MAP PROJECTIONS

Last day we look at the different types of co-ordinate system which could be used to specify the location of features in 2- and 3-dimensional space. Today we will look at how geographic co-ordinates (i.e. latitude and longitude) can be converted to a planar co-ordinate system using map projections.

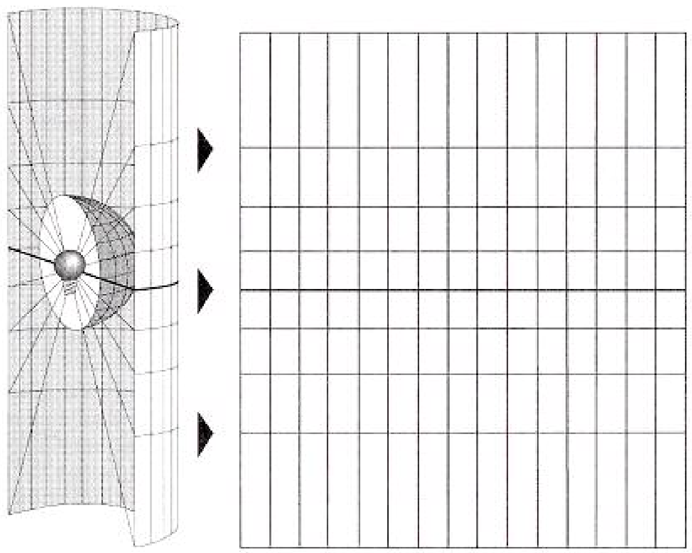
In practice map projections use mathematical formulae to convert from one co-ordinate system to another. This can be expressed in general terms as:

$$x = f(\phi,\lambda)$$
$$y = g(\phi,\lambda)$$

where f and g are the functions for a particular projection. For example, the formulae for the Mercator projection are:

$$x = \lambda$$
$$y = \ln \tan(\phi/2 + \pi/4)$$

Most projections can be envisaged as analogous to shining a light through a scaled-down model of the Earth, known as the **reference globe** (or **generating globe**), onto a surface, referred to as a **developable surface**, which can be 'rolled out' into a flat plane. Different types of projection can be defined depending upon the source of the light (e.g. the centre of the Earth, or a point infinitely far away) and the location and shape of the projection surface (e.g. cone, cylinder or plane). The projected lines of latitude and longitude form a **graticule**. These lines are not necessarily equally spaced, may converge and may be curved, depending upon the projection. A rectangular co-ordinate system, known as a **grid**, is often superimposed for the purpose of providing grid references, etc.



TYPES OF DISTORTION

Projecting from a curved surface onto a plane necessarily involves distortions. Different types of projection result in different types of distortion. If you are using a GIS to make measurements from the map (e.g. of distance or area), it may be important to know what sort of projection you are using so as to gauge the extent of the distortion in the properties you are measuring. It is also important to know what projections are being used if you are trying to overlay data from different sources - if the sources use different projections then the features will be displaced relative to one another. Four properties are generally identified as important:

- Shape
- Area
- Distance
- Direction

Different projections can be selected to ensure the accuracy of the map with regard to one or sometimes more of these properties, but each projection necessarily results in distortions with regard to some of the other properties.

The scale of a map may be thought of as having two components: one identifies the extent to which the reference globe is scaled down relative to the Earth; the other measures the extent to which features on the curved reference globe are distorted when projected onto a planar surface. The first of these, referred to as the **principal scale** (or sometimes simply as the **map scale**, e.g. 1:50,000), is the representative fraction for the reference globe, calculated by dividing the Earth's radius by the radius of the reference globe. The second, referred to as the **scale factor**, is calculated by the dividing the distance between features on the planar surface (i.e. map) by the distance between these features on the reference globe. A scale factor of 1.0 would indicate no distortion.[1] When the reference globe is projected onto a planar surface, the scale factor will tend to vary between different parts of the map – i.e. some parts of the map will have a scale factor of 1.0, whereas other parts will have scale factors larger or smaller than this.

Analysis of the scale factor helps to understand the nature of the four properties more precisely. We will refer to the scale factor in the east-west direction (i.e. along the parallels) as s_x and the scale factor in the north-south direction (i.e. along the meridians) as s_y.

1. **Shape.** Projections which preserve the correct shape for local areas (e.g. Lambert Conformal Conic, Mercator) are referred to as **conformal** (or **orthomorphic**). To preserve shape, the projected map should preserve the angles between lines on the reference globe. The parallels and meridians must therefore remain at 90 degrees to one another. For example, the Mercator projection represents both the lines of latitude and of longitude as straight parallel lines intersecting at right angles. Given that the meridians on a globe converge towards the poles, the distance between them along the lines of latitude in a Mercator projection must obviously be stretched (i.e. s_x must get larger) as you move towards the poles if the meridians are to remain parallel. However, to retain the correct shape, the distance between the parallels must also be stretched by the same amount. For example, if you increase the width of a square by, say, 10 per cent, then you will also need to increase its height by 10 per cent otherwise it will no longer remain square. The lines of latitude are therefore spaced further and further apart as you move towards the poles as s_y increases.

   In any conformal map, at any given point $s_x = s_y$. However, the lines of latitude do not necessarily need to be straight nor do the lines of longitude need to be parallel. For example, the lines of latitude in the Lambert Conformal Conic are curved but parallel, whilst the lines of longitude are straight but converge.

   It should be noted that conformal projections only preserve shape for small areas. No map projection can preserve shape for larger areas. For example, a conformal map of Ireland might show both Dublin Bay and Cork Harbour as the correct shape, but they will have different scale factors. The overall shape of Ireland will therefore be distorted.

2. **Area.** Projections which correctly represent the area of features (e.g. Albers Equal Area Conic; Equal Area Cylindrical) are referred to as **equal area** (or **equivalent**). To preserve areas, equal area projections must compensate for an increase in the scale factor in one direction by a corresponding decrease in the other cardinal direction (i.e. $s_x . s_y = 1.0$). This preserves the measures of area. However, to preserve areas we must distort shape. Shape and area cannot both be preserved at the same time.

3. **Distance.** Projections which attempt to minimise distortions in measures of distance (e.g. Equidistant Azimuthal, Sinusoidal) are referred to as **Equidistant**. There are two ways in which this can be done. One is to maintain a scale factor of 1.0 along one or possibly two lines of latitude (called **standard parallels**). Distance measures will be correct along these lines, but inaccuracies arise along all other lines. The other approach is to maintain a scale factor of 1.0 in all directions from one or sometimes two points. Distances measured from these points will be correct, but errors will be introduced if distances are measured from any other point. Both approaches provide only partial solutions, although the extent of the errors may be insignificant for most purposes (i.e. about 0.1%). No projection can measure distance correctly throughout the entire map. For example, the Sinusoidal projection provides accurate measures along the Equator and all meridians, but distances in all other directions are distorted.

4. **Direction.** Azimuthal projections (see below e.g. Lambert Equal Area Azimuthal, Equidistant Azimuthal) preserve direction on maps (e.g. for navigation purposes). However, they are only accurate for one or two

[1] The scale factor of features on the reference globe may be thought of as 1.0. The scale factor of features on the planar surface therefore measures the extent by which these features are stretched or compressed.

selected points. Directions starting from all other points will be incorrect. Although not azimuthal, the Mercator projection also shows the correct compass bearing between any two points, although the line connecting them on a Mercator will rarely follow the shortest (i.e. great circle) route – the shortest route will generally appear on a Mercator map as a curved line.

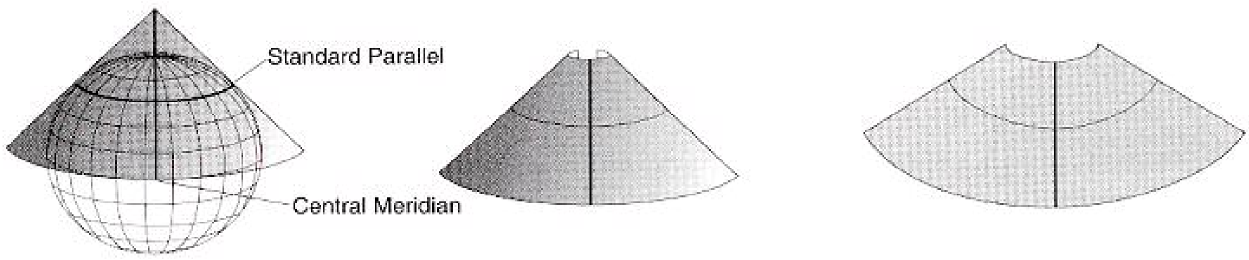
Some map projections, referred to as **Compromise** projections (e.g. Robinson), attempt to strike a balance between objectives without necessarily being perfect with regard to any of them.

An important step in any projection is to identify one or more points of contact - i.e. places where the projection surface touches (**points of tangency**) or intersects (**points of secancy**) the surface of the reference globe. Tangential projections are more common. Planar projections (see below) are tangential at one point only. Conic and cylindrical projections are tangential along a line. If the projection surface intersects the reference globe, rather than merely touching it, then the points of intersection may form a circle (planar projection) or two lines (conic or cylindrical). These projections involve **secant** rather than tangential calculations. The location of the points of contact are important because they define the point or line of no distortion (i.e. true scale, where the scale factor is 1.0), often referred to as the **standard line** (usually a **standard parallel** or a **standard meridian**, but occasionally another line). Distortion generally increases with distance from the point or line of contact.

PROJECTION TYPES

The three main types of projection are **conic**, **cylindrical** and **planar**.

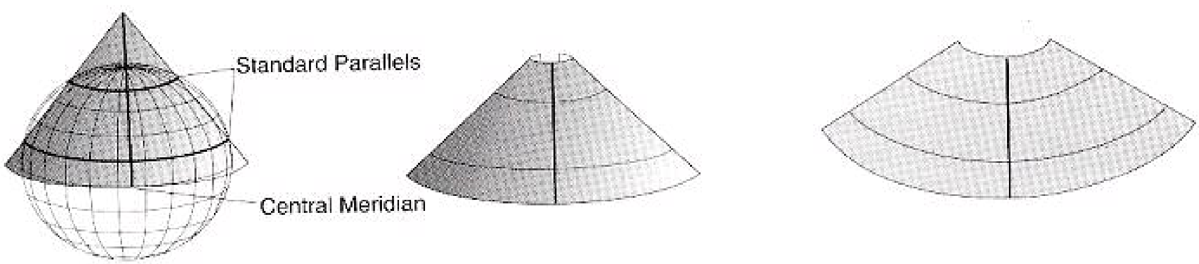
Conic Projections



The simplest conic projection is tangential to the globe along a line of latitude (known as the **standard parallel**). The lines of latitude and longitude are projected onto the cone, and the cone is then metaphorically cut along any meridian. The opposite meridian becomes the **central meridian** in the map. The other meridians converge towards a point, and the lines of latitude appear as concentric circular arcs.

Distortion increases north and south of the standard parallel, so the top of the cone (corresponding to polar areas) is usually cut off.

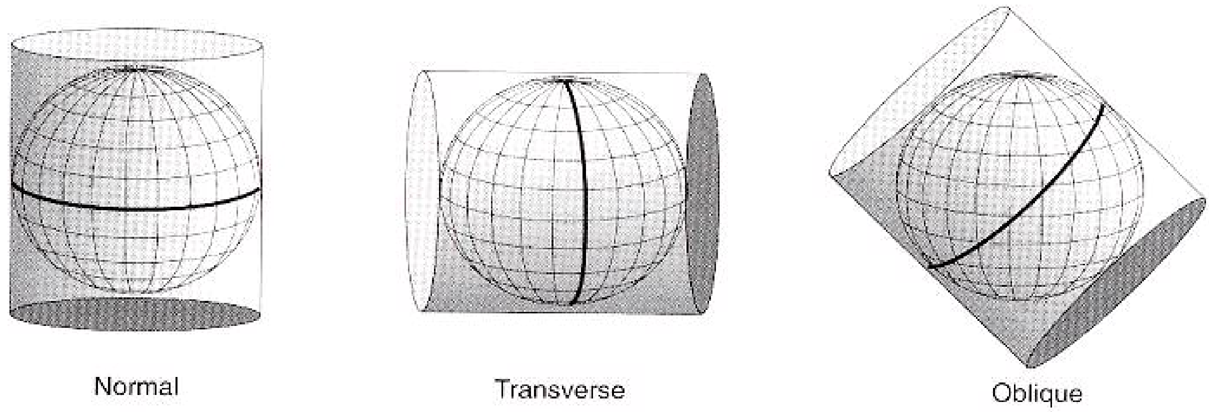
Secant conic projections use two lines of contact (i.e. the projection surface intersects the globe along two lines of latitude – i.e. two standard parallels). The cone may also sit on the globe 'crookedly' to form an **oblique conic projection** (i.e. the line of contact does not lie along a line of latitude, but intersects it at an angle).



The lines of latitude are arc shaped, whilst the meridians are straight but radiate from a common point (corresponding to the tip of the cone). The spacing between the parallels (i.e. lines of latitude) can be adjusted (analogous to moving the light source closer or further away). **Equidistant Conic** projections, which may be either tangential or secant, have evenly spaced parallels, which means the projection is equidistant in a north-south direction (i.e. the meridians), but it is neither conformal nor equal-area. The **Lambert Conformal Conic** projection is a secant projection designed to preserve shape. It is characterised by an increasing distance between parallels beyond the standard parallels as you move towards the top or bottom of the map. The scale factor is less than 1.0 between the standard parallels, and more than 1.0 outside the standard parallels. The Lambert Conformal Conic is particularly useful in mid-latitude areas with a large east-west extent. The **Albers Equal-Area Conic** projection is also suited to mid-latitude areas with a large east-west extent. It is also a secant projection, but the objective in this case is to accurately represent areas. Shape is only distorted slightly between the central parallels, but the distortion increases towards the top and bottom of the map. The parallels between the standard parallels are more widely spaced than those at the top or bottom.

Cylindrical Projections

Cylindrical projections may either have one line of tangency or two of secancy. The projection may be **normal** (using lines of latitude as the lines of contact - e.g. Mercator), **transverse** (using meridians - e.g. Transverse Mercator) or **oblique** (using any other great circle line).



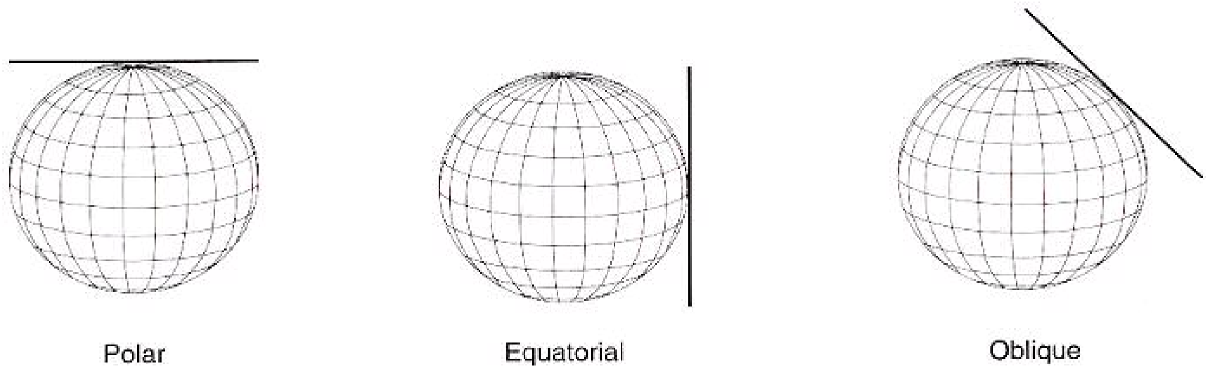
The **Mercator** projection is probably the best known projection of all. It produces a graticule with 90 degree angles. The meridians are equally spaced, whilst the spacing of the parallels (i.e. lines of latitude) increases towards the poles. The Mercator projection preserves direction, but results in massive distortions of some of the other properties (especially area in the polar regions).

The **Universal Transverse Mercator** (UTM) is also very widely used. The world is divided into 60 zones with central meridians at 6 degree intervals. The zones start at 180 degrees West, and the zone numbers rise as you travel east. Ireland is in zone 29 and Britain is in zone 30. Each zone between 84° N and 80° S is divided into 20 rows of 8 degrees each, except for the most northerly one which is 12 degrees (i.e. a total of 60x20=120 'regions'). Two lines of secancy run parallel to and approximately 180 km on either side of the central meridian. The scale factor along these lines is 1.0, but is 0.9996 along the central meridian and 1.0004 on either edge of the zone. The UTM is a conformal projection (i.e. small features have the correct shape, and the scale is the same in all directions). Lines of latitude and longitude both appear curved in the projection, except for the central meridian and the equator. The co-ordinates are expressed in metres. The origin is the intersection of the central meridian and the equator, but to ensure all values are positive the origin is given a false easting of 500,000. In the southern hemisphere the origin is given a false northing of 10,000,000, but no adjustment is necessary in the northern hemisphere. One of the distinguishing characteristics of the UTM is that the x co-ordinates have 6 digits, but the y co-ordinates have 7 digits. Because each zone is in effect a different projection, maps do not fit together across zone boundaries.

Planar Projections

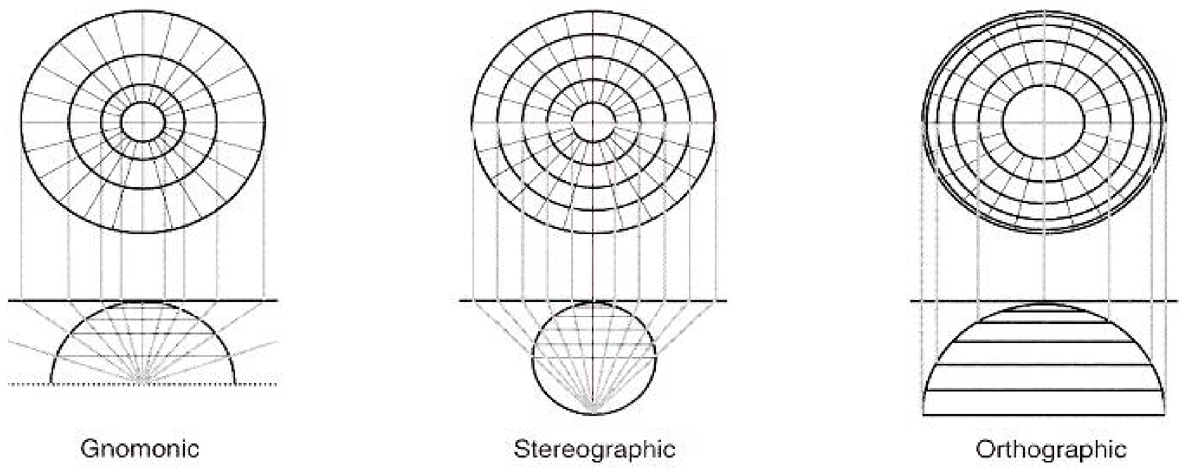
Planar projections, sometimes referred to as **azimuthal** or **zenithal** projections, project map data onto a flat surface which normally touches the globe at a single point. The point of contact defines the **aspect** of the projection. The aspect can be **polar** (using either the north or south pole), **equatorial** (using a point somewhere on the Equator) or

oblique (using any other point). Planar projections provide accurate representations of direction from their **focus** (i.e. point of contact), where the graticular angles are correctly represented as 90 degrees, but not from any other point. Areal and conformal distortion increases with increasing distance from the focus, so planar projections are best suited for displaying circular regions.



Polar projections have a dartboard arrangement of concentric lines of latitude and radiating meridians.

The spacing between the lines of latitude depends upon the location of the hypothetical light source. **Gnomonic** projections view the surface data from the centre of the Earth, **stereographic** projections view the data from the point on the Earth's surface directly opposite the point of contact, and **orthographic** projections view the data from a point at infinite distance (analogous to a very distant planet).



Other Projections

There are various other projections which do not fit into any of the above categories. **Modified** projections use various mathematical devices to modify one of the standard projections to reduce distortion (e.g. **Space Oblique Mercator**). **Pseudo** projections are totally artificial. The **Sinusoidal** projection is a pseudocylindrical projection which preserves area, but distorts shape away from the central meridian and the equator. To minimise these distortions the map is split into several discontinuous segments, each with its own central meridian. The parallels are straight, but the meridians are sinusoidal curves. The **Robinson** projection is also a pseudocylindrical projection which does not accurately preserve any properties, but it results in only minimal distortions in both shape and area in the central parts of the map.

NATIONAL GRIDS

The UTM is conformal (i.e. preserves local shape) and is suitable for medium scale mapping in most areas, especially where the north-south extent exceeds the east-west extent, but it is not sufficiently accurate for some large scale uses (e.g. accurate surveying). Many national mapping agencies have therefore adopted a co-ordinate system which is particularly suited to their own local conditions (e.g. shape, latitude, etc.). For example, a Transverse Mercator would not be suitable for Canada, because of its large east-west extent, therefore Canada uses a Lambert Conformal Conical projection which is especially suitable for mid to high latitudes.

In the US, each state adopted its own projection and co-ordinate system in the 1930s, referred to as **State Plane Co-ordinates** (SPC). Some of the larger states are further subdivided into zones for greater accuracy. Texas, for example, is divided into five 'horizontal' zones, each of which use a Lambert Conformal Conical projection. Hawaii is also divided into 5 zones, but they are 'vertical' and are based on a Transverse Mercator projection. The SPCs had to be revised following the adoption of the North American Datum in 1983 (NAD83).

The 'traditional' British and Irish national grids use Transverse Mercator projections, but with a different central meridian and a different spheroid. The Irish Grid was based on a modified Airy spheroid, whereas the British grid used an unmodified Airy spheroid. Ordnance Survey Ireland defined a new co-ordinate system in 2001, referred to as ITM (Irish Transverse Mercator), to be more compatible with GPS. This is based on the GRS80 spheroid, but it uses the same origin as the Irish Grid. As the co-ordinates would have been similar, but not identical, the false origin in ITM was shifted further into the Atlantic to avoid possible confusion – i.e. a co-ordinate for any point on the landmass is unique to one grid or the other.

OSI will also supply data using a UTM (zone 29) co-ordinate system for customers who may need to operate at an international level. This has a different origin, different false origin and different scale factor to ITM.

Reference System	**Irish Grid**	**ITM (Irish Transverse Mercator)**	**UTM (Universal Transverse Mercator)**
Projection :	Transverse Mercator	Transverse Mercator	Transverse Mercator
Spheroid :	Airy (Modified 1849)	GRS80	GRS80
Semi-major Axis :	6,377,341.89 m	6,378,137.00 m	6,378,137.00 m
Semi-Minor Axis :	6,356,036.143 m	6,356,752.3141 m	6,356,752.3141 m
Central Meridian :	-8 degrees	-8 degrees	-9 degrees
Reference Latitude :	53.5 degrees	53.5 degrees	+0 degrees
Scale Factor (i.e. error at central meridian) :	1.000035 [2]	0.999820	0.999600
False Easting :	200,000 m	600,000 m	500,000 m
False Northing :	250,000 m	750,000 m	0

[2] This was revised following new triangulation in 1975.