Experiments with the drinking bird

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We present a simple model of the dynamics of the drinking bird and relate its period to the properties of its internal and external liquids. The effect of humidity on the motion is studied and it is shown that there are two evaporation regimes. The results of the model are in agreement with observations.
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I. INTRODUCTION

The drinking bird (dunking duck or dipping bird)[1] is not only a toy, but also a demonstration apparatus on liquid–vapor equilibrium and evaporation.[2] Figure 1 shows a schematic and a photo of this intriguing thermodynamic device.[3,4] The bird consists of two spherical glass bulbs connected by a glass tube that enters well inside the lower bulb. The bottom bulb (the bird's body, hereafter simply referred to as the body) is almost filled with a highly volatile liquid, normally methylene chloride (CH_2Cl_2), whose normal boiling point is close to room temperature. There is no air inside the bird, but only this internal liquid in thermal equilibrium with its vapor.

The top bulb (the bird's head, hereafter simply called the head) is covered with a porous tissue. It has a small plastic hat and a long beak which is covered with the same tissue as the head. The bird can oscillate around a horizontal metallic bar attached to the tube at the middle. When the bird leans completely forward, it ''drinks'' water from a glass, although other external liquids may be used as well. We call this motion a dip.

The drinking bird undergoes a cycle, which at first sight might seem to exhibit perpetual motion. At the beginning of the cycle, the bird is upright, with all the internal liquid in the lower sphere. The water on the head is in contact with its vapor at a given (room) temperature. If the vapor pressure is smaller than its saturation (or equilibrium) value, evaporation occurs spontaneously. The evaporation cools the head outside, so that the CH_2Cl_2 vapor inside also has to cool. The vapor in the head condenses in very small drops, remaining in equilibrium with the internal liquid as the temperature decreases. The CH_2Cl_2 vapor pressure inside the head becomes smaller than that in the body according to the Clausius–Clapeyron equation, and this pressure gradient forces the internal liquid to rise up in the tube. As the liquid rises, the center of mass of the system also rises, and the momentum produced by the weight eventually forces the bird to tip forward and to dip its beak in the glass, keeping its head wet. When the bird is almost horizontal, the lower end of the tube emerges above the internal liquid surface and some vapor passes from the body to the head (see Fig. 1). While drinking, the bird remains horizontal for a short time. Then, part of the liquid drains back into the body and the bird returns to its upright position. As water evaporation continues from the head, the internal liquid comes up again, starting a new cycle.[3] From the temperature difference between head and body, work can be produced so that the drinking bird is in fact a thermal engine.

A liquid and its saturated vapor in thermal equilibrium at constant temperature and pressure have the same chemical potential. But when the vapor partial pressure is smaller than its saturated pressure (for water, when the humidity is less than 100%), the chemical potential of the liquid is higher than that of the vapor and spontaneous evaporation occurs. This evaporation can be used to do work. Thus, at the most fundamental level, the ability to produce work lies in the difference between the chemical potentials of the external liquid and its nonsaturated vapor. However, the evaporation from the head is not the only possible mechanism for the drinking bird. A temperature gradient between the body and the head may simply be obtained by heating the body, for example, by illuminating a black painted body with a light bulb or solar light.[4] Our experiments with the ''sunbird'' (a drinking bird that does not drink) are described in a companion paper.[6]

Because we could not find a quantitative description of the drinking bird, we present a model that relates its period, that is, the time between consecutive dips, to the properties of the internal and external liquids and to the bird's dimensions. In Table I we list the geometrical and physical data for the toy used in our experiments.[5] We performed various experiments with a drinking bird to verify our model. We also developed a computer simulation of the bird's dynamics.

In Sec. II we present our model and compare it with experiment, using various liquids as cooling agents. In Sec. III we analyze the influence of the humidity on the period when water is used as the external liquid. In Sec. IV we report the results of the numerical integration of the equations of motion. Our conclusions are given in Sec. V. In the Appendix we evaluate the drinking bird moment of inertia and torque, which are needed in Sec. IV.

II. PERIOD OF OSCILLATION

The cooling of the head during one period is directly related to the evaporation of a certain mass of external liquid outside the head, which we denote by Δm_E ($\Delta m_E > 0$). The period temperature decrease inside the head during one cycle, ΔT, is related to the energy loss in one period:

$$C\Delta T = -\Delta m_E \Delta h_E, \tag{1}$$

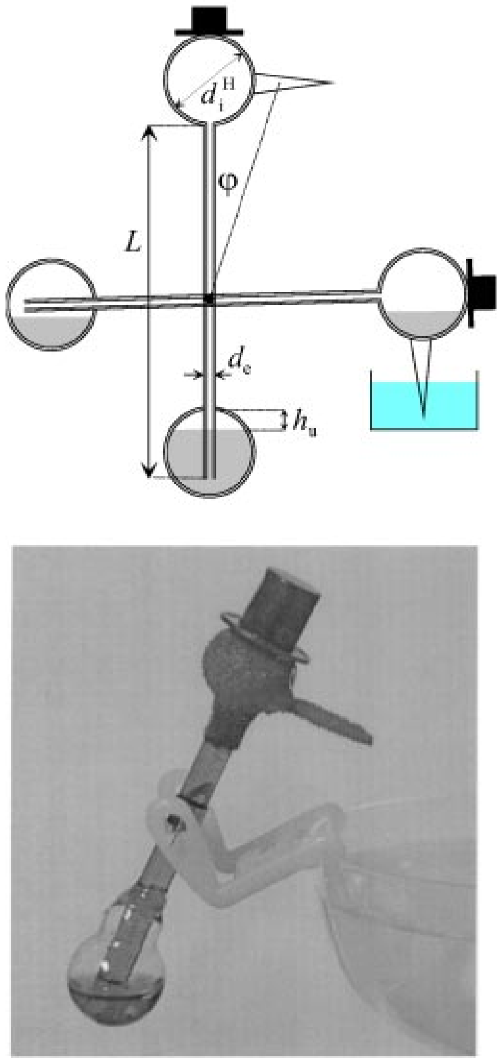

Fig. 1. Drinking bird scheme and photo. For dimensions see Table I.

where C is an effective heat capacity of the head, which characterizes a given bird, and Δh_E is the specific evaporation enthalpy of the external liquid. The minus sign in Eq. (1) makes ΔT a negative quantity.

Table I. Geometrical and physical data for the drinking bird (Ref. 5), with CH_2Cl_2 as the internal liquid (see Fig. 1).

Tube length	$L=6.68$ cm
Tube external diameter	$d_E=0.57$ cm
Tube internal diameter	$d_i=0.40$ cm
Head external diameter	$d_E^H=1.47$ cm
Head internal diameter	$d_i^H=1.41$ cm
Body external diameter	$d_E^B=1.79$ cm
Body internal diameter	$d_i^B=1.73$ cm
Height of upper spherical calotte (empty)	$h_u\approx 0.3$ cm
Angle of Fig. 1	$\varphi\approx 20°$
Hat mass	$m_h=0.81$ g
Beak mass	$m_b\approx 0.2$ g
Glass density	$\rho_g=2.10$ g/cm^3
Glass specific heat	$c_g=0.837$ J g^{-1} °C^{-1}
CH_2Cl_2 density	$\rho_I=1.336$ g/cm^3
CH_2Cl_2 normal boiling point	$T_{I,b}=313.15$ K
CH_2Cl_2 vaporization enthalpy	$\Delta h_I=28094.50$ J/mol

According to the Clausius–Clapeyron equation,[7] the temperature decrease ΔT is related to the pressure decrease inside the head in one period, ΔP, by

$$\Delta T=\frac{\Delta P}{B}, \tag{2}$$

with

$$B=\frac{\Delta h_I P_I(T_R)}{RT_R^2}, \tag{3}$$

where Δh_I is the molar vaporization enthalpy of CH_2Cl_2, $P_I(T_R)$ is the vapor pressure at room temperature, T_R, and R is the ideal gas constant. Equation (2) requires that ΔT be sufficiently small.

The pressure difference between the head and the body (a negative quantity) is given by $-\rho_I g z$, where ρ_I is the internal liquid density and z is the height of the internal liquid level in the tube with respect to the surface level in the body. The total pressure decrease inside the head in one period (a negative quantity) is given by

$$\Delta P=-\rho_I g\Delta z, \tag{4}$$

where Δz is the change of z in one period. If we substitute Eq. (4) in Eq. (2), the temperature change in one period is

$$\Delta T=-\frac{\rho_I g\Delta z}{B}. \tag{5}$$

A similar relation holds between any small change of z and the corresponding change of T.

If the evaporation rate of the external liquid (a negative quantity), $\dot{m}_E$, is approximately constant, the period is

$$\tau=-\frac{\Delta m_E}{\dot{m}_E}=\frac{C\Delta T}{\dot{m}_E\Delta h_E}, \tag{6}$$

where we have used Eq. (1). The numerator $C\Delta T$ is characteristic of a drinking bird operating with a given external liquid. Although the evaporation rate does not remain constant for many external liquids (it changes due to the structure of the felt that covers the head), it may still be considered constant for small time intervals (say, a few periods; see, Fig. 3 and Table II).

To verify Eq. (6), we measured periods and evaporation rates not only for water—the usual liquid drunk by the bird—but also for other liquids whose partial pressures in the air are zero. When water is taken as external liquid, the air humidity directly affects the bird's dynamics: the period is longer in humid days than in dry ones. For very high humidities, the toy does not even work. This problem does not occur for other liquids.

We observed that the period, excluding the short time taken by the dip, is the time needed for the internal liquid to reach the top of the tube and fill approximately half of the head: $z_{\max}\approx L+d_i^H/2$, where L is the tube length and d_i^H is the head internal diameter (see Table I). We also observed that after a dip, the internal liquid does not completely return to the body; the tube is half filled when a new cycle starts. In Fig. 2(a) we show the dependence of z with time obtained by a simulation of the bird's motion. (The details of the simulation will be given in Sec. IV.) In the steady regime, the minimum height of the internal liquid is $z'=L/2$. In Fig. 2(b) we show the temperature change (obtained in the same

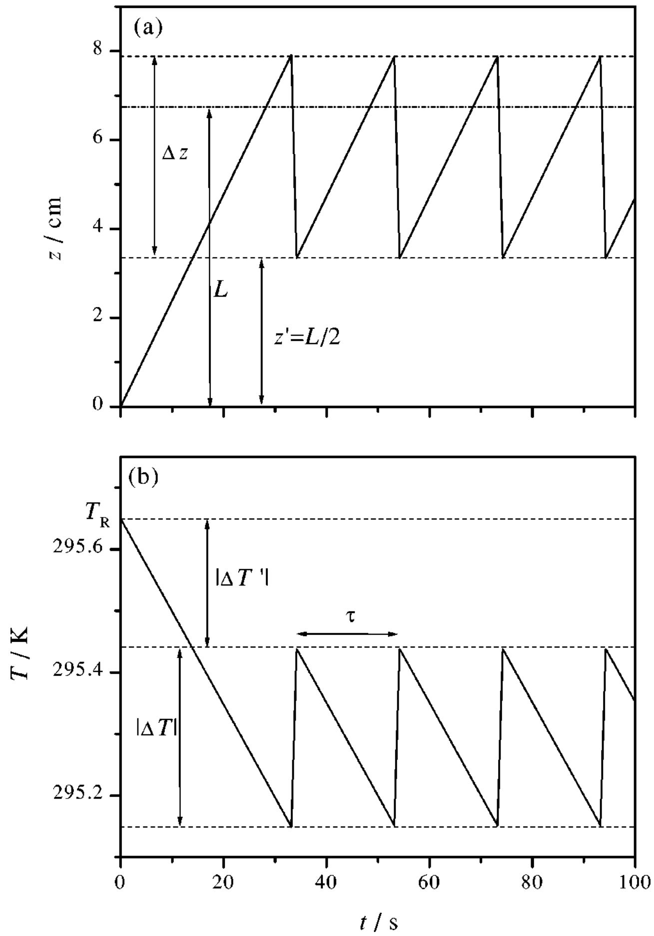

Fig. 2. The evolution of (a) the internal liquid height, z, and (b) the temperature inside the bird's head, T. These results were obtained by the simulation described in Sec. IV, with the parameters indicated in Fig. 5(a).

simulation), which is related to the change of z through Eq. (5). After the dip the temperature inside the head rises only to $T_R - |\Delta T'|$ ($\Delta T'$ is the temperature change corresponding to z'—see Fig. 2), and along the cycle it drops by an amount $|\Delta T|$ (with $|\Delta T| \approx |\Delta T'|$).

For our bird (see Table I) we have $B = (2.07 \pm 0.01) \times 10^3\ \mathrm{Pa\,K^{-1}}$. This value is obtained by substituting $\Delta h_I = 28\,094.50$ J/mol, $R = 8.3145\ \mathrm{J\,K^{-1}\,mol^{-1}}$, and $T_R = (22.5 \pm 0.1)$°C into Eq. (3). The liquid–vapor pressure of the internal liquid $P_I(T_R)$ in Eq. (3) is obtained from the solution of the Clausius–Clapeyron equation:

$$P_I(T_R) = P_0 \exp\left[-\frac{\Delta h_I}{R}\left(\frac{T_{I,b} - T_R}{T_R T_{I,b}}\right)\right]. \tag{7}$$

For $P_0 = 1.013 \times 10^5$ Pa (normal atmospheric pressure) and $T_{I,b} = 313.15$ K (the normal boiling point of the internal liquid,[4] which is close to T_R), we obtain $P_I(T_R) = (0.535 \pm 0.002) \times 10^5$ Pa. For our bird we measured $\Delta z = (4.2 \pm 0.1)$ cm, so that from Eqs. (4) and (5), $\Delta P = -(5.5 \pm 0.1) \times 10^2$ Pa and $\Delta T = -(0.266 \pm 0.005)$ °C. This value of ΔT is much smaller than $T_{I,b} - T_R$ and hence Eq. (2) is valid. Similarly, ΔP is much smaller then P_I. These values for ΔT and ΔP are specific to a given drinking bird. A small ΔT (and, therefore, a small ΔP) facilitates the operation of the bird.

To measure evaporation rates, we placed the drinking bird on a digital balance (whose precision is ± 0.001 g). After pouring a few drops of different liquids on the head, we measured the evolution of both the mass of external liquid and the period.[8] In these experiments the bird does not drink when it dips, so that the external liquid that evaporates on the head is not replaced. Figure 3, which presents results for ethylic alcohol, allows us to confirm the inverse proportionality between period and evaporation rate given by Eq. (6).

Table II displays the evaporation enthalpies for various external liquids as well as our measured quantities (time, evaporation rate, and period) using different external liquids. We note that the evaporation rate is lower for water and, consequently, the initial period is much larger. But it is interesting that $C\Delta T = \dot{m}_E \tau \Delta h_E$ is approximately constant for a given drinking bird operating with a given external liquid. Moreover, for liquids with lower evaporation enthalpies (the organic liquids in Table II, which are also less dense than water), it turns out that the constant is practically the same for all of them: $C\Delta T \approx -1$ J. The partial pressure in the air is zero for all organic liquids, and the lower the evaporation enthalpies, the higher the evaporation rates. Because ΔT is known from Eq. (5), we may extract the value of C from the experimental value for $C\Delta T$. On the other hand, for water $C\Delta T \approx -1.26$ J, so that $C = 4.75\ \mathrm{J\,°C^{-1}}$, a value we will use in Sec. IV.

III. INFLUENCE OF HUMIDITY

As mentioned, air humidity affects the bird's period if water is the external liquid. At room temperature, when the vapor partial pressure, P^E, is equal to the liquid–vapor equilibrium pressure at that temperature, $P_E(T_R)$, the water on the head no longer evaporates and the bird stops. If the humidity is close to saturation, the evaporation rate is low and the periods would be large. On the other hand, low partial pressure of water on dry days leads to a large evaporation rate and a short period.

According to Fick's law, the evaporation rate of water is given by[10,11]

$$\dot{m}_E = -\eta'(100 - H), \tag{8}$$

where $\eta' > 0$ is a diffusion coefficient depending on the substance and on the evaporation area and $H = 100 P^E / P_E(T_R)$ is the relative humidity.[12] We substitute Eq. (8), which is valid for normal convection, in Eq. (6) and find that the period becomes $\tau = \kappa'(100 - H)^{-1}$, where $\kappa' = \Delta m_E / \eta'$. However, due to the bird's motion, this relation is not exactly observed. Our experimental results (described below) suggest a different power dependence of the period on the humidity, namely

$$\tau(H) = \kappa(100 - H)^{-\beta}, \tag{9}$$

where κ and β are phenomenological parameters.

To study the evaporation rate we placed the drinking bird in a closed chamber (Fig. 4) and measured the periods for various relative humidities. Figure 4 shows the logarithm of the period as a function of the logarithm of $(100 - H)$. The experimental results clearly show two linear dependencies of $\ln \tau$ on $\ln(100 - H)$ in different ranges of the humidity. For $\ln(100 - H) \leqslant 3.2$, that is, for up to $\approx 75\%$ relative humidity, the linear fit to the data leads to $\beta = 1.82$ and $\ln \kappa = 9.44$. We interpret this power law behavior as due to forced air con-

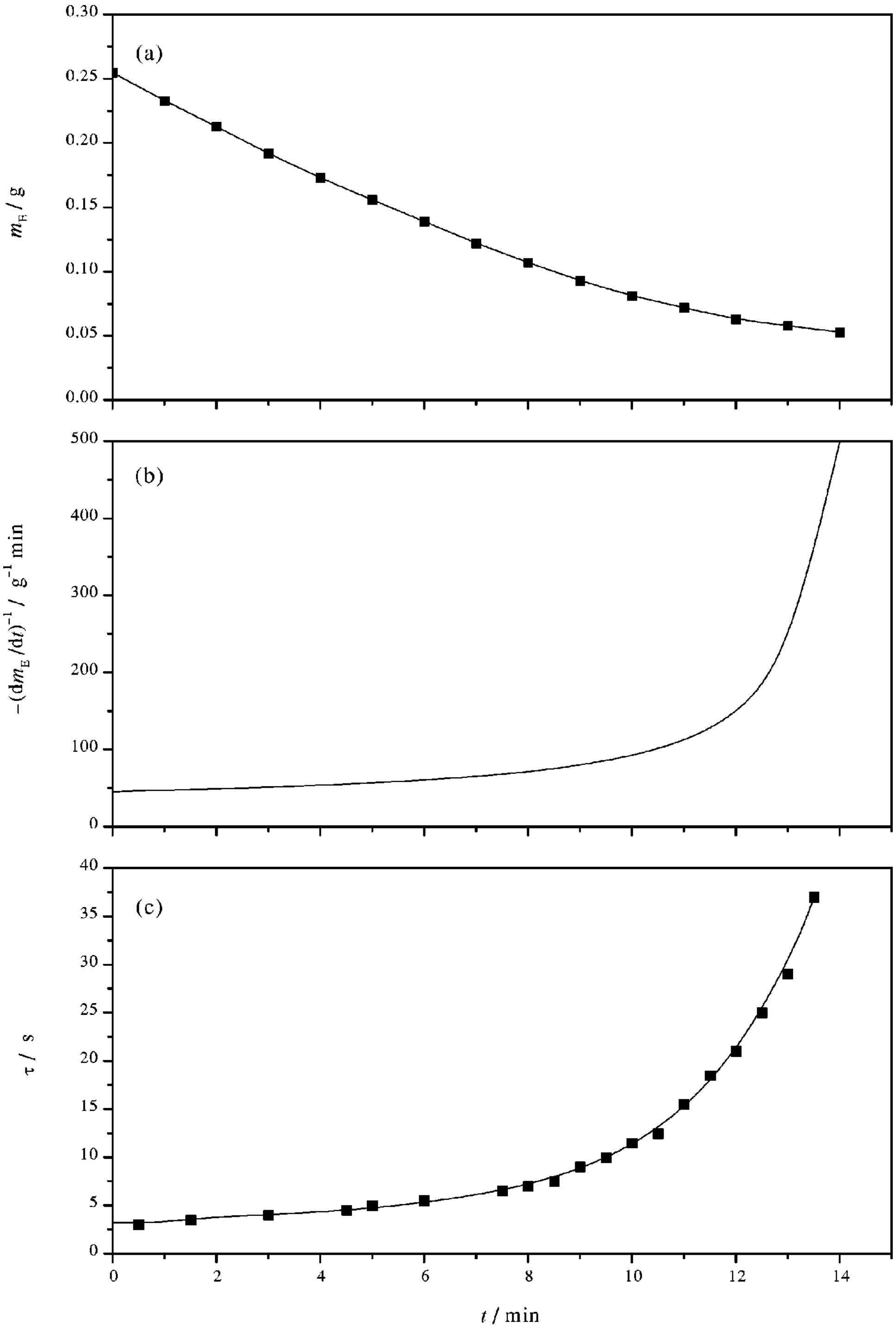

Fig. 3. (a) The mass m_E of ethylic alcohol on the bird's head, (b) the evaporation rate $\dot{m}_E$ obtained from the derivative of the fit to m_E, and (c) the period τ as a function of the time. For time intervals less than 1 min (a few periods, see Table II), the evaporation rate is approximately constant. Similar results were obtained for methylic alcohol, chloroform, and ethyl acetate; n-hexane evaporates too quickly. For water, the evaporation rate is practically constant in the time scale considered here.

vection. Above that humidity the linear fit to the data yields $\beta=1.24$ and $\ln\kappa=7.69$. Normal evaporation ($\beta=1$) would only occur if the bird were at rest or slowly moving.

IV. SIMULATION OF THE DYNAMICS

We now present a model for the dynamics of the bird based on the previous description of water evaporation. We use as the dynamical variable $z(t)$, the column height of the internal liquid at time t. From Eqs. (6) and (9) the mass of the water, dm_E, evaporated during the time interval dt is given by

$$\frac{dm_E}{dt}=-\eta(100-H)^{\beta}, \tag{10}$$

where $\eta=\Delta m_E/\kappa$. The values for κ and β are found in Sec. III. From the data of Table II, we find $\Delta m_E=6.2\times10^{-4}$ g, so that $\eta=4.92\times10^{-11}$ kg/s for $H<75\%$ and $\eta=28.3\times10^{-11}$ kg/s for $H>75\%$.

The evaporation of mass dm_E leads to a temperature decrease dT given by an equation similar to Eq. (1): $dT=-dm_E\Delta h_E/C$ (the heat capacity C was obtained in Sec. II). The temperature decrease dT in the time interval dt leads, in turn, to a pressure gradient dP inside the head given

Table II. The specific evaporation enthalpies Δh_E, (Ref. 9), the times, the evaporation rates, the periods, and $C\Delta T = \dot{m}_E \tau \Delta h_E$ for several external liquids. From top to bottom, the liquids are: chloroform, *n*-hexane (its evaporation is so quick that the bird stops after 1 min), ethyl acetate, ethylic alcohol, methylic alcohol, and water at 50% humidity.

Liquid	Δh_E (J/g)	t (min)	$\dot{m}_E \times 10^4$ (g/s)	τ (s)	$C\Delta T$ (J)
CH_3Cl	247.021	0	−21.6	2 ± 0.5	−1.1 ± 0.3
		3	−4.0	10 ± 0.5	−0.99 ± 0.05
C_6H_{14}	330.757	0	−19.6	1.5 ± 0.5	−1.0 ± 0.3
$C_4H_8O_2$	368.438	0	−5.67	5 ± 0.5	−1.04 ± 0.13
		6	−2.33	12.5 ± 0.5	−1.07 ± 0.04
C_2H_6O	841.547	0	−4.40	3 ± 0.5	−1.1 ± 0.2
		10	−0.90	12.5 ± 0.5	−0.95 ± 0.04
CH_4O	1101.128	0	−5.91	1.5 ± 0.5	−1.0 ± 0.3
		7	−0.96	9.5 ± 0.5	−1.00 ± 0.05
H_2O	2257.104	0	−0.644	9 ± 0.5	−1.30 ± 0.07
(H = 50%)		45	−0.541	10 ± 0.5	−1.22 ± 0.06

by an equation similar to Eq. (2), that is, $dP = B dT$. As a consequence, during the interval dt, the liquid rises in the tube by $dz = -dP/\rho_I g$. The evolution of z is given by $z(t + dt) = z(t) + dz$ with $z(0) = 0$.

With no friction, the angular acceleration is $\alpha = M(z)/I(z)$, where $I(z)$ is the moment of inertia and $M(z)$ the torque with respect to the rotation axis; both depend on the level $z(t)$. The quantities $I(z)$ and $M(z)$ are evaluated in

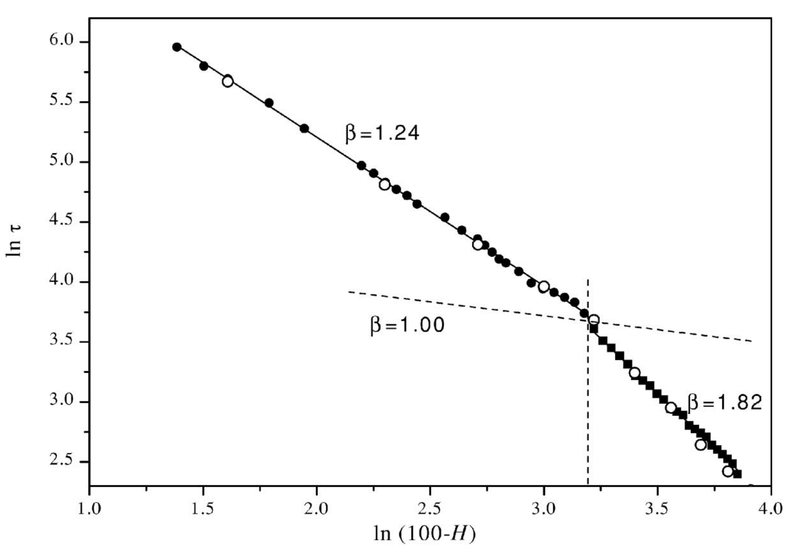

Fig. 4. Top: setup for our experiments on the humidity dependence of the drinking bird period. The bird is placed inside a closed transparent chamber. A digital chronometer and a digital hygrometer complete the setup. The humidity, which started around 50%, increased during the experiment. Bottom: the logarithm of the period, ln τ, vs ln$(100-H)$. Two different evaporation regimes were observed, one for high (closed circles) and the other for low (closed squares) humidities. The change from one regime to the other occurs at ln$(100-H)=3.2$. The least squares fit yields $\ln \tau = 9.44 - 1.82 \ln(100-H)$ for low humidities and $\ln \tau = 7.69 - 1.24 \ln(100-H)$ for high humidities. The open circles are the results of the simulation presented in Sec. IV.

the Appendix.

Because a realistic description of the bird's motion requires including frictional effects, we add to the torque a term proportional to the angular velocity, that is, we define $\mathcal{M}(z)=M(z)-b\omega$, where b is a phenomenological friction coefficient and ω is the angular velocity. In our model we take $b=7.5\times10^{-7}$ J s to account for the experimental damping, although the period is not very sensitive to this parameter. The angular acceleration $\alpha(t)$ is then given by

$$\alpha=\frac{\mathcal{M}(z)}{I(z)}, \tag{11}$$

and the angular velocity and the angle between the tube and the vertical direction are given by

$$\omega(t+\Delta t)=\omega(t)+\alpha\Delta t, \tag{12}$$

$$\theta(t+\Delta t)=\theta(t)+\omega(t+\Delta t)\Delta t, \tag{13}$$

according to the Euler–Cromer algorithm with Δt a finite but small time step.[13] For the numerical integration of Eq. (12) we used a time step $\Delta t=0.001$ s.

When $\theta=90°$, the internal liquid partly returns from the head to the body, and the angular velocity and height of the internal liquid are set to $\omega=0$ and $z=L/2$, respectively, before the new cycle starts. Accordingly, the temperature when a new cycle starts is $T_R-|\Delta T'|$, with $|\Delta T'|=0.211\,°\mathrm{C}$ calculated from Eq. (5) with $\Delta z=L/2$ (see Fig. 2). The value $\Delta T=-0.283\,°\mathrm{C}$ found in our simulation is consistent with that evaluated from Eq. (5) in Sec. II. Also the maximum z obtained in the simulation agrees with the experimental observation that the head was half-filled right before the dip.

Figure 5(a) shows the angle θ as a function of time, for an initial quasi-vertical and motionless bird and humidity $H=65\%$. The period is 19.0 s, in agreement with the experimental value 19.5 ± 0.5 s obtained from the data fit in Fig. 4. Figure 5(b) shows the same quantity for 85% humidity. The period is 74.5 s, close to the experimental value, 76.1 ± 0.5 s, also obtained from the data fit in Fig. 4. We conclude that our model reproduces the data very well.

The drinking bird is sometimes incorrectly presented as a perpetual motion machine and the graphs of Fig. 5 might cause the same misleading impression that the motion continues forever. In our model we assume that the head is wet, that is, there is external liquid in the head. For our drinking bird the water reservoir was large enough to keep it moving for days, so that the results of a model that takes into account the exhaustion of water in the reservoir should be presented using a very different time scale. There are other effects that are not accounted for by our simple model. The cooling mechanism inside the head is admittedly naive, probably the friction varies with angular velocity in a more complicated way than we have assumed, the evaporation rate changes with the angular velocity, and the internal liquid cannot be modeled as a solid.

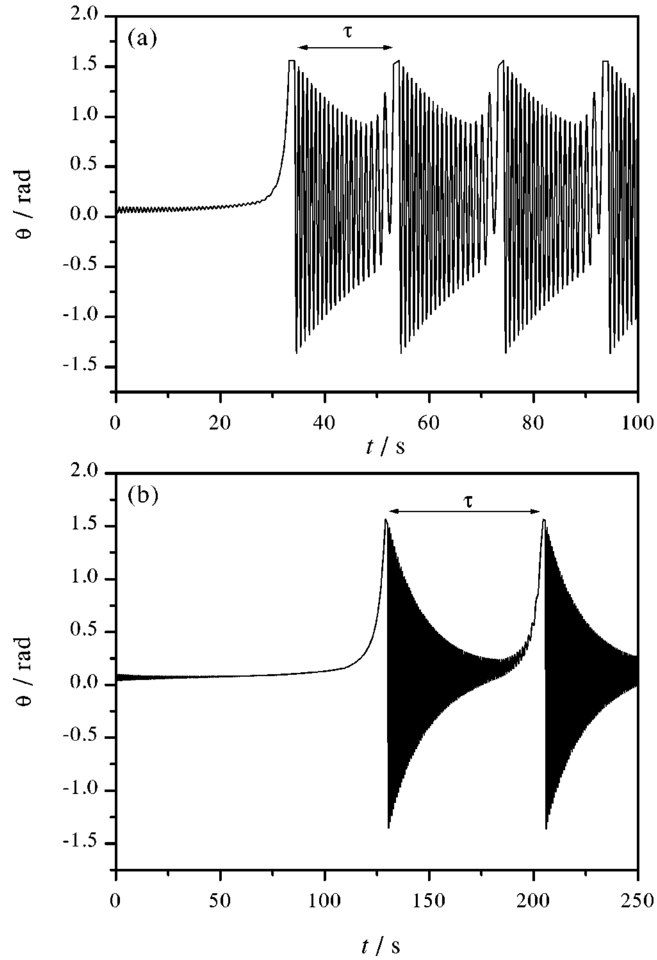

Fig. 5. Simulation of the evolution of the angle for a drinking bird with $m_E=0.4$ g and $m_{wb}=0.1$ g. (See the Appendix for the use of these parameters in the moment of inertia and torque.) (a) Humidity $H=65\%$ ($\tau=19.0$ s, the experimental value is 19.5 s) and (b) $H=85\%$ ($\tau=74.5$ s, the experimental value is 76.1 s). In both cases the initial conditions are $\theta=0.1$ rad and $\omega=0$ rad/s. Note that the time scale is not the same in (a) and (b). The period is almost four times longer for the higher humidity.

V. SUMMARY

The drinking bird is a thermal engine because it can be used to produce work from a temperature difference. The ability to produce work has its origin in the difference between the chemical potentials of the external liquid and its vapor. There are many thermal engines that operate directly by a temperature difference (for example, Carnot and Stirling cycles) or chemical nonequilibrium (internal combustion engines). But there are not many examples in which a chemical potential difference produces work without a chemical reaction.[14]

Although the drinking bird is a well-known device, only qualitative descriptions of its operation are available in the literature. In this work we presented a quantitative model of the bird's motion. We studied the influence of different external liquids on the bird's behavior, confirming that the period was smaller for more volatile liquids such as some organic liquids. For water as the external liquid, we observed how the period depends on the relative humidity and found two evaporation regimes: one at low humidities ($H<75\%$) and the other at high humidities ($H>75\%$). Finally, we presented a numerical integration of the equations of motion for a bird that drinks water. Our results reproduce well the measured periods. The simulation is pedagogically interesting for checking the importance of different parameters and for seeing the role of initial conditions (horizontal or vertical initial position).

ACKNOWLEDGMENTS

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APPENDIX: BIRD'S MOMENT OF INERTIA AND TORQUE

When the internal liquid reaches the height z inside the internal tube, the moment of inertia $I(z)$ with respect to the rotation axis may be approximated by (for notation, see Table I and Fig. 1)

$$I(z)=I_0+\left\{\left[V_I-z\pi\left(\frac{d_i}{2}\right)^2\right]\rho_I-m_I^H(z)\right\}\left(\frac{L}{2}\right)^2+z\pi\rho_I\left(\frac{d_i}{2}\right)^2\left[\frac{1}{12}z^2+\left(\frac{L}{2}-\frac{z}{2}\right)^2\right]+m_I^H(z)\left(\frac{L}{2}\right)^2, \quad (A1)$$

where we have used the parallel axis theorem. In Eq. (A1), I_0 is the moment of inertia for everything but the internal liquid, V_I is the internal liquid volume,

$$V_I=\frac{4}{3}\pi\left(\frac{d_i^B}{2}\right)^3-\frac{1}{3}\pi h_u^2\left(3\frac{d_i^B}{2}-h_u\right), \quad (A2)$$

and $m_I^H(z)$ is the mass of the internal liquid in the head, which depends on $z(t)$:

$$m_I^H(z)=\begin{cases}[z(t)-L]\pi\left(\frac{d_i}{2}\right)^2\rho_I & \text{if } z(t)\geqslant L\\ 0 & \text{if } z(t)<L.\end{cases} \quad (A3)$$

The second term of Eq. (A1) refers to the liquid that remains in the body, the third term describes the liquid in the tube, and the last term the liquid in the head.

The moment of inertia I_0 may be written as

$$I_0=m_g^B\left(\frac{L}{2}\right)^2+\frac{2}{3}m_g^B\left(\frac{d_i^B}{2}\right)^2+\frac{1}{12}m_tL^2+(m_g^H+m_h+m_E)\times\left(\frac{L}{2}\right)^2+\frac{2}{3}m_g^H\left(\frac{d_i^H}{2}\right)^2+(m_b+m_{wb})L_b^2. \quad (A4)$$

The masses in Eq. (A4) are $m_g^B=(4\pi\rho_g/3)[(d_E^B/2)^3-(d_i^B/2)^3]$, the mass of glass in body; $m_g^H=(4\pi\rho_g/3)\times[(d_E^H/2)^3-(d_i^H/2)^3]$, the mass of the glass in the head; $m_t=\pi\rho_g[(d_E/2)^2-(d_i/2)^2]$, the mass of the tube; m_h, the mass of the hat; m_E, the mass of the water on the head except for the beak; m_b, the mass of the beak; and m_{wb}, the mass of the water in the beak. In the last term of Eq. (A4),

$$L_b^2=l_b^2(1+\text{tg}^2\varphi), \quad (A5)$$

with $l_b\approx(d_E^H+L)/2$, and φ the angle shown in Fig. 1. The parallel axis theorem was used and, for the glass tube, the rotation axis was assumed to be at the tube's center.

The torque with respect to the fixed rotation axis is given by

$$M(z)=M_0-\left\{\left[V_I-z\pi\left(\frac{d_i}{2}\right)^2\right]\rho_I-m_I^H(z)\right\}\frac{L}{2}g\sin\theta-\rho_I\left[z\pi\left(\frac{d_i}{2}\right)^2\right]\frac{1}{2}(L-z)g\sin\theta+m_I^H(z)\frac{L}{2}g\sin\theta, \quad (A6)$$

with θ the angle between the vertical and the tube. In Eq. (A6) the moment of the ''fixed'' parts is

$$M_0=-m_g^B\frac{L}{2}g\sin\theta+(m_g^H+m_h+m_E)\frac{L}{2}g\sin\theta+(m_b+m_{wb})L_bg\sin(\theta+\varphi). \quad (A7)$$

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