Simulating Drag Crisis for a Sphere Using Skin Friction Boundary Conditions

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Abstract

In this paper we use a General Galerkin (G2) method to simulate drag crisis for a sphere, where the unresolved turbulent boundary layer is modeled as a decreasing skin friction.

1 Introduction

A major challenge of turbulence simulation is the excessive number of degrees of freedom needed to represent the velocity scales of the flow in space and time, which may be estimated to be of the order Re^3, with the Reynolds number defined by $Re = UL/\nu$, where U and L are characteristic scales of velocity and length, and ν is the viscosity. Many turbulent flows of interest have $Re > 10^6$, and thus representation of all scales in the flow is impossible using any computer of today, or tomorrow.

Fortunately it appears that many quantities of interest are less dependent of the Reynolds number, typically various mean values of the flow variables. An example of such a quantity is the total dissipation of energy in fully developed turbulence, for example in a turbulent wake behind a moving object, for which there is both experimental and computational evidence [2, 1] supporting a *Law of finite energy dissipation*. Thus for sufficiently high Reynolds number the total energy dissipation would be constant, irrespectively of on what scale the actual viscous dissipation takes place. Other examples are drag and lift for a body in a flow, which appear only weakly dependent on the Reynolds number in certain regimes. Here it appears that the main dependence of the Reynolds number is through the separation of the flow, corresponding to different sizes of the turbulent wake attached to the rear of the body.

Separation of the flow is determined by the flow in the boundary layer at the surface of the body, with separation if the flow momentum in the streamwise direction near the surface is reduced to zero by an adverse pressure gradient and the loss of kinetic energy in the boundary layer. The energy dissipation in the boundary layer is related to the skin friction, which is proportional to $Re^{0.5}$ or $Re^{0.2}$ in a laminar and a turbulent boundary layer respectively (for a flat plate), according to boundary layer theory and experimental observations [13].

Thus for a high Reynolds number turbulent flow, it appears that the dependence of the Reynolds number for certain quantities of interest, such as drag, is restricted to the energy dissipation in the boundary layer, which is given by the skin friction of the boundary layer. This is apparent when studying the turbulent flow past a circular cylinder or a sphere, where the separation of the flow is more or less constant in the range $Re = 10^3 - 10^5$, corresponding to a similarly more or less constant drag. Near $Re = 10^5$ the laminar boundary layer undergoes transition to turbulence, leading to increased momentum near the boundary resulting in a delayed separation, corresponding to a smaller wake and lower drag, referred to as *drag crisis* [13]. As the Reynolds number is further increased the skin friction decreases and separation is further delayed. Although for the cylinder when increasing the Reynolds number, the small wake is not stable and starts to oscillate, corresponding to a higher drag [4, 11].

A question is then if it is possible to model high Reynolds number turbulent flow solely by modeling the turbulent boundary layer as a skin friction? In [4] drag crisis is modeled for a circular cylinder using a *General Galerkin (G2) method* [2, 8, 3, 5, 6] with a friction boundary condition, and in this paper we address the problem of modeling drag crisis for sphere, for which we present some preliminary results.

2   General Galerkin Methods

In [2, 8] a new framework for computation of mean value output in turbulent flow is presented, based on stabilized Galerkin finite element methods with adaptive mesh refinement based on a posteriori output error estimates using duality, which we refer to as *General Galerkin methods*, or *G2 methods*. In G2 we compute approximate weak solutions [7] to the Navier-Stokes (NS) equations directly, and we do not apply any filter to obtain averaged NS equations as in RANS or LES [12]. G2 was first used to compute drag of a surface mounted cube and a square cylinder [2, 8], and was then extended to

flow past a circular cylinder, a sphere, and a cylinder rolling along ground [3, 5, 6].

3   Friction Boundary Conditions

For the square shapes the separation is given by the geometry, but for the circular shapes the separation depend on the Reynolds number. For the problems in [3, 5, 6] the Reynolds number is sub-critical, corresponding to laminar boundary layer separation, where the laminar boundary layer is resolved by the adaptive mesh refinement. Although, a turbulent boundary layer is too expensive to resolve, and therefore alternative strategies are needed.

Various types of wall models are proposed in the literature, see [12] for an overview. Typically a wall model corresponds to solving a simplified NS equation for the boundary layer which is then coupled to the rest of the flow in some way.

In [4] we propose a very simple wall model based on a friction boundary condition on a part of the boundary Γ_{slfr} with normal n and two orthogonal tangential vectors τ_1, τ_2:

$$u \cdot n + \alpha \; n^T \sigma n = 0, \tag{1}$$

$$u \cdot \tau_k + \beta^{-1} n^T \sigma \tau_k = 0, \quad k = 1, 2, \tag{2}$$

for a solution $\hat{u} = (u, p)$ with velocity u and pressure p, and with the stress tensor $\sigma = \sigma(\hat{u})$, and where we use matrix notation with all vectors v being column vectors and the corresponding row vector is denoted v^T.

Here α is a penetration parameter and β is a friction parameter, both positive functions defined on the boundary. In principle, $(\alpha, \beta) \to (0, \infty)$ corresponds to a penalty imposition of a no slip boundary condition, and $(\alpha, \beta) \to (0, 0)$ corresponds to a penalty imposition of a slip boundary condition. By increasing β we increase the resistance at the boundary, and by increasing α we increase the penetration of the boundary.

The idea here is to model the influence on the global flow of the unresolved turbulent boundary layer as a skin friction given by the function β acting as a friction parameter.

This wall model is partly inspired by the work in [9, 10], where such a boundary condition is used to study reattachment of a low Reynolds number flow past a surface mounted cube.
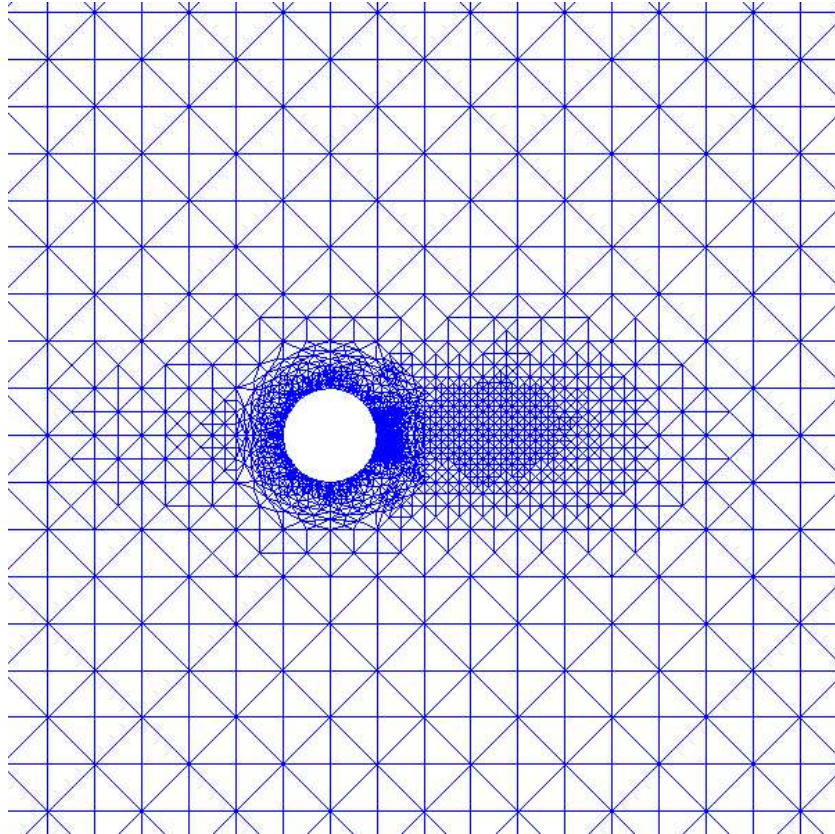
Figure 1: Section of the computational mesh.

4 Flow Past a Sphere

Drag crisis for a sphere occurs near $Re = 10^5$, and thus we may drop the viscous term from the discrete equations since it is dominated by the numerical dissipation of G2 in the form of a weighted least squares stabilization of the residual. We assume the boundary to be non-penetrable so that $\alpha = 0$, and we are thus left with a model containing only the friction parameter β and the discretization parameter h (with h the local mesh size).

Using the locally refined mesh in Fig. 1, with mesh size $h = h(x)$ for x in the computational domain $\Omega \subset \mathbb{R}^3$, we let β vary linearly from the constant value 0.1 to 0 on a time interval $I = [0, 10]$.

We find that the drag coefficient c_D drops linearly as a function of β until reaching a value $c_D \approx 0.2$ at $\beta \approx 0.02$, which is stable until β is below

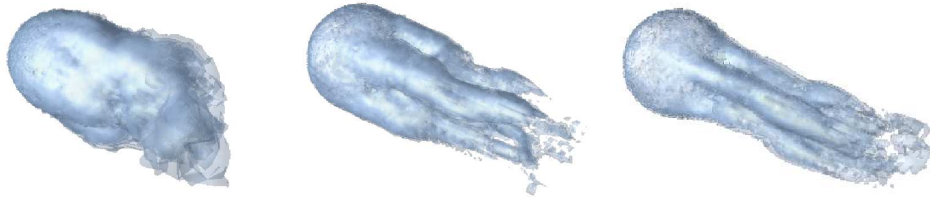Figure 2: Vorticity for the sphere corresponding to $\beta = 0.1, 0.025, 0.01$.

0.01, as c_D drops linearly towards 0 as $\beta \to 0$.

The reduction of c_D for β in the interval $[0.1, 0.01]$ corresponds to the scenario of drag crisis, whereas the scenario for c_D corresponding to β lower than $\beta_0 = 0.01$ is unknown experimentally to the knowledge of the author. Preliminary results indicate that value for β_0 depend on the mesh resolution, with a finer resolution corresponding to a smaller β_0, and thus that the results corresponding to $\beta < \beta_0$ may be non-physical corresponding to insufficient mesh resolution.

In Fig. 2 we show snapshots of the vorticity for 3 different values of β, where we find that as β is reduced, separation is delayed and the turbulent wake seems to approach a configuration with 4 tubes of streamwise vorticity attached to the rear of the sphere.

In Fig. 3 we show snapshots of the velocity field in a section just downstream the sphere, where we can follow the development of the 4 vorticity tubes. In particular, we find that the stable value of $c_D \approx 0.2$ corresponds to the fully developed configuration of 4 tubes of streamwise vorticity.

5 Discussion and Future Directions

Using a G2 method and friction boundary conditions, we have simulated drag crisis for a sphere. We noted that since the viscosity parameter is small for the high Reynolds numbers corresponding to drag crisis, we may drop the viscous term from the discrete G2 equations, and thus the only dissipation in G2 is due to the least squares stabilization of the residual and the friction boundary conditions. In particular, the only parameters left in the model are the discretization parameter h and the friction parameter β.

We found that as β is reduced we are able to follow the scenario of drag crisis for the sphere corresponding to a drop in the drag coefficient of more than 50%. We also found that for β less than a value β_0, the computational results appear to be non-physical judging from available experiments,
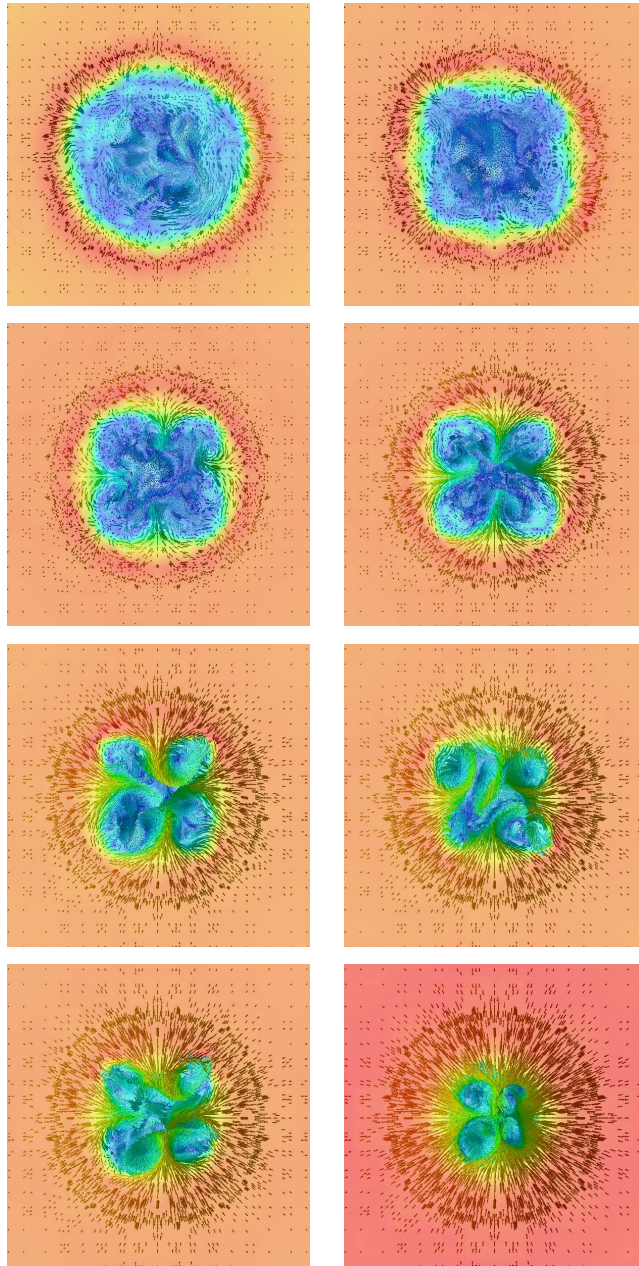
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Figure 3: Section of the velocity field just downstream the sphere, for friction parameters $\beta = 0.082, 0.032, 0.022, 0.018, 0.013, 0.012, 0.011, 0.0097$, corresponding to drag coefficients $c_D = 0.5, 0.3, 0.2, 0.2, 0.2, 0.2, 0.2, 0.1$.

possibly due to insufficient mesh resolution as β_0 is a function of h.

We plan to further investigate simulation of flows with turbulent boundary layers using G2 with friction boundary conditions, and in particular to study the dependence of the mesh resolution.

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