STRUCTURAL OPTIMIZATION OF FELIX CANDELA'S CHAPEL LOMAS DE CUERNAVACA

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Abstract: Felix Candela designed and built many thin-shelled concrete roof structures, most during the 1950s and 60s in Mexico. His most sophisticated structures, and those that are pertinent to our analysis, are of the hyperbolic paraboloid (hypar) form. Using a finite element method that solves for large-scale optimization problems, one of Candela's structures (Chapel Lomas de Cuernavaca) is modeled, meshed, and analyzed. The structure is optimized for thickness distribution and structure shape. In each optimization an attempt is made to reduce overall tensile stress and deflection to the system while stretching the limits of the structure's physical shape. The computerized analysis shows that a distributed concrete thickness reduces shell stress and deflections and the span of the structure could have been larger without compromising its structural integrity.

Keywords: *thin walled structures, structural optimization, shell structures, concrete structures.*

1. INTRODUCTION

Felix Candela (1910-1997) is known as one of the world's leading designers and builders of thin-shell concrete structures. Although he was trained as an architect in Spain, he considered himself foremost a builder and engineer of structures. In 1939, Candela was exiled to Mexico, where he was self-educated in the theory and design of thin-shell concrete structures. To gain experience with such construction, he built several experimental shell structures in Mexico. Working with his brother, he founded Cubiertas ALA, a company dedicated to the construction of thin-shell concrete structures in Mexico. A detailed description of this background can be found in Faber (1963) and Garlock and Billington (2008).

While Candela at first experimented with more traditional thin-shell forms such as barrels and funicular vaults, the geometric shape that he used in the vast majority of his works was the hyperbolic paraboloid ('*hypar'*). Such a form adds strength to the structure through double curvature and has the advantage of generating curved surfaces with straight lines, thus leading to economy of construction through the elimination of curved forms. A more thorough discussion of the hypar is provided later. The first hypar structure built by Candela was The Cosmic Rays Pavilion, built on the campus of the Universidad Nacional Autónoma de Mexico (UNAM) in 1951. This was one of the thinnest structures ever built, at only 1.5 centimeters (5/8 inch). The Pavilion was well received by the general public as well as the architectural and engineering profession, and it

Figure 1: Chapel Lomas de Cuernavaca

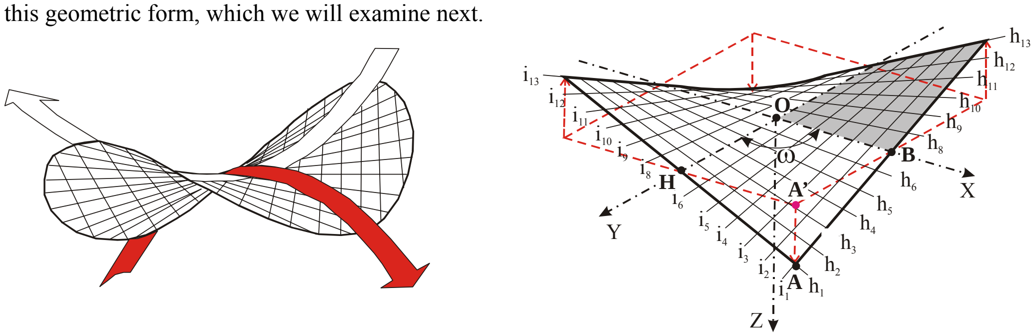
was widely published.

This structure was the beginning of Candela's explosive career as a designer and builder of thin-shell concrete hypar structures, which were typically 4 centimeters (1.5 inches) thick. Following the construction of the Cosmic Rays Pavilion, Cubiertas ALA received several commissions for works. A progression in maturity and style is seen as one traces Candela's structures chronologically (Holzer 2007). Like the leading structural engineers that came before him, he went from *imitation* (of sketches and structures he had read about in articles or books), to *innovation* (using the hyperbolic paraboloid in ways that had not been attempted before), to *inspiration*. Towards the end of his life, when asked what his favorite structures were, he replied Iglesia de la Virgen Milagrosa (1955), Restaurant Los Manantiales (1958), Chapel Lomas de Cuernavaca (1958) (Figure 1), and Bacardi Rum Factory (1960). These structures represent his inspiration following many years of dedicated study and experience with construction of thin shells.

Candela did not use computers to analyze or design these structures, but instead used membrane theory, which he found compared well to structural analysis methods for determinate structures (using simplifying assumptions for determinate conditions). Candela optimized his designs through his experience of building them, not through sophisticated computerized structural analyses of the indeterminate structure as we would do today. The objective of this paper is to (1) review Candela's approach to structural optimization, and (2) examine one structure in particular, the Chapel Lomas de Cuernavaca, and optimize the shape and thickness using the most sophisticated structural optimization techniques available today. By comparison to Candela's as-built design, we evaluate whether computerized structural optimization would have led to a more efficient design of the Chapel and whether such a tool is useful for designing thin shell concrete structures today.

2. CANDELA'S OPTIMIZATION OF FORM

Candela did not use structural optimization techniques, as defined today, to develop the form and dimensions of his thin shell designs; however, we find evidence that he did optimize his forms for efficiency of construction, for deflection control, for consideration of energy costs, and for fluidity of the form. Candela's reputation came not only from the aesthetic quality of his work, but also from his ability to construct his structures economically. Since the quantity of both concrete and steel material is minimal in shells, their economy lies in reducing the cost of forming. To understand this process of forming a hyperbolic paraboloid structure in construction, one must first understand the inherent geometric attributes of this geometric form, which we will examine next.



(a) curved boundaries

(b) straight boundaries

Figure 2: The hyperbolic paraboloid (hypar)

2.1 The Hyperbolic Paraboloid (Hypar)

Candela stated that "of all the shapes we can give to the shell, the easiest and most practical to build is the hyperbolic paraboloid." We can understand this shape best as a saddle (Figure 2a) in which there are a set of arches in one direction, and a set of cables, or inverted arches, in the other. The shape is defined by straight lines. The boundaries, or edges, of the hyperbolic paraboloid can be straight as shown in (Figure 2b), or curved as shown in Figure 2a. The edges in the latter case are developed by planes 'cutting through' the hypar surface. Candela used both straight edges and curved edges to create his designs.

Figure 2b shows that the hypar surface is formed by two systems of straight line generators: lines h_n, and i_n. Line h_n is parallel to the director plane xOz, and line i_n is parallel to the director plane yOz. xOy can be any angle, ω. xOz and yOz must both be at right angles. In this case, the surface of the hypar can be defined by the simple equation $z = kxy \sin \omega$. k is the warping of the hypar defined by the distance AA′ divided by the multiplication of distances OB and OH.

2.2 Optimization Through Building

Candela's 'bread-and-butter' structures, that is, the ones that kept his company in business and were repeated quite often, were "umbrella" types, which have straight edges. One quadrant (called *tympan*) of an umbrella structure is shaded in Figure 2b. By placing together four tympans, an umbrella structure is formed, as shown in Figure 3. Candela created large roof coverings by placing these umbrellas in sequence. Candela optimized the dimensions of the umbrellas using his experience with full-scale experiments. His first experimental umbrella was built around 1952, and in 1953 Candela constructed a second experimental umbrella (Figure 3). The corner deflection of the second umbrella reached several centimeters under its own weight after a few weeks. Several men standing on the umbrella did not increase the deflections, so Candela assumed that those deflections were due to the rise of the umbrella being too small.

Candela refers to these experiments "as a lesson to find the optimum rise, which depends on the area covered by the umbrellas. Of this simple proportion depends the success in the design of these structures, since the necessary calculations are elementary." He considered that since the rise is proportional to the area, if the area becomes larger, the height of the umbrella becomes larger, thus leading to a larger volume (space), which requires more heating or air conditioning. In addition, deflections in larger umbrellas may be difficult to control.

With the Bacardi Rum Factory, Candela pushed the limit of scale to create an aesthetically sensitive solution to the problem of covering industrial space. When Candela was asked what he learned from the Bacardí project he responded "I learned what is the limit from these types of structures… if one increases the scale, the size increases and the volume gets larger, for example in the site where one has to place air conditioning. This constitutes a serious limitation… Since in Mexico we do not have that problem, well that did not affect us much, but, on the other hand, the forms, they were more expensive every time. And the problems of deformation and of other things that can also occur also became more critical in larger scale… That is to say there is always a limit beyond which one should not pass."

In addition to consideration of safety and energy, Candela 'optimized' his forms for visual appeal. On site visits to Candela's structures reveal a forward progression, i.e., a developing maturity, of his forms. As his works progress, fluidity in form, for example through the elimination of visually disrupting stiffening beams, is observed (Holzer 2007). One of his more mature forms is that of Chapel Lomas de Cuernavaca, which is what we examine in detail next.

3. CHAPEL LOMAS DE CUERNAVACA

Chapel Lomas de Cuernavaca was built in 1958 in the town of Cuernavaca, Mexico. Only one 'exaggerated' hypar was used to generate the form for this Chapel (Figure 1). By cutting the front of a hypar surface with an oblique plane, the form of the 'mouth' of Cuernavaca was formed. Another plane, parallel to the ground formed the hyperbolic sides of the church. Figure 4 shows Candela's drawing of the Chapel where one can see the straight line generators forming the curved surface. Figure 5 shows the straight line forms used to build the saddle-shaped Chapel Lomas de Cuernavaca, which clearly reflect the straight line generators of the hypar shown in Figure 4(b).

Figure 3: Experimental Umbrella

Candela was able to construct most of the structure out of concrete that is only four centimeters (one and one-half inches) thick (Basterra 2001). At the side foundations, Candela recognized that the structure would need to be strengthened, and sufficiently thickened these sides to reduce stresses. The exact gradation of thickness is not documented but an onsite visit reveals a thickness of about 52 cm (20 inches) at the base of the front mouth (Draper et al 2007). Candela reinforced the chapel with a large beam on the shorter back mouth of the structure, a small compression lip on the front mouth and buttresses on the front and back edges of the chapel. In addition, he added extra reinforcement at the base of the front mouth where analyses show the highest compressive stresses and the largest susceptibility to buckling.

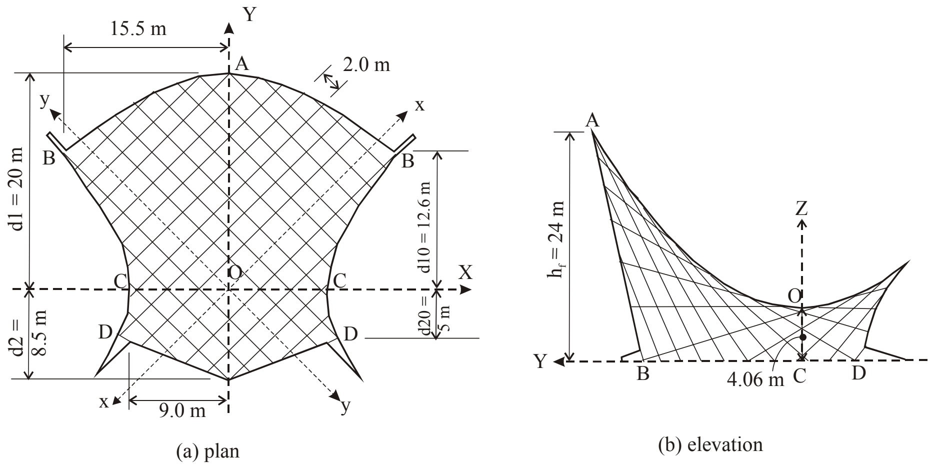


Figure 4: Generation and dimensions of original design for Chapel Lomas de Cuernavaca.

The first design of the Chapel had a height at the mouth of 24 meters (79 feet), which collapsed during decentering. The proposed redesign of the Chapel had a rise of the mouth of 18 meters (59 feet). Design drawings for both the 24 meter and 18 meter designs are in the Candela archive in the Avery Library of Columbia University. In the end, the structure was built with a height of approximately 21 meters (based on Faber (1963) and another set of drawings in Avery). Instead of using a stiffening arch at the mouth, as proposed in the 18 meter redesign, the Chapel was built with a thickening at the edge. The back opening of the shell has a deep beam, which analyses show is unnecessary and perhaps more harmful than helpful to the structure since it creates tensile forces (Basterra 2001, Holzer 2007).

Recent visits to the site indicate that the current structure is still in excellent shape, 50 years later, and displays no threat of failure. The analyses that follow are of the 24 meter original design. These analyses will demonstrate whether the stresses in this shell were large enough to have caused failure, or whether the failure was due to other factors, such as flawed construction. Furthermore, optimization of this design will show if Candela's design could have been improved for reduced material and improved safety.

4. STRUCTURAL OPTIMIZATION OF THE CHAPEL

Our optimization studies on the chapel at Cuernavaca fall into two categories: (1) A thickness optimization study that compares a shell with uniform thickness to one in which the thickness is optimally distributed over the area and (2) a shape optimization study that examines the height of the front rise. The maximum deflections and tensile stresses are evaluated. The maximum compressive stresses are not discussed but shown to be well within the limits of concrete strength (Draper et al. 2008).

The concrete material properties assume a unit weight of 23,520 N/m^3, a Young's Modulus of 21.5 GPa, and a Poisson's ratio of 0.2. The reinforcing steel is conservatively neglected. The load on the structure is its self weight. While Candela considered wind in his design (Faber 1963), no such calculations are found. An analysis of the Chapel using wind loads (assuming wind on half of a bluff semi-circular shell (a quarter-circular shell) shows that wind would not have controlled the design shown in Figure 4 (Holzer 2007).

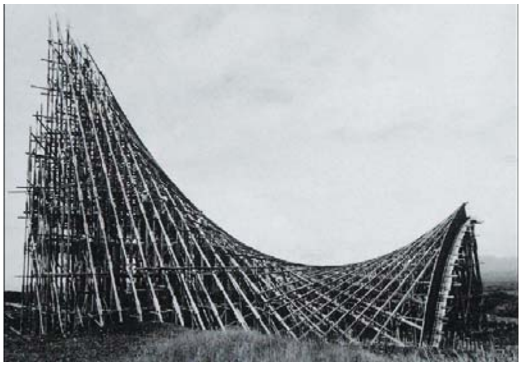


Figure 5: Form boards for construction of the Chapel

4.1 Mesh, Programs, and Optimization

The bounding curves and straight line generators of the Chapel are shown in Figure 4. The grid of straight line generators is formed with one set of parallel lines perpendicular to another (ω = 90º). The straight line generators are used to create the mesh for the Chapel, which was refined to an optimum level. The center spine, which runs along the Y-axis, divides the structure into two symmetrical parts. Due to this shell symmetry, our mesh only considers the right half of the structure with appropriate symmetry boundary conditions placed at the line of symmetry (i.e., the center spine). In addition, the mesh does not consider the buttress elements at the front and back of the structure as they are not part of the shell, but are essentially heavy beam elements.

The Chapel boundaries, boundary restraints, straight line generators, and number of divisions in the generators (which affects the coarseness of the mesh) are defined using a program written in Matlab®. This data is then read by FEMGEN, a component of FEMGV, which is a general purpose pre-processing tool for model building and mesh generation. (Femsys 2007). FEMGEN creates the mesh and outputs files for coordinates, connectivity and boundary conditions, which are read by Dynaflow, a nonlinear transient finite element program (Prevost 2006).

The optimization solution uses as an objective function, the energy E, which must be minimized with respect to displacement to satisfy equilibrium, and maximized with respect to thickness to stiffen the structure where needed. The total energy is the strain energy minus the potential energy and therefore:

$$\left[\max_{\underline{\rho}}\left[\min_{\underline{d}}\left[\frac{1}{2}\underline{d}^T K \underline{d} - \underline{f}^T \underline{d}\right]\right]\right] \quad (1)$$

where $\underline{\rho}$ is the thickness coefficient vector, $\underline{d}$ is the displacement vector, $\underline{f}$ is the applied load vector, and K is the global stiffness matrix, which is a function of the thickness cubed.

A constraint of minimum and maximum thickness is imposed through a minimum and maximum thickness coefficient, ρ_{min} and ρ_{max}, respectively. When the thickness coefficients are multiplied by a reference thickness, t_0, the minimum and maximum thickness, t_{min} and t_{max}, respectively, are defined, which bound the thickness of any given element. Furthermore, an overall volume constraint is imposed, which is defined through a mean thickness coefficient, ρ_{mean}, and t_0 so that the mean thickness of all the elements, t_{mean}, cannot exceed ρ_{mean} times t_0.

Picard iterations (Picard 1891) are used to solve the optimization equation. These are called fixed-point iterations (also known as successive substitutions) that fix one set of variables while varying the other. The fixed set of variables is then updated based on the changes made to the other set of variables, and the process is repeated until the problem converges to an answer. This method is appropriate because K , $\underline{d}$ and $\underline{f}$ are all functions of $\underline{\rho}$. The input file contains a value of the maximum number of steps so that the solution converges without using too many steps (this value was studied and confirmed through experimentation). An outline of the optimization steps is as follows:

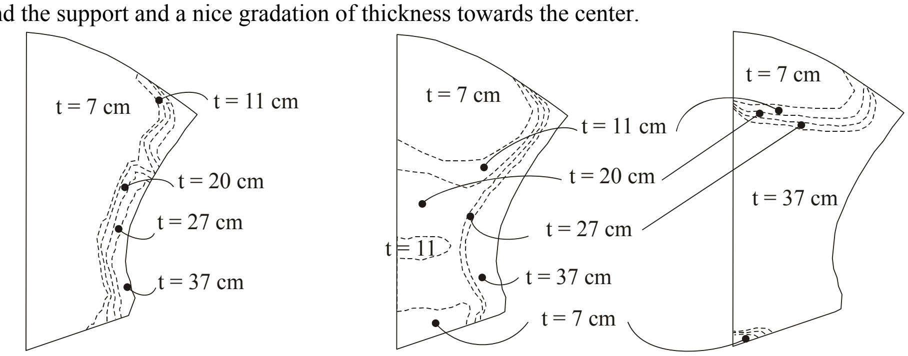
- *Step 1 – Initial Assumptions:* assume an initial thickness coefficient, ρ, for each element (typically $\rho = \rho_{mean}$).
- *Step 2 – Minimization Problem:* fix $\underline{\rho}$ and compute $\underline{d}$ by solving the equilibrium equation $K\underline{d} = \underline{f}$ using Dynaflow.
- *Step 3 – Maximization Problem:* fix $\underline{d}$ and maximize E over $\underline{\rho}$ (where E is the equation inside the brackets of Eqn (1)) using SNOPT.
- *Step 4 – Iteration:* repeat steps 2 and 3 until results converge.

The SNOPT optimization software (Gill et al. 2006) is used to solve step 3. In this step, the use of fixed point iterations is not sufficient to solve the problem. Because of the cubic non-linearity of the maximization problem we must use a method called 'move limits'. To be able to solve the maximization problem we cannot vary $\underline{\rho}$ too much; that is $\underline{\rho}$ cannot be allowed to move freely between ρ_{min} and ρ_{max}. Move limits restricts the variation of $\underline{\rho}$ in each iteration, and thus allows SNOPT to arrive at a solution.

4.2 Thickness Optimization

For the thickness optimization, a reference thickness, t_0, equal to 40 cm is selected and t_{min} and t_{max} are set equal to 4 cm and 40 cm, respectively. It is assumed that each element has the same initial thickness (Step 1) equal to ρ_{mean}. The selection of ρ_{mean} requires careful consideration because it is proportional to the volume constraint, which determines how much material will be distributed throughout the structure. If the constraint is too high, there will be too much material and it will not distribute to the correct areas (the areas with the greatest stress) but rather will be forced to distribute over portions of the shell which should be left with the minimum amount of material. On the other hand, if the volume constraint is too low, there will not be enough material to distribute to areas with higher stresses.

A range of ρ_{mean} values (from 0.20 to 0.70) was studied and a ρ_{mean} value of 0.375 (t_{mean} = 15 cm) was selected. Figure 6 shows the thickness optimization solutions for ρ_{mean} equal to 0.20, 0.375, and 0.70. It is seen that ρ_{mean} = 0.20 (t_{mean} = 8 cm) is too low since there is not enough material to distribute over the support. On the other hand, ρ_{mean} = 0.70 (t_{mean} = 28 cm) is too high and provides too much material in unnecessary areas. The analysis with ρ_{mean} = 0.375, however, shows a clear distribution of larger thickness around the support and a nice gradation of thickness towards the center.



(a) ρ_{mean} = 0.20 (t_{mean} = 8 cm) (a) ρ_{mean} = 0.375 (t_{mean} = 15 cm) (a) ρ_{mean} = 0.70 (t_{mean} = 28 cm)

Figure 6: Thickness optimization results.

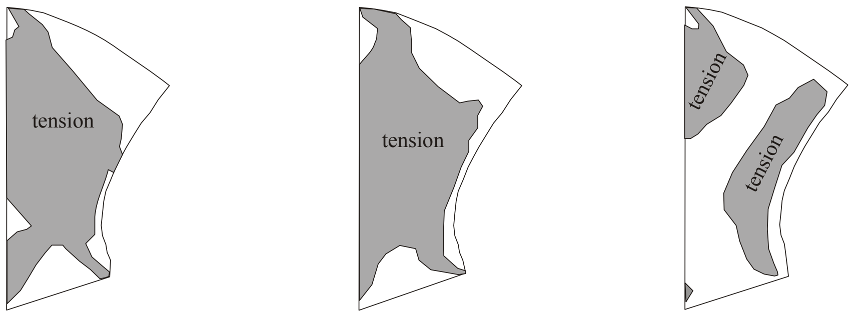
The results of the optimization study described above were verified by comparison to another analysis that assigned a random thickness to each element as the initial condition (Step 1). Both analyses had essentially the same optimized thickness distribution result, thus verifying the optimization solution.

The deflections and maximum tensile stresses of the shell with an optimized thickness *distribution* (t_{mean} = 15 cm) are compared to the same shell with a *uniform* thickness of *4 cm* (typical of Candela structures) and another shell with a *uniform* thickness of *15 cm* (thus having the same volume of concrete as the distributed thickness).

The deflections of the shell reflect stiffness and safety. Heinz Isler, another respected designer of thin shell concrete structures, set a maximum accepted deflection to span ratio (Δ/L) equal to 1/300 for his shells (Billington 2003). We can use this value as a reference for examining the maximum downward deflection in our shell, which occurs at the high point of the front mouth. The span, L, in this case is the cantilevered portion of the shell. The Δ/L values are equal to 1/1626, 1/1799, and 1/3012 for the 4 cm shell, 15 cm shell, and distributed thickness shell, respectively. These ratios are significantly smaller than that imposed by Isler. In addition, it is seen that the deflection for the distributed thickness shell is much smaller than the uniform thickness shells. Examination of the contours of maximum downward displacement show that the uniform thickness of 4 cm shell has not only a larger maximum, but that maximum extends over a greater area than the other two shells. The contours of the distributed thickness shell show smaller overall deflections and the larger deflections contained in a smaller area.

The maximum tensile (principal) stresses are equal to 12 kPa, 56 kPa, and 54 kPa for the 4 cm shell, 15 cm shell, and distributed thickness shell, respectively. However, a direct comparison of these stresses is not appropriate since the 4 cm shell has a smaller volume and thus smaller load (self weight). The tensile stresses are all well within the tensile capacity of even the weakest concrete. Candela did not document the concrete strength that he used, but we can conservatively assume a compressive strength of 13,800 kPa (2,000 psi), which will lead to a conservative tensile strength estimate of 1,500 kPa (220 psi).

For a comparison of stresses between the three shell designs, we examine the amount of tensile area in each shell with an understanding that tension is undesirable in thin shells concrete structures. Figure 7 shows the areas of tension and compression maximum principal stresses for the three shell designs. The figures clearly show that the distributed thickness shell develops less tensile area, and thus is a more efficient design.



(a) 4 cm shell (b) 15 cm shell (c) distributed thickness shell (t_{mean} = 15 cm)

Figure 7: Areas of tension (based on principal stress) in the shell.

4.2 Shape Optimization

While the distribution of thickness was obtained via computerized optimization, the effects of the shell *rise* is obtained via *free* optimization. Rise refers to the peak elevation of the front mouth of the shell (h_f in Figure 4(b)). Free optimization refers to creating the shape of the shell free hand, that is, without a computer algorithm. The values for t_0, t_{min}, t_{max}, and ρ_{mean} are the same as in the thickness optimization study described previously, and are kept constant in the shape optimization designs. The only variable that is changed is d1 (Figure 4(a)), which affects the rise of the shell as shown in Figure 8. d1 ranges from 15 to 25 meters in this study.

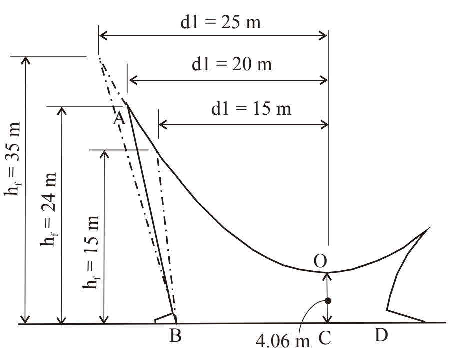


Figure 8: Variation in rise with changing d1.

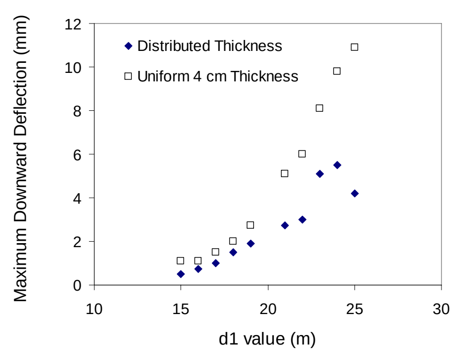


Figure 9: Effect of d1 on maximum deflection.

Figure 9 shows the effects of d1 on the maximum downward deflection for a shell with a uniform thickness of 4 cm and for a shell with an optimized distributed thickness. It is seen that the maximum deflection increases at a greater rate with d1 if the shell has a uniform thickness. Increasing the rise by 25% increases the deflections by a factor of about 2.

The shell with d1 = 25 meters has Δ/L values are equal to 1/1677, 1/1106, and 1/1757 for the 4 cm shell, 15 cm shell, and distributed thickness shell, respectively. These are still within the limits used by Isler, but, as expected, larger than in the design with d1 = 20 discussed previously. The maximum tensile (principal) stresses are equal to 32 kPa, 137 kPa, and 86 kPa for the 4 cm shell, 15 cm shell, and distributed thickness shell, respectively, well within the conservative estimate of tensile strength discussed previously. An examination of maximum principal stresses of the d1 = 25 meter shell reveals that the distribution of tension and compression is about the same as that discussed for the d1 = 20 meter design (Figure 7).

5. SUMMARY AND CONCLUSIONS

This study examined specifically one structure designed and built by Felix Candela in 1958: Chapel Lomas de Cuernavaca. Without the aid of a computer, Candela understood where the forces would be greatest and 'optimized' his designs by increasing the thickness in these locations, i.e., the supports. A detailed finite element structural optimization study shows that varying the thickness of the shell, with the largest thickness at the support, leads to the most effective design in terms of reduced deflections, reduced tensile stresses, and most efficient use of material. Since Candela did not document the variation of thickness in his design, we cannot make a direct comparison of the built structure and our analyses. We can conclude, however, that a distributed thickness is appropriate for this design.

Our results also indicate that the shell could have been designed with an even larger rise than the 24 meter original design that collapsed during decentering of the forms. Since in all of our studies the stresses are well below a conservative estimate of the concrete strength, collapse during decentering is attributed to a problem related to construction; for example the concrete may not have reached an appropriate strength before decentering, or the decentering was not done properly so that the forces in the shell did not distribute evenly as the temporary supports (scaffolding) were removed. Note that the effects of wind load were not considered other than in the 24 meter design. Wind will have a larger effect on the designs with a larger rise, and a distributed thickness will thus lead to the most efficient design.

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