The Effect of Sidecut Radius on the Dynamics of Alpine Skiing.

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Abstract.
The invention and inception of so-called "parabolic skis" has led to what has been called a revitalization of the ski industry in the face of increasing interest in snowboards. Here, the effect of the inner curvature (side-cut) on the ability to make shorter carved turns is tested. The outcome at this point is inconclusive, due mainly to the difficulty of performing a purely carved turn.

Keywords: ski, carve, dynamics, side-cut, parabolic

1. Introduction

The "parabolic" ski, perhaps more aptly named the side-cut ski, as will be used for the rest of this work, is defined by a thinner middle (waist), than tip (shovel) or tail. The magnitude of this thinning can be several centimeters or more, and is thus easily visible when viewing the footprint of a the ski.

The first side-cut ski was introduced by Elan in 1993, just over a decade ago. To alpine skiers this fact can seem surprising as, since then, the market for recreational alpine skiing has been completely taken over by this variety of ski. It has reached the point where most major ski manufacturers no longer even *make* straight skis, and those that are still floating around the market often sell for under $20 Canadian.

There is little question, as a skier, that there is a marked improvement in the ability to make turns with a side-cut ski over a straight ski. This applies to both the effort needed to get the skis to change direction and the facilitating of many higher-end techniques. But, while many ski companies will boast the research put into their particular footprint, there is surprisingly little research into the actual mechanics of how it is a skier interacts with the slope. That having been said, there are certainly some sources available. In particular, the book "The Physics of Skiing" provides some excellent insight (Lind and Sanders, 2004).

In particular, within "Technote #5," the possible radius of a purely carved turn is determined using several simplifying approximations. As one might suppose, this radius is largely dependent upon the radius of side-cut of the ski, as it is that edge that pulls one around the curve. This paper will put these predictions to experimental tests using skis of several side-cut radii.

2. Computational Model

Before describing the forces present on skis during a carved turn, it is first necessary to describe the geometry of the ski itself. Please refer to Figure 1 for the definition of variables that will be necessary hereafter.

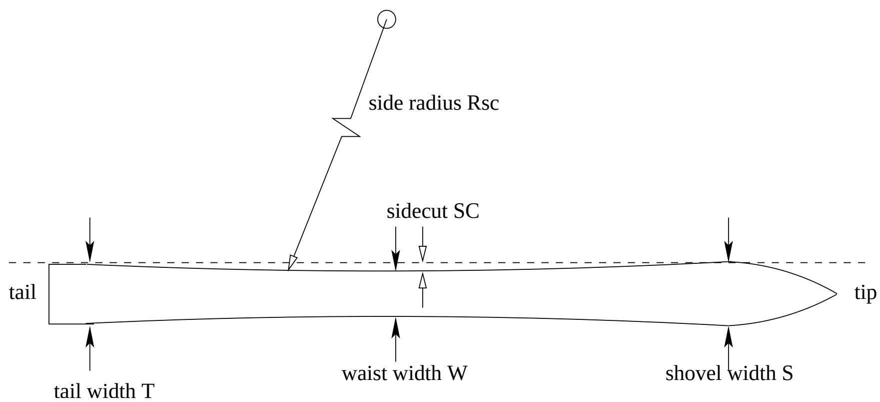

Figure 1. A top view of a typical side-cut ski.

There is one length, other than those defined in the figure, that will be of importance. Contact length, C, refers to the amount of ski that will be in contact with the hill during normal performance. For a typical ski this will be close to the total length, as typically the only part of a ski that does not come in contact with the slope is the tip.

For the present analysis, the most important property that affects ski performance is the side geometry of the ski. The side-cut radius, R_{sc}, that can be defined by the shovel, waist point, and tail of the ski. It can be given by the following relationship,

$$R_{sc} = \frac{C^2}{8SC}, \tag{1}$$

where the side-cut SC can be given by the relation $SC = \frac{1}{4}(S - 2W + T)$, and is the physical indent of the ski from widest to thinnest.

The carved turn itself simply consists of generating the force necessary to change the direction of the skier's momentum. This force is typically generated by simply pressing the ski's edge into the snow and letting the friction present pull the skier around a curved arc by slowing the inner edge of the ski. In true skiing situations this arc is nearly never a circle, due to purposeful skidding that occurs during the course of the turn. It is, however, much simpler to examine the turn formed by a turn made up entirely of carving, as will be performed here.

There are a number of forces to consider for the carving of a turn by a skier. First and foremost there are the forces pertaining to the weight of the skier on the slope. These will be referred to as W, for the weight straight down, F_N as the component perpendicular to the hill, and F_{load} as the force passing through the skis onto the snow itself. The reason that both F_{load} and F_N are necessary is due to the fact that a skier can, and will, move their center of mass away from directly above their skis, a technique known as "angulation." Next, the transverse forces, those in the plane of the slope, are F_{lat}, the component of weight along the hill, and F_C, the centrifugal force. The total transverse force can be written as $F_{tl} = F_C - F_{lat}$.

Finally, it is necessary to define several angles. It is possible to define all directions using two angular coordinates, α and β. The first of these α is simply the angle between the slope and the horizontal. The second, β, is the angle formed between the skier's direction and an equipotential line. It is simplest to imagine this on a uniform slope, where β will be the angle downslope from a horizontal line drawn *across* the slope, see Figure 2.

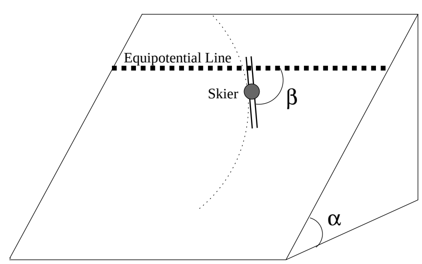

Figure 2. Definition of the angles α and β. During a turn on a uniform slope α will stay constant while β changes.

With all of these definitions in place the remaining forces can be written

$$F_C = Wv^2/gR_T \quad (2)$$
$$F_{lat} = W \sin\alpha \cos\beta \quad (3)$$
$$F_N = W \cos\alpha \quad (4)$$
$$F_{load}^2 = (F_C - F_{lat})^2 + F_N^2 \quad (5)$$

from the geometry of the situation. For a more detailed derivation of these, see Technote #5, and Chapter 4 of Lind (Lind and Sanders, 2004). If we assume the skier is in dynamic equilibrium such that the radius of contact of the turn can be defined as,

$$R_{con} = R_{SC} \cos \Omega, \tag{6}$$

with Ω taken to be formed between the ski and the slope, see Figure 3.

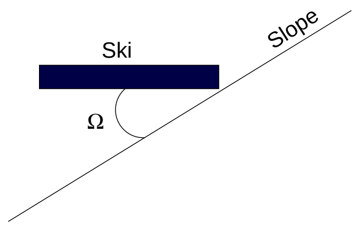

Figure 3. As a ski is placed at some angle to the slope an angle Ω forms between the ski and the slope. In reality, the edge of the ski would press into the snow but this does not alter the definition of Ω.

Furthermore, recalling that F_N is the force normal to the slope and F_{load} is equal and opposite to the snow reaction force (F_{reac}) then the relation $R_T F_{reac} = R_{SC} F_N$ follows directly. Squaring both sides and subtracting one from the other yields the relation,

$$F_{load}^2 R_T^2 - F_N^2 R_{SC}^2 = 0 \tag{7}$$

By substituting in the above expressions for forces this becomes a quadratic expression for R_T as follows,

$$a R_T^2 + b R_T + c = 0 \tag{8}$$

where the coefficients are,

$$a = (\sin\alpha \cos\beta)^2 + \cos^2\alpha \tag{9}$$

$$b = (-2v^2/g) \sin\alpha \cos\beta \tag{10}$$

$$c = (v^2/g)^2 - R_{SC}^2 \cos^2\alpha \tag{11}$$

and the solutions will be the positive solutions of the quadratic formula,

$$R_T = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}. \tag{12}$$

3. Experiment and Data

To test equation 12, it was decided that it should be possible to isolate two of the four variables, α and β, simply by examining the tracks left behind an actual carved turn. The velocity v was taken by measuring the time taken and distance covered, and R_{SC} was easy to calculate if one measures the dimensions of the ski.

The first measurement performed was that of the slope of the hill, to determine alpha. The results of four measurements at different locations around the slope are as follows,

α (radians)
0.297
0.384
0.305
0.262

The average is $\alpha = 0.312 \pm 0.05$.

Next three points on the curve were chosen, and at each the angle from the fall line, β, was measured. Finally, a line was drawn into the center of the circle from each of these points. Of course, originally there was no determined center of the circle so the first line was simply drawn long enough that it would be certain to intersect with another later. The second line was drawn back until it intersected the first and the third was drawn straight back and found to intersect simultaneously with the other two. The intersection point was taken to be the center of the circle, which allowed the radii to each point to be measured.

Having all of this data it was possible to predict the radius that a turn should have, using (12), and compare it to R_{meas}, the radius actually measured using the method mentioned in the previous paragraph. These measurements were performed on two pairs of skis with different side-cuts, the values are given below.

Less Side-Cut Skis	$R_{SC} = 14.8m$		
R_{meas} (m)	δR_{meas} (m)	R_T(m)	δR_T(m)
7.92	2	14.6	0.5
7.87	2	14.8	0.5
7.67	2	14.7	0.5

More Side-Cut Skis	$R_{SC} = 13.32m$		
R_{meas} (m)	δR_{meas} (m)	R_T (m)	δR_T (m)
16.1	2	13.29	0.5
16.5	2	13.17	0.5
16.17	2	13.00	0.5

4. Discussion

By comparing the actual radius of turn performed, R_{meas}, and the calculated turn radius, R_T, one can see that the measurements performed on the less side-cut (LSC) skis, do not agree with the predictions of equation 12 at all. The disagreement is on the order of a factor of two. The second set, however, while not in perfect agreement, are near experimental error, although it must be admitted that the error is fairly significant.

The only notable difference between these two runs, other than the different R_{SC} is that the skier was changed due to equipment difficulties. It is suspected that the skier for the second set of runs, being the more experienced of the two, was better able to perform a good approximation to a purely carved turn. As mentioned earlier, a typical ski turn is made up of a good deal of control skidding, and is not nearly as easy to predict as the model given here.

The experimental error present in the R_{meas} of the data was actually very difficult to determine. The method of drawing perpendicular lines from several points on the curve seems simple enough in one's mind; however, to keep the velocity somewhat constant throughout the section of curve examined, it was best to choose a somewhat short region. This implies that if the curves are drawn incorrectly in any way the point of intersection could vary greatly, and is the reason for the reported $\pm 2\ m$ error.

These results indicate that that it is, indeed, extremely difficult to perform a purely carved turn. It does appear, however, that it appears that a properly performed (purely carved) turn will be reasonably predicted by the above equations. Little can be said about the effect of the side-cut radius, due to the difficulty in performing a repeatable turn.

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References

Lind, D. and Sanders, S.P., The Physics of Skiing: Skiing at the triple point *Springer*, 2004.