NANO INDENTER ANGULAR MEASUREMENTS

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Diamond nano indenters are made with precise angular geometry in order to achieve the highly accurate readings required in nano indentation. They are very small, some less than 50 microns, because low mass is an important requirement. Instruments that measure angles on larger objects such as protractors or comparators are not practical nor precise enough to measure nano indenter angles even with the help of microscopes.

Micro Star Technologies (MST) has developed a laser goniometer especially designed to precisely measure diamond nano indenter angles. Nano indenter faces are highly polished and reflective which is the basis for the laser goniometer measurements. MST has built several of these instruments and uses them to monitor production and provide accurate angular measurements of finished nano indenters.

Complementing the goniometer measurements are mathematical formulations that provide derived angles and other functions needed to precisely interpret nano indentation results.

The principles of this paper apply to 3 and 4 sided pyramids, made of diamond, sapphire or other hard materials. The discussion is centered on 3 sided pyramids but the same measurements and calculations can be extended to 4 sided pyramids. Here a 3 sided pyramid is also called a “tip”.

THE LASER GONIOMETER

This instrument uses a visible light laser and high resolution angle encoders to measure tip angles. A goniometer measurement gives the angular position of the line ‘A’ with respect to a frame of reference as shown on Fig 1. ‘A’ is the perpendicular to a tip face. Two angles are determined from each measurement, a tilt angle ‘a’ and a rotation angle ‘b’.

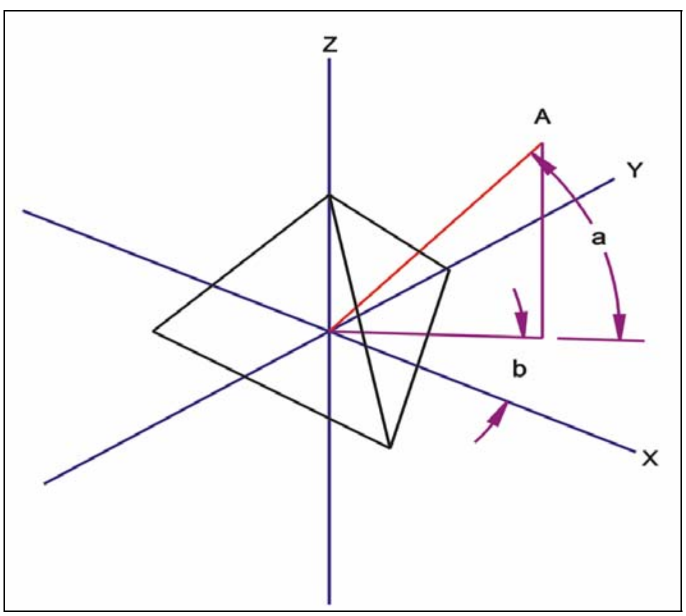

Fig. 1. Angular measurement of line ‘A’ (red) in a XYZ coordinate frame. Two angles are defined: a is the angle between A and the XY plane; b is the angle between the projection of A on the XY plane and X. Angle ‘a’ is A’s tilt angle; b is A’s rotation angle.

The goniometer measurements resolution is 0.001°. The accuracy of the measurements depends on the goniometer calibration which is a comparison with a gauge whose angular dimensions are known. At MST two gauges are used for goniometer calibration.

The first gauge is a pellicle which is a stretched membrane 2µ thick[1]. Thickness variations are less than 400nm across 25mm. This membrane is useful as a calibration gauge because the perpendicular A_F is 180.000° from A_R , (Fig. 2) with an error considerably smaller than the goniometer resolution of 0.001°[2].

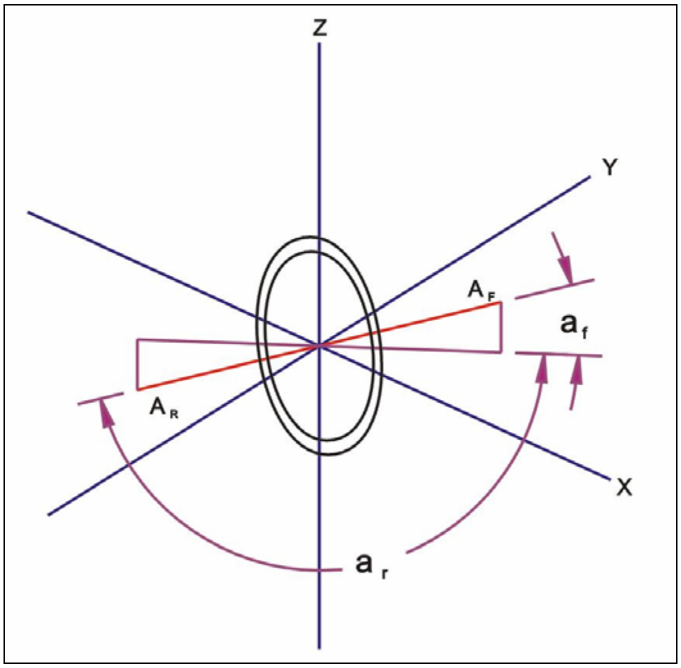

Fig. 2. A pellicle used as a gauge. The sum of angles a_f and a_r is 180.000°. A_F is the front perpendicular line and A_R the rear perpendicular.

The second gauge is a cube corner prism whose angular dimensions have been measured and certified by NIST[3]. See Appendix C for data on NIST certification of this prism.

The accuracy of the goniometer measurements ultimately rest on the demonstrated uncertainty of the measurements performed with the instrument. The following definition is used to calculate the uncertainty value for a given set of measurements. Suppose a number of measurements N of an angle 'a' $a_1, a_2, a_3 \ldots a_N$ is taken. Each measurement taken a_i may be slightly different from the others. The mean value $\bar{a}$ of the measurements is:

$$\bar{a} = \frac{(a_1 + a_2 + a_3 \cdots a_N)}{N} \qquad (1)$$

The standard deviation σ_a of the measurements is:

$$\sigma_a = \sqrt{\frac{1}{N-1}\sum_1^i (a_i - \bar{a})^2} \qquad (2)$$

and the uncertainty $\sigma_{\bar{a}}$ is:

$$\sigma_{\bar{a}} = \frac{\sigma_a}{\sqrt{N}} \qquad (3)^4$$

A numerical example of these formulas is shown on Table 1 where 8 angle measurements were taken. The actual uncertainty calculated with formula (3) is 0.0025°. MST follows the common practice of multiplying it by 2 to enhance the level of confidence on the uncertainty. Therefore the value given is 0.005°.

MEASURM. 1	**35.271°**
2	**35.290°**
3	**35.285°**
4	**35.271°**
5	**35.279°**
6	**35.280°**
7	**35.271°**
8	**35.283°**
AVERAGE:	**35.279°**
ST.DEV:	**0.007°**
UNCERTAINTY (x2):	**0.005°**

Table 1. Example of uncertainty calculation on 8 angle measurements.

To summarize, after measuring an angle 8 times, the accepted value becomes 35.279° with a 0.005° uncertainty. This means that the actual angle could have any value between 35.282° and 35.276°.

NIST has given 0.000125° as the uncertainty on their measurements of the prism certified for MST.

3 SIDED PYRAMID GEOMETRY

As shown on Fig. 3, the theoretical geometry of a regular 3 sided pyramid can be defined by the angle between a face and the center line (blue) which is called the half angle 'hlf'. The 3 half angles are identical. The base is an equilateral triangle and the 3 angles are 60°. The front projection angle 'fr' and side angle 'sd' are unique for a given half angle.

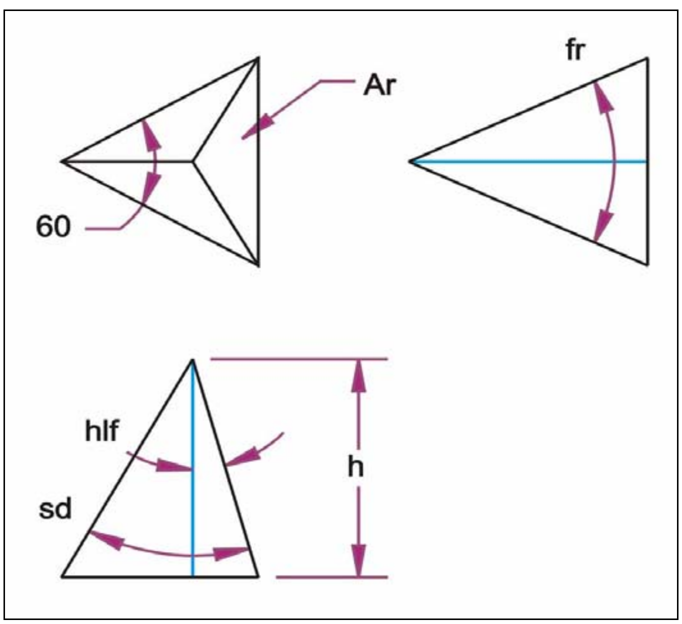

Fig. 3. Orthogonal views of a regular 3 sided pyramid.

There is an important parameter derived from a tip geometry called the area function coefficient f:

$$f = \frac{A_r}{h^2} \tag{4}$$

$$A_r = f \times h^2 \tag{5}$$

Where Ar is the area of the tip base and h is the tip height. This function comes into play when an indentation is made and the depth 'h' of the indentation is known. The area of the indenter footprint is calculated from h.
For instance for a cube corner tip with a half angle of 35.264°, fr = 101.537°, sd = 90.000° and f = 2.598. In this example, if h = 50nm, Ar = 6,495.0nm^2. These values are readily calculated with computer graphics or trigonometry.

A real tip indenter on the other hand, can never be made to exact theoretical dimensions. The 3 half angles may differ from the nominal and from each other. Similarly the base may not be an equilateral triangle and its angles may differ from 60°. On a theoretical tip, the center line passes through the tip apex and through the center of the base, both well defined points. On a real tip, the location and orientation of the base and the center line are not readily defined.

A real tip presents additional variables regarding its center line or axis. All indenter tips are attached to a holder whose axis is presumed to coincide with tip center line or at least be parallel to it. This would allow the tips to exhibit the same properties when transferred from one measuring or indenting instrument to another.

One possible variable is the attachment of the tip to its holder by the manufacturer. There may be a small angle between the actual tip center line and the holder's center line. This angle is designated 'e'.

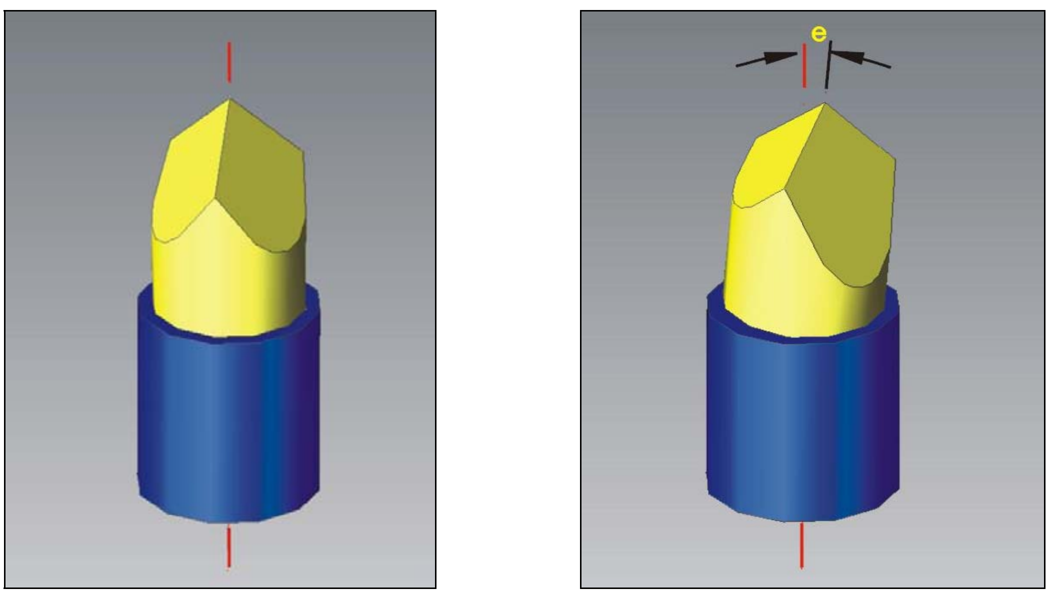

Fig. 4a (left), a theoretically perfect tip. Fig. 4b (right) a real tip.

Fig. 4 illustrates a theoretical and a real tip. Fig. 4a shows a theoretically perfect tip where the tip axis coincides with and holder axis and the 3 faces are identical. Fig. 4b shows an exaggerated view of a real tip where the tip axis differs from the holder axis by angle 'e' and the faces are uneven.

GONIOMETER MEASUREMENTS

To measure real tip angles, the tip is attached to the goniometer with its holder axis aligned with the goniometer Z coordinate. The laser goniometer determines the perpendicular vector to each one of the tip faces and measures its angular position with respect to the XYZ coordinate system. As illustrated on Fig. 5, each perpendicular vector A_1, A_2 or A_3 has two associated angles, 'a' and 'b'. Vector A_1 tilt angle is a_1 and its rotation angle is b_1. See Fig. 1 for definitions of these angles.

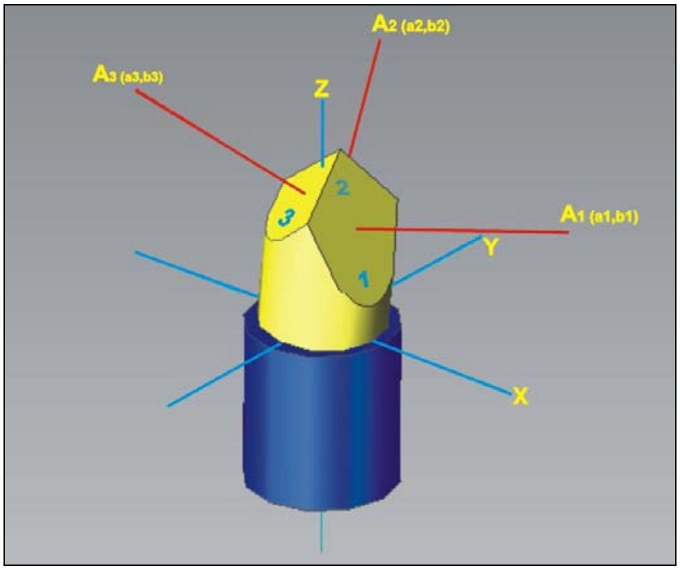

Fig. 5. Three vectors perpendicular to the tip faces and their associated angles. The tip holder axis is aligned with the Z coordinate direction.

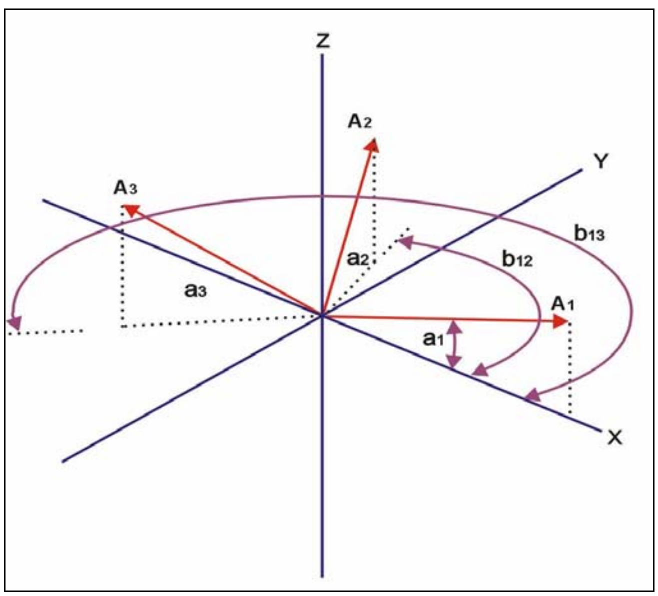

Fig. 6. The three face vectors, their tilt and rotation angles in the goniometer coordinate system.

The tip vectors shown on Fig. 6 illustrate the tilt and rotation angles on the XYZ coordinate system. Notice that the face designated as A_1 is aligned with the X axis so its rotation angle b_1 is set to 0°. This leaves 5 angles to be measured, three tilt angles and two rotation angles (a_1, a_2, a_3 ,b_{12}, b_{13}). These are the five basic measurements performed with the goniometer on a 3 sided pyramid. A four sided pyramid requires four tilt and three rotation angles.

Fig. 7 shows that the face tilt angle is the same as the angle between the face and the tip axis. On a theoretically perfect tip the three tilt angles are the same and identical to the tips nominal half angle. On a real tip the three tilt angles may differ and are measured separately.

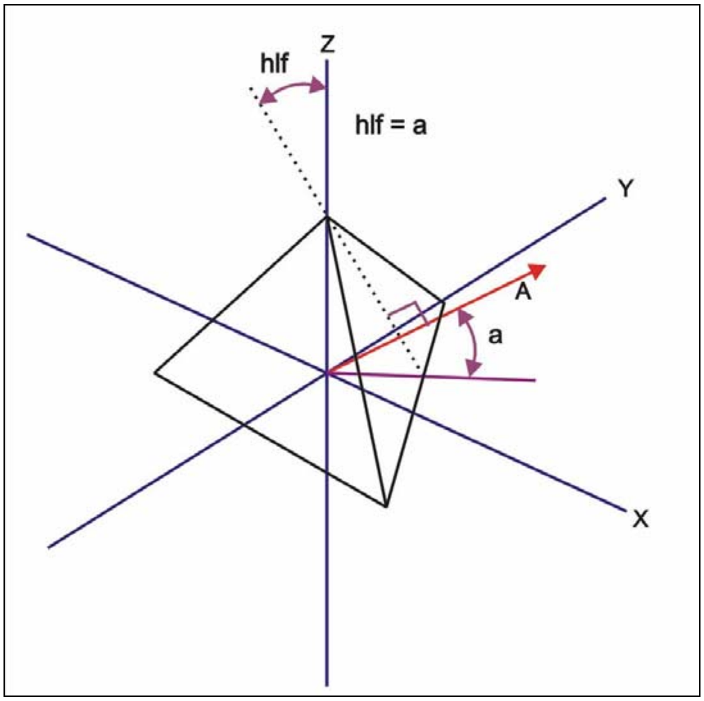

Fig. 7. The face tilt angle 'a' is the same as the angle between the face and the tip axis or half angle 'hlf'.

TABULATION AND ANALYSIS

Tables 2 and 3 are examples of the spread sheet where tip measurements are recorded and mathematical formulations are applied to analyze the tip geometry and its deviations from theoretical or nominal values. Table 2 shows the measurements of a cube corner tip and Table 3 of a Berkovich tip. The following is a detailed description of these tables.

ROW 5 contains the symbols for all the angles measured or calculated, plus the area function coefficient 'f'.

ROW 6 contains the nominal tip values to be measured and calculated. The half angle nominal value of a cube corner tip is 35.264° which is also the nominal value of the three tilt angles a_1, a_2 and a_3. A Berkovich tip half angle is 65.270° The nominal rotation values of any three sided tip are 120.000° for b_{12} and 240.000° for b_{13}.

The units of all numerical values in the table are in decimal degrees, except 'f' which is unitless. All angle values, measured or calculated, are given with three decimal places. This stems from the goniometer resolution which is 0.001°. The coefficient 'f' is given with six decimals places, compatible with the area calculation.

ROWS 7 to 16 at COLUMNS B to F contain all the tip measurements taken with the goniometer. In the examples shown six sets of measurements were taken.

When measuring, the tip is installed in the goniometer and a set of measurements are taken, then the tip is removed from the goniometer. This procedure is repeated for each set of measurements. This ensures that mechanical, electronic, environmental, and procedural inconsistencies appear as measurement variations, as can be seen on the measurements values.

ROW 17, COLUMNS B to F contain the measurements averages according to formula (1) on page 3 with N = 6. These are the most accurate angle values assigned to the tip based on the measurements. All the calculated values on the rest of the table are calculated from these five average angle values.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1							3 SIDED INDENTER MEASUREMENTS AND ANALYSIS											
2	SERIAL. #:	TC-0102	DESCRIPTION:				CUBE CORNER (TC)							INI:		BM	DATE:	2/18/2007
3				MEASURED VALUES							CALCULATED RESULTS							
4			TILT ANGLES		ROTATION ANGLES		E ANGLES			E TILT ANGLES		E ROTATION ANGLES			FACE TO FACE ANGLES		AREA FUNCT.	AREA FUNCT.
5		a_1	a_2	a_3	b_{12}	b_{13}	e_T	e_R	a_{e1}	a_{e2}	a_{e3}	b_{e12}	b_{e13}	c_{12}	c_{23}	c_{31}	f	fe
6	NOMINAL VALUES:	35.264	35.264	35.264	120.000	240.000	0.000		35.264	35.264	35.264	120.000	240.000	90.000	90.000	90.000	2.598001	2.598001
7	MEASUR 1	35.337	34.930	35.538	119.667	239.599												
8	2	35.363	34.936	35.516	119.665	239.557												
9	3	35.323	34.945	35.539	119.703	239.621												
10	4	35.354	34.908	35.540	119.651	239.581												
11	5	35.354	34.936	35.517	119.674	239.546												
12	6	35.331	34.925	35.537	119.692	239.601												
13	7																	
14	8																	
15	9																	
16	10																	
17	AVERAGE:	35.344	34.930	35.531	119.675	239.584	0.356	101.824	35.270	35.268	35.267	119.506	239.507	89.992	89.996	89.994	2.598899	2.598792
18	DIFF FROM NOMINAL:	0.080	-0.334	0.267	-0.325	-0.416	0.356		0.006	0.004	0.003	-0.494	-0.493	-0.008	-0.004	-0.006	0.000897	0.000791
19	ST.DEV:	0.016	0.013	0.011	0.019	0.029												
20	UNCERTAINTY (X2):	0.013	0.010	0.009	0.016	0.023												

TABLE 2. CUBE CORNER INDENTER

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1							3 SIDED INDENTER MEASUREMENTS AND ANALYSIS											
2	SERIAL. #:	TC-0102	DESCRIPTION:				BERKOVICH (TB)							INI:		BM	DATE:	4/12/2007
3			MEASURED VALUES								CALCULATED RESULTS							
4			TILT ANGLES		ROTATION ANGLES		E ANGLES			E TILT ANGLES		E ROTATION ANGLES		FACE TO FACE ANGLES			AREA FUNCT.	AREA FUNCT.
5		a_1	a_2	a_3	b_{12}	b_{13}	e_T	e_R	a_{e1}	a_{e2}	a_{e3}	b_{e12}	b_{e13}	c_{12}	c_{23}	c_{31}	f	fe
6	NOMINAL VALUES:	65.270	65.270	65.270	120.000	240.000	0.000		65.270	65.270	65.270	120.000	240.000	42.482	42.482	42.482	24.494345	24.494345
7	MEASUR 1	65.340	65.284	65.135	120.121	239.725												
8	2	65.320	65.296	65.125	120.132	239.777												
9	3	65.325	65.281	65.139	120.099	239.723												
10	4	65.339	65.294	65.116	120.141	239.743												
11	5	65.328	65.282	65.125	120.115	239.727												
12	6	65.333	65.283	65.130	120.094	239.709												
13	7																	
14	8																	
15	9																	
16	10																	
17	AVERAGE:	65.331	65.287	65.128	120.117	239.734	0.098	200.288	65.239	65.303	65.204	119.833	239.795	42.443	42.502	42.610	24.445127	24.445003
18	DIFF FROM NOMINAL:	0.061	0.017	-0.142	0.117	-0.266	0.098		-0.031	0.033	-0.066	-0.167	-0.205	-0.039	0.020	0.129	-0.049218	-0.049342
19	ST.DEV:	0.008	0.007	0.008	0.018	0.024												
20	UNCERTAINTY (X2):	0.006	0.005	0.007	0.015	0.019												

TABLE 3. BERKOVICH INDENTER

ROW 18 contains the difference between the nominal values and the measured angles or calculated results through the rest of the table. This row shows at a glance how much the tip varies from nominal.

ROW 19 contains the standard deviation calculated from the measured values according to formula (2).

ROW 20 contains the calculated uncertainty based on formula (3) and multiplied times two (see page 3).

ROW 17, COLUMNS G to R contains values calculated from the measurement averages of columns B to F. Cells G7 to R16 are blank because no calculations are done on the individual measurements, only on the resulting averages of Row 17.

In Figs. 5 and 6, tip angle measurements result in three vectors. Each vector is defined by two angles, tilt and rotation, a and b. The vectors magnitude is not defined by the measurements since the only consideration is angular direction. In order to perform other calculations the vectors magnitude is set = 1 and then converted to Cartesian coordinates.

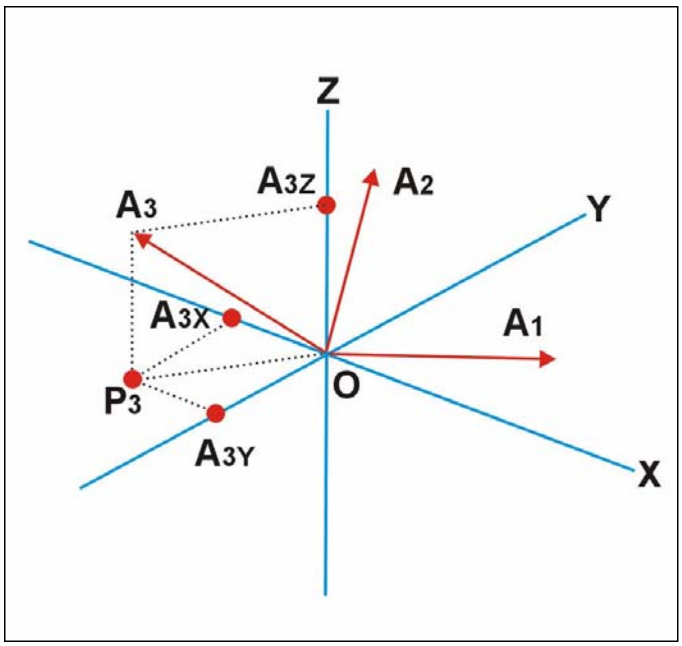

Fig. 8. Vector A3 projected coordinates.

Fig. 8 shows as example vector A_3 with its projections A_{3X}, A_{3Y} and A_{3Z}. P_3 is the projection of A_3 on the XY plane. Vector A(a,b) is converted to A: (A_X, A_Y, A_Z) as follows:

$$A_X = A\cos(a)\times\cos(b) \tag{6}$$

$$A_Y = A\cos(a)\times\sin(b) \tag{7}$$

$$A_Z = A\sin(a) \tag{8}$$

In particular for each vector,

$$A_{1X} = \cos(a_1)\cos(b_1), \quad A_{1Y} = \cos(a_1)\sin(b_1), \ldots \text{ etc}$$

The magnitude of the coefficient ‘A’ of equations (6), (7) and (8) is =1. The remaining calculations on the Table are performed using these Cartesian coordinates of the measured vectors A_1, A_2 and A_3.

COLUMNS G and H pertain to the calculated angles e_T and e_R. As illustrated on Fig. 4, a real tip axis may not coincide with the holder axis. The angular position of a real tip axis (vector 'E') is calculated and given on cells G17 and H17. The angle between E and the holder axis is e_T. The rotational position of E is given by e_R. The nominal value of e_T is 0° and there is no nominal value for e_R.

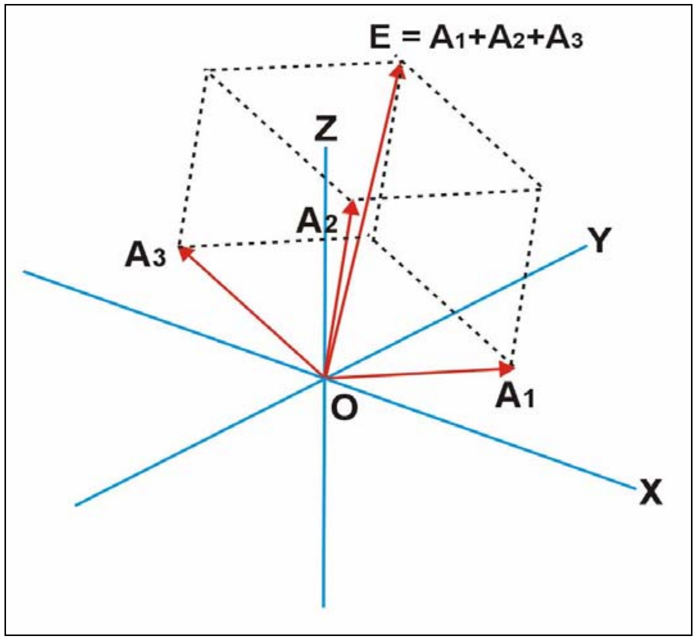

Fig. 9. Vector E obtained by adding the three measured vectors.

On Table 2 vector E is obtained by adding the three measured vectors A_1, A_2 and A_3 (Fig. 9). Trigonometric calculations provide the resulting angles e_T and e_R.

All calculations and formulae are verified by building virtual three dimensional computer models of both theoretical and real tips. For instance, a virtual tip was built with the exact angles of Row 17 B to F. The virtual tip axis angles were measured and verified to be identical with the calculated values of G17 and H17.

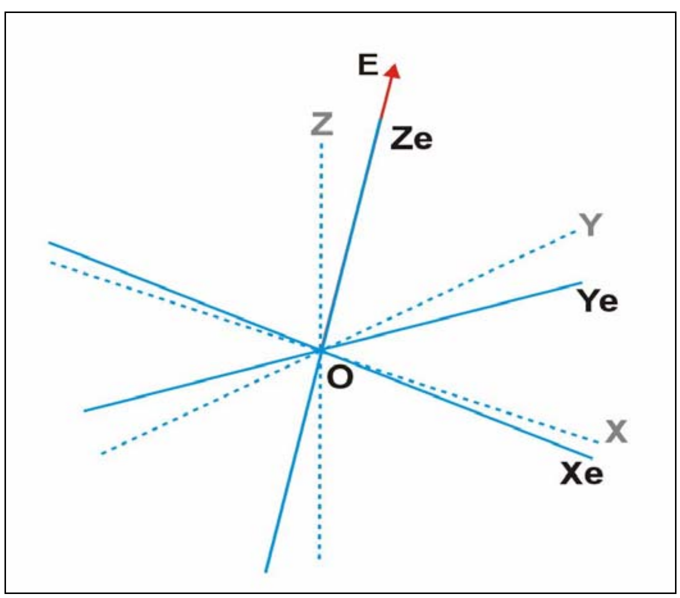

Fig. 10. The 'e' coordinate system aligned with the real tip axis E.

COLUMNS I, J and K pertain to the calculated angles a_{e1}, a_{e2} and a_{e3}. These are the face vectors tilt angles measured on a new coordinate system where the X, Y and Z axis are replaced by Xe, Ye, and Ze axis. This 'e' coordinate system is constructed by aligning the Ze axis with vector E, as shown on Fig. 10. See Appendix A for the mathematical formulae to derive the 'e' coordinates.

The axis of an irregular three sided pyramid is a line passing through the apex and a point at the centroid (G) of a the triangular base defined by the ends of the three edge lines (L) of equal length (Fig. 11). Since the pyramid is irregular this triangle may not be equilateral.

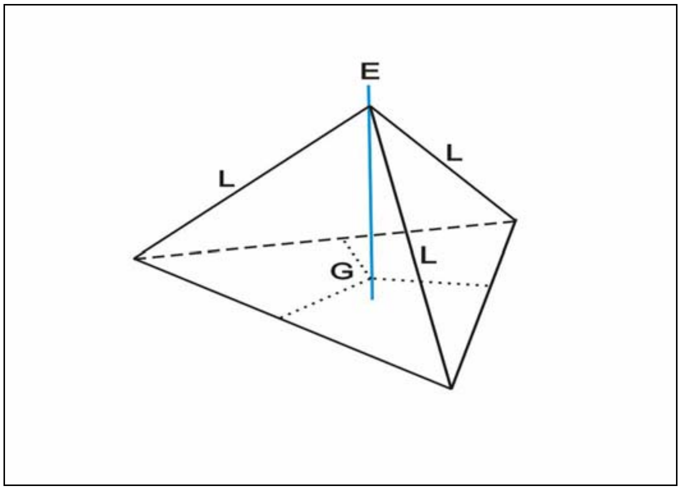

Fig. 11. Axis of an irregular three sided pyramid.

COLUMNS L and M pertain to the calculated angles b_{12} and b_{13}, which are the rotation angles measured on the 'e' coordinate system. The tilt and rotation angles calculated on columns I to M are obtained using the reverse trigonometric procedure of formulae (6), (7) and (8), translated to the 'e' coordinate system. These are the real tip angles based on its true axis E. If the tip was perfectly aligned with its holder, e_T would be = 0 on G17, and the 'e' angles on G17 to M17 would be identical to the measured angles on B17 to F17.

COLUMNS N, O and P pertain to the three face to face calculated angles c_{12}, c_{23} and c_{31}. These are the angles in three dimensional space between vectors A_1, A_2 and A_3. The angle between A_1 and A_2 is $c_{12.}$ C_{23} and c_{31} are defined similarly (see Fig. 12).

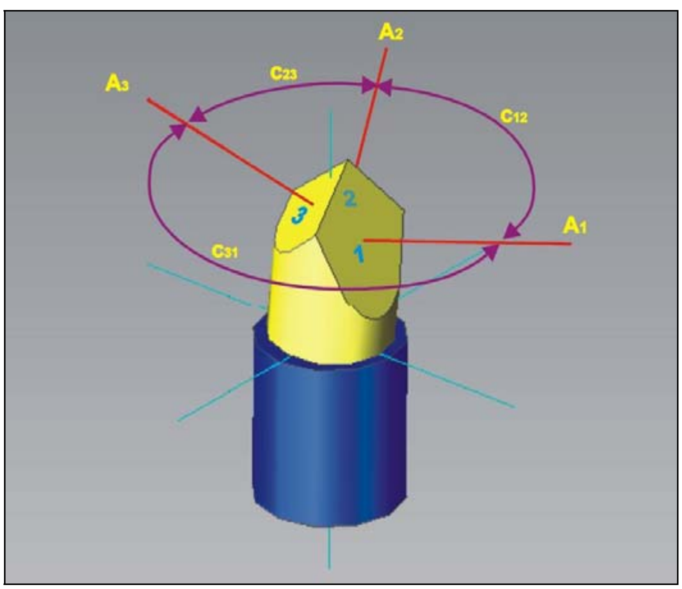

Fig. 12. Face to face angles 'c'.

The face to face angles are independent of a coordinate system and serve as a verification of the tip angular geometry. For instance, the 'c' angles of a cube corner tip, such as the one measured on Table 2, are nominally 90.000°. The calculated angle values are on N17 to P17 and their difference from the nominal are on N18 to P18.

<u>COLUMNS Q and R</u> pertain to the area function calculations using formulae (4) and (5). The nominal area function coefficient 'f' of a perfect tip is readily calculated on Q5 and R5. On Q17 and R17 are the values of 'f' for the measured tip.

Column Q refers to 'f' calculated from the measured angles from B17 to F17. This means that the coefficient is calculated in the XYZ coordinate system and not in the Xe Ye Ze. As shown on Fig. 13, the height 'h' is parallel to the Z axis (same as the holder axis) and the base triangle is perpendicular to it.

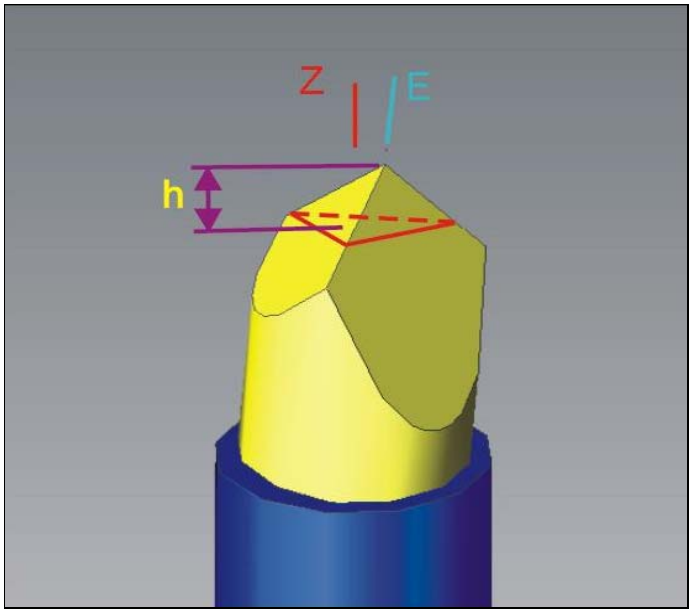

Fig. 13. Area function based on the holder axis 'Z'.

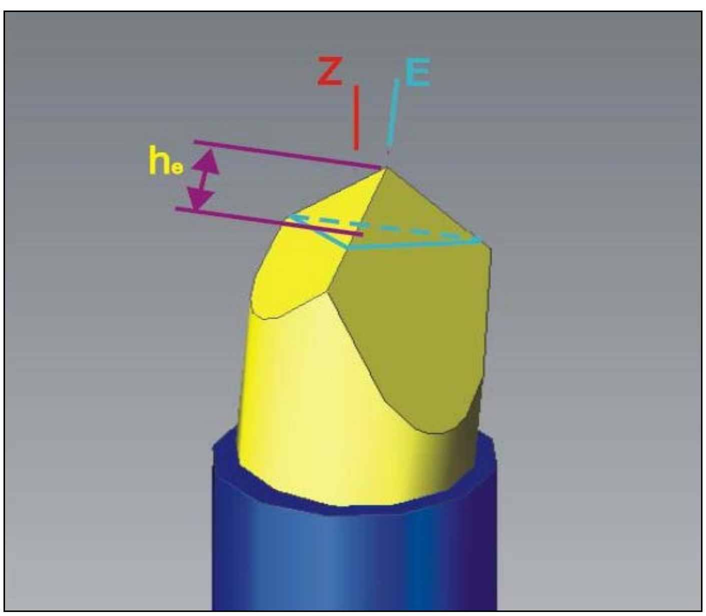

Fig. 14. Area function based on the tip axis 'E'.

On column R, the calculations are based on the ‘e’ coordinate system and stem from the angles on I17 to M17. The value of this coefficient ‘f_e’ will vary depending on how large the angle ‘e’ is. If e = 0 then $f = f_e$. As shown on Fig. 14, the height ‘h’ is aligned with the tip axis ‘E’ and the base triangle is perpendicular to it.

In practice when the tip is used as an indenter, ‘f’ is the coefficient of interest, not ‘f_e’, because the indentation can only be made in alignment with the holder. The value of ‘f_e’ is calculated as a reference.

The mathematical derivation of ‘f’ is given on Appendix B.

SUMMARY

The laser goniometer enables precise nano indenter angle measurements. Nano indenters are usually sub millimeter in size therefore other measuring technologies can not be applied with precision.[5]

Mathematical formulations, complementing the measurements, are used to calculate additional geometrical nano indenter parameters. These are the true indenter axis and its related angles which may differ from the measured ones. The area function coefficient (usually derived empirically) is also mathematically calculated from the goniometer measurements. The calculations on Appendixes A and B, plus others not shown, were derived with trigonometric and vector analysis.

All the mathematical formulae used in the calculations have been verified with computer three dimensional models. A tip virtual model is built based on its measured angles. The derived parameters are measured on the model and verified identical to the mathematically calculated parameters.

Several laser goniometers are used at Micro Star Technologies for nano indenter manufacturing and testing. Goniometer measuring and data analysis are available as a service for most types of 3 and 4 sided indenters, whether made by Micro Star or by other manufacturers..

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[1] The pellicle is a nitrocellulose membrane, supplied by “THOR LABS”, www.thorlabs.com.

[2] A pellicle thickness change of 400nm in 25mm results in a 0.0000016° error.

[3] NIST: National Institute of Standards and Technology.

[4] Taylor, John R. (1996) ‘Introduction to Error Analysis, the study of uncertainties in physical measurements’, 109-110, University Science Books, Sausalito, California.

[5] To accept it for certification, NIST required a large size prism (50mm). Their instrument, a “High Accuracy Coordinate Measuring Machine” (CMM) could not be used effectively on smaller prisms.

APPENDIX A

COORDINATE TRANSFER FUNCTION

Let there be a point 'A' to be transferred from a coordinate system XYZ to a new coordinate system XeYeZe.

$$A: (X, Y, Z) \rightarrow Ae: (Xe, Ye, Ze)$$

As illustrated on Fig. 1C, the coordinate system XeYeZe has the same origin as XYZ but its Ze axis has been tilted an angle e_T with respect to the Z axis. The direction of tilt of Ze is defined by the angle e_R measured on the X Y plane.

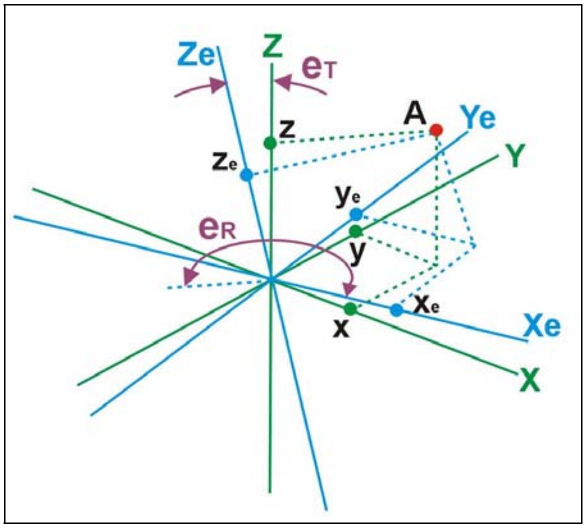

Fig. A1. Transfer of point A from the Z (green) to the Ze (blue) coordinates.

The following equations calculate (Xe, Ye, Ze) from (X, Y, Z) and angles e_T and e_R.

$Xe = Xex + Xey + Xez:$

$$Xex = X - X \cos e_R^{\,2} (1 - \cos e_T)$$

$$Xey = -Y \cos e_R \sin e_R (1 - \cos e_T)$$

$$Xez = -Z \sin e_T \cos e_R$$

$Ye = Yex + Yey + Yez:$

$$Yex = -X \cos e_R \sin e_R (1 - \cos e_T)$$

$$Yey = Y - Y \cos e_R^{\,2} (1 - \cos e_T)$$

$$Yez = -Z \sin e_T \sin e_R$$

$Ze = Zex + Zey + Zez:$

$$Zex = -Z \sin e_T \cos e_R$$

$$Zey = -Z \sin e_T \sin e_R$$

$$Zez = Z \cos e_T.$$

APPENDIX B

AREA FUNCTION DERIVATION
3 SIDED TIP

Given an irregular three sided pyramid, with the three measured vectors A_1, A_2 and A_3, find the coefficient 'f' such that the area 'Ar' of the pyramid base can be calculated with the formula:

$$A = f \times h^2$$

Let a pyramid with apex at 'O', be defined by the height 'h' and base sides 'm', 'n' and 'p', as in Fig B1. O is at the origin of the coordinate system XYZ. The three unit vectors A_1, A_2 and A_3 are perpendicular to the three pyramid faces. The three edges, not part of the base are vectors V_1, V_2 and V_3.

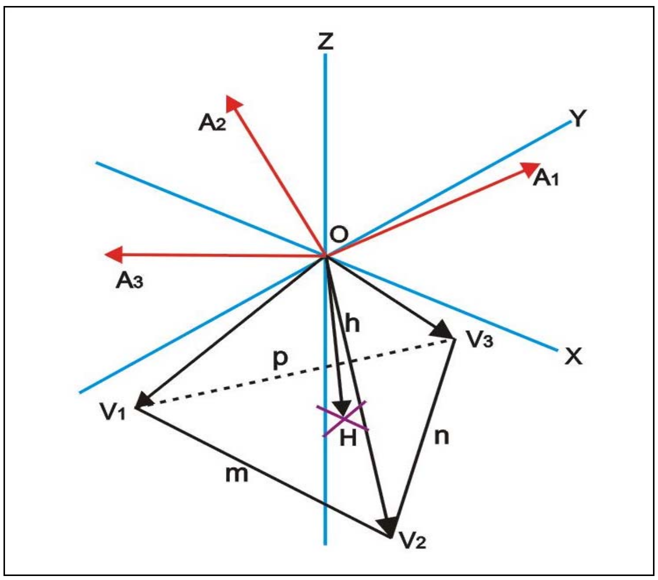

Fig. B1 Irregular three sided pyramid.

The three unit vectors A1, A2 and A3 are known from the laser goniometer measurements. Their end coordinates are calculated with the formulae of page 9.

A_1: (A_{1X}, A_{1Y}, A_{1Z})

A_2: (A_{2X}, A_{2Y}, A_{2Z})

A_3: (A_{3X}, A_{3Y}, A_{3Z})

The edges V are perpendicular ($\perp$) to vectors A as follows,

$V_1 \perp A_2$ and A_3

$V_2 \perp A_1$ and A_3

$V_3 \perp A_1$ and A_2

The end coordinates of vector V are calculated with the cross product (**X**) of the two vectors A perpendicular to it.

$$V_1 = A_2 \boldsymbol{X} A_3$$

$$V_2 = A_1 \boldsymbol{X} A_3$$

$$V_3 = A_1 \boldsymbol{X} A_2$$

$$V_1 = \begin{vmatrix} A_{2Y} & A_{2Z} \\ A_{3Y} & A_{3Z} \end{vmatrix} X - \begin{vmatrix} A_{2X} & A_{2Z} \\ A_{3X} & A_{3Z} \end{vmatrix} Y + \begin{vmatrix} A_{2X} & A_{2Y} \\ A_{3X} & A_{3Y} \end{vmatrix} Z$$

$$V_1 = \left(A_{2Y} \times A_{3Z} - A_{3Y} \times A_{2Z}\right)X - \left(A_{2X} \times A_{3Z} - A_{3X} \times A_{2Z}\right)Y + \left(A_{2X} \times A_{3Y} - A_{3X} \times A_{2Y}\right)Z$$

Similarly for V2 and V3, giving the V vectors end coordinates,

$$V_1\text{:}\ \ (V_{1X}, V_{1Y}, V_{1Z})$$

$$V_2\text{:}\ \ (V_{2X}, V_{2Y}, V_{2Z})$$

$$V_3\text{:}\ \ (V_{3X}, V_{3Y}, V_{3Z})$$

The area Ar of the base triangle is given by,

$$A_r = r \times s$$

where $$s = \frac{m+n+p}{2}$$

and $$r = \frac{\sqrt{s(s-m)(s-n)(s-p)}}{s}$$

The base sides m, n and p are obtained by subtracting the respective edge vectors V,

$$m = V_1 - V_2 = \sqrt{\left(V_{1X} - V_{2X}\right)^2 + \left(V_{1Y} - V_{2Y}\right)^2 + \left(V_{1Z} - V_{2Z}\right)^2}$$

similarly for $$n = V_2 - V_3 = \cdots$$

and $$p = V_3 - V_1 = \cdots$$

The pyramid height h is the length of vector H,

$$H = \frac{V_1 + V_2 + V_3}{3}$$

And the H coordinates,

$$H : (H_X, H_Y, H_Z)$$

$$H_X = \frac{V_{1X} + V_{2X} + V_{3X}}{3}$$

$$H_Y = \frac{V_{1Y} + V_{2Y} + V_{3Y}}{3}$$

$$H_Z = \frac{V_{1Z} + V_{2Z} + V_{3Z}}{3}$$

and

$$h = \sqrt{{H_X}^2 + {H_Y}^2 + {H_Z}^2}$$

With Ar and h known, the area function coefficient can be calculated:

$$f = \frac{A_r}{h^2}$$

AREA FUNCTION DERIVATION
4 SIDED TIP

Given an irregular four sided pyramid, with the four measured vectors A_1, A_2, A_3 and A_4 find the coefficient 'f' such that the area 'Ar' of the pyramid base can be calculated with the formula:

$$A = f \times h^2$$

Let a pyramid with apex at 'O', be defined by the height 'h' and base sides 'p1', 'p2', 'n1' and 'n2', as in Fig B2. O is at the origin of the coordinate system XYZ. The four unit vectors A_1, A_2, A_3 and A_4 are perpendicular to the four pyramid faces. The four edges, not part of the base are vectors V_1, V_2, V_3 and V_4.

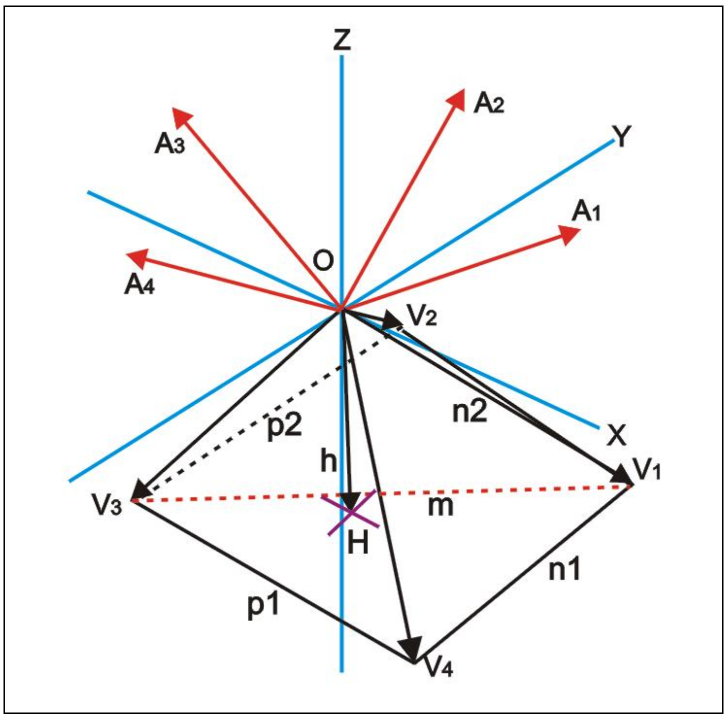

Fig. B2 Irregular four sided pyramid.

The four unit vectors A1, A2, A3 and A4 are known from the laser goniometer measurements. Their end coordinates are calculated with the formulae of page 9.

A_1: (A_{1X}, A_{1Y}, A_{1Z})

A_2: (A_{2X}, A_{2Y}, A_{2Z})

A_3: (A_{3X}, A_{3Y}, A_{3Z})

A_4: (A_{4X}, A_{4Y}, A_{4Z})

The edges V are perpendicular ($\perp$) to vectors A as follows,

$V_1 \perp A_1$ and A_2

$V_2 \perp A_2$ and A_3

$V_3 \perp A_3$ and A_4

$V_4 \perp A_4$ and A_1

The end coordinates of vector V are calculated with the cross product (**X**) of the two vectors A perpendicular to it.

$V_1 = A_1 \,\mathbf{X}\, A_2$

$V_2 = A_2 \,\mathbf{X}\, A_3$

$V_3 = A_3 \,\mathbf{X}\, A_4$

$V_4 = A_4 \,\mathbf{X}\, A_1$

$$V_1 = \begin{vmatrix} A_{1Y} & A_{1Z} \\ A_{2Y} & A_{2Z} \end{vmatrix} X - \begin{vmatrix} A_{1X} & A_{1Z} \\ A_{2X} & A_{2Z} \end{vmatrix} Y + \begin{vmatrix} A_{1X} & A_{1Y} \\ A_{2X} & A_{2Y} \end{vmatrix} Z$$

$$V_1 = \left(A_{1Y} \times A_{2Z} - A_{2Y} \times A_{1Z}\right)X - \left(A_{1X} \times A_{2Z} - A_{2X} \times A_{1Z}\right)Y + \left(A_{1X} \times A_{2Y} - A_{2X} \times A_{1Y}\right)Z$$

Similarly for V2, V3 and V4 giving the V vectors end coordinates,

V_1: (V_{1X}, V_{1Y}, V_{1Z})

V_2: (V_{2X}, V_{2Y}, V_{2Z})

V_3: (V_{3X}, V_{3Y}, V_{3Z})

V_4: (V_{4X}, V_{4Y}, V_{4Z})

The area Ar of the base quadrilateral is given by the sum of the 2 triangles of sides p1 n1 m and p2 n2 m, where m is the common diagonal.

$$A_r = A_{r1} + A_{r2}$$

$$A_{r1} = r_1 \times s_1$$

$$A_{r2} = r_2 \times s_2$$

where $$s = \frac{m+n+p}{2}$$

and $$r = \frac{\sqrt{s(s-m)(s-n)(s-p)}}{s}$$

The base sides n and p, and the diagonal m are obtained by subtracting the respective edge vectors V,

$$m = V_1 - V_3 = \sqrt{(V_{1X} - V_{3X})^2 + (V_{1Y} - V_{3Y})^2 + (V_{1Z} - V_{3Z})^2}$$

$$n2 = V_1 - V_2 = \sqrt{(V_{1X} - V_{2X})^2 + (V_{1Y} - V_{2Y})^2 + (V_{1Z} - V_{2Z})^2}$$

similarly for $$n1 = V_4 - V_1 = \cdots$$

$$p2 = V_2 - V_3 = \cdots$$

and $$p1 = V_3 - V_4 = \cdots$$

The pyramid height h is the length of vector H,

$$H = \frac{V_1 + V_2 + V_3 + V_4}{4}$$

And the H coordinates,

$$H : (H_X, H_Y, H_Z)$$

$$H_X = \frac{V_{1X} + V_{2X} + V_{3X} + V_{4X}}{4}$$

$$H_Y = \frac{V_{1Y} + V_{2Y} + V_{3Y} + V_{4Y}}{4}$$

$$H_Z = \frac{V_{1Z} + V_{2Z} + V_{3Z} + V_{4Z}}{4}$$

and $$h = \sqrt{{H_X}^2 + {H_Y}^2 + {H_Z}^2}$$

With Ar and h known, the area function coefficient can be calculated:

$$f = \frac{A_r}{h^2}$$

APPENDIX C

PRISM GAUGE NIST CERTIFICATION

A cube corner prism was submitted by Micro Star Technologies (MST) to the National Institute of Standards and Technology (NIST) for measurement and certification as a traceable gauge. The prism is one of the gauges used to verify the calibration of MST's laser goniometers.

PRISM SPECIFICATIONS

Supplier	Edmund Optics of Barrington NJ
Description	Cube Corner Tetrahedral Prism
Size	50mm
Material	BK7
Surface Accuracy	1/10 λ
Angle Tolerance	3 arc seconds = 0.00083°
MST number	TC1023

NIST CERTIFICATION

On August 2006 NIST measured the prism and provided a report describing their procedure and the certified angle values, which are shown highlighted on Table C1.

NIST test number	821/273748-06
Date	8/25/06
Standard deviation	0.1 arc seconds = 0.00003°
Measurements uncertainty	0.45 arc seconds = 0.00013°

MST MEASUREMENTS

Table C2 shows 8 sets of measurements taken with the MST laser goniometer. The table is explained in detail starting on page 6.

NIST AND MST DATA COMPARISON

On Table C3 the two sets of data are compared. MST data is highlighted in blue and NIST data in yellow, as well as their difference to the nominal values. The last row (12) shows the difference or discrepancies between the two sources.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1							3 SIDED INDENTER MEASUREMENTS AND ANALYSIS NIST DATA											
2	SERIAL. #:	TC-1023	DESCRIPTION:				CUBE CORNER (TRACEABLE GAGE)							INI:		NIST	DATE:	2/18/2007
3				MEASURED VALUES							CALCULATED RESULTS							
4			TILT ANGLES		ROTATION ANGLES		E ANGLES			E TILT ANGLES		E ROTATION ANGLES		FACE TO FACE ANGLES			AREA FUNCT.	AREA FUNCT. E
5		a_1	a_2	a_3	b_{12}	b_{13}	e_T	e_R	a_{e1}	a_{e2}	a_{e3}	b_{e12}	b_{e13}	c_{12}	c_{23}	c_{31}	f	fe
6	NOMINAL VALUES:	35.264	35.264	35.264	120.000	240.000	0.000		35.264	35.264	35.264	120.000	240.000	90.000	90.000	90.000	2.598001	2.598001
7																		
8	NIST DATA:	35.273	35.250	35.270	120.008	239.986	0.015	152.435	35.259	35.263	35.270	119.998	239.992	90.009	89.993	89.998	2.598060	2.598060
9	DIFF FROM NOMINAL:	0.009	-0.014	0.006	0.008	-0.014	0.015		-0.005	-0.001	0.006	-0.002	-0.008	0.009	-0.007	-0.002	0.000059	0.000059

TABLE C1. NIST DATA AND ANALYSIS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1							3 SIDED INDENTER MEASUREMENTS AND ANALYSIS MST DATA											
2	**SERIAL. #:**	**TC-1023**	**DESCRIPTION:**				**CUBE CORNER (TRACEABLE GAGE)**							**INI:**		**BM**	**DATE:**	**2/18/2007**
3			**MEASURED VALUES**								**CALCULATED RESULTS**							
4			**TILT ANGLES**		**ROTATION ANGLES**		**E ANGLES**			**E TILT ANGLES**		**E ROTATION ANGLES**		**FACE TO FACE ANGLES**			**AREA FUNCT.**	**AREA FUNCT.**
5		a_1	a_2	a_3	b_{12}	b_{13}	e_T	e_R	a_{e1}	a_{e2}	a_{e3}	b_{e12}	b_{e13}	c_{12}	c_{23}	c_{31}	f	fe
6	**NOMINAL VALUES:**	35.264	35.264	35.264	120.000	240.000	0.000		35.264	35.264	35.264	120.000	240.000	90.000	90.000	90.000	2.598001	2.598001
7	**MEASUR 1**	35.271	35.248	35.285	119.995	239.977												
8	2	35.290	35.248	35.269	119.994	239.991												
9	3	35.285	35.265	35.262	119.992	239.969												
10	4	35.271	35.256	35.273	120.008	239.975												
11	5	35.279	35.253	35.263	120.006	239.996												
12	6	35.280	35.261	35.261	119.999	239.979												
13	7	35.271	35.256	35.277	120.008	239.980												
14	8	35.283	35.250	35.277	119.992	239.995												
15	9																	
16	10																	
17	**AVERAGE:**	35.279	35.255	35.271	119.999	239.983	0.015	143.489	35.267	35.268	35.269	119.989	239.987	89.996	89.993	89.995	2.598787	2.598787
18	**DIFF FROM NOMINAL:**	0.015	-0.009	0.007	-0.001	-0.017	0.015		0.003	0.004	0.005	-0.011	-0.013	-0.004	-0.007	-0.005	0.000786	0.000786
19	**ST.DEV:**	0.007	0.006	0.009	0.007	0.010												
20	**UNCERTAINTY (X2):**	0.005	0.004	0.006	0.005	0.007												

TABLE C2. MST DATA AND ANALYSIS

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1							3 SIDED INDENTER COMPARISON											
2	SERIAL. #:	TC-1023	DESCRIPTION:				CUBE CORNER (TRACEABLE GAGE)							INI:		BM	DATE:	2/18/2007
3				MEASURED VALUES							CALCULATED RESULTS							
4			TILT ANGLES		ROTATION ANGLES		E ANGLES			E TILT ANGLES		E ROTATION ANGLES			FACE TO FACE ANGLES		AREA FUNCT.	AREA FUNCT. E
5		a_1	a_2	a_3	b_{12}	b_{13}	e_T	e_R	a_{e1}	a_{e2}	a_{e3}	b_{e12}	b_{e13}	c_{12}	c_{23}	c_{31}	f	fe
6	NOMINAL VALUES:	35.264	35.264	35.264	120.000	240.000	0.000		35.264	35.264	35.264	120.000	240.000	90.000	90.000	90.000	2.598001	2.598001
7	MST:	35.279	35.255	35.271	119.999	239.983	0.015	143.489	35.267	35.268	35.269	119.989	239.987	89.996	89.993	89.995	2.598787	2.598787
8	NIST:	35.273	35.250	35.270	120.008	239.986	0.015	152.435	35.259	35.263	35.270	119.998	239.992	90.009	89.993	89.998	2.598060	2.598060
9																		
10	MST DIFF FM NOM.	0.015	-0.009	0.007	-0.001	-0.017	0.015		0.003	0.004	0.005	-0.011	-0.013	-0.004	-0.007	-0.005	0.000786	0.000786
11	NIST DIFF FM NOM.	0.009	-0.014	0.006	0.008	-0.014	0.015		-0.005	-0.001	0.006	-0.002	-0.008	0.009	-0.007	-0.002	0.000059	0.000059
12	MST - NIST	0.006	0.005	0.001	0.009	0.003	0.000		0.008	0.005	0.001	0.009	0.005	0.013	0.000	0.003	0.000727	0.000727

TABLE C3. MST AND NIST DATA COMPARISON