

## Mathematics Department Workshops

### Topic: Index Numbers

### Resource Sheet HT2.IDX.1c- Chess Rice

#### Equipment

A chess board

An electronic balance or postage scale, capable of weighing in grams

One 2 Kg bag of uncooked rice

Tell the mythical story about the poor peasant Sissa who invented the game of Chess and took it to show the King of Persia. The wealthy King liked the game so much that he promised to give the peasant any gift he asked.

"If it pleases Your Majesty," said Sissa, "I would like to ask simply for one grain of rice to place on the first square of the chess board, then two grains for the second square, four grains on the third, doubling the number each time, and continuing on to the end of the chess board."

The King was furious. "You could have asked for jewels! You could have asked for gold! You could have asked for riches beyond your imagination – yet you ask for rice!"

So the King ordered Sissa to leave his presence, and told the servants to give Sissa his rice. It was not long, however, before the King realised he did not have enough rice to give....

Ask the students to estimate how many grains of rice they think it might need to fulfil Sissa's wish. Prompt them further to suggest what this much rice might weigh. As much as your 2 Kg bag perhaps?

Give each pair of students a tablespoon of rice and ask them to count out 100 grains into a pile. Carefully gather in ten of these piles and weigh them.

Ask the students to suggest how to complete the calculation in order to find the mass of one grain of rice. Depending on the rice you are using, the pile of 1000 grains will weigh about 20 to 50 grams.

Now invite one student to the chess board and ask them to count out grains of rice for the first row on the chess board.

1  
2  
4  
8  
16  
32  
64  
128

*(The other students can help with these last two by preparing piles of 10 grains)*

In Row Two onwards, use pieces of paper as "IOUs" instead of piles of rice, so the first piece of paper will say "256 grains" and so on. At Row Two Square Three (1012) you may suggest to the class that you simplify to 1000 grains for easier calculation.

256  
512  
1000  
2000  
4000  
8000  
16000  
32000

Remind the class that they know the mass (weight) of 1000 grains, so they can work out the mass of 32 000 grains. (About 800 – 1000 g)

In Row Three switch to grams and kilograms instead of grains. Encourage the students to use rounding as this is only estimation.

2 kg (hold up your original 2 Kg bag at this point!)

4 kg

8 kg

16 kg

32 kg

64 kg

128 kg

256 kg

So by the end of Row Three we have reached one quarter of a metric tonne. By now we would be using a JCB to lift the rice onto the square.

At the end of Row Four we find we reach 32 tonnes. We are using fleets of freight lorries at this point.

At the end of Row Five we reach 8 000 tonnes on the last square. We will need to store the rice for this square temporarily in a large aircraft hangar.

At the end of Row Six we require 2 million tonnes of rice, equivalent to the annual rice production of Korea. We still have two more rows to go.

By the end of Row Seven we have exceeded 500 million tonnes of rice on a single square. This is approximately the amount of rice produced by the whole of Asia in a year.

By the end of Row Eight, at over 100 billion tonnes, we would need over 150 years' worth of worldwide rice production on a single square. And it all started with just one grain...

#### **APPENDIX: Grains – Actual numbers**

If you have access to a specialist mathematical software program, it is possible to calculate the number of grains exactly using powers of 2. Such software can be set to display exact answers without resorting to standard form.

One free example of a powerful integer calculator is WinCalc, which can be downloaded from <http://math.exeter.edu/rparris/wincalc.html>. Another more sophisticated program, although not as straightforward to use, is called Calc and is available here: <http://www.isthe.com/chongo/tech/comp/calc/index.html>

Here then are the end-of-row results for the Grains On A Chess Board problem, since many students are very interested in seeing them! Students can be encouraged to try these calculations on their own calculators to give them a feel for exponential growth. Mass calculations are given for a 25mg grain, which is a mid-range value.

End of Row 1:  $N = 2^7$

128 or 3.2g

End of Row 2:  $N = 2^{15}$

32768 or 819.2g

End of Row 3:  $N = 2^{23}$

8388608 or 209.7kg

End of Row 4:  $N = 2^{31}$

2147483648 or 53.7 tonnes

End of Row 5:  $N = 2^{39}$   
549755813888 or 13744 tonnes

End of Row 6:  $N = 2^{47}$   
140737488355328 or 3.5 million tonnes

End of Row 7:  $N = 2^{55}$   
36028797018963968 or 901 million tonnes

End of Row 8:  $N = 2^{63}$   
9223372036854775808 or 231 billion tonnes

Total Grains Of Rice Used:  $N = 2^{64} - 1$   
18446744073709551615 or 462 billion tonnes

According to the International Rice Research Institute, world rice production in 2007 was “just” 645 million tonnes.  
<http://www.irri.org/statistics>