# Non-logarithmic slide rules

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## 1 Introduction

Slide rules and logarithms are synonymous. Thus, it may come as a surprise to discover that to produce a slide rule capable of multiplying two numbers is is not necessary to use logarithms. This idea is not new, but dates to a somewhat obscure note [1]. This I discovered by chance and have not seen cited elsewhere. I do not believe this is well know. Thus, I have taken the trouble to reproduce an explanation and prototype rule. Furthermore, it is worth noting that Nelson's rule works for arbitrary positive numbers and not simply whole numbers. Surely this is obvious, however [1] explains the theory in terms of triangular numbers, whereas this paper uses a more general setting. Furthermore, we simplify the rule significantly, which I believe is novel, and provide a simple geometric interpretation for this simplification.

The reasons for the obscurity of this technique are clear. The logarithmic rule is far superior for a number of obvious reasons – division on a logarithmic rule is a natural process for example. Nevertheless, this demonstrates an interesting numerical relationship and that logarithms are not necessary for slide rule production.

## 2 Nelson's Rule

Given a natural (whole) number $n$, the $n$th *triangular number* is the sum of 1 to $n$ inclusive. ie $1 + 2 + \cdots + n$. It is readily verified that

$$\sum_{n=1}^{n} i = \frac{n(n+1)}{2}.$$

Define, for *any positive number* $x$, the function

$$\Delta(x) := \frac{x(x+1)}{2}.$$

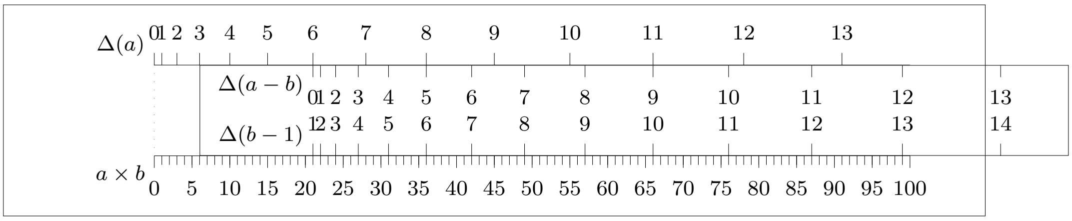



Figure 1: Nelson's rule

Then, by elementary algebra, we have that

$$\begin{aligned}\Delta(a) - \Delta(a-b) + \Delta(b-1) \\ &= \frac{1}{2}\left(a^2 + a - (a-b)(a-b+1) + (b-1)b\right) \\ &= \frac{1}{2}\left(a^2 + a - (a^2 - 2ab + a + b^2 - b) + b^2 - b\right) = ab.\end{aligned}$$

Given $a$ and $b$, by calculating $\Delta(a) - \Delta(a-b) + \Delta(b-1)$ we may calculate $ab$. On this observation the Nelson rule relies. By drawing scales proportional to $\Delta(x)$ (instead of $\log(x)$ as in a logarithmic slide rule) we may design a slide rule.

The scale are drawn as follows: the top scale is proportional to $\Delta(a)$, the top scale of the slider is proportional to $\Delta(a-b)$, the bottom scale of the slider is proportional to $\Delta(b-1)$ and the bottom scale on the stock is a simple proportional (equal division) scale.

Such a rule, set up to multiply 8 and 3, is shown in Figure 1. Set up the rule with the $a - b = 8 - 3 = 5$ on the top slider scale adjacent to the $a = 8$ on the top scale. Look at the $b = 3$ on the lower scale. This is above the answer on the proportional scale – 24.

A "cutout and keep" version of the rule may be found on the last page.

## 3 Improvements on Nelson's rule

I believe that Nelson's rule is unnecessarily complex and may be significantly simplified. Trivial algebra confirms that

$$\frac{a^2}{2} - \frac{(a-b)^2}{2} + \frac{b^2}{2} = ab.$$

Comparing areas of triangles and rectangles in Figure 2 confirms this geometrically. By using an almost identical construction, this time using the function

$$S(x) = \frac{x^2}{2}$$

in place of $\Delta(x)$, we arrive at the slide rule shown in Figure 3. This rule is set up to calculate $a \times b = 11 \times 5$ by setting $a = 11$ on the top scale adjacent to the $a - b = 6$ mark on the slide. The answer is read below $b = 5$ on the lower scale. The symmetry between $a$ and $b$ in the product $a \times b$ is more evident in this construction than in Nelson's.

I have little doubt that other relationships along similar lines are possible.

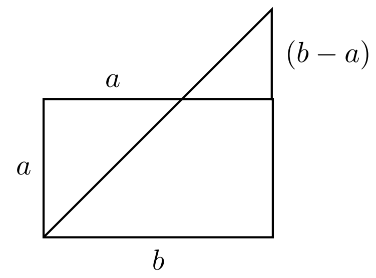



Figure 2: Geometrical interpretation of improvements on Nelson's rule

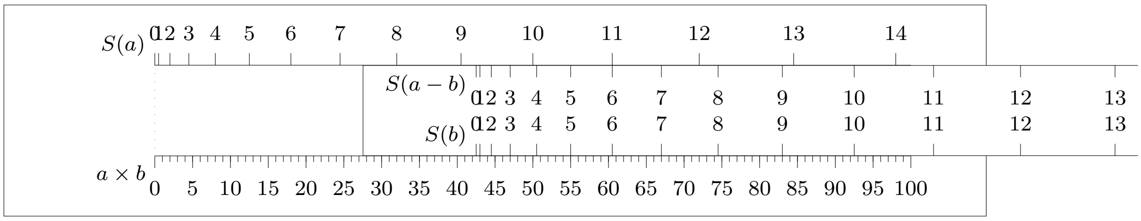
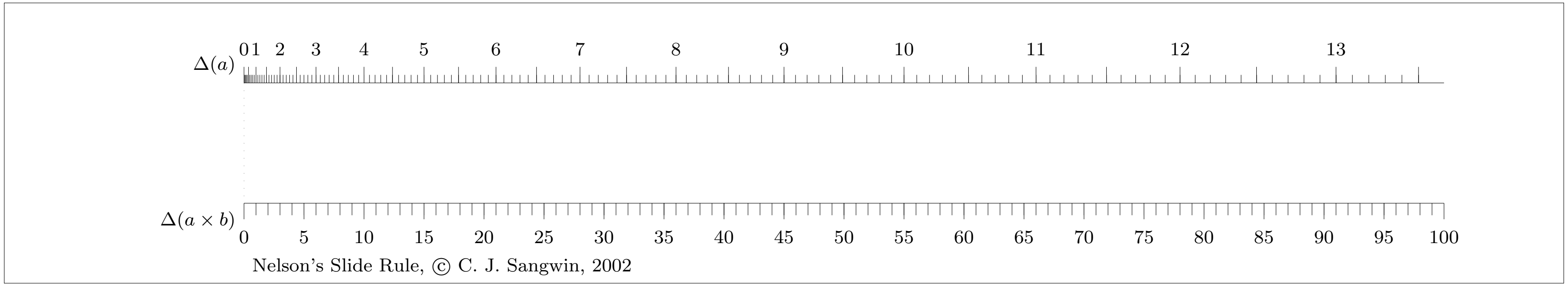
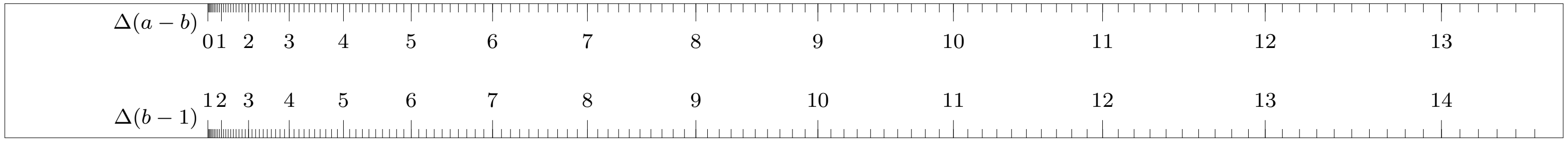



Figure 3: Improvements on Nelson's rule – $11 \times 5$

# References

[1] Mills, B. D., *The Nelson Slide Rule,* American Mathematical Monthly, 65(3), 194–195, (1958).


$\Delta(a)$
0 1 2 3 4 5 6 7 8 9 10 11 12 13
$\Delta(a \times b)$
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
Nelson's Slide Rule, © C. J. Sangwin, 2002


$\Delta(a-b)$
0 1 2 3 4 5 6 7 8 9 10 11 12 13
1 2 3 4 5 6 7 8 9 10 11 12 13 14
$\Delta(b-1)$