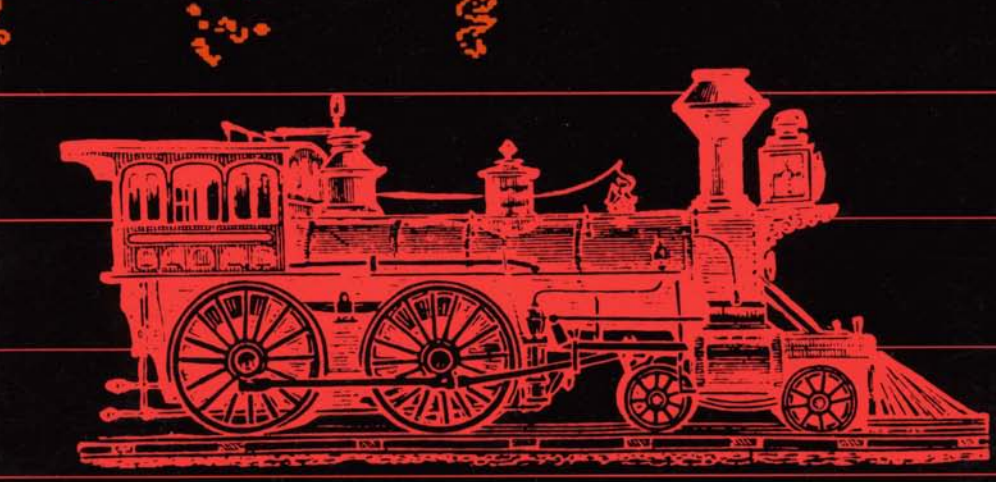
MARTIN GARDNER

WHEELS, LIFE AND OTHER MATHEMATICAL AMUSEMENTS



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THE GAME OF LIFE, PART I

Most of the work of John Horton Conway, a distinguished mathematician at the University of Cambridge, has been in pure mathematics. For instance, in 1967 he discovered a new group—some call it "Conway's constellation"—that includes all but two of the then known sporadic groups. (They are called "sporadic" because they fail to fit any classification scheme.) It is a breakthrough that has had exciting repercussions in both group theory and number theory. It ties in closely with an earlier discovery by John Leech of an extremely dense packing of unit spheres in a space of 24 dimensions where each sphere touches 196,560 others. As Conway has remarked, "There is a lot of room up there."

In addition to such serious work Conway also enjoys recreational mathematics. Although he is highly productive in this field, he seldom publishes his discoveries. One exception was his paper on "Mrs. Perkins' Quilt," a dissection problem discussed in my *Mathematical Carnival.* Another was sprouts, a topological pencil-and-paper game invented by Conway and M. S. Paterson. It is also the topic of a chapter in the same book.

In this chapter we consider Conway's most famous brainchild, a fantastic solitaire pastime he calls "Life." Because of its analogies with the rise, fall and alterations of a society of living organisms, it belongs to a growing class of what are called "simulation games"—games that resemble real-life processes. To play Life without a computer you need a fairly large checkerboard and a plentiful supply of flat counters of two colors. (Small checkers or poker chips do nicely.) An Oriental "go" board can be used if you can find flat counters small enough to fit within its cells. (Go stones are awkward to use because they are not flat.) It is possible to work with pencil and graph paper but it is much easier, particularly for beginners, to use counters and a board.

The basic idea is to start with a simple configuration of counters (organisms), one to a cell, then observe how it changes as you apply Conway's "genetic laws" for births, deaths and survivals. Conway chose his rules carefully, after a long period of experimentation, to meet three desiderata:

(1) There should be no initial pattern for which there is a simple proof that the population can grow without limit.

(2) There should be initial patterns that *apparently* do grow without limit.

(3) There should be simple initial patterns that grow and change for a considerable period of time before coming to an end in three possible ways: Fading away completely (from overcrowding or from becoming too sparse), settling into a stable configuration that remains unchanged thereafter, or entering an oscillating phase in which they repeat an endless cycle of two or more periods.

In brief, the rules should be such as to make the behavior of the population both interesting and unpredictable.

Conway's genetic laws are delightfully simple. First note that each cell of the checkerboard (assumed to be an infinite plane) has eight neighboring cells, four adjacent orthogonally, four adjacent diagonally. The rules are:

(1) Survivals. Every counter with two or three neighboring counters survives for the next generation.

(2) Deaths. Each counter with four or more neighbors dies (is removed) from overpopulation. Every counter with one neighbor or none dies from isolation.

(3) Births. Each empty cell adjacent to exactly three neighbors—no more, no fewer—is a birth cell. A counter is placed on it at the next move.

It is important to understand that all births and deaths occur *simultaneously*. Together they constitute a single generation or, as we shall usually call it, a "tick" in the complete "life history" of the initial configuration. Conway recommends the following procedure for making the moves:

(1) Start with a pattern consisting of black counters.

(2) Locate all counters that will die. Identify them by putting a black counter on top of each.

(3) Locate all vacant cells where births will occur. Put a white counter on each birth cell.

(4) After the pattern has been checked and double-checked

to make sure no mistakes have been made, remove all the dead counters (piles of two) and replace all newborn white organisms with black counters.

You will now have the first generation in the life history of your initial pattern. The same procedure is repeated to produce subsequent generations. It should be clear why counters of two colors are needed. Because births and deaths occur simultaneously, newborn counters play no role in causing other deaths or births. It is essential, therefore, to be able to distinguish them from live counters of the previous generation while you check the pattern to be sure no errors have been made. Mistakes are very easy to make, particularly when first playing the game. After playing it for a while you will gradually make fewer mistakes, but even experienced players must exercise great care in checking every new generation before removing the dead counters and replacing newborn white counters with black.

You will find the population constantly undergoing unusual, sometimes beautiful and always unexpected change. In a few cases the society eventually dies out (all counters vanishing), although this may not happen until after a great many generations. Most starting patterns either reach stable figures—Conway calls them "still lifes"—that cannot change or patterns that oscillate forever. Patterns with no initial symmetry tend to become symmetrical. Once this happens the symmetry cannot be lost, although it may increase in richness.

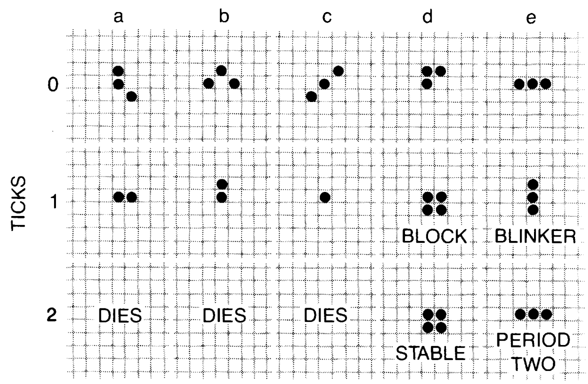
Conway originally conjectured that no pattern can grow without limit. Put another way, any configuration with a finite number of counters cannot grow beyond a finite upper limit to the number of counters on the field. At the time this was one of the most difficult questions posed by the game. Conway offered a prize of $50 to the first person who could prove or disprove the conjecture before the end of 1970. One way to disprove it would be to discover patterns that keep adding counters to the field: A "gun" (a configuration that repeatedly shoots out moving objects such as the "glider," to be explained below) or a "puffer train" (a configuration that moves but leaves behind a trail of "smoke"). The results of the contest for Conway's prize are discussed in the next chapter.

Let us see what happens to a variety of simple patterns.

A single organism or any pair of counters, wherever placed, will obviously vanish on the first tick.

A beginning pattern of three counters also dies immediately unless at least one counter has two neighbors. Figure 126 shows the five connected triplets that do not fade on the first tick.

Figure 126



The fate of five triplets in "life"

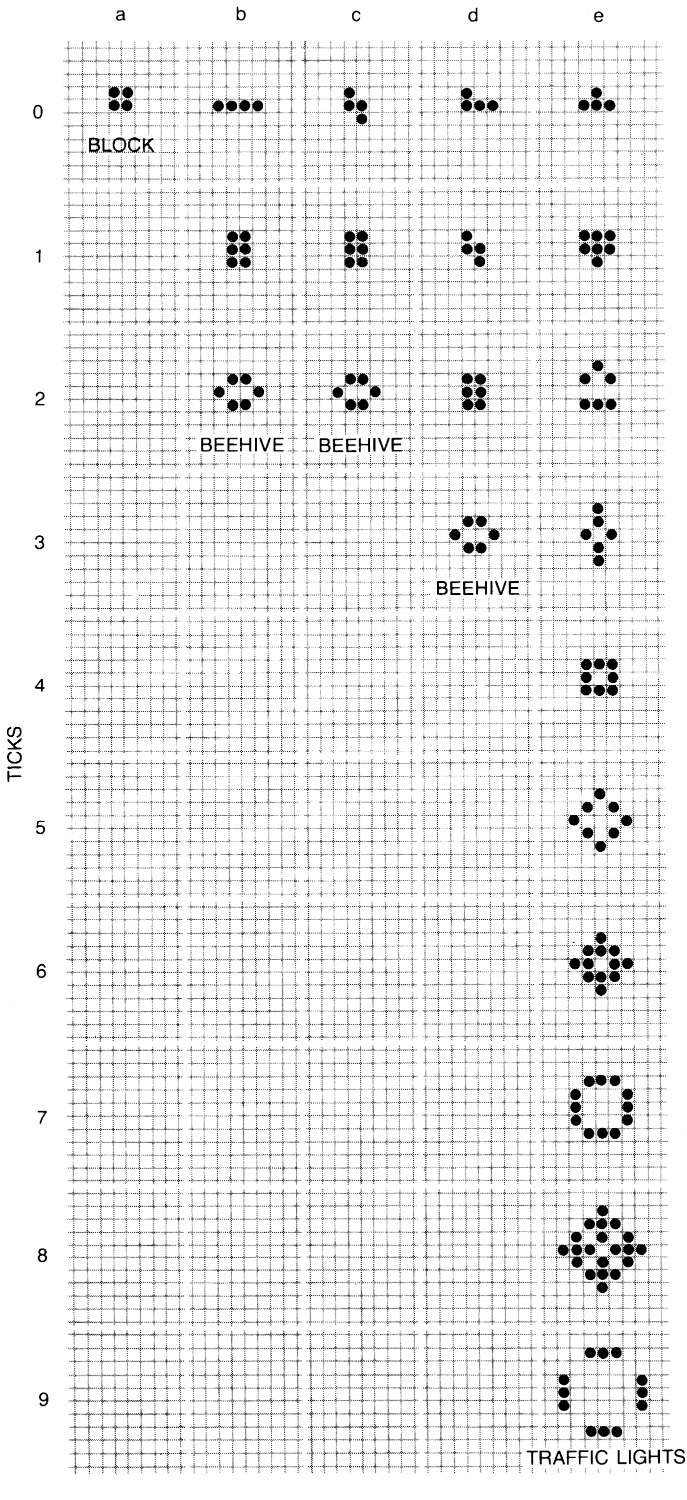
(Their orientation is of course irrelevant.) The first three [*a, b, c*] vanish on the second tick. In connection with *c* it is worth noting that a single diagonal chain of counters, however long, loses its end counters on each tick until the chain finally disappears. The speed a chess king moves in any direction is called by Conway (for reasons to be made clear later) the "speed of light." We say, therefore, that a diagonal chain decays at each end with the speed of light.

Pattern *d* becomes a stable "block" (two-by-two square) on the second tick. Pattern *e* is the simplest of what are called "flip-flops" (oscillating figures of period 2). It alternates between horizontal and vertical rows of three. Conway calls it a "blinker."

Figure 127 shows the life histories of the five tetrominoes (four rookwise-connected counters). The square [*a*] is, as we have seen, a still-life figure. Tetrominoes *b* and *c* reach a stable figure, called a "beehive," on the second tick. Beehives are frequently produced patterns. Tetromino *d* becomes a beehive on the third tick. Tetromino *e* is the most interesting of the lot. After nine ticks it becomes four isolated blinkers, a flip-flop called "traffic lights." It too is a common configuration. Figure 128 shows 12 common forms of still life.

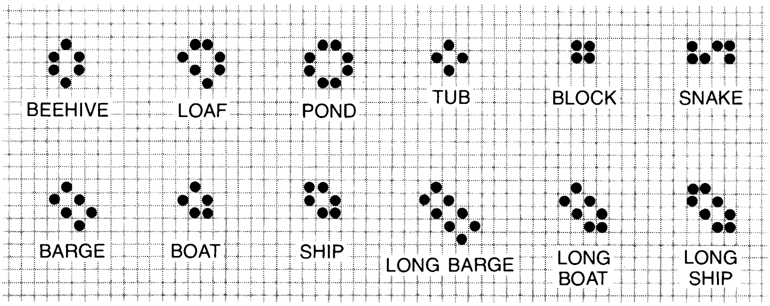
The reader may enjoy experimenting with the 12 pentominoes (all possible patterns of five rookwise-connected counters) to see what happens to each. He will find that five vanish before the fifth tick, two quickly reach a stable loaf, and four in

Figure 127



The life histories of the five tetrominoes

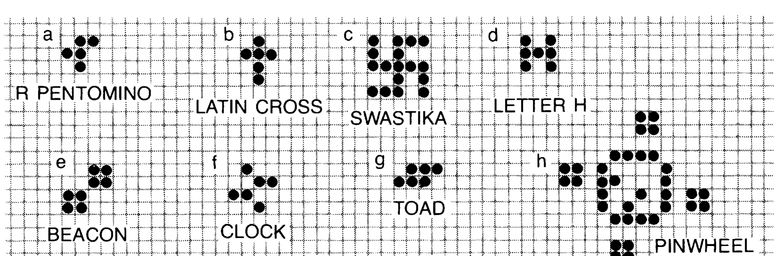
Figure 128



The commonest stable forms

a short time become traffic lights. The only pentomino that does not end quickly (by vanishing, becoming stable or oscillating) is the *R* pentomino [*"a" in Figure* 129]. Conway has tracked it for 460 ticks. By then it has thrown off a number of gliders. Conway remarks: "It has left a lot of miscellaneous junk stagnating around, and has only a few small active regions, so it is not at all obvious that it will continue indefinitely." Its fate is revealed in the addendum to this chapter.

Figure 129



The R pentomino *(a)* and exercises for the reader

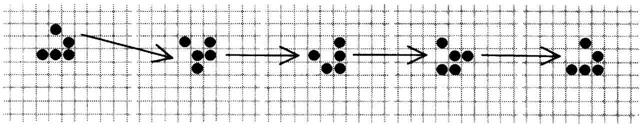
For such long-lived populations Conway sometimes uses a computer with a screen on which he can observe the changes. The program was written by M. J. T. Guy and S. R. Bourne. Without its help some discoveries about the game would have been difficult to make.

As easy exercises the reader is invited to discover the fate of the Latin cross [*"b" in Figure* 129], the swastika [*c*], the letter *H*

[*d*], the beacon [*e*], the clock [*f*], the toad [*g*] and the pinwheel [*h*]. The last three figures were discovered by Simon Norton. If the center counter of the *H* is moved up one cell to make an arch (Conway calls it "pi"), the change is unexpectedly drastic. The *H* quickly ends but pi has a long history. Not until after 173 ticks has it settled down to five blinkers, six blocks and two ponds. Conway also has tracked the life histories of all the hexominoes, and all but seven of the heptominoes. Some hexominoes enter the history of the *R* pentomino; for example, the pentomino becomes a hexomino on its first tick.

One of the most remarkable of Conway's discoveries is the five-counter glider shown in Figure 130. After two ticks it has shifted slightly and been reflected in a diagonal line. Geometers call this a "glide reflection"; hence the figure's name. After two more ticks the glider has righted itself and moved one cell diagonally down and to the right from its initial position. We mentioned earlier that the speed of a chess king is called the speed of light. Conway chose the phrase because it is the highest speed at which any kind of movement can occur on the board. No pattern can replicate itself rapidly enough to move at such speed. Conway has proved that the maximum speed diagonally is a fourth the speed of light. Since the glider replicates itself in the same orientation after four ticks, and has traveled one cell diagonally, one says that it glides across the field at a fourth the speed of light.

Figure 130

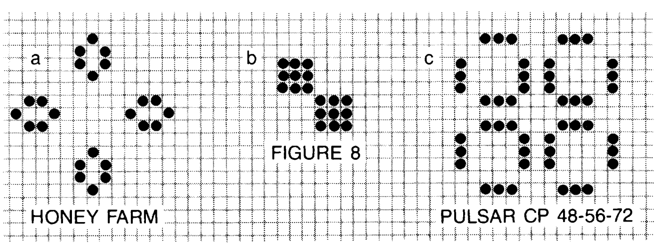


The "glider"

Movement of a finite figure horizontally or vertically into empty space, Conway has also shown, cannot exceed half the speed of light. Can any reader find a relatively simple figure that travels at such a speed? Remember, the speed is obtained by dividing the number of ticks required to replicate a figure by the number of cells it has shifted. If a figure replicates in four ticks in the same orientation after traveling two unit squares horizontally or vertically, its speed will be half that of light. Figures that move across the field by self-replication are extremely hard to find. Conway knows of four, including the glider, which he calls "spaceships" (the glider is a "featherweight spaceship"; the others have more counters). I will disclose their patterns in the Answer Section.

Figure 131 depicts three beautiful discoveries by Conway and his collaborators. The stable honey farm [*a in Figure* 131] results after 14 ticks from a horizontal row of seven counters. Since a five-by-five block in one move produces the fourth generation of this life history, it becomes a honey farm after 11 ticks. The "figure 8" [*b in Figure 131*], an oscillator found by Norton, both resembles an 8 and has a period of 8. The form *c*, in Figure 131 called "pulsar *CP* 48–56–72," is an oscillator with a life cycle of period 3. The state shown here has 48 counters, state two has 56 and state three has 72, after which the pulsar returns to 48 again. It is generated in 32 ticks by a heptomino consisting of a horizontal row of five counters with one counter directly below each end counter of the row.

Figure 131



Three remarkable patterns, one stable and two oscillating

Conway has tracked the life histories of a row of n counters through $n=20$. We have already disclosed what happens through $n=4$. Five counters result in traffic lights, six fade away, seven produce the honey farm, eight end with four beehives and four blocks, nine produce two sets of traffic lights, and 10 lead to the "pentadecathlon," with a life cycle of period 15. Eleven counters produce two blinkers, 12 end with two beehives, 13 with two blinkers, 14 and 15 vanish, 16 give "big traffic lights" (eight blinkers), 17 end with four blocks, 18 and 19 fade away and 20 generate two blocks.

Conway also investigated rows formed by sets of n adjacent counters separated by one empty cell. When $n=5$ the counters interact and become interesting. Infinite rows with $n=1$ or $n=2$ vanish in one tick, and if $n=3$ they turn into blinkers. If $n=4$ the row turns into a row of beehives.

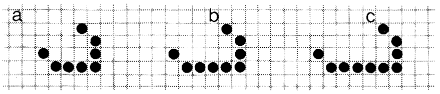
The 5-5 row (two sets of five counters separated by a vacant cell) generates the pulsar *CP* 48-56-72 in 21 ticks. The 5-5-5 ends in 42 ticks with four blocks and two blinkers. The 5-5-5-5 ends in 95 ticks with four honey farms and four blinkers, 5-5-5-5-5 terminates with a spectacular display of eight gliders and eight blinkers after 66 ticks. Then the gliders crash in pairs to become eight blocks after 86 ticks. The form 5-5-5-5-5-5 ends with four blinkers after 99 ticks, and 5-5-5-5-5-5-5, Conway remarks, "is marvelous to sit watching on the computer screen." This ultimate destiny is given in the addendum.

ANSWERS

The Latin cross dies on the fifth tick. The swastika vanishes on the sixth tick. The letter *H* also dies on the sixth tick. The next three figures are flip-flops: As Conway writes, "The toad pants, the clock ticks and the beacon flashes, with period 2 in every case." The pinwheel's interior rotates 90 degrees clockwise on each move, the rest of the pattern remaining stable. Periodic figures of this kind, in which a fixed outer border is required to move the interior, Conway calls "billiard-table configurations" to distinguish them from "naturally periodic" figures such as the toad, clock and beacon.

The three unescorted ships (in addition to the glider, or "featherweight spaceship" are shown in Figure 132. To be precise, each becomes a spaceship in 1 tick. (The patterns in Figure 132 never recur.) All three travel horizontally to the right with half the speed of light. As they move they throw off sparks that vanish immediately as the ships continue on their way. Unescorted spaceships cannot have bodies longer than six counters without giving birth to objects that later block their

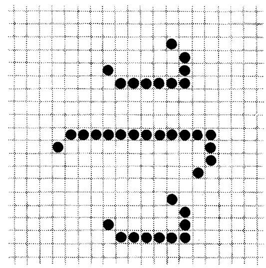
Figure 132



Lightweight *(left)*, middleweight *(center)*, and heavyweight *(right)* spaceships

motion. Conway has discovered, however, that longer spaceships, which he calls "overweight" ones, can be escorted by two or more smaller ships that prevent the formation of blocking counters. Figure 133 shows a larger spaceship that can be es-

Figure 133



Overweight spaceship with two escorts

corted by two smaller ships. Except for this same ship, lengthened by two units, longer ships require a flotilla of more than two companions. A spaceship with a body of 100 counters, Conway finds, can be escorted safely by a flotilla of 33 smaller ships.

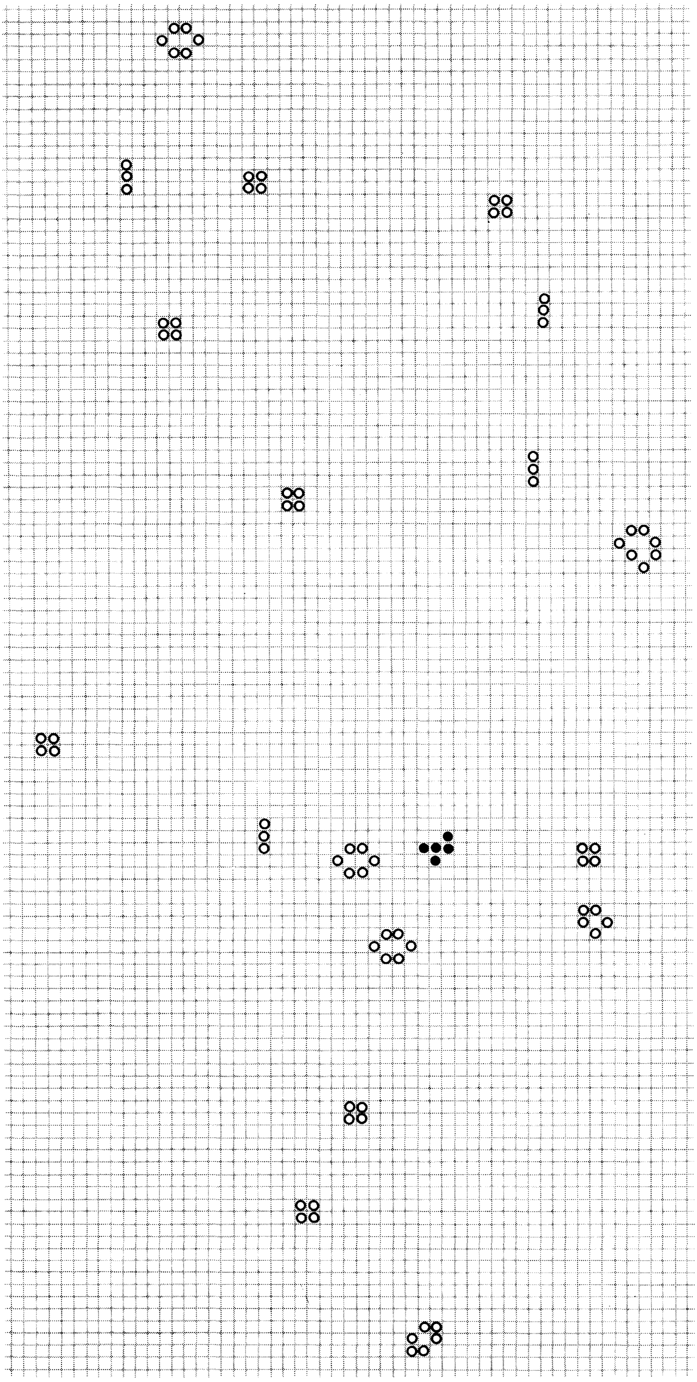
ADDENDUM

My 1970 column on Conway's "Life" met with such an instant enthusiastic response among computer hackers around the world that their mania for exploring "Life" forms was estimated to have cost the nation millions of dollars in illicit computer time. One computer expert, whom I shall leave nameless, installed a secret switch under his desk. If one of his bosses entered the room he would press the button and switch his computer screen from its "Life" program to one of the company's projects. The next two chapters will go into more details about the game. Here I shall comment only on some of the immediate responses to two questions left open in the first column.

The troublesome *R* pentomino becomes a 2-tick oscillator after 1,103 ticks. Six gliders have been produced and are traveling outward. The debris left at the center [*see Figure* 134] consists of four blinkers, one ship, one boat, one loaf, four beehives, and eight blocks. This was first established at Case Western Reserve University by Gary Filipski and Brad Morgan, and later confirmed by scores of "Life" hackers here and abroad.

The fate of the 5-5-5-5-5-5-5 was first independently found by Robert T. Wainwright and a group of hackers at Honeywell's Computer Control Division, later by many others. The pattern stabilizes as a 2-tick oscillator after 323 ticks with four traffic lights, eight blinkers, eight loaves, eight beehives, and

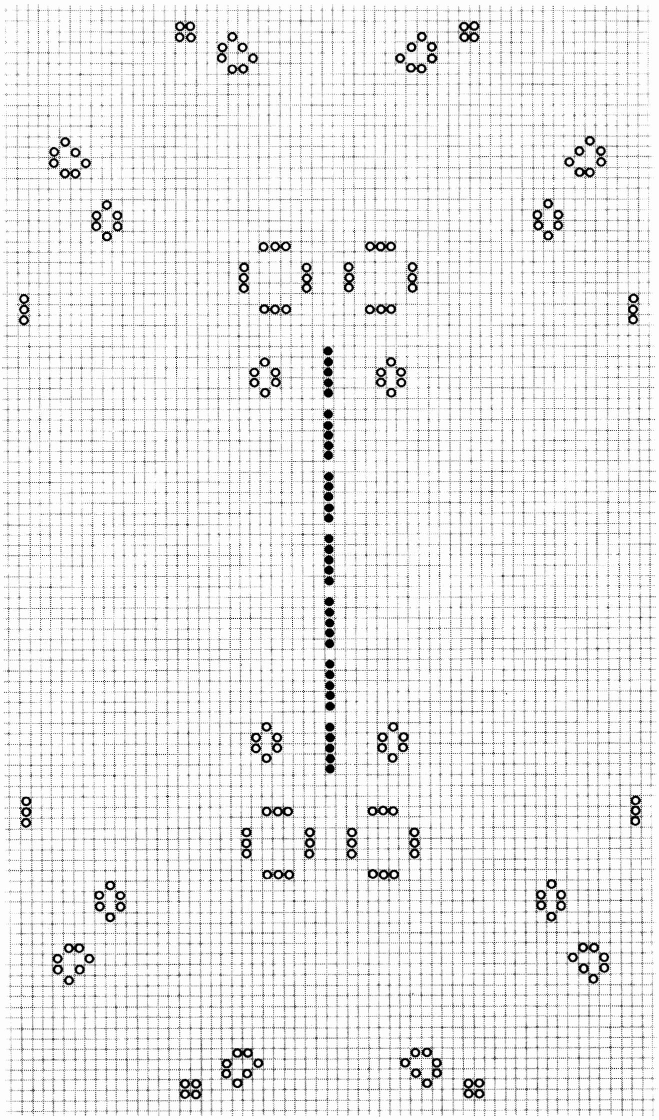
Figure 134



R pentomino's original (black) and final (open dots) state. (Six gliders are out of sight.)

four blocks. Figure 135 reproduces a printout of the final steady state. Because symmetry cannot be lost in the history of any life form, the vertical and horizontal axes of the original symmetry are preserved in the final state. The maximum population (492 bits) is reached in generation 283, and the final population is 192.

Figure 135



Initial pattern and final state of the 5-5-5-5-5-5-5 row

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THE GAME OF LIFE, PART II

Cellular automata theory began in the mid-fifties when John von Neumann set himself the task of proving that self-replicating machines were possible. Such a machine, given proper instructions, would build an exact duplicate of itself. Each of the two machines would then build another, the four would become eight, and so on. (This proliferation of self-replicating automata is the basis of Lord Dunsany's amusing 1951 novel *The Last Revolution.*) Von Neumann first proved his case with "kinematic" models of a machine that could roam through a warehouse of parts, select needed components and put together a copy of itself. Later, adopting an inspired suggestion by his friend Stanislaw M. Ulam, he showed the possibility of such machines in a more elegant and abstract way.

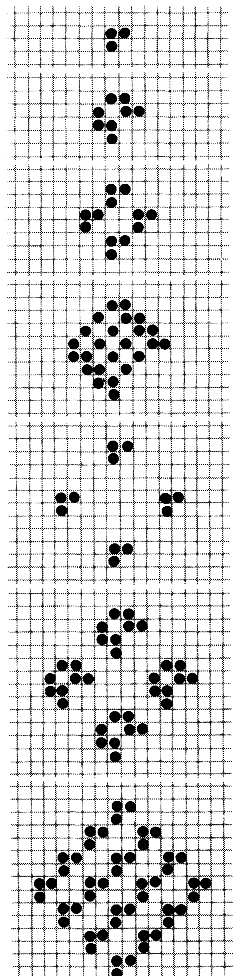
Von Neumann's new proof used what is now called a "uniform cellular space" equivalent to an infinite checkerboard. Each cell can have any finite number of "states," including a "quiescent" (or empty) state, and a finite set of "neighbor" cells that can influence its state. The pattern of states changes in discrete time steps according to a set of "transition rules" that apply simultaneously to every cell. The cells symbolize the basic parts of a finite-state automaton and a configuration of live cells is an idealized model of such a machine. Conway's game of "Life" is based on just such a space. His neighborhood consists of the eight cells surrounding a cell; each cell has two states (empty or filled), and his transition rules are the birth, death and survival rules I explained in the previous chapter. Von Neumann, applying transition rules to a space in which each cell has 29 states and four orthogonally adjacent neighbors, proved the existence of a configuration of about 200,000 cells that would self-reproduce.

The reason for such an enormous configuration is that, for von Neumann's proof to apply to actual automata, it was necessary that his cellular space be capable of simulating a Turing machine: an idealized automaton, named for its inventor, the British mathematician A. M. Turing, capable of performing any desired calculation. By embedding this universal computer in his configuration, von Neumann was able to produce a universal constructor. Because it could in principle construct any desired configuration by stretching "arms" into an empty region of the cellular space, it would self-replicate when given a blueprint of itself. Since von Neumann's death in 1957 his existence proof (the actual configuration is too vast to construct and manipulate) has been greatly simplified. The latest and best reduction, by Edwin Roger Banks, a mechanical engineering graduate student at the Massachusetts Institute of Technology, does the job with cells of only four states.

Self-replication in a trivial sense—without using configurations that contain Turing machines—is easy to achieve. A delightfully simple example, discovered by Edward Fredkin of M.I.T. about 1960, uses two-state cells, the von Neumann neighborhood of four orthogonally adjacent cells and the following parity rule: Each cell with an even number of live neighbors (0, 2, 4) at time t becomes or remains empty at time $t+1$, and each cell with an odd number of neighbors (1, 3) at time t becomes or remains live at time $t+1$. It is not hard to show that after 2^n ticks (n varying with different patterns) any initial pattern of live cells will reproduce itself four times—above, below, left and right of an empty space that it formerly occupied. The four replicas will be displaced 2^n cells from the vanished original. The new pattern will, of course, replicate again after another 2^n steps, so that the duplicates keep quadrupling in the endless series 1, 4, 16, 64, ... Figure 136 shows two quadruplings of a right tromino. Terry Winograd, in a 1967 term paper written when he was an M.I.T. student, generalized Fredkin's rule to other neighborhoods, any number of dimensions and cells with any prime number of states.

Ulam investigated a variety of cellular automata games, experimenting with different neighborhoods, numbers of states and transition rules. In a 1967 paper "On Recursively Defined Geometrical Objects and Patterns of Growth," written with Robert G. Schrandt, Ulam described a number of different games. Figure 137 shows generation 45 of a history that began with one counter on the central cell. As in Conway's game, the cells are two-state, but the neighborhood is that of von Neumann (four adjacent orthogonal cells). Births occur on cells

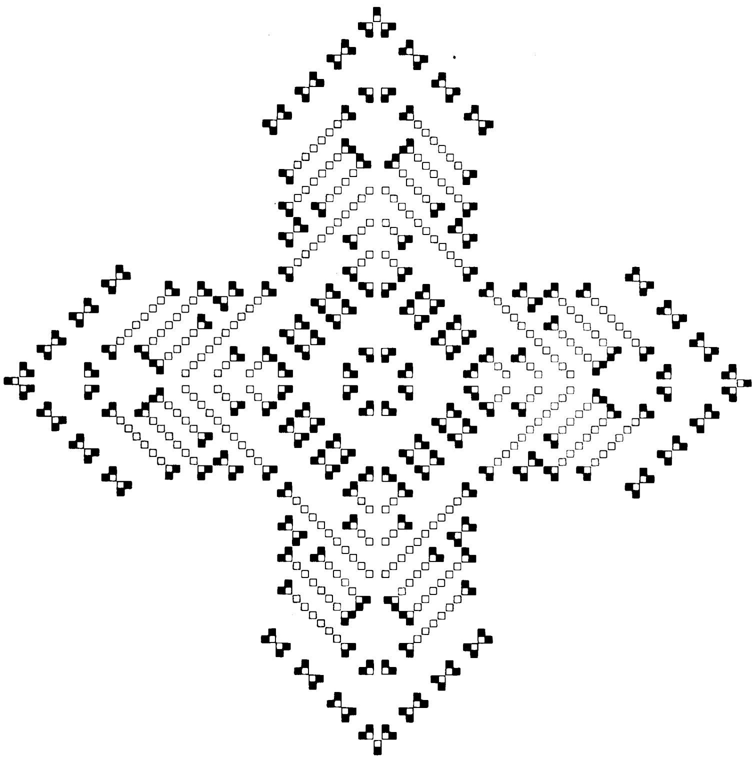
Figure 136



The replication of a tromino

that have one and only one neighbor, and all live cells of generation n vanish when generation $n+2$ is born. In other words, only the last two generations survive at any step. In Figure 137 the 444 new births are shown as black cells. The 404 white cells of the preceding generation will all disappear on the next tick. Note the characteristic subpattern, which Ulam calls a "dog bone." Ulam experimented with games in which two configurations were allowed to grow until they collided. In the ensuing

Figure 137



Generation 45 in a cellular game devised by Stanislaw M. Ulam

"battle" one side would sometimes wipe out the other; sometimes both armies would be annihilated. Ulam also explored games on three-dimensional cubical tessellations. His major papers on cellular automata are in *Essays on Cellular Automata,* edited by Arthur W. Burks.

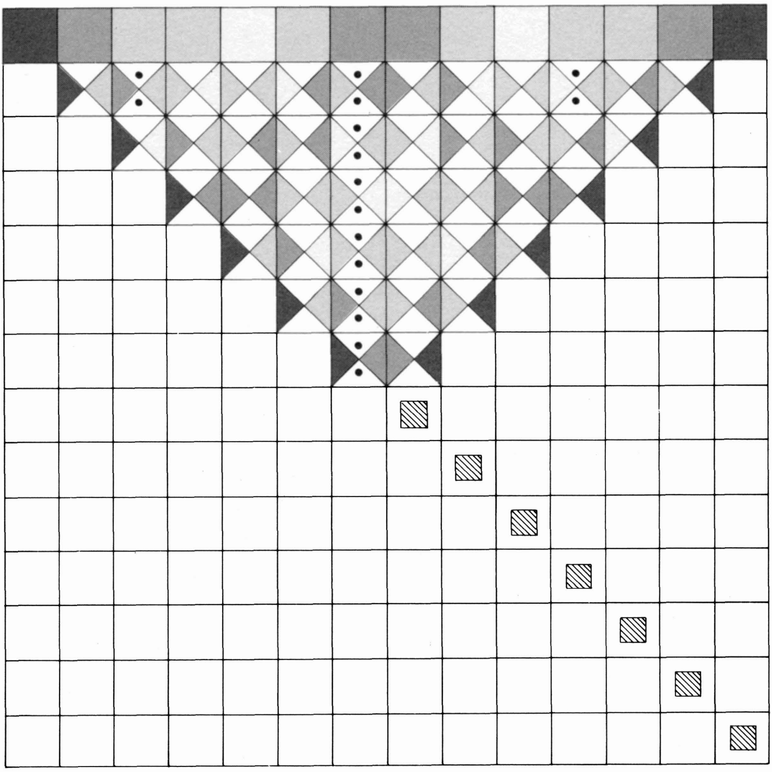
Similar games can be devised for triangular and hexagonal tessellations but, although they *look* different, they are not essentially so. All can be translated into equivalent games on a square tessellation by a suitable definition of "neighborhood." A neighborhood need not be made up of touching cells. In chess, for instance, a knight's neighborhood consists of the squares to which it can leap and squares on which there are pieces that can attack it. As Burks has pointed out, games such as chess, checkers and go can be regarded as cellular automata games in which there are complicated neighborhoods and transition rules and in which players choose among alternative next states in an attempt to be first to reach a certain final state that wins.

Among the notable contributions of Edward F. Moore to cellular automata theory the best-known is a technique for proving the existence of what John W. Tukey named "Garden of Eden" patterns. These are configurations that cannot arise in a game because no preceding generation can form them. They appear only if given in the initial (zero) generation. Because such a configuration has no predecessor, it cannot be self-reproducing. I shall not describe Moore's ingenious technique because he explained it informally in an article in *Scientific American* (see "Mathematics in the Biological Sciences," by Edward F. Moore; September, 1964) and more formally in a paper that is included in Burks's anthology.

Alvy Ray Smith III, a cellular automata expert at New York University's School of Engineering and Science, found a simple application of Moore's technique to Conway's game. Consider two five-by-five squares, one with all cells empty, the other with one counter in the center. Because, in one tick, the central nine cells of both squares are certain to become identical (in this case all cells empty) they are said to be "mutually erasable." It follows from Moore's theorem that a Garden of Eden configuration must exist in Conway's game. Unfortunately the proof does not tell how to find such a pattern and so far none is known. It may be simple or it may be enormously complex. Using one of Moore's formulas, Smith has been able to calculate that such a pattern exists within a square of 10 billion cells on a side, which does not help much in finding one.

Smith has been working on cellular automata that simulate pattern-recognition machines. Although this is now only of theoretical interest, the time may come when robots will need "retinas" for recognizing patterns. The speeds of scanning devices are slow compared with the speeds obtainable by the "parallel computation" of animal retinas, which simultaneously transmit thousands of messages to the brain. Parallel computation is the only way new computers can increase significantly in speed because without it they are limited by the speed of light through miniaturized circuitry. The cover of the February, 1971, issue of *Scientific American* [reproduced in Figure 138] shows a simple procedure, devised by Smith, by which a finite one-dimensional cellular space employs parallel computation for recognizing palindromic symmetry. Each cell has many possible states, the number depending on the number of different symbols in the palindrome, and a cell's neighborhood is the two cells on each side.

Figure 138



Cellular automaton

Smith symbolizes the palindrome TOO HOT TO HOOT with four states of cells in the top row. *T, O* and *H* are represented by blue, red and yellow respectively, and black marks the palindrome's two ends. Here we have indicated the colors by different shadings. The white cells in the other rows are in the quiescent state. The horizontal rows below the top row are successive generations of the top configuration when certain transition rules are followed in discrete time steps. In other words, the picture is a space-time diagram of a single row, each successive row indicating the next generation.

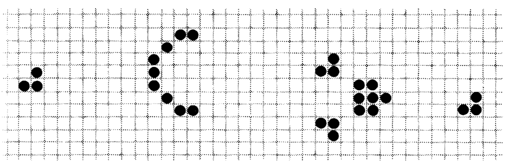
In the first transition each shade travels one cell to the left and one cell to the right, except for the end shadings, which are blocked by black; black moves inward at each step. Each cell on which two shadings land acquires a new state, symbolized by a cell divided into four triangles. The left triangle has the shading that was previously on the left, the right triangle has the shading previously on the right. The result of this first move is shown in the second row. When an adjacent pair of cells forms a tilted square in the center that is a solid shading, it indicates a "collision" of like shadings and is symbolized by black dots in the two white triangles of the left cell. Dots remain in that cell for all subsequent generations unless a collision of unlike shadings occurs to the immediate right of the dotted cell, in which case the dots are erased. When collisions of *unlike* shadings occur, the left cell of the pair remains undotted for all subsequent generations even though like shadings may later collide on its right.

At each move the shadings continue to travel one cell left or right (the direction in which the shaded triangles point) and all rules apply. If the palindrome has n letters, with n even as in this example (the scheme is modified slightly if n is odd), it is easy to see that after $n/2$ moves only two adjacent nonquiescent cells remain. If the left cell of this pair is dotted, the automaton has recognized the initial row as being palindromic. Down the diagram's center you see the colliding pairs of like shadings in the same order as they appear on the palindrome from the center to each end. As soon as recognition occurs the left cell of the last pair is erased and the right cell is altered to an "accept" state, here symbolized by nested squares. An undotted left cell would signal a nonpalindrome, in which case the left cell would become blank and the right cell would go into a "reject" state.

A Turing machine, which computes serially, requires in general n^2 steps to recognize a palindrome of length n. Although recognition occurs here at step $n/2$, the accept state is shown moving in subsequent generations to the right to symbolize the cell-by-cell transmission of the acceptance to an output boundary of the cellular space. Of course it is easy to construct more efficient palindrome-recognizing devices with actual electronic hardware, but the point here is to do it with a highly abstract, one-dimensional cellular space in which information can pass only from a cell to adjacent cells and not even the center of the initial series of symbols is known at the outset. As Smith puts it anthropomorphically, after the first step each of the three dotted cells thinks it is at the center of a palindrome. The dotted cells at each end are disillusioned on the next move because of the collision of unlike shadings at their right. Not until generation $n/2$ does the dotted cell at the center know it *is* at the center.

Now for some startling new results concerning Conway's game. Conway was fully aware of earlier games and it was with them in mind that he selected his recursive rules with great care to avoid two extremes: too many patterns that grow quickly without limit and too many that fade quickly. By striking a delicate balance he designed a game of surprising unpredictability and one that produced such remarkable figures as oscillators and moving spaceships. He conjectured that no finite population could grow (in number of members) without limit, and he offered $50 for the first proof or disproof. The prize was won in November, 1970, by a group in the Artificial Intelligence Project at M.I.T. consisting of (in alphabetical order) Robert April, Michael Beeler, R. William Gosper, Jr., Richard Howell, Rich Schroeppel and Michael Speciner. Using a program devised by Speciner for displaying life histories on an oscilloscope, Gosper made a truly astounding discovery: he found a glider gun! The configuration in Figure 139 grows into such a gun, firing its first glider on tick 40. The gun is an oscillator of period 30 that ejects a new glider every 30 ticks. Since each glider adds five more counters to the field, the population obviously grows without limit.

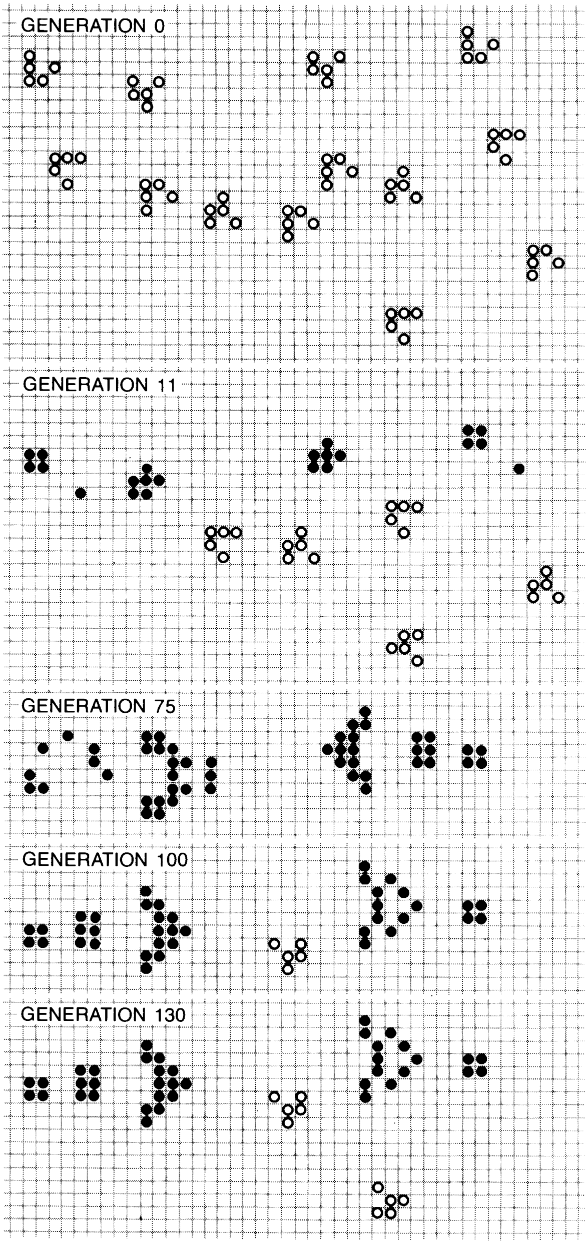
Figure 139



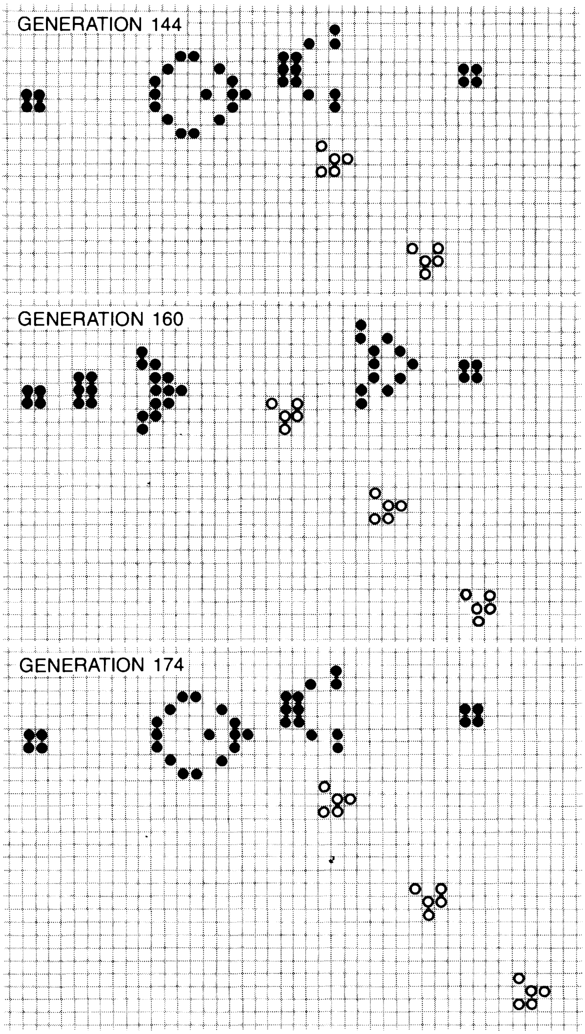
A configuration that grows into a glider gun

The glider gun led the M.I.T. group to many other amazing discoveries. A series of printouts (supplied by Robert T. Wainwright of Yorktown Heights, N.Y.) shows how 13 gliders crash to form a glider gun [*see Figure* 140]. The last five printouts show the gun in full action. The group also found a way to position a pentadecathlon [*see Figure 141*], an oscillator of period 15, so that it "eats" every glider that strikes it. A pentadecathlon can also reflect a glider 180 degrees, making it possible for two pentadecathlons to shuttle a glider back and forth forever. Streams of intersecting gliders produce fantastic results. Strange patterns can be created that in turn emit gliders. Sometimes collision configurations grow until they ingest all guns. In other cases the collision mass destroys one or more guns by shooting back. The group's latest burst of virtuosity is a way of placing eight guns so that the intersecting streams of gliders build a factory that assembles and fires a middleweight spaceship about every 300 ticks.

Figure 140



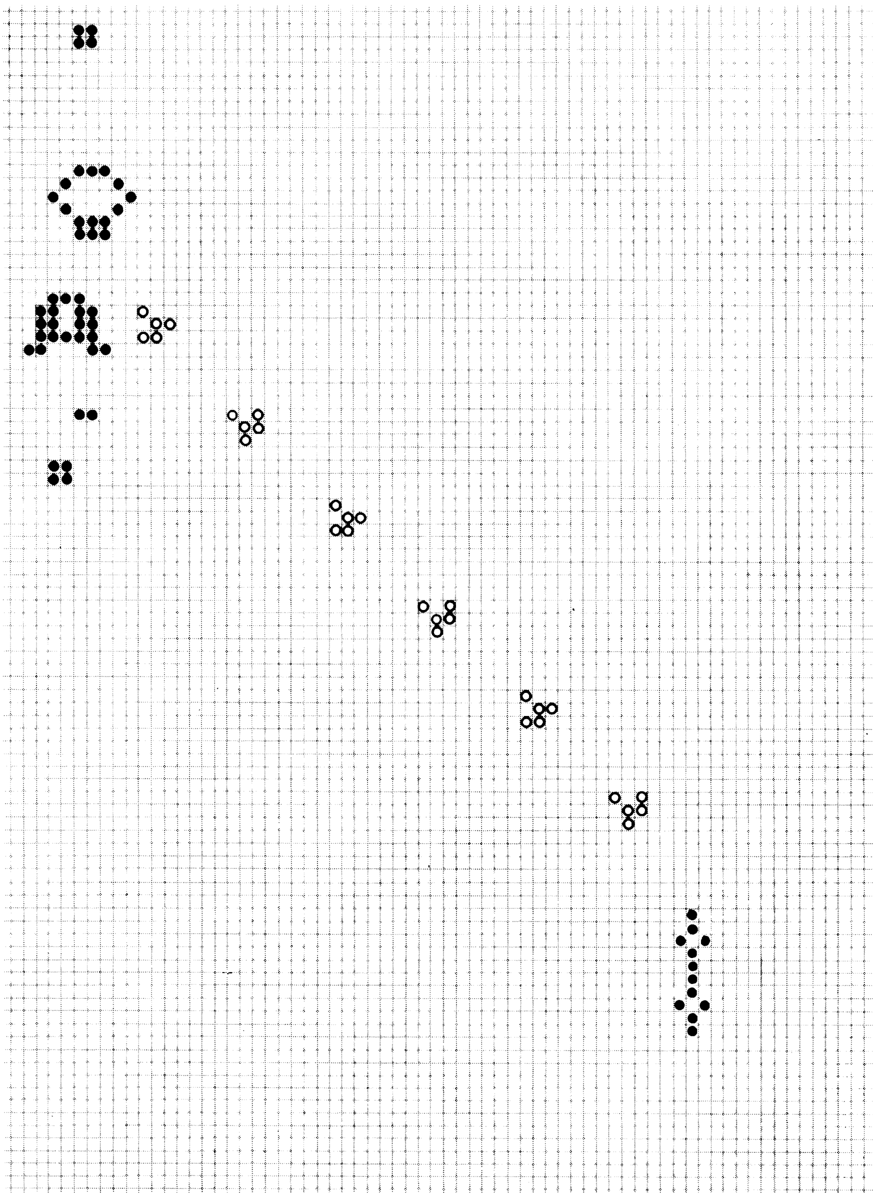
Here and on the facing page 13 gliders crash to form a glider gun (generation 75) that oscillates with a period of 30, firing a glider in each cycle



b

The existence of glider guns raises the exciting possibility that Conway's game will allow the simulation of a Turing machine, a universal calculator capable in principle of doing anything the most powerful computer can do. The trick would be to use gliders as unit pulses for storing and transmitting information and performing the required logic operations that are handled in actual computers by their circuitry. If Conway's game allows a universal calculator, the next question will be

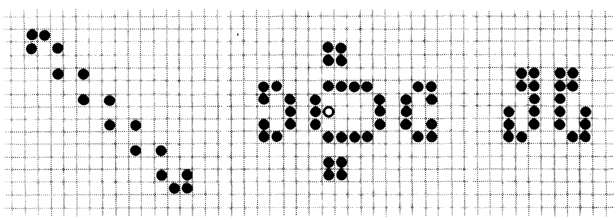
Figure 141



Pentadecathlon *(bottom right)* "eats" gliders fired by the gun whether it allows a universal constructor, from which nontrivial self-replication would follow. So far this has not been achieved with a two-state space and Conway's neighborhood, although it has been proved impossible with two states and the von Neumann neighborhood.

The M.I.T. group found many new oscillators [*see Figure* 142]. One of them, the barber pole, can be stretched to any length and is a flip-flop, with each state a mirror image of the other. Another, which they rediscovered, is a pattern Conway's group had found earlier and called a Hertz oscillator. Every four ticks the hollow "bit" switches from one side of the central frame to the other, making it an oscillator of period 8. The tumbler, which was found by George D. Collins, Jr., of McLean, Va., turns upside down every seven ticks.

Figure 142

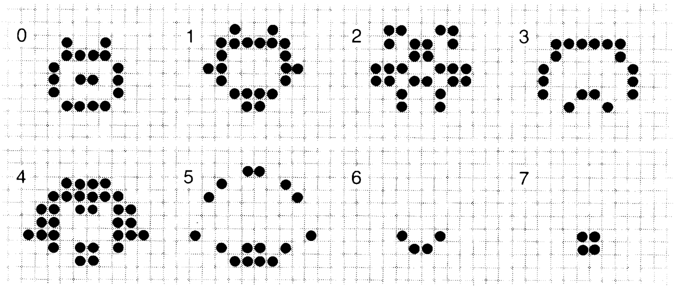


Barber pole *(left)*, Hertz oscillator *(middle)*, and tumbler *(right)*

The Cheshire cat [*see Figure* 143] was discovered by C. R. Tompkins of Corona, Calif. On the sixth tick the face vanishes, leaving only a grin; the grin fades on the next tick and only a permanent paw print (block) remains. The harvester was constructed by David W. Poyner of Basildon in England. It plows up an infinite diagonal at the speed of light, oscillating with period 4 and ejecting stable packages along the way [*see Figure* 144]. "Unfortunately," writes Poyner, "I have been unable to develop a propagator that will sow as fast as the harvester will reap."

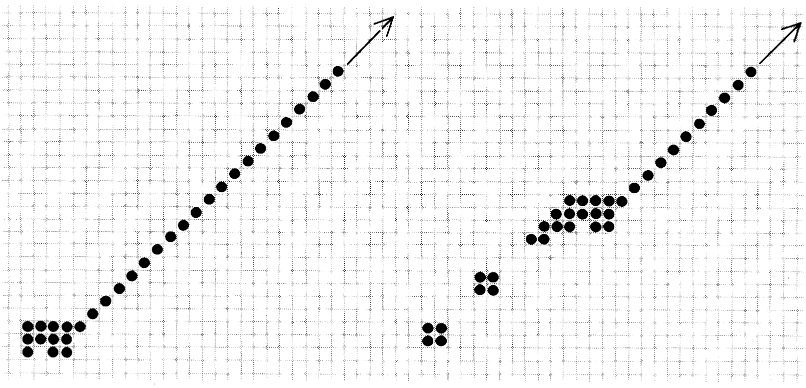
Wainwright has made a number of intriguing investigations. He filled a 120-by-120 square field with 4,800 randomly placed bits (a density of one-third) and tracked their history for 450 generations, by which time the density of this primordial soup, as Wainwright calls it, had thinned steadily to one-sixth.

Figure 143



The Cheshire cat (0) fades to a grin (6)
and disappears, leaving a paw print (7)

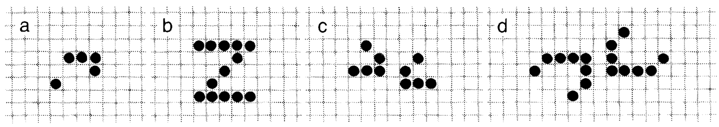
Figure 144



The harvester, shown at generations (0) *left* and 10 *(right)*

Whether it would eventually vanish or, as Wainwright says, percolate at a constant minimum density is anybody's guess. At any rate, during the 450 generations 42 short-lived gliders were formed. Wainwright found 14 different patterns that became glider states on the next tick. The most common pattern to produce a glider on the next tick is shown [*a in Figure* 145]. A Z-pattern found by Collins and by Jeffrey Lund of Pewaukee, Wis., after 12 ticks becomes two gliders that sail off in opposite directions [*b in Figure* 145]. Wainwright and others set two gliders on a collision course that causes all bits to vanish on the fourth tick [*c in Figure* 145]. Wallace W. Wagner of Ana-

Figure 145

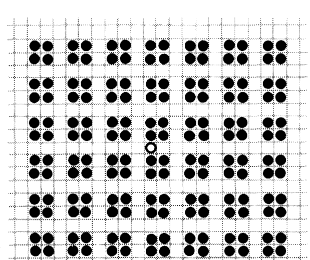


Two spawners of gliders and two collision courses

heim, Calif., found a collision course for two lightweight spaceships that also ends (on the seventh tick) in total blankness [*d in Figure* 145].

Wainwright has experimented with various infinite fields of regular stable patterns, which he calls agars—rich culture mediums. When, for instance, a single "virus," or bit, is placed in the agar of blocks shown in Figure 146 so that it touches the corners of four blocks, the agar eliminates the virus and repairs itself in two ticks. If, however, the alien bit is positioned as shown (or at any of the seven other symmetrically equivalent spots), it initiates an inexorable disintegration of the pattern. The portion eaten away contains active debris that has overall bilateral symmetry along one axis and a roughly oval border that expands, probably forever, in the four compass directions at the speed of light.

Figure 146



Agar doomed by a virus

The most immediate practical application of cellular automata theory, Banks believes, is likely to be the design of circuits capable of self-repair or the wiring of any specified type of new circuit. No one can say how significant the theory may eventually become for the physical and biological sciences. It may have important bearings on cell growth in embryos, the replication of DNA molecules, the operation of nerve nets, genetic changes in evolving populations and so on. Analogies with life processes are impossible to resist. If a primordial broth of amino acids is large enough, and there is sufficient time, self-replicating, moving automata may result from complex transition rules built into the structure of matter and the laws of nature. There is even the possibility that space-time itself is granular, composed of discrete units, and that the universe, as Fredkin and others have suggested, is a vast cellular automaton run by an enormous computer. If so, what we call motion may be only simulated motion. A moving spaceship, on the ultimate microlevel, may be essentially the same as one of Conway's spaceships, appearing to move on the macrolevel whereas actually there is only an alteration of states of basic space-time cells in obedience to transition rules that have not yet been discovered.

22

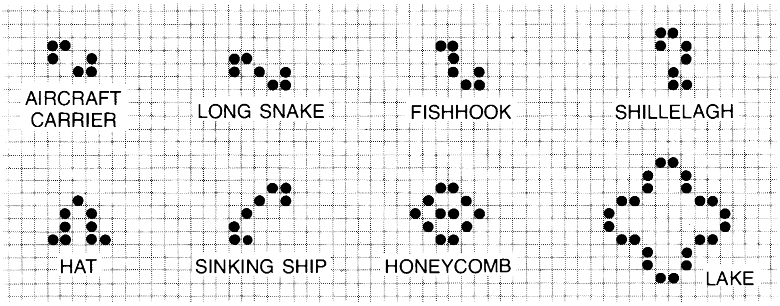
THE GAME OF LIFE, PART III

So much has been discovered about Conway's "Life" since I first wrote the last two chapters, that it was impossible to summarize the highlights in an addendum. A book could and should be written about the game, an *Encyclopedia of Life,* or a *Handbook of Life,* that would put all the important known Life forms on record and thereby save Lifenthusiasts the labor of rediscovering them. The eleven issues that appeared of Robert Wainwright's periodical *Lifeline* continue to be the main repository of such data. Wainwright is said to be working on a book, and there are rumors of other books about "Life" that are in the making. In the meantime, I will try in this chapter to pull together some of the significant developments in "Life" since my second column on the game ran in *Scientific American* in 1971. Because so many basic forms were independently discovered by many people, I shall not often attempt to credit first discoverers.

The earliest and most important group of Lifenthusiasts was at M.I.T., centering around William Gosper who is now working for Xerox at their Stanford research headquarters. In the mid-70s the most active "Life" group was in the computer control division of Honeywell, Inc., Framington, Mass. It included (alphabetical order) Thomas Holmes, Keith McClelland, Michael Sporer, Philip Stanley, Donald Woods, and his father William Woods. In the late seventies, an active group of "Life" hackers formed at the University of Waterloo, in Canada, with John Abbott, David Buckingham, Mark Niemiec, and Peter Raynham as the leaders. Most of what I shall report comes from these three groups.

All still lifes with 13 or fewer bits have long been known. The block and tub are the only 4-bit stable forms, and the boat

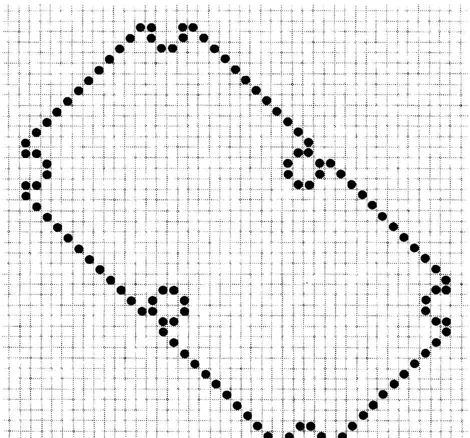
Figure 147



More still lifes

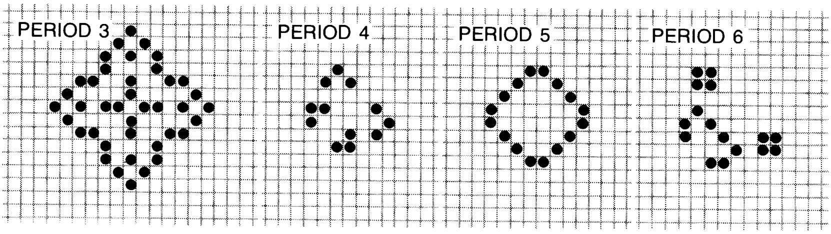
is the only one with 5 bits. Figure 128 caught four of the five 6-bit still lifes, missing only the aircraft carrier shown in Figure 147. There are four 7-bit stable forms: the loaf, long boat, long snake, and fishhook. The fishhook or "eater" is the smallest still life lacking any kind of symmetry. Note that forms such as the boat, barge, ship, and sinking ship can be stretched to any length, and lakes can be made as large as you like, with any number of barges, boats, and ships at anchor on the water. There are nine 8-bit still lifes, ten 9-bit forms, 25 with 10 bits, 46 with 11 bits, 121 with 12 bits, and 149 with 13 bits. The stable pool table in Figure 148 was constructed out of long sinking ships and parts of ponds by William Woods.

Figure 148



The stable pool table

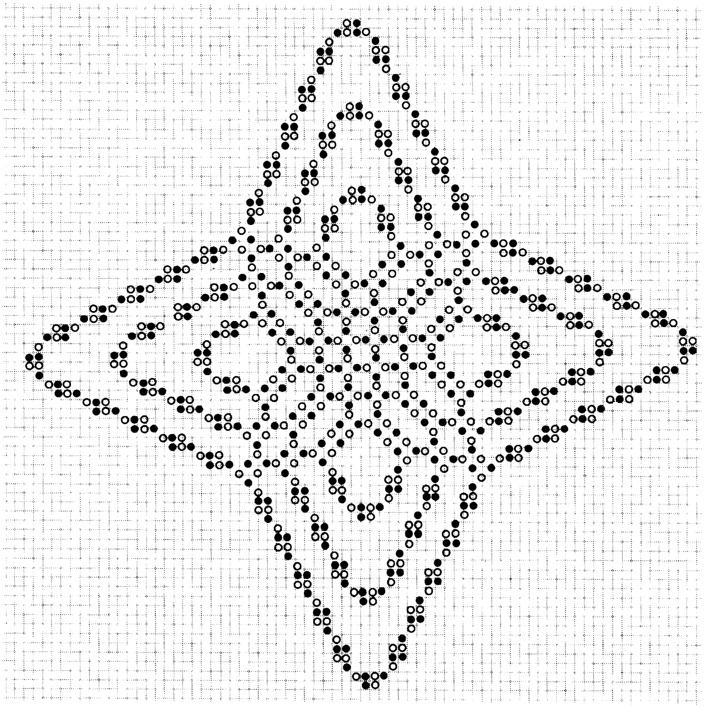
Figure 149



Low-period oscillators

Hundreds of elegant oscillators have been found. Figure 149 shows a few of small size, with short periods. The M.I.T. group, early in the history of "Life," found easy ways to construct giant flip-flops (period-2 oscillators) such as the one shown in Figure 150. It oscillates between the patterns shown in black dots and circles.

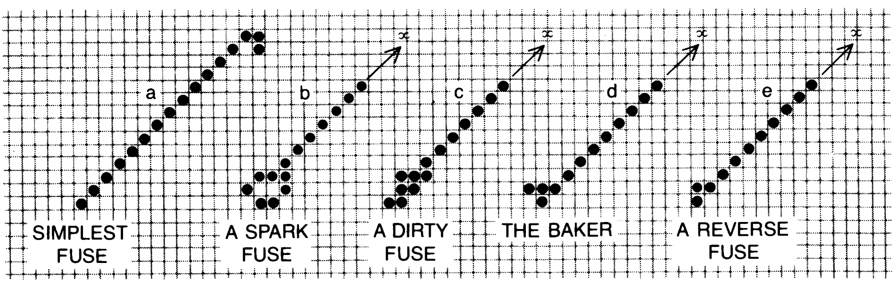
Figure 150



A flip-flop pattern that alternates between states shown in black and with circles

Another large class of "Life" forms that have been intensively investigated are what the Honeywell group named the fuses. These are stems one or more bits wide, either diagonal or orthogonal, usually infinite in length, that burn steadily from one end toward the other. The simplest is the fuse shown in Figure 151 *a*, a diagonal of bits that either rises to infinity or has a stable top as shown. It simply burns itself out without producing any sparks or stable smoke. If you put another bit to the left of the lower end, it forms a tiny flame that travels along with the burning.

Figure 151



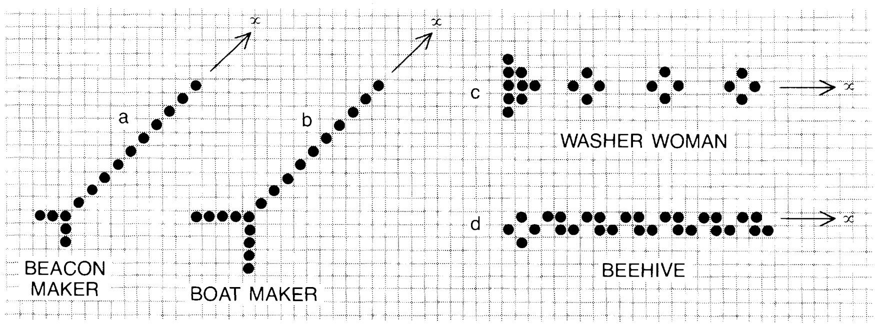
Five fuses

Fuse *b* in Figure 151 oscillates with a period of 4, giving off sparks that fade quickly. A "dirty fuse," like the one shown in *c* in Figure 151, leaves clouds of debris behind as it burns. At one point it shoots off a glider. Fuse *d* in Figure 151, named the "baker" by its discoverer, McClelland, is a confused fuse that bakes a string of stable loaves while it burns. The last three fuses all oscillate with periods of 4, and all four burn with the speed of light.

Fuse *e* in Figure 151 eventually becomes a clean fuse of period 4, but leaves behind a cloud consisting of three blocks, three beehives, two blinkers, a ship, and four gliders. William Woods calls it a "reverse fuse" because it explodes first, then burns quietly for the rest of its endless life. The harvester, described in the previous chapter, is of course a fuse.

Other unusual fuses are shown in Figure 152. Fuse *a,* found by Steve Tower, has a period of 8. It leaves behind a trail of beacons. Fuse *b* abandons a twin pair of boats every four ticks. Orthogonal fuse *c,* which burns with a speed slower than light, consumes two tubs every 18 ticks, then changes them to traffic lights (four blinkers). It was discovered by Earl Abbe. Wain-

Figure 152

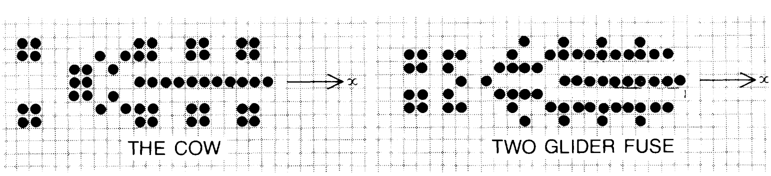


More fuses

wright's fuse *d* consumes three fenceposts every 12 generations, and turns them into a beehive.

Two fuses of a more complicated nature, discovered by Don Woods, are shown in Figure 153. The cow burns at light speed, with period 8, slowly "chewing its cud" by eating the blocks on either side, bringing them back again, then eating them a second time. The two-glider fuse throws off two gliders every 12 ticks. I resist the impulse to describe two close relatives of fuses, the wicks (infinite in both directions) and the kinkbombs. Kinkbombs come in three varieties: duds, firecrackers, and bombs, as detailed by Mark Horton in the 11th issue of *Lifeline*.

Figure 153



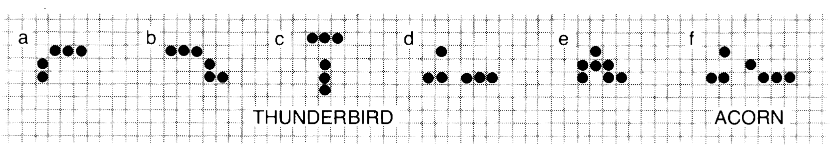
Two remarkable fuses

There are 102 distinct patterns of bits within a 3 × 3 square (excluding rotations and reflections, but including the patterns consisting of nine bits and no bits). Some of these are polyominoes, some not. All the letters of the alphabet in Braille are among the 102. The fates of all 102 are known. Also known are the fates of all polyominoes through the order-7 heptominoes.

Methuselah patterns are those of fewer than 10 bits which do not stabilize until after more than 50 generations. Two examples were given in the previous chapter: The 5-bit *R*-pentomino and the pi-heptomino of 7 bits. The first generation of the pi-heptomino, by the way, reappears in tick 31, but shifted 9 cells. Because of interaction with its exhaust, in generation 61, it fails to make it as a spaceship.

Other examples of Methuselahs are shown in Figure 154. The first one, *a* is the smallest known. It becomes the *R*-pentomino in two ticks, giving it a life of 1,105 generations. Methuselah *b* stabilizes (six blocks, twelve blinkers, one loaf) after 608 generations, *c* (the thunderbird) lasts 243 ticks, and *d* goes to 1,108. The heptomino *e* stabilizes after 148 ticks, having produced three blocks, a ship, and two gliders. The acorn *f*, found by Charles Corderman, is the most amazing Methuselah known. It lives for 5,206 generations! When it stabilizes as an "oak" of 633 bits, it has produced numerous gliders, 13 of which escape.

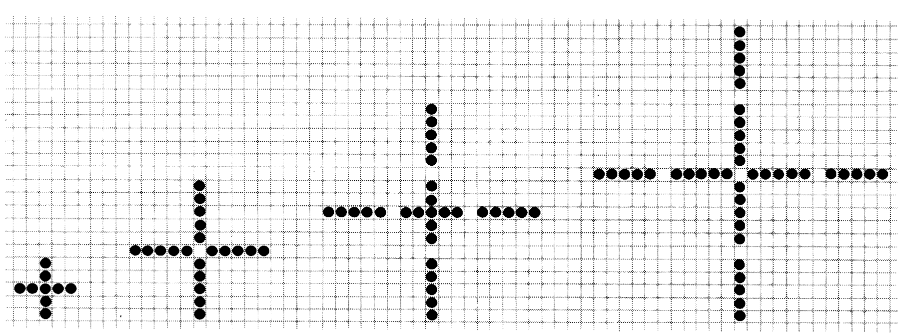
Figure 154



Methusalehs

The Honeywell group tracked the life histories of the first nine members of the 5-cell crosses, of which the simplest are shown in Figure 155. The first is a portion of an infinite trellis

Figure 155

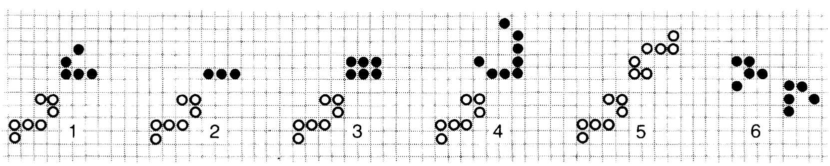


The five-cell cross series

consisting of solid horizontal and vertical rows, two cells apart, that surround an infinity of empty 2×2 squares. Like the infinite trellis, this cross vanishes in one tick. The next cross disappears in 8 ticks. The third ends with many traffic lights in 6 ticks, and the fourth stabilizes after 34 ticks with eight blinkers, having produced a truly spectacular display of fireworks along the way. (Its 19th generation is a beautiful ring of blocks with a checkerboard in the center.) Order-5 and order-7 crosses in this sequence stabilize as four pulsars in 36 and 21 ticks respectively, orders 6 and 8 go to four pulsars and a tub in 36 and 21 ticks respectively, and order-9 ends after 42 ticks with 16 blocks and 8 blinkers.

William Gosper, in 1971, found the eater (fishhook), the incredible 7-bit stable form shown with circles in Figure 156. It has the ability to consume an enormous variety of "Life" forms, then quickly repair itself. The first four pictures show the eater about to ingest a glider, blinker, pre-beehive, and a lightweight spaceship. In the fifth picture two eaters are poised to devour one another. This is prevented by their amazing ability to self-repair, so the pattern oscillates with period 3. The last picture shows how two gliders collide to produce an eater on the 13th tick. In recent years eaters of larger size have been discovered, with a variety of bizarre feeding habits.

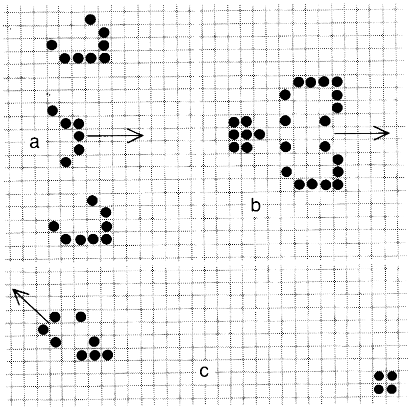
Figure 156



The eater (circles) and some of its prey

Extensive investigations have been made of different kinds of agars (regular patterns that are infinite in two dimensions), the procrastinators (forms that take more than 50 ticks to become a single simple stable form), and puffer trains. The puffers leave a trail of permanent smoke. Three are shown in Figure 157. The first, discovered by Gosper, is an engine escorted between two lightweight spaceships. It puffs along at half the speed of light until after more than 1,000 ticks it develops a period of 140. Paul Schick discovered an entire family of puffer trains, the simplest of which, shown in *b*, leaves nothing behind. The pair of mirror-image lightweight spaceships pull along the symmetrical heptomino engine with a period of

Figure 157

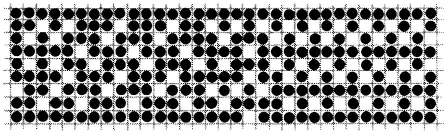


Puffer trains

12. The switch-engine puffer train *c* in Figure 157, moves too slowly (one-twelfth the speed of light) to be of much use. It travels diagonally like a glider, eventually producing eight blocks every 288 generations. No escorting spaceships are needed, but without the stabilizing block its smoke catches up with the engine and destroys it.

The first Garden of Eden pattern, reproduced in Figure 158, was found by Roger Banks in 1971. It required an enormous computer search of all possible predecessor patterns. The confining rectangle (9 × 33) holds 226 bits. The only other Garden of Eden pattern known was found by a French group in 1974, led by J. Hardouin-Duparc, at the University of Bordeaux. It is inside a rectangle of 6 × 122.

Figure 158



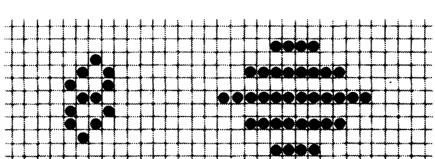
A garden of Eden

Although any "Life" pattern generates only one successor, the converse is not true. A given pattern may have two or more predecessors. This is why searching for Garden of Eden patterns is so difficult—the computer has to look at all possible predecessors at each backward tick. If the universe eventually turns out to be one monstrous cellular automaton, one may reasonably ask whether there is an initial Garden of Eden state that required a creation because it has no predecessor pattern. By the way, the fact that a "son" of a Garden of Eden pattern may have more than one "father" has led Conway to offer $50 to the first person who can find a pattern that has a father but no grandfather. The existence of such a pattern is still an open question.

The most spectacular of the new developments in "Life" involve gliders and their collisions. Gosper's group found new types of glider guns, more compact spaceship factories produced by glider crashes, and innumerable "Life" forms that eat gliders or reflect them back at different angles. Before its members broke up to go their separate ways, the M.I.T. group managed to complete a 17-minute film about their discoveries that has become a classic.

A pure glider generator is one that generates one or more gliders with no debris left over. Two elegant ones found by the Honeywell group are shown in Figure 159. The biloaf *left* in four ticks produces two gliders going opposite ways. The 4-8-12 diamond *right* in 15 ticks forms four gliders headed in four different directions. Half a dozen 5-bit forms turn into a single glider, and more than a hundred 6-bit forms do the same. A search for predecessors of the original Gosper glider gun turned up a pattern of 21 bits that is the smallest known, though it seems possible there may be a way of positioning just four gliders (20 bits) so that they crash and form a gun.

Figure 159



Two glider-generators

I mentioned earlier Gosper's way of placing eight guns so that their gliders crash to form a spaceship factory which fires off a middleweight spaceship about every 300 generations. Gosper soon improved this to four guns and one pentadecathlon. This pattern produces a factory that shoots off lightweight or middleweight spaceships (depending on the timing) every

60 ticks. Wainwright positioned three "newguns" that generate a middleweight spaceship every 46 generations.

Lifenthusiasts have investigated thousands of ways that gliders and spaceships can collide to produce an incredible variety of stable patterns (including the null pattern of nothing at all), as well as patterns that change, and patterns that produce new gliders and/or spaceships. Figure 160 shows some unusual collisions found by the Waterloo group. On the left is the pattern just before the crash; on the right, the outcome after the indicated number of ticks (t = ticks).

The breeder is one of the most remarkable life forms found by the M.I.T. group; remarkable because its population growth is so rapid. Figure 161 is a photograph of a computer scope that shows the breeder breeding gliders. The little dots are gliders, about 1,000 of them inside the triangular region. The breeder consists of ten puffer trains moving east, their exhaust carefully controlled so that they generate gliders that crash to form guns that instantly spring into action along the horizontal axis. The picture shows the breeder at generation 3,333. Thirty guns are firing northeast at a rate of one glider per tick. The firing rate increases without limit until at about tick 6,500 the number of gliders starts to exceed the age of the breeder. Seeing the breeder in action was one of the most awesome high points of my visit to M.I.T.

When I wrote the previous chapter for the February 1971 issue of *Scientific American,* I raised the question of whether the rules of "Life" permit the construction of a universal computer. I had the pleasure of reporting the next month that "Life" is indeed universal. Gosper at M.I.T. and Conway at Cambridge independently "universalized" the "Life" space by showing how gliders could be used as pulses to simulate a Turing machine. Exactly how this is done is too complicated to go into here, but you will find it clearly outlined by Conway in the second volume of *Winning Ways,* the book he coauthored with Elwyn Berlekamp and Richard Guy.

The universality of "Life" means that it is possible in principle to use moving gliders to perform any calculation that can be performed by the most powerful digital computer. For example, one can arrange a formation of glider guns, eaters, and other "Life" forms so that a stream of gliders, with gaps in the right places, will calculate pi, e, the square root of 2, or any other real number to any number of decimal places. Of course, it would be a very inefficient way to do such calculations, nonetheless they are possible if you have a large enough field and sufficient ingenuity to build the needed "machine."

In *Winning Ways* Conway uses Fermat's last theorem to illus-

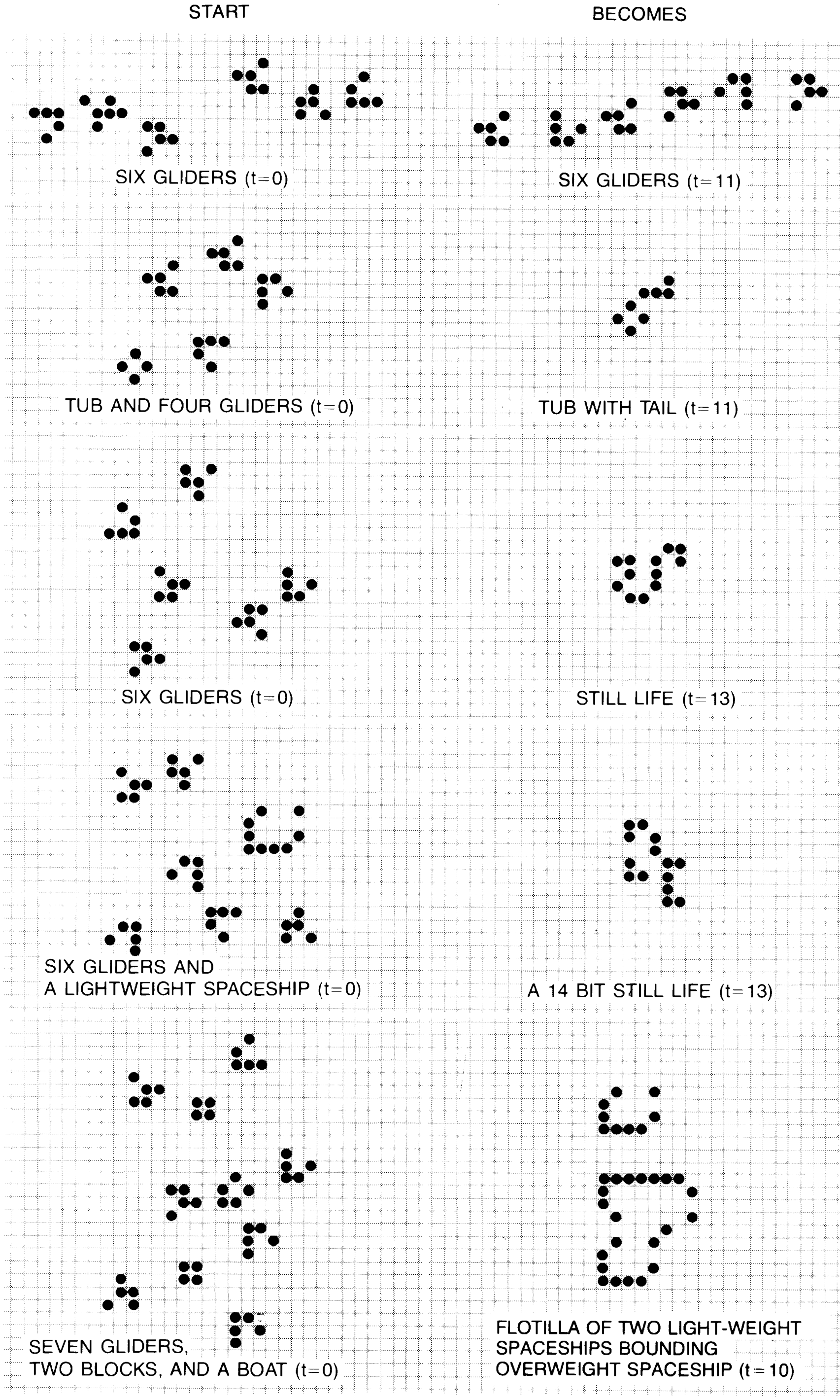
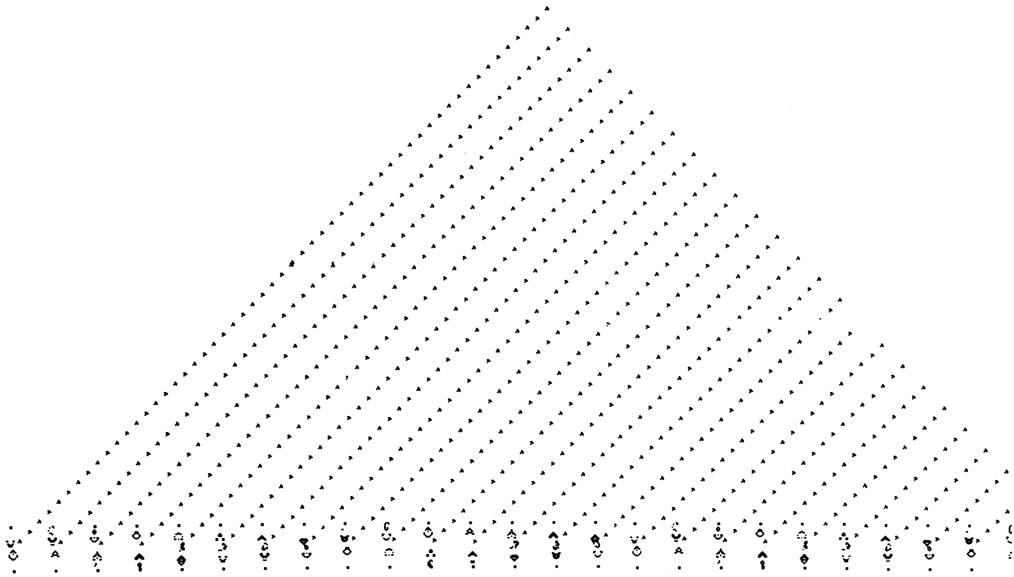

START
BECOMES
SIX GLIDERS (t=0)
SIX GLIDERS (t=11)
TUB AND FOUR GLIDERS (t=0)
TUB WITH TAIL (t=11)
SIX GLIDERS (t=0)
STILL LIFE (t=13)
SIX GLIDERS AND
A LIGHTWEIGHT SPACESHIP (t=0)
A 14 BIT STILL LIFE (t=13)
SEVEN GLIDERS,
TWO BLOCKS, AND A BOAT (t=0)
FLOTILLA OF TWO LIGHT-WEIGHT
SPACESHIPS BOUNDING
OVERWEIGHT SPACESHIP (t=10)

Figure 160

Figure 161



The breeder

trate "Life's" computing power as well as its limitations. A "Life" machine can be constructed that will steadily test the values of the four variables in Fermat's famous formula. The program could be designed to halt, say by fading away, if it found a counterexample to Fermat's conjecture. On the other hand, if the conjecture is true, the "Life" machine will keep searching forever for the right combination of values. We know from undecidability theory that there is no way to know in advance whether any given problem is solvable, therefore there is no way to know in advance whether any given pattern in "Life" will continue to change or to reach a stable end.

In 1981, in a letter telling me he had found "Life" to be universal, Conway added a note on the back of the envelope. "If

(ask Gosper) gliders can crash to form a pentadecathlon, then I can produce self-replicating machines, and it's undecidable whether a given machine is self-replicating."

I cannot remember if I asked Gosper this question, but at any rate, gliders *can* crash to form pentadecathlons, and Conway states flatly, in *Winning Ways*, that self-replicating machines can be constructed in "Life" space. We are not speaking now of moving forms like spaceships, but of machines that will build exact copies of themselves. The original machine may either remain in the space or it can be programmed to self-destruct after it has replicated itself. So far as I know no one has built such a machine, but if Conway is right (his proof has not been published), it is possible to build them.

Conway also asserts in *Winning Ways* that he has proved that "Life" patterns exist which move steadily in any desired rational direction, recovering their initial forms after a fixed number of moves. As for spaceships (which move without producing smoke), no new ones have been discovered other than those already known to Conway in 1970.

Conway goes on to speculate that if you imagine a sufficiently large broth of randomly placed bits, one would expect that by pure chance self-replicating creatures would arise, and those best adapted to survive would live longer than the others. Interactions with the environment would produce mutations. As in organic evolution, most mutations would be harmful, but some would have survival value. "It's probable," Conway writes, "given a large enough 'Life' space, initially in a random state, that after a long time, intelligent self-reproducing animals will emerge and populate some parts of the space."

I would prefer the word "possible" here to "probable," but there is no question that "Life's" analogy with biological evolution on earth is remarkable. One science fantasy writer, the widely read Piers Anthony, plays with this theme in his 1976 novel, *Ox*. Diagrams of "Life" patterns head each chapter, and the book's plot involves intelligent, sentient beings called "pattern entities" or "sparkle clouds" that have evolved by just the process Conway imagines, in a cellular space of dimensions higher than our spacetime. Their behavior is totally determined by transition rules, but like us they imagine themselves to have free wills. There is an amusing Chapter 11 in which Cal explains the rules of "Life" to Aquilon and she experiments with some simple patterns.

"Try this one," Cal suggests, giving her the *R*-pentomino:

> "That's similar to the one I just did. You've just tilted it sideways, which makes no topological difference, and added one dot."
>
> "Try it," he repeated.
>
> She tried it, humoring him. But soon it was obvious that the solution was not a simple one. Her numbered patterns grew and changed, taking up more and more of the working area. The problem ceased to be merely intriguing; it became compulsive. Cal well understood this; he had been through it himself. She was oblivious to him now, her hair falling across her face in attractive disarray, teeth biting lips. "What a difference a dot makes!" she muttered.

In Chapter 13 Aquilon, still tracking the pattern's fate, exclaims: "This *R*-pentomino is a menace! I'm getting a headache! It just goes on and on." Gosper once said that to him the most impressive aspect of Conway's game is how it demonstrates the impossibility of predicting the outcome of processes that are rigidly determined by extremely simple rules of change. After Aquilon has learned about gliders and glider guns, she remarks: "If I were a pattern, I'd be very careful where I fired my gliders! That game plays a rough game!"

"It does," Cal replies. "As does all nature."

Much work has been done on variants of "Life": playing by other rules, and on other lattices such as triangular or hexagonal, and in dimensions higher than two. One-dimensional "Life" has also been explored—see the articles by Jonathan Millen and Munemi Miyamoto. "Life" has been investigated on wraparound fields that are cylinders and toruses, and even Moebius surfaces and Klein bottles. Some interesting results have emerged, but nothing compares with "Life" in the combination of richness of interesting forms with such simple transition rules. This is a tribute to Conway's intuition, and to the thoroughness with which he and his friends initially explored hundreds of alternate possibilities, including games with two or more sexes. Attempts have also been made to invent competitive games based on "Life," for two or more players, but so far without memorable results.

"Life" may have some practical uses. There have been attempts to apply it to socioeconomic systems, and a generalization of "Life" has been suggested as an explanation of why some nebulas have spiral arms (see the article by Kenneth Brecher). Arthur Appel and Arthur Stein, at IBM, found a way of applying rules similar to "Life's" in programs designed to identify the hidden edges in computer drawings of solid shapes.

I spoke earlier of the possibility that the universe is a vast cellular automaton, operated by the movements of ultimate particles (perhaps not yet discovered) according to unknown transition rules. Physicists are now searching for a GUT (Grand Unification Theory) that will bring together all the forces of nature into one unified theory based on a gauge structure. As physicist Claudio Rebbi explained in his article on "The Lattice Theory of Quark Confinement" (*Scientific American,* February 1983), a popular approach is to think of the gauge game as being played by particles on an abstract lattice of four-dimensional cubes—a sort of spacetime "Life." This suggestion was made in 1974 by Kenneth Wilson, and is now known as lattice gauge theory.

The game metaphor for GUT carries with it the implication that the basic particles of the universe (pieces), the fundamental laws (transition rules), and spacetime (board) are not logical necessities. They are simply given. It is folly, as Hume and the positivists have taught us, to ask why they are what they are. Like chess players, physicists should accept the game and enjoy their (endless?) task of trying to guess how it is played, not waste energy speculating on why the game is designed the way it is. Now we are back to Leibniz and his stupendous vision of a transcendent Mind, contemplating all possible games, then choosing for our universe the Game that best suits the Mind's incomprehensible purposes.

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Postscript 1994

All readers of Chapter 2 surely know that in 1993 Andrew Wiles announced a proof of Fermat's last theorem. After much fanfare in the media, and articles in science and math periodicals, a serious gap was found in the proof. Wiles is confident he can overcome the snag, but at the time of this postscript the flaw had not been corrected. In September of 1994 Fermat's conjecture became a theorem at last when R. Taylor and A. Wiles announced they had found a way around the difficulty of Wiles's earlier approach.

Noam P. Elkies found an infinity of fourth powers that equal the sum of three distinct fourth powers. See his paper "On $A^4 + B^4 + C^4 = D^4$," in *Mathematics of Computation,* Vol. 51 (1988), pages 825–835. A later computer search by Roger Frye found only one solution for D^4 less than one million: $95800^4 + 414560^4 + 217519^4 = 422481^4$. No wonder the question remained so long unanswered!

Minimum lengths for Golomb rulers through 19 marks have now been proved, extending the chart shown on page 163. The shortest 12-ruler is 2,4,18,5,11,3,12,13,7,1,9 (length 85), and the shortest 13 ruler is 2,3,20,12,6,16,11,15,4,9,1,7 (length 106). Both were proved unique by Douglas Robertson. The shortest rulers of 14 and 15 marks are 127 and 151. The latter was shown unique in 1985 by James B. Shearer. The shortest ruler of 16 marks has a length of 179.

In 1993 W. Olin Sibert proved that 199 and 216 are optimal lengths for 17 and 18 marks. Apostolos Dollas has shown that the shortest length for 19 nodes is 246. He is planning to work on 20 marks, for which the conjectured minimum length is 283. More than 200 papers on Golomb rulers have appeared since I first introduced them in the column here reprinted as Chapter 15.

The total number $C(n)$ of $n \times n$ Costas arrays (two-dimensional analogs of Golomb rulers) has now been enumerated for all n equal or less than 22. Much of this work has been done by Oscar Moreno and his group at the University of Puerto Rico, at Rio Piedras. It is still not known whether there are Costas arrays for $n = 31$ and 32.