Scheduling of inspectors for ticket spot checking in urban rail transportation

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Abstract

A central issue for operators of passenger transportation in urban rail is balancing the income from tickets against the cost of the operation. The main part of the income except for governmental subsidies comes from sales of tickets. There are various ways to ensure that all passengers carry valid tickets, i.e. to avoid so called *fare evasion*. Many European companies use spot checking of passengers and among these is DSB S-tog.

The current paper describes a decision support tool developed at DSB S-tog. Based on historical data regarding when penalty fares are claimed and based on the schedules of the inspectors, this tool enables the construction of new schedules for tickets inspectors, so that the income from penalty fares claimed from passengers without a valid ticket is maximised. Other tools to increase income from ticket sales and penalty fares are also discussed.

1 Introduction

Many major cities throughout the world have extensive public transportation systems based on railways, subways, trams, buses etc. The operators of these transportation systems are usually contractually obliged to provide transportation services both with respect to quality of service and with respect to capacity. The operators may be public or private companies, however, in both cases a cost-efficient operation is of prime importance. On the cost side, this calls for efficient planning and operation. On the income side, the focus is on increasing the use of public transportation and on ensuring a sufficiently large income in terms of sold tickets. Note that a trade-off exists between attracting passengers through low prices and ensuring a sufficiently high income from tickets.

Most train companies operate with several different approaches to ensure that all passengers buy valid tickets for their travel. The approaches include a ticket check when passengers enter and/or leave the transportation network and spot check of tickets carried out by ticket inspectors. For the latter approach there is again a trade-off between expenses for salaries etc. for the ticket inspectors and increased income from ticket sales and penalty fares due to intensified inspection activities.

In the Greater Copenhagen region, the major part of the passenger transportation by rail is carried out by the operator DSB S-tog. In the DSB S-tog network, it is the responsibility of each passenger to be in possession of a valid ticket - no check is carried out neither on entry to nor on exit from the network. Spot checking of tickets is the only instrument used to ensure that all passengers carry valid tickets. The spot checks are carried out by personnel mainly dedicated to this task - the ticket inspectors. Several questions regarding the corps of ticket inspectors and its use are immediate: For a given number of employees in the corps, how should the spot checking be carried out both regarding the temporal (when), the spatial (where) and the methodological (how) dimension?

DSB S-tog is in the process of developing IT-based decision support tools to address these questions. In the current paper we describe the system developed to optimise the income from penalty fares in the temporal dimension given the number of ticket inspectors available and the rules and regulations regarding the activities of the inspectors defined by union agreements and organisational aspects of the company.

The paper is organised as follows: In section 2 we describe the situation in DSB S-tog and three other European operators with respect to spot checking of tickets. The available data and the mathematical model on which the decision support tool is based are the topics of section 3. Section 4 reports experimental results, and section 5 discusses further development of the tool as well as tools to address the other questions raised previously. Finally, conclusions of the work are presented in section 6.

2 Ticket spot checking at different rail operators

This section compares the overall procedures for the spot check inspections of tickets for different operators. All of the operators operate 'open' systems, where no checking of tickets is performed upon entry to or exit from the system.

2.1 DSB S-tog, Copenhagen

At DSB S-tog, ticket inspections are mainly performed in two different ways. Most of the inspections are performed in the driving trains, usually by teams of 1 - 4 ticket inspectors. In larger control raids on stations, 10 - 15 ticket inspectors may participate. The ticket inspectors mainly perform the inspections wearing uniform, however since September 2008, inspections using plain clothes are also performed. The experiences using plain clothes are good, there is less trouble with unruly passengers. Approx. 8% of all 91 million passengers per year have their tickets inspected. Of the *inspected* passengers, approx. 2% travel without a ticket. We believe that the *total* number of passengers travelling without a ticket is approx. 4%. The difference between these two figures lies in the fact that many passengers travelling without at ticket are able to escape inspection. An independent investigation [1] shows similar percentages for *questioned* passengers, but these numbers are probably biased in the same way as the ones observed by the ticket inspectors themselves. Presently, DSB S-tog has approx. 160 ticket inspectors employed. The train units operated by DSB S-tog are relatively long: 84 and 43 meters, respectively. The value of a penalty fare is currently 750 DKK equivalent to 100 EUR.

2.2 T-banedrift, Oslo

Compared to DSB S-tog, the network of T-banedrift [4], [14], [7] is smaller and so is the volume of passengers. T-banedrift has a relatively small number of ticket inspectors employed, and thus the fraction of inspected passengers is considerably smaller than that of DSB S-tog. The train units operated by T-banedrift are shorter compared to the ones operated by DSB S-tog. This, in combination with large teams of 7 - 8 ticket inspectors at a time, makes it much harder to escape inspection. Thus the fraction of inspected passengers not holding a ticket is as high as 4%, twice the number of DSB S-tog. Ticket inspection is mainly conducted in plain clothes, however, in the weekend uniforms are used to provide more authority. Presently the value of a penalty fare is 900 NOK or 750 NOK upon direct payment. This is equivalent to 100 and 85 EUR, respectively.

2.3 SBB S-Bahn, Zürich

SBB S-Bahn Zürich [18], [17], [5], [15] operates a much larger network than that of DSB S-tog. The volume of passengers is also much higher. SBB has considerably more inspectors employed than DSB S-tog, however, relative to the network size and passenger volume, DSB S-tog has more inspectors employed. The percentage of *inspected* passengers of the *total* amount of passengers is thus less than half of that of DSB S-tog. The fraction of the *inspected* passengers *travelling without a ticket* is similar to that of DSB S-tog. Presently the value of a penalty fare is 80, 120 and 150 CHF for the first, second and third time a passenger is encountered without a ticket in a two year period. This is equivalent to 50, 80 and 100 EUR. The train units operated by SBB are of similar length to those of DSB S-tog, however some have two stories and toilets, which complicates the inspection process considerably. Presently, SBB almost exclusively uses uniformed personnel for ticket inspections. Earlier, ticket inspectors also operated in plain clothes, however, contrary to the experience of DSB S-tog, it is the experience of SBB that inspectors in plain clothes result in more trouble with unruly passengers. We believe this is a cultural difference between the two countries. The Swiss legislation states that an operator may not profit from collecting penalty fares [6]. This puts some constraints upon the effort to optimise the spot check inspection of tickets.

2.4 ÖBB S-Bahn, Vienna

ÖBB S-Bahn Wien [11], [12], [3] operates a network that is larger and has more passenger volume than DSB S-tog, however, ÖBB has much fewer ticket inspectors employed. Some of the trains operated by ÖBB have traditional train conductors controlling the tickets of all passengers in the train. Others are spot check inspected. With regard to optimisation, the planning policy of ÖBB until now has been to inspect as many passengers as possible. However, through cooperation with DSB S-tog, ÖBB is presently investigating methods similar to those described in the current paper and in [9]. Presently the value of a penalty fare is 60 EUR. The fraction of *inspected* passengers travelling without a ticket is similar to that of DSB S-tog. Also, the characteristics of the rolling stock are similar.

3 Decision support for ticket inspector scheduling - the mathematical model

The overall purpose of the decision support model is to find out at which time it is best to perform spot check ticket inspections in order to maximise the net revenue gained from the penalty fares.

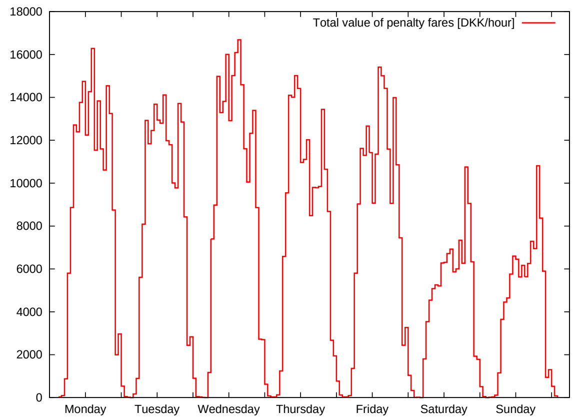

Figure 1: Total value of penalty fares as a function of the time of week the penalty fares were claimed.

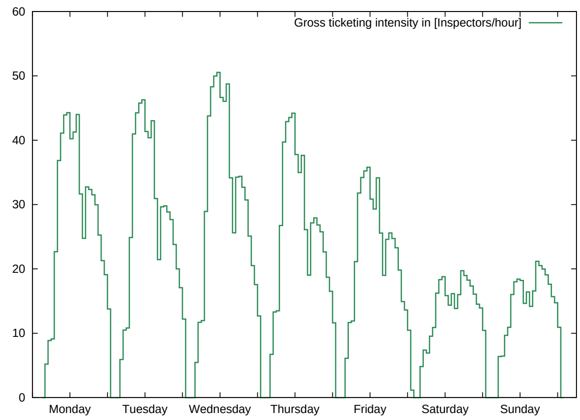

Figure 2: Gross ticketing intensity in inspectors per hour as a function of time of week.

3.1 Available data for decision making

When a passenger not holding a valid ticket is encountered by one of DSB S-togs ticket inspectors, a penalty fare is claimed. The penalty fare is a paper form filled out by the ticket inspector. One copy of the form is given to the passenger not holding a valid ticket, the other copy is kept by the ticket inspector. The data from the form is later entered into a database. From this database, the following data is available for decision making: Date and time of issuance, place of issuance (the last station), the train line that the passenger without a ticket was travelling on, the destination station of the passenger, and, finally, the monetary value of the penalty fare claimed. Consequently, with regards to where and when penalty fares have been claimed, a very good data basis for modelling already exists. Figure 1 shows how the value of the claimed penalty fares are distributed throughout the week.

Unfortunately, DSB S-tog does not presently record in detail when and where ticket inspections have taken place. One reason is that the ticket inspectors perform other ad-hoc service-related tasks during their duty and that the detailed recording of the different tasks is time consuming. However, detailed records of the actual working schedule (check-in, check-out) for each ticket inspector do exist. These data can be aggregated to the so-called *gross ticketing intensity*, i.e. the number of inspectors on duty per hour (figure 2). As mentioned earlier, the ticket inspectors also perform service-oriented ad-hoc tasks apart from ticket inspections. For the modelling purposes described in this paper it is assumed that the ticket inspectors in general perform ticket inspections with a constant intensity over the entire active duty. Unfortunately, it is not recorded in detail when the individual ticket inspectors take their breaks. Consequently, we assume that half of the inspectors take their break of one hour duration in the third hour of their duty and the rest in the forth hour of their duty. Applying this assumption to the data from figure 2 yields the *break intensity* as shown in figure 3. The validity of all of the assumptions has been thoroughly asserted through analysis of the data.

Subtracting the data on breaks in figure 3 from the gross ticketing intensity data of figure 2 yields the *net ticketing intensity* as shown in figure 4.

The *penalty fare intensity* (figure 5) is calculated as the total penalty fare value (figure 1) divided by the net ticketing intensity (figure 4). The penalty fare intensity is a measure of how much revenue one can expect to gain from a ticket inspector per inspection hour.

On the expense side, the salary of the ticket inspectors is composed of the following terms: Base salary, night-time bonus, weekend bonus and per-duty bonus. The sum of the base salary,

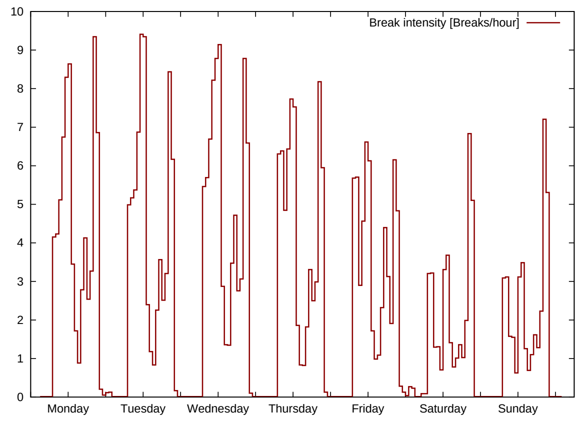

Figure 3: Break intensity in total breaks per hour as a function of time of week.

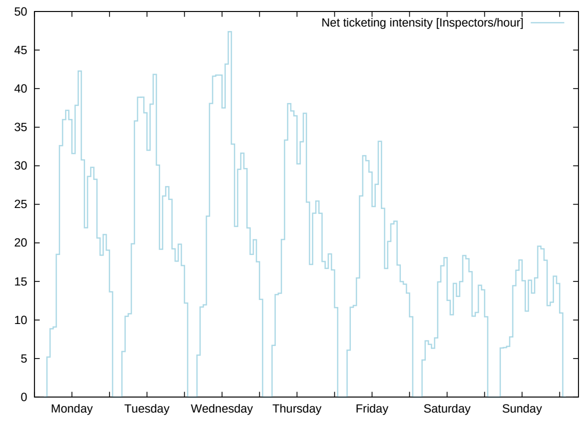

Figure 4: Net ticketing intensity in inspectors per hour as a function of time of week.

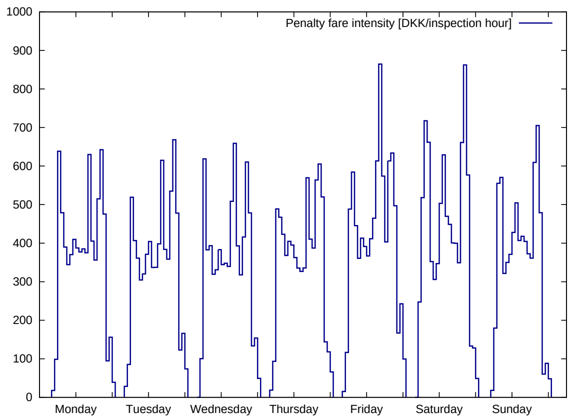

Figure 5: Penalty fare intensity in DKK per inspection hour as a function of time of week.

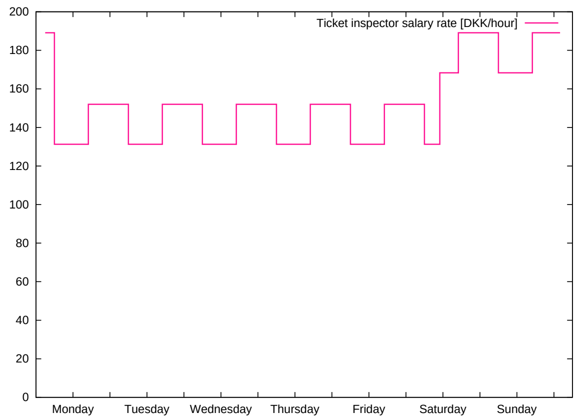

Figure 6: Ticket inspector salary rate in DKK per hour as a function of time of week.

night-time and weekend bonuses may be seen from figure 6. The per-duty bonus is a constant bonus given for each duty.

3.2 The mathematical model

The decision support tool for scheduling the corps of ticket inspectors is a so-called mathematical programming model [8], [16]. Similar models to optimise the scheduling of the ticket inspection process have not been found in literature. However, the model has some similarity to the work force approximation model described in [2].

The goal of the model is to maximise the net income from a given corps of ticket inspectors, i.e. to maximise the differences between the incomes from penalty fares and the expenses used to obtain that income. The planning period for the model is a week in order to take into account that activities during week-ends differ from activities on working days. Therefore, the model generates generic plans which can be used in a multi-period planning context.

A mathematical programming model is based on a set of decision variables which together express the potential decisions. In addition, parameters describing the problem to be solved are parts of the model. Each potential decision may in practice be feasible or infeasible, and a set of constraints expressed in terms of equations and inequalities constrains the set of potential decisions to the set of feasible decisions. Finally, an objective function expresses the value of a particular set of decisions. The goal of solving such a model is to find that set of feasible solutions which maximises the value of the objective function.

At DSB S-tog, the size of the inspector corps, the possible *duty profiles* for an inspector duty (when are breaks taken, how intensively is the ticketing carried out, see figure 10), check-in and check-out, and the minimum and maximum number of inspectors on duty for each time interval over each day of the week are examples of parameters for the problem. For each time interval during the week, the resulting plan must indicate exactly how many inspectors start their duties at that particular time interval, and which duty profiles are to be used. The constraints define the sets of feasible solutions, for instance by ensuring that the number of inspectors on duty at a particular time interval during the week is between the minimum and the maximum number allowed, and the objective function measures the net income for a given feasible set of decisions.

The model is developed with the time intervals over a week as central concepts. The set of these is denoted T and indexed by $t \in \{0, ..., |T| - 1\}$. We do not allow duties to start in all intervals during the week. By $C \subset T$ and indexed by c we denote the set of time intervals, in which it is possible for an inspector to start a duty, i.e. to check-in. Duties may have different profiles regarding breaks and ticket checking intensities. The set of duty profiles (see figure 10) is denoted V and indexed by $v \in \{0, ..., |V| - 1\}$. Each duty profile v covers a set of time intervals S_v indexed by $s \in \{0, ..., |S_v| - 1\}$.

Thus, when describing the parameters and constraints of the model, we in general use t to indicate the weekly time interval currently under consideration, c to indicate the check-in interval of the current duty under consideration, and s to indicate the current time interval relative to the start of the duty.

As an example, consider that we want to indicate whether a particular duty with profile $v \in V$ started at time interval $c \in C$ covers the time interval $t \in T$ under consideration. The parameter $\rho(v, c, t) \in \{0, 1\}$ takes the value 1 if the duty covers t and 0 otherwise:

$$\rho(v, c, t) = \begin{cases} 1 & \text{if } t \in [c; c + |S_v| - 1] \\ 0 & \text{otherwise} \end{cases}$$

i.e. if an $s \in S_v$ exists such that $t = c + s$.

The duty gross ticketing intensity (figure 2), denoted by $a(v, s) \in [0..1]$, in time interval s of the duty profile v, is the fraction of the time in that interval spent on ticket inspection as well as on breaks. The duty break intensity $p(v, s) \in [0..1]$ is the fraction of time in the interval spent on breaks. Thus, the duty net ticketing intensity (figure 4), denoted $b(v, s) \in [0..1]$, is defined as the difference between the duty gross ticketing intensity and the duty break intensity: $b(v, s) = a(v, s) - p(v, s)$. Since a ticket inspector is not claiming penalty fares while having a break, the amount of penalty fares a ticket inspector can be assumed to claim is related to the net ticketing intensity.

The penalty fare intensity (figure 5), denoted $g(t) \in \mathbb{R}$, is defined as the monetary value of the penalty fares claimed in time interval t divided by the net number of inspectors on duty in the time interval.

Since the time index s (denoting the current time interval relative to the check-in time c of the relevant duty profile v) equals 0 for the time c of check-in, the following relation holds with regards to the general time index t for the current time interval: $s = t - c$. Hence, $g(t) = g(c + s)$, and one can infer whether a duty with profile v started in time interval c has a break in interval t from $p(v, t - c) = p(v, s)$.

For a duty profile v starting in time interval c we denote by $i(v, c, s) \in \mathbb{R}^+$ the *income from penalty fares* claimed by a ticket inspector starting his duty of type v in time interval s of the duty:

$$i(v,c,s) = g(c+s) \cdot b(v,s), \quad s \in \{0,\dots,|S_v|-1\}$$

For each time interval t we denote the *salary rate* (figure 6) of an inspector by $l(t) \in \mathbb{R}^+$. The salary rate in the time interval is composed of a base salary rate with night-time and weekend bonuses. The *salary expenses* $u(v,c,s) \in \mathbb{R}$ in time interval s of the duty for an inspector starting his duty of type v in time interval c is $u(v,c,s) = l(c+s), \quad s \in \{0,\dots,|S_v|-1\}$. In addition, an inspector receives a *per-duty bonus* $m \in \mathbb{R}$.

The number of inspectors available in the inspector corps is denoted $r \in N$. The total amount of hours inspectors are available on duty per week is $n \in \mathbb{R}$. The number of inspector duties available per week is $d \in \mathbb{N}$. $r_{min}(t) \in \mathbb{N}$ and $r_{max}(t) \in \mathbb{N}$ is the minimum and maximum number of inspectors required to be on duty in time interval t. $p_{max}(t) \in \mathbb{N}$ indicates the maximum allowed number of coinciding breaks in time interval t. $x_{max}(c) \in \mathbb{N}$ is the maximum number of allowed ticket inspector check-ins in time interval c. Note that these parameters are conceptually different from those previously described, as these represent strategic decisions to be taken by the operator prior to solving the model. An extended model may include these as variables in the optimisation.

The decision variables of the model are $x(v,c)$, $v \in V$, $c \in C$ denoting the number of inspectors using duty profile v who start their duty (check-in) in time interval c. $x(v,c) \in \mathbb{N}_0$ and hence these variables are integer variables.

The objective function $z \in \mathbb{R}$ of the model calculates the net income from the complete weekly duty scheme given by the values of the decision variables and has to be maximised.

$$z = \sum_{v \in V} \sum_{c \in C} x(v,c) \Big(\sum_{s \in S_v} \big(i(v,c,s) - u(v,c,s) \big) - m \Big)$$

Thus, the objective function is the difference between the sum of income from penalty fares $i(v,c,s)$ and the sum of salary expenses $u(v,c,s)$ and the per-duty bonuses for all combinations of duty profiles V, check-in intervals C and duty profile time intervals S.

The constraints of the model cover seven issues: The available number of inspectors, the available amount of inspector time per week, the available amount of inspector duties per week, for each time interval the minimum and maximum number of inspectors allowed on duty, the maximum number of inspectors concurrently having a break, and the maximum number of inspectors starting their duty in a particular time interval. The constraints are:

$$\begin{array}{lll}
\forall t \in T: & \sum_{v \in V} \sum_{c \in C} x(v,c) \cdot \rho(v,c,t) & \leq r \\
& \sum_{v \in V} \sum_{c \in C} x(v,c) \sum_{s \in S_v} a(v,s) & \leq n \\
& \sum_{v \in V} \sum_{c \in C} x(v,c) & \leq d \\
\forall t \in T: & \sum_{v \in V} \sum_{c \in C} x(v,c) \cdot \rho(v,c,t) & \geq r_{min}(t) \\
\forall t \in T: & \sum_{v \in V} \sum_{c \in C} x(v,c) \cdot \rho(v,c,t) & \leq r_{max}(t) \\
\forall t \in T: & \sum_{v \in V} \sum_{c \in C} x(v,c) \cdot p(v,t-c) & \leq p_{max}(t) \\
\forall c \in C: & \sum_{v \in V} & \leq x_{max}(c)
\end{array}$$

Note that in general, the constraints regarding maximum number of inspectors concurrently on duty makes the constraints regarding the available number of inspectors superfluous.

3.3 Output from the model

One set of all the necessary parameters for a calculation is handled collectively as a so-called scenario. This enables us to easily compare the results of different calculations using different sets of the parameters in the model.

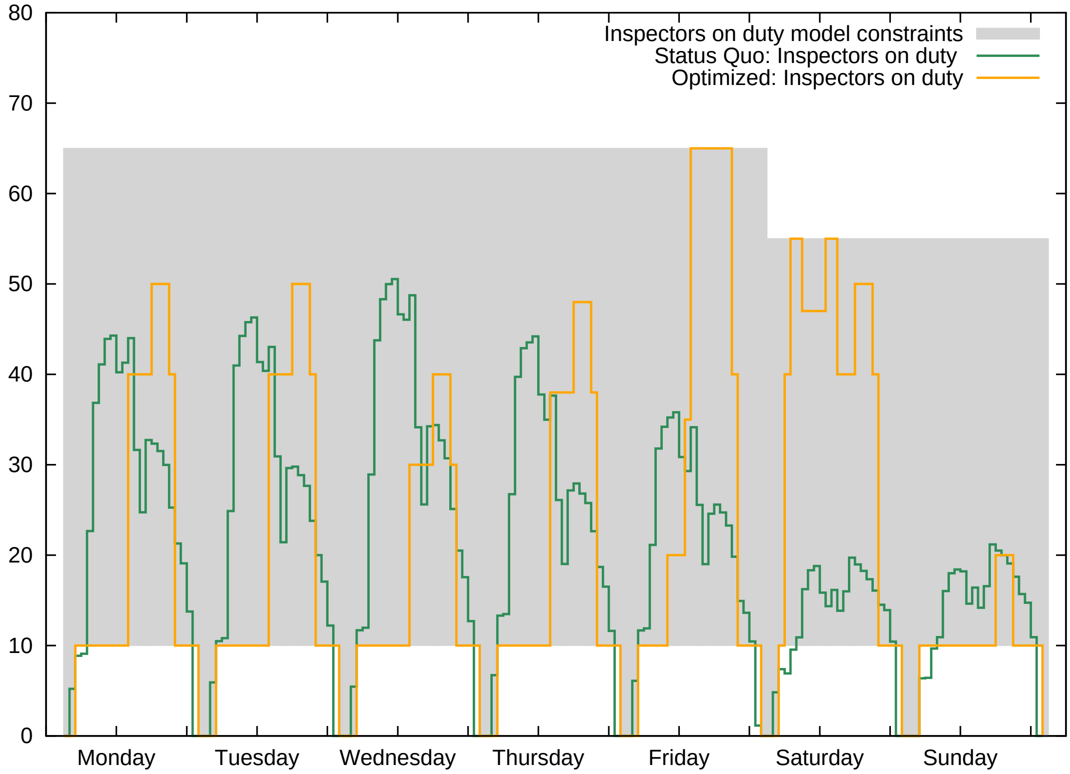

Figure 7: Parameters and optimisation results of the model: The number of allowed inspectors on duty, the present (status-quo) number of inspectors on duty and the number of inspectors on duty as optimised by the model.

The output from the model is in the form of a an optimisation report, one report for each scenario. An optimisation report contains a tabular overview of the parameters describing the scenario, the calculated optimal duty plan for the given scenario, as well as key economic figures of the calculated duty plan. The key economic figures include the expected net income, the expected expenses and the expected balance per week and year. Furthermore the optimisation report also includes several graphs. Examples of the graphical output in the optimisation reports are given in figures 7 to 10.

In the example in figure 7, the parameters of the scenario are set to demand a minimum of 10 inspectors on duty on all days from early in the morning to late at night and to allow a maximum of 65 inspectors on duty Monday to Friday and a maximum of 55 inspectors in the weekend. In the example in figure 8 the parameters of the scenario are set to allow a maximum of 15 coinciding breaks at all times. In figure 9 the parameters of the scenario are set to allow a maximum of 30 simultaneous check-ins at all times. Figure 10 shows the four different duty profiles in the scenario, three of standard duration and one of short duration. The ones with standard duration have the breaks in the middle, early and late in the duty respectively.

4 Experimental results

The results from the model are promising. Using a parameter set-up that is considered realistically implementable with regard to union agreements etc. a revenue increase in the order of 15% may be gained from changing the present duty schedules to the ones optimised by the model.

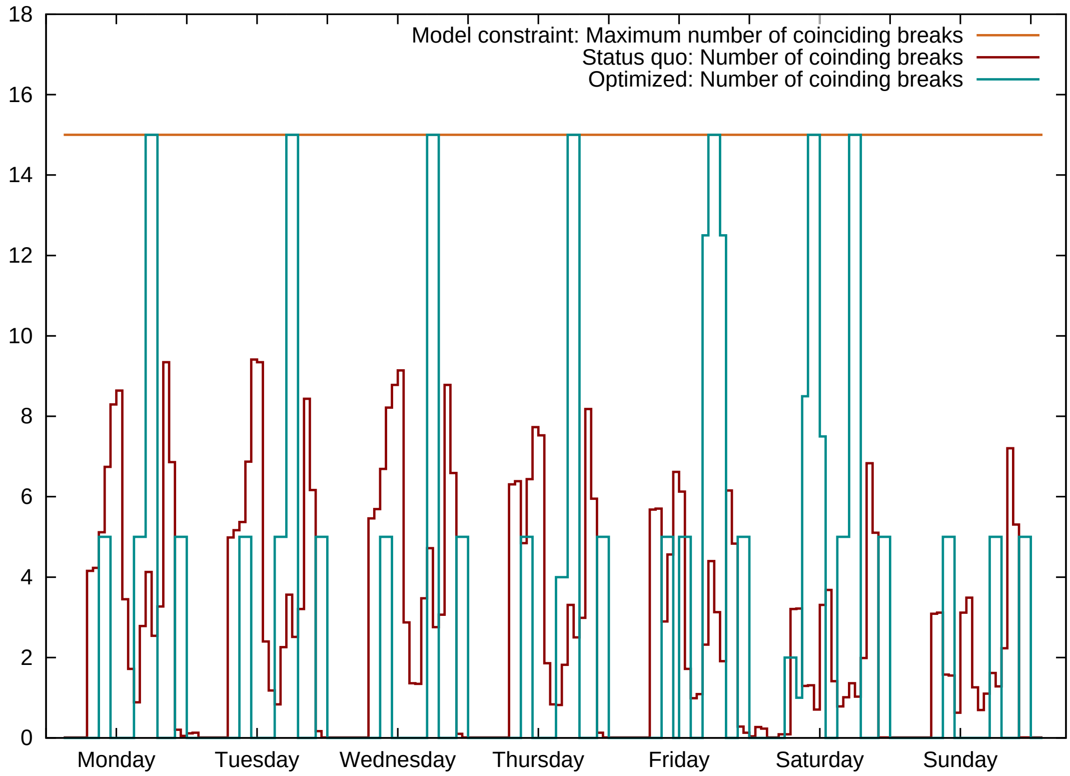

Figure 8: Parameters and optimisation results of the model: The model constraints with regard to maximum number of coinciding breaks, the present (status-quo) number of coinciding breaks, and the number of coinciding breaks in the duty plan optimised by the model.

One form of output from the model is the set of coefficients to the objective function, as shown in figure 11. These figures show how much net revenue may be gained from a ticket inspector starting his duty at a certain time of the week using a certain duty profile. The difference between the value of the graph for two points in time is thus the amount one may gain (or lose) by moving a duty of a ticket inspector between those two points in time. The difference between two duty profiles is equivalently the amount one would gain (or lose) if one would change the duty profile of a ticket inspector. Figure 11 may be used to create rules-of-thumb on how the ticket inspectors should schedule their duties. As may be seen from figure 11, the values are high for all days just after midday. This has to do with the fact that inspectors starting their duty at this time are able to inspect tickets in the afternoon rush-hour (where the passenger volume is high and where many passengers travel without a ticket), to take their break at a time where the passenger volume is low (few passengers without a ticket), and to return to inspections when the fraction of passengers travelling without at ticket is rising in the early hours of the evening (compare to the penalty fare intensity shown in figure 5). A rule of thumb may thus be to make afternoon duties start at 1400h. Similarly other rules of thumb may be deducted: Favour duties on Fridays and Saturdays, favour morning duties at 0600h Monday to Thursday, at 1000h on Friday and at 0700h in the weekend. From figure 11 it may also be observed that the differences between the duty profiles with early, standard and late time of break are negligible. This clearly shows that the ticket inspectors should be allowed to decide for themselves when they want to take their break.

The current model is, as already mentioned, a mixed integer programming model, a MIP model. However, solving the model as an linear programming model without the integrality constraint on the variables leads to quite similar results. The results from the LP model include

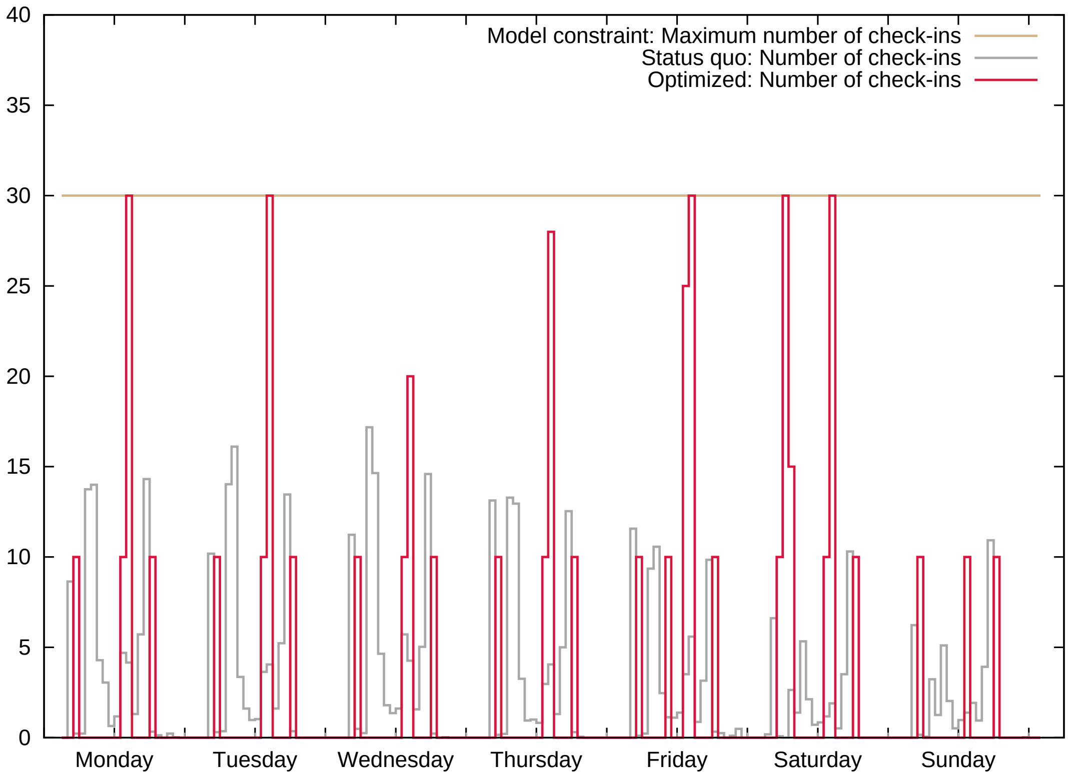

Figure 9: The model constraints with regard to maximum number of coinciding check-ins, the present, status-quo number of check-ins, and the number of check-ins in the duty plan optimised by the model.

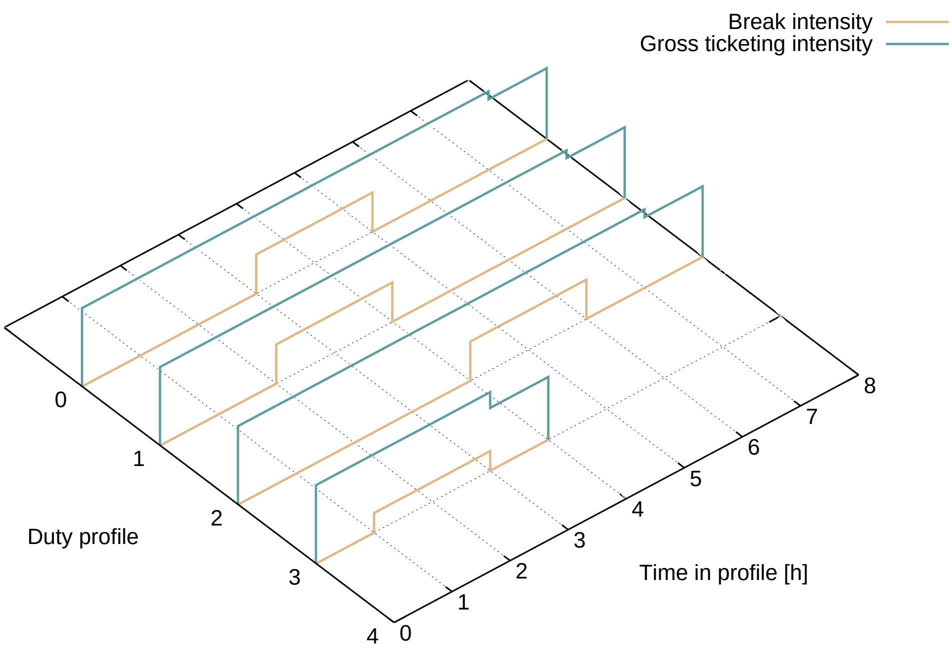

Figure 10: Examples of model duty profiles with corresponding gross ticketing and break intensities.

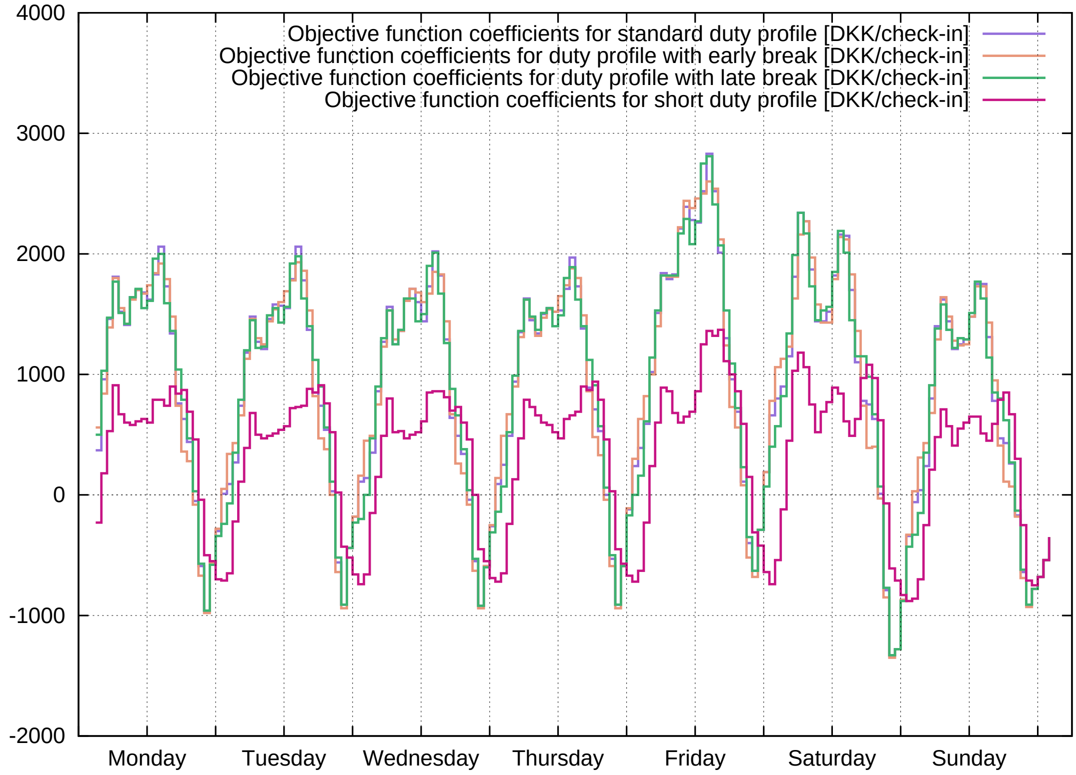

Figure 11: The value of the coefficients to the objective function of the model (MIP) for the time of week and the four different duty profiles corresponding to the profiles shown in figure 10.

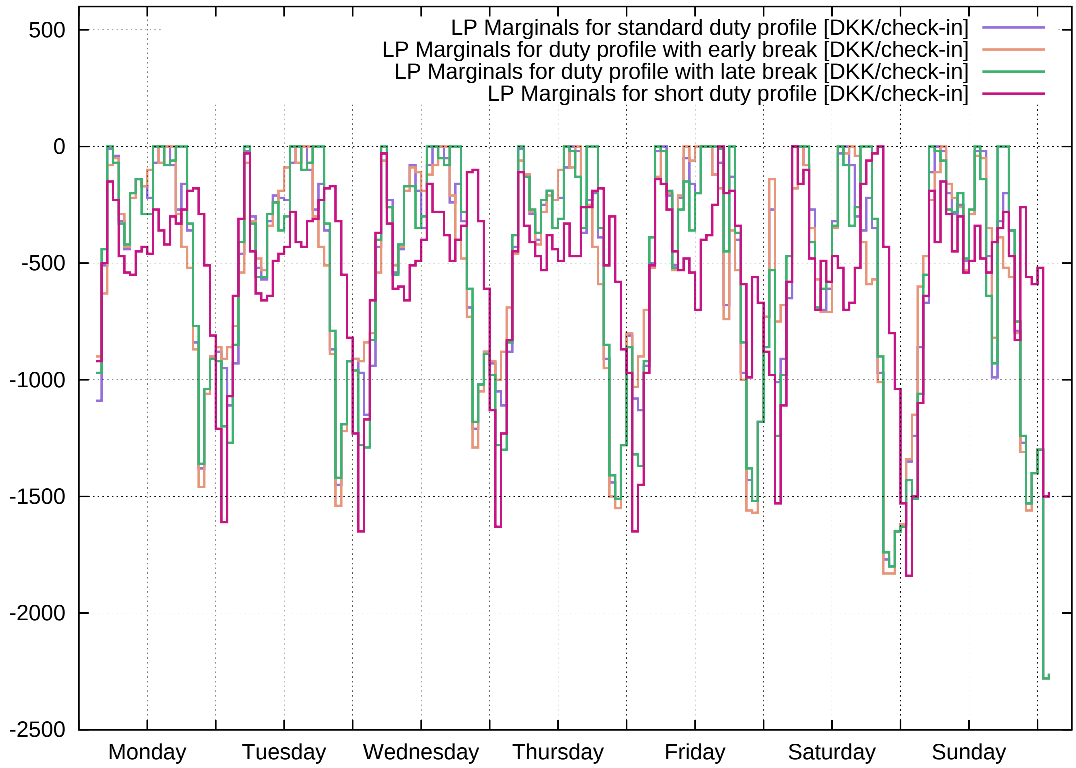

Figure 12: The value of the marginals in the corresponding LP formulation of the model for the time of week and the four different duty profiles corresponding to the profiles shown in figure 10.

so-called marginals for variables and constraints. Each marginal shows, how much the objective function would change, if the bounds on the variable/constraint is increased one unit. Since the MIP and the LP solutions are similar, these marginals are also expected to contain information valid for the MIP solution. Figure 12 shows the marginals for the decision variables. It may be seen that forcing a positive number of duties to start early is very costly. The marginals for constraints may be used to evaluate the cost of the lower bound on the number of ticket inspectors on duty in a time interval - the marginals for these constraints indicate the increase in the objective function, if the lower bound is decreased.

5 Further development and other models

The model described in this paper represents just one way of analysing the problem of optimising spot check ticket inspections, namely in the time (when) dimension. DSB S-tog is currently exploring three other ways of analysing the problem, namely in the spatial (where) dimension, in the quantitative (how much) dimension and in the methodological (how) dimension. These models are all still in the development stage.

The spatial model is envisioned as a statistical, real-time, tactical decision support model, producing a map of where the most passengers not having a ticket may be expected, e.g. for the next two hours. This map may then be used to dispatch ticket inspectors to the areas where most revenue may be gained from ticket inspection. The map can even be shown to the ticket inspectors in the field using their mobile PDA equipment. The further development of the model at DSB S-tog currently awaits new GPS equipment for better localisation of the ticket inspectors to produce data for the modelling purposes.

The quantitative model is envisioned as a statistical, strategic model answering the question: “What is the optimal number of ticket inspectors to be employed?” The more spot checks are carried out, the less passengers are likely to find it attractive to not have a ticket. Key aspects of the model thus include the relation between the number of ticket inspectors (i.e. the volume of ticket spot check inspections) and the volume of passengers not having a ticket. A major challenge will be the calibration of the model since the number of ticket inspectors has been quite constant over the years. One way of overcoming this problem would be to slowly hire more ticket inspectors and to follow the development in passenger reactions. Another way would be to experiment with more spot checking on certain lines and less on others, then reversing the inspection intensity on the lines for a period and following the passenger reactions carefully. Others have proposed similar models to the one we are currently exploring, see [10] and [6]. The latter explores game theory to produce a simple decision model.

The methodological model is envisioned as a strategic, agent-based simulation model [13] answering the question: “How should the spot check ticket inspection be performed in and around the train?” The purpose of the model is to explore different physical ways of conducting the spot check ticket inspections by calculating how many of the passengers not having a ticket we are able to inspect. Key aspects of the model include: Size of inspection team, type of clothing worn (uniform, plain clothes) and movement pattern of inspection team. In order to illustrate these aspects a simple animation has been produced. Showing this animation to management and to the ticket inspectors have already started a change in awareness of the physical way the ticket inspections are carried out. This awareness may prove it unnecessary to complete the model, since the changes are already widely implemented.

For a further discussion of all of these proposed models, please refer to [9].

The scheduling model described in this paper and the proposed models described above all interact with each other. However, the individual models are models each with their own limited domain. Future development includes investigation of the possibilities of integrating all models and taking into account the interaction between the different decisions the models represent.

6 Conclusions

We have described ongoing developments in DSB S-tog on decision support tools to be used in the process of maximising the income from penalty fares and increased ticket sales. One of these tools is ready to use - the others are still in the development phase.

The tools are based on IT and different mathematical models and clearly demonstrate the value of using methods from Operations Research in the process of analysing and planning the activities of ticket inspectors. Furthermore, the discussion spawned by the modelling activity does in itself influence both the planning process and the operation due to increased awareness of the possibilities and their consequences in the planning department and in the operational department.

References

[1] COWI. *HUR, DSB og ØSS: Rejsehjemmelsundersøgelse 2006. Årsrapport*. 2007.

[2] Michael Folkmann, Julie Jespersen, and Morten Nielsen. *Estimates on Rolling Stock and Crew in DSB S-tog Based on Timetables*. Lecture Notes in Computer Science. Springer, 2004.

[3] ÖBB. *Home Page*. http://www.oebb.at/.

[4] T-banedrift Oslo. *Personal communication*. 2008.

[5] SBB. *Home Page*. http://www.sbb.ch/en/index.htm.

[6] Patrick Schuler. *Schwarzfahren: Eine ökonomische Betrachtung*. Lehrstuhl für Industrieökonomik, Sozialökonomisches Institut, Universität Zürich. 2006.

[7] Oslo T-Banedrift. *Home Page*. http://www.tbane.no/.

[8] Hamdy A. Taha. *Operations Research*. Prentice-Hall, 2003.

[9] Per Thorlacius. *Optimering af stikprøvekontrol af billetter: Hvordan foretages dette?* DSB S-tog, Production Planning, Kalvebod Brygge 32, DK-1560 København V, Denmark. 2008.

[10] Verband Deutscher Verkehrsunternehmen (VDV). *Maßnahmen zur Einnahmensicherung, Teil II: Kennzahlen der Fahrausweisprüfung und optimaler Kontrollgrad*. Number 9708 in VDV Mitteilung. 2001.

[11] ÖBB Vienna. *Personal communication*. 2009.

[12] Schnellbahn Wien. *Home Page*. http://www.schnellbahn-wien.at/index.htm.

[13] Wikipedia. *Agent-based Model*. http://en.wikipedia.org/wiki/Agent_based_model.

[14] Wikipedia. *Oslo Metro*. http://en.wikipedia.org/wiki/Oslo_Metro.

[15] Wikipedia. *Zürich S-Bahn*. http://en.wikipedia.org/wiki/Zürich_S-Bahn.

[16] Laurence A. Wolsey. *Integer Programming*. Wiley-Interscience Series in Discrete Mathematics and Optimization. John Wiley and Sons, 1998.

[17] ZVV. *Home Page*. http://www.zvv.ch/en/.

[18] SBB Zürich. *Personal communication*. 2007.