matrix of size k; $\mathbf{Q}_k f_k = 0$ and $\mathbf{H}_k \in \mathbb{C}^{k\times k}$ is an upper Hessenberg matrix with non-negative subdiagonal elements. Matrix $\mathbf{H}$ is the orthogonal projection of $\mathbf{S}$ into the generated Krylov subspace.

Since $\mathbf{S}$ is normal (i.e., $\mathbf{S}\mathbf{S}^* = \mathbf{S}^*\mathbf{S}$), all its eigenvalues are well conditioned [14] and all eigenvectors are orthogonal [13]. In fact, every normal matrix is unitarily diagonalizable. Therefore, if the residual of the Arnoldi computation is small (i.e., $\|\mathbf{S}\cdot v - \mu\cdot v\|_2$), then the Ritz pair (μ, v) is an exact eigenpair of a matrix that is close to $\mathbf{S}$ (Ritz pairs can be computed as eigenvalues of the obtained Hessenberg matrix). Thus, (μ, v) is a good approximate eigenpair of $\mathbf{S}$ in the sense of backward error.

Moreover, several Arnoldi algorithms use a reorthogonalization process in order to ensure the orthogonality of the vectors [14]. In particular, the complete orthogonalization of the Arnoldi vectors can be achieved using the DGKS correction [15], which avoids the appearance of spurious or "ghost" eigenvalues [19], as in the traditional Lanczos process [13].

Further improvement can be achieved using the IRA process, which also solves the problem on the starting vector [14], [19].

ACKNOWLEDGMENT

The authors would like to thank Dr. Van Loan, Cornell University, Ithaca, NY, for the precious discussions on numerical methods and all the reviewers for their valuable comments.

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