A Backward-Wave Antenna Based on Negative Refractive Index L-C Networks

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1 Introduction

In the past few years, there has been significant interest in materials with simultaneously negative permeability and permittivity, also referred to as negative refractive index meta-materials [1], [2], [3], [4]. In 1968, Veselago theoretically investigated the propagation of electromagnetic waves in a substance with $\epsilon < 0$ and $\mu < 0$ [5]. He termed these materials "left handed media" (LHM) since the wave vector, $\vec{k}$, forms a left- handed triplet with the vectors $\vec{E}$ and $\vec{H}$. That is, phase velocity and the Poynting vector are antiparallel, which consequently results in counter-intuitive phenomena such as reversals of the conventional Doppler shift and Cherenkov radiation as well as reversed refraction. Subsequent work by Pendry proposes these negative refractive index meta-materials as unconventional lenses with interesting sub-wavelength resolving properties [1].

Recently, it has also been recognized that such LHM are related to guiding structures that support backward waves [1], [3], [4]. In a LHM, electromagnetic waves accumulate positive phase as they propagate away from a source. This phase advancement (backward-wave propagation) results in a backfire beam or reversed Cherenkov radiation as it is referred to by Veselago. Backward-wave (BW) radiating structures have been known for some time. Early examples include the bifilar helix antenna, corrugated surfaces or periodic arrays of radiating elements fed by slow wave transmission line (TL) sections of large periodic spacing ($d > \lambda/2$) [6], [7]. These structures operate above the Bragg frequency and cannot be treated using an effective medium approach. This is due to the fact that the structures do not operate in the long wavelength regime where the wavelength is much longer than both the element dimension and periodic spacing. As a result, effective material constants such as a refractive index are not typically defined. On the other hand, log-periodic dipole arrays and related uniform dipole arrays with a transposed feed have shown to produce backward-waves even for small longitudinal periodic spacing [8]. Nevertheless, the dipoles are resonant so the element dimension still remains electrically large. Therefore, the structure cannot be viewed as homogeneous.

2 Negative Refractive Index Transmission Line Networks

In [2], meta-materials comprised of straight metal wires and split ring resonators operating in the long wavelength regime are presented exhibiting a microwave frequency range with simultaneously negative values of effective permeability and permittivity. Recently, it has also been suggested that structures possessing an incremental length equivalent circuit which is the dual of that for a transmission line (see Figure 1(b)) exhibit propagation characteristics that resemble those within a medium with negative permittivity and permeability [4]. Since these structures operate in the long wavelength regime, they can be treated as

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an effective medium, unlike the backward-wave radiating structures mentioned earlier [6]. In [4], slow-wave 2-D L-C loaded transmission line networks in a high-pass configuration, supporting propagating backward-waves, are used to demonstrate negative refraction and focusing. In this paper, a 1-D L-C loaded transmission line network supporting fast backward-wave propagation is used to demonstrate characteristics analogous to "reversed Cherenkov radiation".

An analogy is readily drawn between TEM propagation on transmission lines and plane-wave propagation in a homogeneous isotropic medium with positive material parameters, ϵ and μ. Comparing the differential equations governing the propagation characteristics in both cases, the distributed L,C parameters of a conventional transmission line (see Figure 1(a)) can be related to the permitttivity and permeability of the medium in the following manner:

$$\epsilon = C \, , \; \mu = L \tag{1}$$

This transmission line representation of a medium with positive material parameters, ϵ and μ, offers insight into devising materials with negative ϵ and μ. Intuition would suggest that in order to achieve such a medium ($\epsilon < 0$ and $\mu < 0$), the series reactance and shunt susceptance shown in Figure 1(a) should become negative given that the material parameters are directly proportional to these circuit quantities. This change in sign implies the dual transmission line representation shown in Figure 1(b) for a medium with negative material parameters.

As for the conventional transmission line, analogous expressions for permittivity and permeability can be derived for the dual transmission line structure. The equivalent expressions for permittivity and permeability of the dual structure are indeed negative quantities:

$$\epsilon = \frac{-1}{\omega^2 L \quad x} \, , \; \mu = \frac{-1}{\omega^2 C \quad x} \tag{2}$$

The characteristic impedance of the dual structure shown in Figure 1(b), however, remains the same as in the case of an ordinary transmission line:

$$Z_o = \sqrt{L/C} \tag{3}$$

Simple circuit analysis of the dual transmission line structure shows that given positive power flow (in the positive x direction), the propagation constant, β or equivalently the refractive index, n, must be a negative quantity. The propagation constant and group velocity (v_g) for the dual transmission line structure are given by the following expressions [9]:

$$\beta = \frac{-1}{\omega\sqrt{LC} \quad x} \, , \; v_g = \omega^2 \sqrt{LC} \quad x \tag{4}$$

Since the propagation constant is negative and the group velocity is positive, the structure supports backward-waves. In addition, as is expected for media with negative material constants, the dual transmission line structure exhibits frequency dispersion.

3 Design

By appropriately choosing the circuit parameters of Figure 1(b) such that $|\beta| < k_o$, a basically fast-wave structure can be designed that supports a fundamental TM spatial harmonic that radiates toward the backfire direction. A physically realizable antenna structure would, however, include interconnecting transmission lines between adjacent lumped element series capacitors and shunt inductors. The corresponding dispersion relation for such a structure is presented graphically by means of the Brillouin diagram shown in Figure 2 for representative transmission line and L,C lumped element loading parameters. From Figure 2, it is evident that above the lowest cutoff frequency, the phase velocity is negative for positive group velocities (i.e. positive slopes), along the loaded transmission line structure. Therefore, backward waves are supported in the lowest passband and the structure resembles a medium with negative refractive index.

The array is a fast-wave structure of a high-pass configuration with similar dispersion characteristics to that of a rectangular waveguide periodic slot array with reversed elements [7]. With a progressive increase of $k_o d$, the low frequency stopband is overcome and the phase delay per cell increases from $-\pi$ at the bottom of the lowest passband. Eventually the fundamental n=0 harmonic enters the fast-wave region and a beam appears in the visible space in the backfire direction. As $k_o d$ is further increased, the n=0 harmonic scans from backfire through to the stopband at broadside.

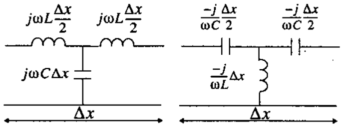
The proposed coplanar waveguide implementation of a radiating structure based on Figure 1(b) is shown in Figure 3. The dimensions are given for a printed structure at 15 GHz on a 20 mil substrate of $\epsilon_r = 3$. It is important to note that the unit cell dimension is much smaller than a free space wavelength for this structure. The commercial method of moments software HP-ADS (Hewlett-Packard Advanced Design System) was used in designing the unit cell layout. The gaps in the CPW feed line serve as the series capacitors and the narrow lines connecting the centre conductor to the coplanar ground planes act as the shunt inductors depicted in Figure 1(b). It is in fact the capacitive gaps that radiate in this structure while the inductive lines are non-radiating due to the anti-parallel currents flowing on each pair of inductive lines. This odd symmetry causes cancellation in the far-field and leads to low cross-polarization levels.

4 Simulation Results

A 16 element array of the unit cells shown in Figure 3 has been simulated using HP-ADS. The simulated E-Plane pattern at 15 GHz is shown in Figure 4. Radiation towards the backfire direction in the far-field pattern clearly indicates the excitation of a backward wave. Since the transverse dimension of the array is electrically short, the structure is backed by a long metallic trough as was demonstrated in [10]. The trough acts as a waveguide below cut-off and recovers the back radiation resulting in unidirectional far-field patterns.

5 Conclusion

A novel backward-wave radiating structure inspired by negative refractive index L-C materials has been proposed. A CPW-based printed configuration is presented with preliminary method of moments simulation results. Experimental results will be shown at the time of the symposium.



(a) TL (b) Dual TL

Figure 1: Incremental Length Equivalent Circuits

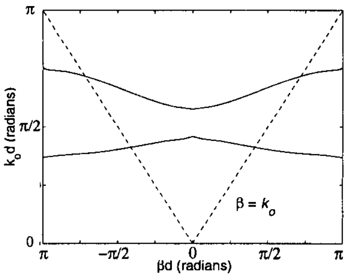


Figure 2: k_o - β Diagram

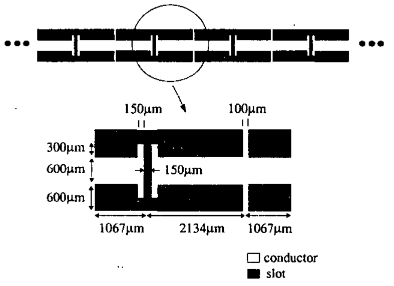


Figure 3: CPW-based BW Antenna

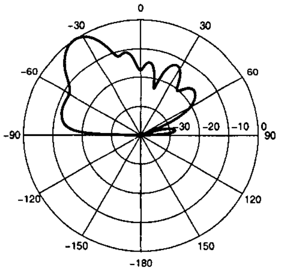


Figure 4: Simulated E-Plane at 15 GHz

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