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CALIBRATION OF AIR-DATA SYSTEMS AND FLOW DIRECTION SENSORS

by

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CONTENTS

LIST OF SYMBOLS

a	Speed of sound, or specific force (i.e. force per unit mass), due to gravity or inertial forces relative to g_o
C_L	Lift coefficient
f	Sampling frequency
g_o	Acceleration due to gravity
g	Representative value of acceleration due to gravity, used in definition of International Standard Atmosphere, or in calibration of accelerometers (9.80665 m/s^2)
h	Geometric height
H or H_a, H_s, H_i, H_R	Pressure altitudes; the values of geopotential altitude in ISA at which p, p_s, p_i, p_R occur (note that pressure altitudes are measures of pressure, not of altitude per se). See note at subscript a
k	Temperature sensor recovery factor
K	Flow direction sensor calibration factor ($\alpha_{sensor}/\alpha_{true}$ or $\beta_{sensor}/\beta_{true}$)
L	Pipe length (from sensor to instrument, associated with acoustic and pressure lag), or, with subscripts r, p, or y, the longitudinal, lateral and vertical distances respectively from the aircraft CG to the inertial platform
M	Mach number ($= V/a$)
n	Load factor normal to flight path
p	Pressure or roll rate
q	Pitch rate
q_c	Impact pressure ($p_p - p$)
r	Distance, yaw rate, or distance from sensor centre of pressure to the centre line of boom or fuselage
R	Equivalent radius of boom or fuselage
s	Laplace operator
S	Wing area
t	Time
T	Absolute temperature
V	True airspeed
V_e	Equivalent airspeed ($= V\sqrt{\sigma}$)
V_R	Indicated airspeed (IAS), corresponding to pressures P_{pR} and p_R
V_i	Airspeed corresponding to pressures p_{ps} and p_s, i.e. the IAS in the absence of, or corrected for, lag errors
V_c	Calibrated airspeed, corresponding to pressures p_p and p, i.e. the IAS corrected for lag and pressure errors
u, v, w	Body axis components of velocity
W	Aircraft weight
x	Horizontal distance between sideslip sensor and CG (positive when sensor is forward of CG)

z	Vertical distance between sideslip sensor and CG (positive when sensor is below CG)
ΔC_p	$(p - p_s)/(\frac{1}{2}\rho V^2)$ } pressure error coefficients
ΔC_{pp}	$(p_p - p_s)/(\frac{1}{2}\rho V^2)$ } pressure error coefficients
α	Angle of attack (with respect to body datum)
β	Sideslip angle, or $\sqrt{(1 - M^2)}$
ϕ	Bank angle
γ	Ratio of specific heats of air at constant pressure and constant volume, or angle of climb
ϵ	Induced upwash at sensor location
ψ	Heading angle
ρ	Air density
σ	relative density (σ/σ_{SL}), or standard deviation
δ	Relative pressure (p/p_{SL}), or bias error in observation
θ	Relative temperature (T/T_{SL}), or aircraft pitch angle
λ	Time constant for pressure lag
μ	Coefficient of viscosity
τ	Acoustic lag time
ω_n	Natural frequency of vane (rad/s)
Ω	Angle between sensor axis of rotation and the O_{xz} or O_{xy} planes for angle of attack or of sideslip sensors respectively
ρ	Damping ratio of vane

Subscripts (except where otherwise specified in the text)

a	Ambient value. Used only where the text requires it; otherwise symbols without suffix indicate ambient values
b	Value indicating bias error
i	Values corresponding to pressures at the sensor, p_{ps} and p_s, applied to V and M
j	Value at j^{th} point in a manoeuvre
ISA	Value of parameter in International Standard Atmosphere
LE, TE	At leading and trailing edges respectively
nom	Parameter calculated from observations of the aircraft state
o	Angle at which sensor is aligned with the O_x axis, or value of parameter at beginning of manoeuvre
p	Value for pitot pressure or total temperature. For pitot pressure this indicates stagnation pressure after passage through a normal shock if there is one, which distinguishes suffix p from suffix t (used in Ref.1 but not necessary here), indicating isentropic stagnation pressure with no shock loss. Without further suffix, the true value; with further suffix, as defined by that suffix
R	Value output by transducer or instrument or, when applied to pressures, the value at the instrument which motivates its output; may also refer to the value which an instrument would indicate, derived from applied pressures, e.g. M_R derived from p_{pR} and p_R

s	Sensed value. The pressure inside the sensing orifice (pitot tube, static orifice, trailing static), or the temperature recorded at the temperature sensor
w	Due to wind
x, y, z	In direction of body axes O_x, O_y, or O_z respectively

LIST OF UNITS

ft	feet
K	kelvin
km	kilometre
kn	knot
m	metre
mb	millibar
N	newton
s	second

LIST OF ABBREVIATIONS

A&AEE	Aeroplane and Armament Experimental Establishment, UK
AFFTC	Air Force Flight Test Center, US
ASI	Airspeed indicator
CG	Centre of gravity
IAS	Indicated airspeed
ISA	International Standard Atmosphere
NATC	Naval Air Test Center, US
NLR	National Aerospace Laboratory, Netherlands
RMS	Root mean square
TAS	True airspeed
VMC	Visual meteorological conditions

CALIBRATION OF AIR-DATA SYSTEMS AND FLOW DIRECTION SENSORS

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SUMMARY

This volume in the AGARD Flight Test Techniques Series deals with the practical aspects of calibrating air-data and flow direction measurement systems. The available flight test calibration methods are described and their applicability, accuracies and limitations are reviewed. The volume is complementary to Volume 11 of AGARDograph 160 in the Flight Test Instrumentation Series which presents a comprehensive review of the theory of pressure and flow measurement and of instrumentation requirements.

1 INTRODUCTION

In aircraft flight testing the definition of the aircraft state at any instant is one of the most important aspects. The steady or "quasi-steady" elements such as ambient pressure and velocity of the air relative to the aircraft (the latter inclusive of magnitude and flow direction) are obtained from the air-data system and from flow direction measurement systems such as vanes and probes. The full dynamic aspects of the state, that is linear and angular accelerations, are normally obtained by inertial methods which are not the concern of this volume. In flight analysis of aircraft performance and handling characteristics the air-data pressures (static and dynamic) and the flow incidence angles are the primary parameters in terms of which the aircraft behaviour is defined and described. Forces and moments are non-dimensionalised against kinetic or impact pressure, and the resulting coefficients are found to be inter-related functions of flow incidence and Mach number and (sometimes) Reynolds number, the last two themselves derived from air-data measurements. Angle of attack and sideslip angle are often used when defining aircraft limiting conditions, angle of attack being the dominant parameter when considering aircraft stalling behaviour and sideslip angle when considering fin loads at given dynamic pressure (but accuracies required in angle measurement are somewhat lower in this latter application than for performance analysis or derivative extraction). It is therefore evident that these air-data and flow direction parameters must be obtained with requisite accuracy, and consequently accurate calibration of the measurement system is a necessary part of flight testing. Theoretical aspects of these calibrations have been given in Ref (1). The present volume deals primarily with practical aspects of these calibrations, though some further necessary theory is given also. The requirements of the practical user have been borne in mind, and the equations are presented with numerical constants evaluated so far as possible.

In the case of air-data systems the evaluation of these constants requires the assumption that the ratio of specific heats, γ, is a constant, as it is, with value 1.4, for most practical flight regimes; this is the value used in this volume. Outside certain extreme values of altitude and velocity this value may no longer apply (see Section 2).

The term "pressure error" is used to describe the difference between the ambient pressure or the correct pitot pressure and the values which are measured at the aircraft instruments, corrected for instrument error - that is, between p and p_R and between p_p and p_{pR}; the pressure error therefore includes the error due to the aerodynamic characteristics of the aircraft in modifying the pressure existing at the sensor inlet, and also the system error which may exist in the transmission of the sensed pressure p_s or p_{ps} to the instrument, where it is measured as p_R or p_{pR}; this latter error is usually due to viscous resistance (and sometimes acoustic lag also) in pressure transmission, and is described as the "lag error". However when a pressure error is non-dimensionalised as a "pressure error coefficient", ΔC_p or ΔC_{pp} the inclusion of quite separate lag effects in an aerodynamic coefficient is inappropriate and the coefficients are defined in terms of $p-p_s$ and p_p-p_{ps}. The expression "position error" has not been used in this context of errors in the sensed pressures because current usage is to prefer the term "pressure error", "position error" being appropriate to errors, due to aerodynamic effects within the aeroplane flow field arising from sensor positioning, in other quantities such as flow direction. The authors are aware that this usage is inconsistent with the definition of "position error" in the AGARD Multilingual Aeronautical Dictionary Ref (2), but they find that particular definition to be too narrow.

The two aspects of calibration dealt with, of air-data systems and flow direction measurement, have little in common until the stage is reached of their ultimate application in defining the aircraft state; therefore in this volume they are treated quite separately, air-data systems in Part 1 and flow direction calibration in Part 2.

It is assumed throughout that instrument calibration corrections have been applied, and all the symbols used indicate values so corrected where that is appropriate.

PART 1 CALIBRATION OF AIR-DATA SYSTEMS

2 RELATIONSHIPS BETWEEN PRESSURE, AIRSPEED, MACH NUMBER AND PRESSURE ALTITUDE

The relevant relationships have been developed in Ref (1) and are reproduced here.

The relationships between pressure and geopotential altitude are given for altitudes in feet, the existing unit in practice. They are given here in terms of δ and H but may be applied also with subscripts s, i, or R on the parameters and are given in a slightly different and more convenient form than Ref (1).

$$\delta = (1 - 6.875\,585\,6 \times 10^{-6} \times H)^{5.255\,88} \qquad H \leqslant 36\,089 \qquad (1)$$

$$\delta = 1.265\,674\,8 \times \mathrm{Exp}(-4.806\,346 \times 10^{-5} \times H) \qquad 36\,089 \leqslant H \leqslant 65\,616 \qquad (2)$$

$$\delta = (0.988\,625\,85 + 1.532\,332\,3 \times 10^{-6} \times H)^{-34.163\,22} \qquad 65\,616 < H < 105\,000 \qquad (3)$$

and the inverse relationships are readily developed as follows:

$$H = (1 - \delta^{0.190\,263\,1})/(6.875\,585\,6 \times 10^{-6}) \qquad \delta \geqslant .223\,361 \qquad (4)$$

$$H = 4902 - 20\,805.83 \times \ln\delta \qquad .223\,361 \geqslant \delta \geqslant .054\,033 \qquad (5)$$

$$H = 645\,177.2 \times (1.011\,505\,1 \times \delta^{-0.029\,271\,25} - 1) \qquad .054\,033 \geqslant \delta \geqslant .008\,566\,6 \qquad (6)$$

The following are the relationships between pressure, airspeed and Mach number as in Ref (1), but in some instances in a slightly different form. The relationships are given for V_c, M, p_p' (corrected if necessary) and p, but apply also with consistent sub-subscripts s, i, or R. The constants shown are for a value of γ of 1.4, the value which applies up to high Mach number (>2) or very high altitude (>100 000 ft). Outside these limits the constants of Eqs (7), (8) and (11) may be changed (though the subsonic Eq (7) is unlikely to apply); Eqs (9), (10) and (12) are those linking airspeed indicator reading with applied pressure and will apply as stated unless (which appears improbable) ASIs are calibrated for these extreme conditions. The application of the methods here described in the extreme conditions causing change of γ are outside the scope of this paper.

In these equations the letter A is used to represent the recurrent quantity $1.2^{3.5} \times (6/7)^{2.5}$

$$q_c = p_p - p = p\left[\left(1 + M^2/5\right)^{3.5} - 1\right] \qquad M \leqslant 1 \qquad (7)$$

$$= p\left[AM^2 \Big/ \left(1 - \frac{1}{7M^2}\right)^{2.5} - 1\right] \qquad M > 1 \qquad (8)$$

and (in order that $V_c = V$ when $p = p_{SL}$ and $a = a_{SL}$)

$$q_c = p_{SL}\left[\left[1 + \left(\frac{V_c}{a_{SL}}\right)^2 \Big/ 5\right]^{3.5} - 1\right] \qquad V_c \leqslant a_{SL} \qquad (9)$$

$$= p_{SL}\left[A\left(\frac{V_c}{a_{SL}}\right)^2 \Big/ \left(1 - 1\Big/\left[7\left(\frac{V_c}{a_{SL}}\right)^2\right]\right)^{2.5} - 1\right] \qquad V_c > a_{SL} \qquad (10)$$

Inverse expressions for M and V_c are readily obtained from Eqs (7) and (9) but from Eqs (8) and (10) an iterative form is required as follows.

$$M^2 = \left[\left(\frac{q_c}{p} + 1\right)\Big/ A\right]\left(1 - \frac{1}{7M^2}\right)^{2.5} \qquad \frac{q_c}{p} > .892\ 929 \qquad (11)$$

$$\left(\frac{V_c}{a_{SL}}\right)^2 = \left[\left(\frac{q_c}{p_{SL}} + 1\right)\Big/ A\right]\left[1 - 1\Big/\left[7\left(\frac{V_c}{a_{SL}}\right)^2\right]\right]^{2.5} \qquad \frac{q_c}{p_{SL}} > .892\ 929 \qquad (12)$$

and the Eqs (11) and (12) converge rapidly from an initial value of 1 for M^2 or $\left(\frac{V_c}{a_{SL}}\right)^2$. A convergence standard of 10^{-4} between successive values is appropriate.

$$V_e = V\sqrt{\sigma} = Ma\sqrt{\sigma} = Ma_{SL}\sqrt{\delta} \qquad (13)$$

The "Compressibility" or "scale-altitude" correction $V_e - V_c$ is obtained, for given V_c and H by calculating in succession δ (Eqs (1) to (3)), $\frac{q_c}{p_{SL}}$ (Eqs (9) or (10), hence $\frac{q_c}{p}$, M (Eqs (7) or (11), V_e (Eq (13), and, M being known, $V = Ma = Ma_{SL}\sqrt{\theta}$

3 PRESSURE ERRORS – THEIR ORIGIN AND NEED FOR CALIBRATION AND CORRECTION

The "pressure error" arises when, wishing to measure true pitot and ambient pressures p_p and p, we in fact measure incorrect values p_{pR}, p_R, with consequently incorrect evaluation (if no correction is applied) of airspeed, Mach number and pressure altitude; errors in pitot pressure contribute to errors in airspeed and Mach number only; those in the static system contribute to those errors and also to that in pressure altitude. The errors may be due to aerodynamic effects at the sensor, to delays in pressure transmission to instruments (lag), or both; lag is considered in Section 5, while the aerodynamic effects (differences between p_p and p_{ps}, and between p and p_s) are considered here.

The measurement of p_p in general presents little difficulty since it requires that the flow along a streamline is brought to rest at the pitot tube and, with a well-designed pitot, this is easily achieved. As shown in Ref (1), pitot tubes may be designed in which the pressure error $p_{ps} - p_p$ is sensibly zero for large angles of incidence between the pitot and the local flow. Therefore in most of this volume it will be assumed that p_{ps} is equal to p_p, that is, there is no error in the pitot. When an aircraft will operate over very wide ranges in angle-of-attack or sideslip, or in any case where there is reason to believe that a pitot error exists, the method of calibration is to install, as a test instrument, a source of "true" pitot pressure. This may be a pitot-in-venturi such as "5" of Figure 25 of Ref (1), for which the "region of insensitivity to angle of attack" is very wide, or a swivelling, self adjusting pitot (eg Figure 73 of Ref (1)) such that p_p can be measured correctly for all likely flight conditions. The pitot error can then be calibrated as desired as a function of angle of attack or of sideslip, or of lift coefficient and Mach number, using a sensitive differential pressure gauge between "true" and aircraft sources.

The measurement of p presents greater difficulty because we are here concerned with the Bernoulli relationship (as for the pitot also) between pressure and velocity, and we will sense true static pressure only if the local velocity at the sensor (outside the boundary layer) has the value which, in that relationship, corresponds to free-stream static pressure. For subsonic speeds that will be the undisturbed free-stream velocity; supersonic it may have another, lower value to compensate for pressure loss through shock waves. In either case it is very unlikely that these conditions can be achieved throughout the flight envelope, though they may occur at some conditions within it. The approach must be to accept that there will be a static pressure error, and to calibrate it so that corrections may be applied. Part 1 of this volume is devoted primarily to that calibration.

The need for such calibration and correction is that of obtaining, in all regimes of flight, correct evaluation of airspeed, Mach number and pressure altitude. Correct values are required in many applications; in research and in aircraft evaluation, in which comparison is made with data from other sources such as wind tunnels, the basic parameters of ambient, kinetic and dynamic pressure, and Mach number, must be evaluated with sufficient accuracy to permit valid cross-relation of data; in performance evaluation, particularly when data from several sources such as inertial and air-data are obtained, often with "mathematical modelling", accurate data are required; in air traffic control, accurate altitude determination, independent of the characteristics of particular aircraft, is required.

It is clearly desirable that the calibration, when complete, should be readily applicable for all likely flight conditions; therefore the calibration should not be sensitive to a large number of independent variables, giving complexity in application and requiring an extensive calibration programme to establish it. The calibration is likely to be responsive to the flow pattern about the aircraft - that is, to angles of attack and sideslip, Mach number, and possibly Reynolds number. It should not be responsive to engine flow rates, so sensors should not be positioned where they may be influenced by intakes or exhausts. So far as possible it should not be responsive to aircraft control surfaces, or to the undercarriage, though to the extent that controls may change lift-incidence relationships such a response may be unavoidable; positioning of sensors in the region of the direct effect on local flow of a control surface, undercarriage or other appendage should be avoided.

The calibration must cover the full range of conditions and variables for which it will be applied; it is a matter of judgement whether effects of engine flow, control position etc, should be included in the range of calibration tests. Measurements in ground effect should be made if accurate readings will be required in flight within this effect (ie within 1.5 wing spans of the ground surface approximately).

In general the calibration of the static pressure system is achieved by making a measurement of ambient pressure p at the aircraft, independently of the aircraft system, and comparing this with p_s as recorded on the aircraft (using calibrated test instrumentation for the pressure measurement and taking precautions to ensure synchronisation of the measurements of p and p_s). A static pressure comparison rather than a speed comparison is almost invariably used, although a method which compares a measure of true airspeed with that of the aircraft system is given in Section 4.5; even in this case the speed comparison leads ultimately to a pressure derivation in terms of radar measured altitude, which is used for calibration at other regions of the flight envelope. Thus the essential problem is the measurement of the ambient pressure at the aicraft, undisturbed by the aircraft itself, with sufficient accuracy; close to ground level fairly easy solutions are available, but at high altitude more substantial difficulties are encountered. The methods used for this determination of ambient pressure are described below. Further details and test techniques may be obtained from Ref (3).

4 PRESSURE ERROR CALIBRATION METHODS

4.1 Tower Flypast

This, the traditional method for pressure error calibration, depends upon a measurement of pressure at a reference point on the ground, and of the vertical height difference between the reference point and the aircraft at the time when pressures are recorded in the aircraft. Derivation of the pressure at the aircraft is then achieved by applying to the reference point pressure an increment due to the height difference ΔH using the relationship $\Delta p = -\rho g_0 \Delta H$ or, if pressures are expressed as pressure altitudes, $H_{aircraft} = H_{reference} + \Delta H \frac{T_{ISA}}{T}$. If the temperature factor on H were ignored, the consequent error would be $\Delta H(1-\frac{T_{ISA}}{T})$. If a maximum limit E is set for this error, we have that $\left| T_{ISA}-T \right| < \frac{TxE}{\Delta H}$; if E has the value 1 ft (a rigorous case) and we take the lowest likely value of T we have that $\left| T_{ISA}-T \right| < \frac{270}{\Delta H}$, so that the temperature correction may be omitted for moderate ΔH (up to 50 ft) if the temperature is within 5K of standard, and the effect of possible errors in measuring temperature (due to convection effects etc which are difficult to eliminate entirely) is unlikely to be significant.

The height difference ΔH can be obtained by any suitable method, and different methods may be appropriate for different sites. The most usual is by photography on a horizontal sight line as the aircraft passes on a known track such as a runway centre-line; height is then readily available from image position, lens focal length and camera-to-track distance. In some cases (usually at low speed only, on particular sites) photography on a vertical line from below the track may be used, with height derivations from lens focal length, image size and aircraft dimensions or, as a variant on this method, lines of known spacing on the ground may be photographed from the aircraft. Other possible methods for determination of ΔH are kinetheodolite, radio altimeter or visual observation.

Using a horizontal sightline, accuracy to within 1 foot is possible if a camera with graticule screen is used on a carefully surveyed site; there should be a reference point of known height within the frame, to which the graticule scale can be related to eliminate changes in camera setting. The reference point on the aircraft should be marked so that it is identifiable on the film image (and this should be at the position of the measuring instrument, not of the sensor, since a pressure change with vertical height takes place within the aircraft pipework just as in the atmosphere). For photography from the aircraft of fixed lines on the ground, a number of parallel lines along the ground parallel to the direction of flight may be used so that images near either edge of the film frame can be used and related to known distance on the ground; a 1% accuracy in film measurement will give height to within 1 ft for an aircraft height above ground of 100 ft; if the ground height is not constant longitudinal markers will be required to relate to local ground height. The height thus obtained will be that of the camera lens, from the relationship $H = L \times f/l$ (where L and l are ground distance and film image distance, and f is the camera focal length), and must be adjusted to the height of the pressure instrument. For photography of the aircraft from a vertical camera on the ground, a wider angle of view at the camera (ie a shorter focal length) will be necessary to ensure acquiring the aircraft image, so the film image will be smaller and achievable accuracy will be less, but accuracy within 5 ft should be attainable; this is so much a matter of the particular dimensions used that the user must assess accuracy according to the detail of the method he has chosen. At sea level, height determination to within 1 ft, corresponding to 0.035 mb, is well within the accuracy required.

It is usually convenient, but not essential, to have the ground reference point for pressure at the position of the observation point for measuring ΔH; pressure can then be measured for each aircraft observation. This does require two sensitive pressure transducers which must be carefully calibrated to eliminate discrepancy between them. Another approach, which minimises the effect of instrument calibration error, is to measure ambient pressure on the test aircraft transducer while it is stationary before and after the trial, the height difference $(\Delta H)_{ref}$ between the aircraft transducer and the reference point being accurately measured. The corresponding pressure at the reference point can then be obtained by adding $(\Delta H)_{ref}\frac{T_{ISA}}{T}$ to the pressure altitude measured on the aircraft (or the equivalent conversion $-\rho g_0(\Delta H)_{ref}$ if pressure units are used). At the time when pressures are measured on the stationary aircraft, pressure readings are taken also on a monitoring transducer on which pressures will be measured on each test run of the trial; the readings of this transducer then provide the basis for interpolation between the reference point pressures derived, as described above, from the aircraft transducer. Pre- and post- trial pressures as measured on the two transducers must obviously be consistent, and the cause investigated if they are not.

The presence of a wind does not of course have a direct effect on the static source calibration. However unless the trial is conducted in low-wind conditions, turbulence can be expected in flight close to the ground such as this method requires; this militates against accurate observation and should be avoided. The trial should not be made in wind velocities greater than 10 kn, and much preferably not greater than 5 kn. If, for any reason, higher wind velocities are accepted it should be remembered that, in addition to the adverse effect of wind upon the conditions for accurate measurement, the wind itself has a dynamic pressure which may be developed at the transducer where the ground observation of ambient pressure should be measured. For example the dynamic pressure of a 20 kn wind is equivalent to about 17 ft of pressure altitude, or about 0.65 mb at sea level.

The tower flypast method is not primarily intended for measurement of pressure error in ground effect, although with suitable safety precautions it might be so used. Unless ground effect results are required, ground clearance must be at least 1.5 wing spans.

For accurate results it is important that fully stabilised conditions are achieved before the aircraft passes through the observation point. No significant height, speed or direction change should take place in the 10 seconds preceding that; this means, at high subsonic speed, steady conditions over a distance of 2 km, and the run in will begin at a range of about 12 km. At lower speeds shorter approach distances will apply; these features will depend on aircraft characteristics (including for example the shorter time required for speed stabilisation on propeller-driven aircraft).

Both at very high speeds and at speeds close to the stall (nature of approach terrain being also relevant) pilots are likely to prefer greater altitudes than at medium speeds, and provided that the height flown is within range for accurate measurement the pilot should be briefed to operate at the height which he regards as optimum from considerations of safety and good speed stabilisation.

The method provides the most accurate available method of measuring the pressure error of the static system, and it was a sufficient method until Mach numbers became high enough for the pressure error to become a function of two variables, angle of attack (or lift coefficient) and Mach number.

Accuracy: ±0.25 mb (dependent on transducer quality).

Advantage: The most accurate method, involving only height difference measurement by intrinsically accurate methods, and pressure measurement by high-accuracy transducers.

Limitations:

(i) Only 1g level flight calibration is possible.

(ii) Ground level only (but available up to any pressure altitude obtainable by use of a high-altitude site and choice of low atmospheric pressure conditions).

(iii) Requires calm-air conditions and good visibility.

(iv) Supersonic conditions cannot usually be tested, because of limits imposed for environmental reasons; where such limits do not apply the method is applicable supersonic.

4.2 Trailing Static or Trailing Cone

This method is intended to overcome the low-altitude limitation of the tower flypast method by providing a sensor for ambient pressure which the aircraft itself can carry for test purposes, and which senses pressure at a point sufficiently remote from the aircraft pressure field for its effect to be minimal. This sensor is trailed on a long tube, which also serves to transmit the sensed pressure to the aircraft; a towing cable is passed through the tube to relieve the tubing itself of tension. There are two main types of towed static pressure sensor (which are described and illustrated in Section 3.4.2.4 of Ref (1)). The first is the "trailing bomb", which is a fairly long static tube with a stabilising tail; this performs best at low speed when its weight causes it to fly below the pressure field and flow disturbance of the aircraft, but at higher speeds when it comes into the aircraft flow field it may become unstable. The second type is the "trailing cone", in which the sensor is a machined and drilled cylindrical insert in the tube itself, of the same diameter as the tube and bonded into it to give a smooth contour; a drag-producing device in the form of a perforated cone is attached at the end of the tube to cause a stable trail. The sensing element should preferably be at least 8 cone diameters forward of the cone apex (although with "tower flypast" calibration of the trailed static system itself, so that errors can be calibrated out, cone-sensor separations down to 5 diameters may be used). The trailing cone, which is usable in principle at speeds throughout the flight range (though it may have some regions of instability) is now the preferred form of trailing sensor.

The trailing sensor is itself subject to a pressure error, but if it is flown (as it must be) outside the pressure field of the aircraft this should be small, particularly for the trailing cone which has no change of local dimension adjacent to the sensor. The pressure error of a sensor thus trailed outside the influence of the aircraft will be independent of aircraft parameters such as lift coefficient and should be a function of Mach number only when expressed as a pressure coefficient. It is preferable to tow the sensor from a wing-tip or other outboard position, or failing that a high tow position such as a fin-tip; positions liable to effect by jet efflux must be avoided. The optimum length of trail can be found by increasing the length of trail at fixed height and airspeed until the pressure difference between trailed and aircraft sensors does not change significantly as total length is changed. The length thus determined depends substantially on aircraft configuration and on installation, but may typically be 2 wing spans. The pressure error of the trailed sensor, inclusive of residual aircraft pressure field effects and of those due to the trailing sensor itself, can then be determined by the tower flypast method.

In principle a trailing sensor may be used at supersonic speed, with suitable choice of drag generating cone, but the measurement of its own calibration supersonically is usually impracticable and it is preferable to use the method in combination with other methods such as radar tracking (Section 4.4) to obtain supersonic pressure error calibrations. The trailing sensor method presents substantial difficulties in installation, which for some aircraft types may be insuperable. It is best suited for calibrating a pacer aircraft (Section 4.3), where the effort and cost of installation may be balanced against the need for repeated use. Although it is possible to use a trailed sensor with a fixed trail length, using some release system to trail the sensor after take-off and accepting some damage to the system, particularly the cone, on landing, it is much preferable to have a means of extension and retraction in flight. The sensor may not trail in a smooth way, free of oscillation, at all flight regimes and ability to retract allows measurement of pressure, by the sensor, at appropriate regimes and retraction and use of radar measured height to infer the pressure at other regimes, including the supersonic. There are not at present adequate data for forecasting the trail characteristics of trailing sensors but they have been successfully flown in several installations. See for example Ref (4).

Accuracy:	Error from flypast calibration.	±0.2 mb
	Error from differential pressure measurement.	±0.1 to 0.2 mb
	Resultant mean error.	±0.2 to 0.3 mb

Advantages: Fairly accurate method applicable at all altitudes. Independent of a ground installation.

Limitations:

(i) Only 1g level flight calibration is possible.

(ii) Subject to large pneumatic lag because of the long tubing length required (see Section 5), which imposes the level-flight limitation (i).

(iii) Cumbersome and often expensive installation, which may be impossible on some aircraft.

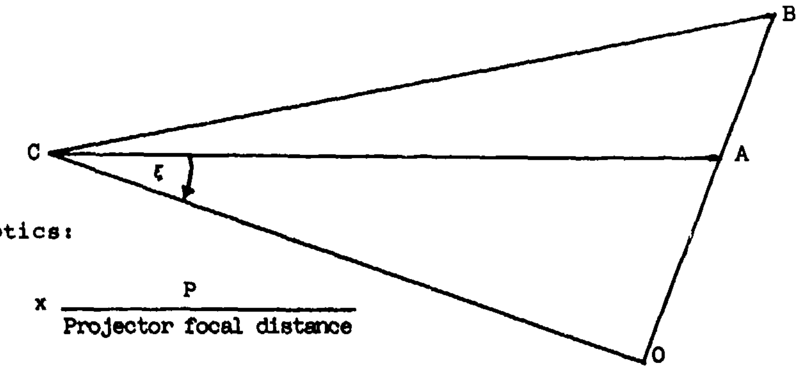
(iv) May not be available, because of unstable flight, at some flight regimes.

4.3 Pacer Aircraft

4.3.1 Calibration of Static Pressure Error by Pacer Aircraft

This method is simple in principle and can to some extent be regarded as the tower flypast method transferred to higher altitude; an observation point (in this case the pacer) at which the ambient pressure is known or may be inferred is established and the subject aircraft ambient pressure is then inferred by measurement of height difference. The pacer's calibration being known, the ambient pressure is derived from its air data system and pressure error correction, and the ambient pressure at the subject is obtained from $\Delta p = -\rho g_0 \Delta H$, or $H_{subject} = H_{pacer} + \Delta H \frac{T_{ISA}}{T}$, and again particularly since ΔH is usually quite small (<10m) the ratio $\frac{T_{ISA}}{T}$ can be ignored unless T_{ISA} and T differ widely (or, if the $-\rho g_0$ form is used, ρ need not be corrected for $\frac{T_{ISA}}{T}$ and ρ_{ISA} may be used).

As in the case of tower flypast, any adequate technique of height difference measurement may be used, depending upon available facilities. Usually the observation is made from the pacer aircraft which can be equipped for the purpose. The simplest method, but not appropriate for high accuracy, is visual observation, preferably in terms of a dimension of the observed aircraft such as fuselage height, and with care an accuracy in estimation to within 20 ft which, depending on altitude may be equivalent to 0.1 to 0.5 mb, may be attainable. The most convenient recording method is photography by a sideways-looking camera in the pacer aircraft, aligned and calibrated so that the "wings level" plane is identifiable on the film. The measurement of height difference then depends upon how well the angle of bank of the pacer aircraft is known, or on whether a reference is identifiable on the film image from which the true horizontal plane can be determined. In favourable conditions a horizon can be observed on the film, and if the altitude of the horizon surface - earth surface or an identifiable cloud layer - is known, the corresponding depression angle and hence the true horizontal on the film may be assessed from simple trigonometry; (depression angle = arccos $\frac{r}{r+h}$ where h is the altitude difference between aircraft and horizon surface and r is the earth radius, 6.357×10^6 metres). Frequently however the horizon is not identifiable and assessment of height difference requires knowledge of the bank angle of the pacer when the observation is made. If instrumentation to measure bank angle is not available, one may have to rely on the assumption that it was zero, in which case a random error of up to 20 feet, depending on actual bank angle and separation, may be introduced; for a typical separation of 300 feet and an actual, but unknown, bank angle of 1°, the error would be 5 feet which in turn may cause a pressure determination error between 0.2 and less than 0.01 mb. If the bank angle can be measured, for example by an inertial system on which the time of camera observation is recorded, a more accurate inter-aircraft height observation can be made. Let the pacer be banked by an angle ξ, wing down towards the subject aircraft, and the subject aircraft be observed at a position B on the film or on a projection of the film on a screen. Then if O is the point representing the wings-level plane, the horizontal plane will be represented on film or projection by the point A. Let f_c and f_p be the focal lengths of camera and projector respectively, D the distance from camera to subject aircraft, and P the projection distance (the two last both measured from the lens centre). Then with ξ small and expressed in radians we have from simple optics:



$$OA = \xi \times (\text{Camera focal distance}) \times \frac{P}{\text{Projector focal distance}}$$

$$= \xi \times \left(\frac{Df_c}{D-f_c}\right) \times \left(\frac{P}{f_p} - 1\right)$$

The first factor in brackets may be taken to equal f_c (since $D >> f_c$), but the second should be evaluated (P may not be $>> f_p$). If we measure directly on the film, $OA = \xi \times f_c$. The most convenient way of obtaining ΔH is then to mark A on the film or projected image and to measure AB in terms of a known dimension of the subject aircraft which can be measured on the image; this gives height difference without need for finding the distance of subject aircraft from the camera. The point to be measured is that of the pressure recording instrument.

The two aircraft operate in formation with a separation of at least 3 wing spans (and safety considerations may dictate greater separation, particularly when supersonic). The important condition for accuracy is that both aircraft, at the moment of reading pressures, should be in steady level flight. Speeds are not directly compared (except in the special case of a pitot calibration test) and it is not important that the two aircraft should be at exactly the same speed; attempts to maintain formation in speed, inevitably diverting pilot attention from maintenance of steady unbanked conditions and requiring engine setting adjustment, should be avoided. Pilots should if necessary allow the formation to "drift apart" longitudinally provided separation does not become so great as to prevent inter-aircraft height observation; in that case however the orientation of the observation camera in the pitch plane should be assessed and the line representing the horizontal should be drawn at that angle. Another acceptable technique is a "slow overtake" in which a speed differential of 10 to 20 kn is maintained and observations are taken at the time of passing; this may be preferable, in establishing steady conditions on the two aircraft, to setting of formation speeds. If the test speed required of the subject aircraft is below the minimum or above the maximum of the pacer, this speed differential method may be extended to greater differences in speed so that the pacer respectively overtakes, or is overtaken by, the subject aircraft. The limit on speed differential being that imposed by the need to make the inter-aircraft height observation, synchronised with the pressure observations; with high speed photography and data acquisition, quite high differences in speed will be practicable. As speed difference increases, pilots may increase separation distances on safety grounds. If practicable, the technique can be assisted by release of a smoke trail from the overtaken aircraft.

For the pacer method to be used as described above, the two aircraft must be able to operate at the required speeds throughout the test envelope (with such additional capability as overtaking, at acceptable and practical speed difference, can give), and the calibration of the pacer aircraft must be well established throughout the speed range. The endurance of the pacer, particularly if supersonic tests are required, may limit the amount of data which can be acquired on one sortie. It may therefore be preferable to "calibrate" an airspace using the pacer, tracked by radar, to obtain the relationship between radar-measured height and pressure, and then to use the same radar to obtain the height of the subject aircraft, and so to infer its ambient pressure, throughout the range of test conditions. This variant of the method allows the pacer to operate at one optimum speed, normally subsonic. It is then possible to change conditions more rapidly on the subject aircraft, to the point if desired of using steady accelerations or decelerations, or climbs or descents, thus obtaining much accelerated data acquisition. Attainment of this benefit requires close time correlation of radar and pressure readings, by synchronising signals. The technique requires correlation of pressure readings taken at different times; care should be taken that rapid ambient pressure changes do not occur within the test period, and the pacer should do calibration runs before and after those of the subject. Similar considerations apply to geographic separation, particularly if the subject aircraft runs are long (for instance at supersonic speed); pacer aircraft runs should cover the geographic range of the subject. Again however, as for the tower flypast method, it is undesirable to make pressure error measurements in the high wind conditions associated with rapid meteorological change of pressure. Note also that any systematic error of the radar (eliminated by the method if the error is constant and applies to both aircraft) may in fact be a function of radar range (if the error is in fact one of angle) so for this reason also the pacer should cover the geographic range of the subject.

Another variant in application of the pacer aircraft is the use of a smoke trail (or, in favourable conditions, contrails) to indicate the height of the pacer aircraft to the subject aircraft. The pacer flies at constant airspeed and pressure altitude, maintained to ±10 ft, leaving a trail which is followed by the subject aircraft, the true ambient pressure being obtained from the pacer aircraft calibration. When the trail has been established, the subject aircraft accelerates from some distance astern of the pacer, flying alongside the trail until it approaches the pacer, and then decelerates, taking readings throughout the acceleration and deceleration which are related to the known pressure altitude of the trail. This method depends upon the trail remaining at the pressure altitude of the pacer, which it will do to sufficient accuracy during the short period of such a trial if meteorological conditions are calm and provided that the pacer operates at a lift coefficient sufficiently low to avoid generation of a downwash which will affect the height of the visible trail. In planning the trial the distance vs time profile of the subject aircraft should be calculated by integration of the intended speed vs time profile, and the distance covered from the start of the run when the speed on deceleration is equal to that of the pacer can be obtained. The pacer should start its run with a lead time such that, when speeds are equal, it will have covered a slightly greater distance than the subject, so that the subject approaches but does not overtake it.

Although the pacer aircraft is usually used as described above to obtain ambient pressure via the static pressure source of the pacer, it is possible to use a speed comparison method and indeed the name "pacer" derives from such use. The two aircraft fly in close formation so that it is established, within close limits of accuracy, that their speeds are identical at the moment of taking readings. The value of V_c for the pacer may then be derived from its calibration, and this therefore must be the value of V_c for the subject aircraft also. The difference between V_i of the subject aircraft and V_c derived from the pacer provides the data from which the pressure error may be obtained,

$\frac{p_p - p}{p_{SL}}$ being derived from V_c and $\frac{p_p - p_s}{p_{SL}}$ from V_i, using Eqs (9) or (10). The static pressure error $p_s - p$ is thus available and associated errors ΔH and ΔC_p may be obtained.

The use of a pacer aircraft obviously requires accurate knowledge of the calibration of the pacer itself, to a standard above that required of the subject aircraft, and this must be established by another method such as trailing static or radar/radio-sonde (Section 4.4). To use the radio-sonde for this purpose, particular care to obtain high accuracy is required. A multi-sonde survey before and after calibration runs will be essential - but expensive and not practicable on all sites; the radar speed method (Section 4.5) may be appropriate when experience is gained in its use; the trailing sensor (if it can be fitted) is at present the most convenient way of calibrating a pacer aircraft at higher altitude.

Accuracy:	Calibration error of pacer	±0.4 mb ("tower flypast" accuracy not available at altitude).
	Error of pressure measurement on pacer	±0.2 mb.
	Error of pressure measurement on subject	±0.2 mb.
	Error of measurement of height difference	±0.1 to ±0.2 mb (error decreases with altitude).
	Overall mean error	±0.6 mb.

Advantage: Fairly accurate method usable at all altitudes within pacer and subject range.

Limitations: (i) Requires two aircraft and is therefore an expensive form of trial.

(ii) Limited to 1g level flight (or small accelerations within which formation can be maintained) for formation or flypast. Limitation does not apply if radar is used.

4.3.2 Calibration of Pitot Error by Pacer Aircraft

As described above, the pacer aircraft provides an ambient pressure as a reference for that sensed by the subject aircraft, either directly measured by the pacer or by speed comparison if that method is used. However if the speeds of the two aircraft are the same, as required in the speed comparison method, a comparison of pitot pressures gives a means of calibrating the pitot system of the subject aircraft, or at least of investigating whether there is a pitot error, for subsequent calibration by pitot-in-venturi (Section 4.6). It is preferable in this application to use absolute pressure instruments, so confining the investigation to pitot pressure only, and since pitot pressure is insensitive to small changes in aircraft flow field, such as that induced by a formating aircraft, the two aircraft should fly in close formation to ensure that speeds are closely matched when data are acquired.

4.4 Methods Making Use of Radar Altitude Measurement

The section above describing the use of pacer aircraft included a method by which the pacer, radar tracked, is used to establish a relationship between radar-measured altitude and ambient pressure, which is then applied at a radar-measured altitude of the subject aircraft. This is one of several applications of the concept of a radar-altitude vs pressure relationship. These are:

(i) Use of a radar-tracked calibrated aircraft as already described.

(ii) Tracking of radio-sondes and derivation of pressure from data transmitted by them.

(iii) Use of the subject aircraft itself, in a condition in which its calibration is known, to establish the radar-altitude vs pressure relationship which is then used at other conditions in which the calibration is not known.

(iv) True airspeed measurement by radar, together with radar altitude, to derive ambient pressure from on-board pressure measurement. This method is usable only at low Mach number (<0.6) and is therefore used, as in (iii), to determine ambient pressure, related to radar-measured altitude, in the range in which it can be used.

These various methods are described in further detail below:

4.4.1 Radar-Tracked Calibrated Aircraft

This method, included here as it is a radar-tracking method, has been described in Section 4.3.

4.4.2 Tracking of Radio-Sondes

A radio-sonde balloon transmits pressure, or more commonly temperature, as it ascends; from the temperature vs altitude profile thus obtained the pressure may be obtained by integration of the hydrostatic equation $dp = -\rho g_0 \rho dH$. A relationship between pressure and radar-measured altitude is thus obtained which can be applied to the radar-measured altitude of the aircraft (using the same radar) to derive pressure.

The practical application of this very simple concept requires consideration of the following error sources:

(i) Measurement of altitude by radar.

(ii) Derivation of ambient pressure at the sonde.

(iii) Change of derived ambient pressure vs radar altitude relationship with time.

(iv) Change of derived ambient pressure vs radar altitude relationship with geographic position.

These error sources are considered below:

4.4.2.1 Measurement of Altitude by Radar

The error of a radar unit is not necessarily represented by an altitude error as a function solely of altitude. The radar's initial data is of range and elevation angle, and error may occur in either or both, elevation errors being usually the more significant; errors in radar altitude may therefore be a function of distance from the radar, which may lead to error in derived H if distances of aircraft and sonde are widely different. For this reason, as well as for true meteorological change with geographic position, the reference from radio-sonde to aircraft should be geographically close to the sonde position. Quantitative assessment of radar altitude error is not readily available since an independent altitude measurement for such an assessment cannot be made with sufficient accuracy over the required range of distance and elevation. In a particular case 2σ values of 35 ft bias error and 25 ft random error have been quoted at a range of 30 km, and 180 ft and 90 ft respectively at 100 km.

4.4.2.2 Derivation of Ambient Pressure at a Radio-Sonde

The decision on transmission of temperature, pressure, or both from the sonde during its ascent is concerned with the accuracy obtainable using "loss in use" equipment, and different sonde systems may adopt different methods. The transmission of both quantities, even if accuracy of pressure transmission is fairly low (because high accuracy here requires expensive equipment not readily thrown away) permits a calculation from both quantities which can give good results; this approach is adopted in USA. It is relevant to point out that the requirements of meteorology and of air data calibration are different and the optimum choice of transmitted parameters may therefore be different in the two applications; since the meteorologist tends to be regarded as the expert who advises on radio-sonde methods it may be particularly important to draw attention to these factors. The essential difference is that the meteorologist (who in any case usually requires a lesser standard of accuracy) wants to know the pressure at a geometric altitude, whereas we want to know it at the radar-measured altitude of the aircraft which, if there is an error in the radar, will differ from the geometric altitude. The meteorologist's requirement is best met by transmission of temperature from the sonde because the derivation of pressure from temperature, by integration of $\int dp = -g_0 \int \rho dH$, is much less sensitive to errors in ρ, arising from such displacement of the temperature/altitude profile as radar errors may produce, than it is to incorrect evaluation of the range of H to be used in the integration; it is therefore at an advantage when used with defined geometric altitudes. If the integration is performed to obtain the pressure at a radar-measured altitude which is in error, the range of H will be incorrect and an error in calculated pressure will result. On the other hand if it is pressure which is transmitted from the sonde, that will be attributed directly to the radar measurement of altitude at which it occurs, so that (if the same radar in the same range is used for sonde and aircraft altitudes) radar errors will be self-cancelling in the air-data application (but will give an error for the meteorologist since the measured pressure will be attributed to the radar-measured altitude rather than the geometric altitude at which it occurs). For example if we imagine a radio-sonde calibration by temperature of the International Standard Atmosphere using a radar which has an error which is linear from zero at sea-level to 100 ft at 30 000 ft, the pressure altitude which will be calculated for a geometric altitude of 30 000 ft will be 29 989 ft, an error of 11 ft, but at this geometric value the radar will indicate 30 100 ft and if, believing this to be the geometric altitude, we integrate to that value we obtain a pressure altitude of 30 089 ft, an error of 89 ft. In these considerations the magnitude of possible radar errors is clearly relevant also. At present the consensus, even for aircraft calibration, is that temperature transmission is preferable, but the above argument should be borne in mind if instrument developments move to the advantage of pressure transmission.

The overall accuracy obtainable from a radar-radio-sonde traverse is difficult to establish, since in general no independent measure of pressure against which it can be assessed is available - that would require a further solution of the pressure vs altitude determination problem which is our present concern. Similarly, as noted above, radar equipment is extremely difficult to calibrate by means of an independent measurement, and that is a factor in the overall assessment for a radio-sonde. Varying assessments of accuracy have been given; a 2σ accuracy of 2 mb at 40 000 ft or 1 mb at 70 000 ft for the pressure derived from a single sonde appears likely.

4.4.2.3 Change of Pressure with Geographic Position and with Time

Change of sea-level atmospheric pressure with geographic position is an occurence known to everyone familiar with meteorological isobar charts; similar charts can be drawn for other altitudes, showing isobars at a constant geopotential altitude or, more typically, contours of constant geopotential altitude for a given pressure, ie mapping a constant pressure surface. An account of these meteorological factors is given in Ref (5). Salient features are that wind tends to flow perpendicular to the pressure gradient - that is, along the isobars, and wind speed is proportional to the pressure gradient, or inversely proportional to isobar spacing, at a given latitude, but the proportionality factor varies with latitude, increasing towards the equator. The relationship between pressure changes and wind velocity is complex but a simplified approximation is given in Ref (5) for geostrophic flow (straight and balanced at equilibrium), and Figure 1 below is taken from it.

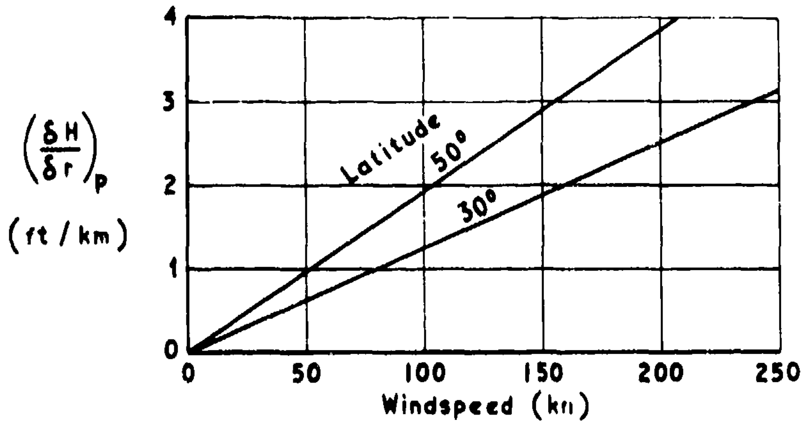


Figure 1   Slope of a Surface of Constant Pressure Altitude, Normal to the Wind Direction

$$\frac{dp}{dr} = \frac{dp}{dH}\left(\frac{\partial H}{\partial r}\right)_p = -\rho g_o\left(\frac{\partial H}{\partial r}\right)_p \qquad (14)$$

H is in this case the altitude of a constant pressure surface of pressure p and r is distance in km, from which we can infer that for a wind of 100 kn at latitude 50°, a distance measured normal to isobars of 100 km corresponds to pressure changes of 7.1 mb, 2.7 mb, and 0.7 mb respectively at constant geopotential altitudes of 0, 30 000 and 60 000 ft. This approximate model gives no adequate basis for correcting measured pressures for change of geographic position and in any case the data of wind velocity are unlikely to be available, or if they are they would probably be accompanied by a pressure survey which would make correction on this basis unnecessary. This model serves rather to help understanding of the requirements for derivation of pressure at an aircraft from atmospheric surveys such as radio-sondes.

Change of pressure with time is a corollary of change with geographic position since the pressure systems of the atmosphere are themselves in motion and changing their form with passage of time. For pressure error calibration trials the following criteria emerge:

(i) Trials should so far as possible be conducted in conditions of steady pressure with low wind velocity.

(ii) Test runs should be made along isobars (ie along the direction of the wind.

(iii) The trial should be conducted so far as possible in the geographic area of the atmospheric survey, or if the trial must be over an extended area (as in high speed trials covering large distances) the atmospheric survey should be extended (by radio-sonde or other means) to include the extremes of the test area.

(iv) An atmospheric survey should be obtained before and after the trial, as close to it in time as possible. If criterion (i) has been observed, the surveys should not differ greatly and linear interpolation with time should be adequate. If it is known before a trial that substantial change with time is likely to occur (and the trial cannot be deferred for better conditions) means should be sought of obtaining intermediate pressure data, for example by repeating a test condition at the beginning, middle and end of the trial so that the pressures recorded on the aircraft provide a basis for interpolation against time.

The most accurate procedure for radio-sondes is to conduct a multi-sonde survey, with sondes spaced geographically to include the trial region and in time to include the duration of the trial. Such a survey was applied in Ref (5) using upper-air data routinely available at 12 hour intervals; this provides data for geographic and temporal mapping of the trial region and, by using a large number of sondes, may improve on the accuracy of pressure determination obtainable by a single sonde or a pair before and after a trial. A 2σ accuracy of 0.5 mb at 40 000 ft at a given position has been estimated using this procedure, but it may be expensive in radio-sonde use if routine upper-air observations do not cover the geographic regions surrounding the test area and therefore may not always be practicable. The procedure is more practicable by nations having a large geographic area so that the whole survey region falls within its boundaries, and within the technical administration of a single organisation.

4.4.2.4 Accuracy, Advantages and Limitations

Accuracy:	Derivation of ambient pressure at the sonde	±0.5 mb to ±3 mb (depending on facilities used).
	Error of pressure measurement on subject aircraft	±0.2 mb.
	Errors due to differences in time and geographic position	Indeterminate - must depend on meteorology, time difference, but should not exceed 1 mb.
	Overall mean error	±0.55 to ±3 mb.

Advantages: Can be an accurate method in favourable conditions.
Can be used at all flight conditions of accelerated flight and climb and descent.

Limitation: Requires external facilities, radio-sonde and radar.

4.4.3 Derivation of Pressure from the Subject Aircraft Itself

An already established calibration of the subject aircraft, obtained for example by flypast calibration, may be used to determine the ambient pressure at an altitude higher than that at which the calibration was obtained, and the ambient pressure obtained, related to a radar measurement of altitude, can give a reference to which further radar altitude measurements, corresponding to aircraft conditions outside the existing calibration, may be referred.

The pre-requisite for this procedure is that it must be known that the existing calibration is applicable at the higher reference altitude and condition. This can apply in the following circumstances:

(i) It is known - usually only in the subsonic range - that the calibration is not affected by angle of attack.

(ii) An angle of attack and Mach number combination of the existing calibration can be repeated at the required altitude.

(iii) The calibration, in the range to be used, is not affected by Mach number.

Condition (i), of insensitivity to angle of attack, is not readily established over an altitude range large enough to be useful. It can be applied only to a nose-boom static sensor probe which can be shown in wind-tunnel calibration to be insensitive to angle of attack, and even then it must be installed sufficiently ahead of the aircraft to give confidence that it will not be affected by changes in the aircraft flow which are due to its angle of attack. The range of angle of attack or lift coefficient required to establish such insensitivity in flight cannot usually be achieved in a flypast calibration in which the lift coefficient range available at any Mach number is limited to that given by the available weight range of the aircraft. If independence of angle of attack is established, in the form that $\Delta C_p = f(M)$ then the value of ΔC_p derived from the calibration can be applied at the same Mach number at any other altitude; in such a case the calibration will be independent of aircraft weight within the range at which independence of angle of attack applies.

Condition (ii), that an angle of attack and Mach number combination from the existing calibration (usually at sea-level) can be repeated at a substantially different altitude, requires a very large weight range for the aircraft, since at a given Mach number the angle of attack is a function of lift coefficient, and we require therefore to obtain the calibration lift coefficient at a different value of p. To do this, $\frac{nW}{p}$ must be kept constant for the calibration and reference altitudes; in level flight a weight range as great as 2:1 corresponds to an altitude of about 18 000 feet for the reference condition using a "known calibration" at sea-level; this technique could be of some value if such a weight range is available, the reference point being observed in relation to radar altitude near the end of a trial when weight is sufficiently reduced.

Alternatively n may be reduced by manoeuvre to less than unity to give a value of $\frac{nW}{p}$,

and hence C_L, corresponding to that of the known calibration, but close correlation of air data and radar data is then required, and accuracy may be impaired by lag effects (even if correction is made for them).

The condition that the calibration is not affected by Mach number should be satisfied for Mach numbers less than 0.5, at which compressibility effects should not occur. In such a case we will relate a calibration value corresponding to one Mach number to a reference point at a different Mach number, whereas for the other conditions as described above we have related at constant Mach number. In assuming that "the calibration is independent of Mach number" we have to decide what parameter it is which has this characteristic of independence. At constant Mach number we can say, as above, $\Delta C_p = f(M)$, or $\Delta C_p = f(M, C_L)$, but in this case the most appropriate rule is that given by the Prandtl-Glauert relationship and we should say that:

$$\Delta C_p \beta = f(M, C_L \beta) \qquad \text{or, for } M < 0.5$$

$$\Delta C_{p\beta} = f(C_L \beta) \qquad \text{where } \beta = \sqrt{(1 - M^2)}$$

noting that at our limiting Mach number of 0.5 the factor β has the value 0.866 whose difference from unity, though small, is not negligibly so.

To obtain the greatest possible altitude, and thus the greatest possible extension, in altitude terms, of the sea-level calibration, the aircraft must be flown at the limiting Mach number of 0.5, the highest lift coefficient which can be flown at that Mach number, and the lowest possible weight (so that again it will be appropriate to obtain the reference data at the end of trial runs).

Then, applying subscripts t and cal to the test and calibration conditions respectively

$$\delta = \frac{W_t/S}{0.7 p_{SL} M_t^2 C_{L_t}}$$

and this defines the altitude which must be specified for the reference point. The required value of $C_L \beta$ is then $C_{Lt} \beta_t$, and if the calibration data are expressed as curves of $\Delta C_p \beta$ plotted against $C_L \beta$ the required point can be found. (If the calibration has been made over a range of weights a range of points having the required value of $C_L \beta$ may occur, but if the method is valid the same value of $\Delta C_p \beta$ will occur at all such points).

From the condition $C_{L_{cal}} \beta_{cal} = C_{Lt} \beta_t$

we have $$\delta \bigg/ \frac{W_t}{W_{cal}} = \frac{M_{cal}^2}{M_t^2} \, \frac{\beta_t}{\beta_{cal}}$$ (assuming the calibration to be at sea level)

In Figure 2, $\frac{C_{Lcal}}{C_{Lt}}$ and $\delta \Big/ \frac{W_t}{W_{cal}}$ are plotted against M_{cal} for the case where $M_t = 0.5$. It can be seen that the value of C_{Lcal} required in the calibration is slightly below the maximum which can be flown at the test altitude, and consequently it will lie within the range of the sea-level calibration. If we take as a possible case, for example, an aircraft of wing-loading 3000 N/m^2 and $C_{Lt} = 1.0$ we have, for $M_t = 0.5$, $\delta = 0.1692$ (H = 41 870 ft) and, if $W_t = W_{cal}$, $M_{cal} = 0.218$ and $C_{Lcal} = 0.887$.

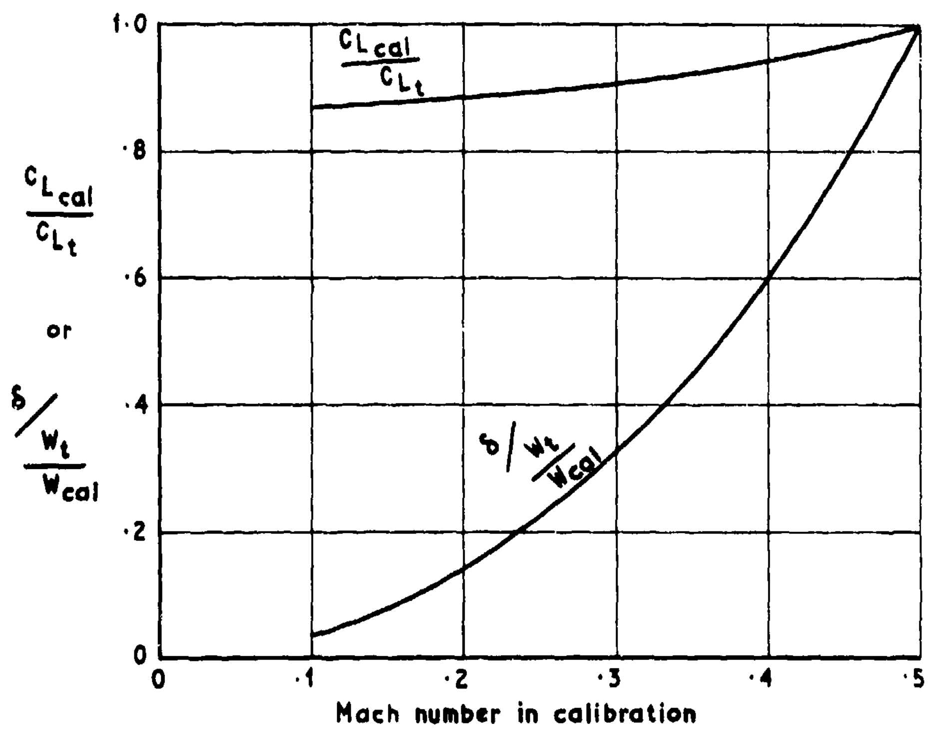


Figure 2 Lift Coefficient and Pressure Altitude Relationships for Constant $C_{L\beta}$

In using this method an iterative calculation is required since, aimed values of δ_t and M_t having been determined as above, the aircraft will be flown at indicated values close to the aimed ones. From indicated M_i and δ_s, C_L and $C_{L\beta}$ can be obtained, and hence, from the low-level calibration, $\Delta C_{p\beta}$, and then ΔC_p. This ΔC_p can then be used to re-evaluate M, δ, $C_{L\beta}$, $\Delta C_{p\beta}$ and so to obtain a revised ΔC_p. This iteration is continued to obtain a final value of ΔC_p and so of p which is attributed to the measured radar altitude.

The method uses some assumptions which cannot readily be verified by other means. The use of $\Delta C_{p\beta} = f(C_{L\beta})$ at low M has some justification in theory but its applicability in practice cannot readily be confirmed; some authorities may prefer $\Delta C_p = f(C_L)$; analysis of an adequately extensive data base of pressure information could be useful in this context but the present authors are not aware of such an analysis. There is a further assumption that Reynolds number is not a significant parameter; this may be true in many instances but there may be some in which it is untrue, and without using a quite separate means of obtaining ambient pressure - which would nullify the purpose of using the method - this cannot be determined.

Accuracy:	Low-level calibration of subject aircraft	±0.25 mb.
	Error of pressure measurement in determining ambient value	±0.2 mb.
	Error of pressure measurement at test condition	±0.2 mb.
	Overall mean error.	±0.4 mb.

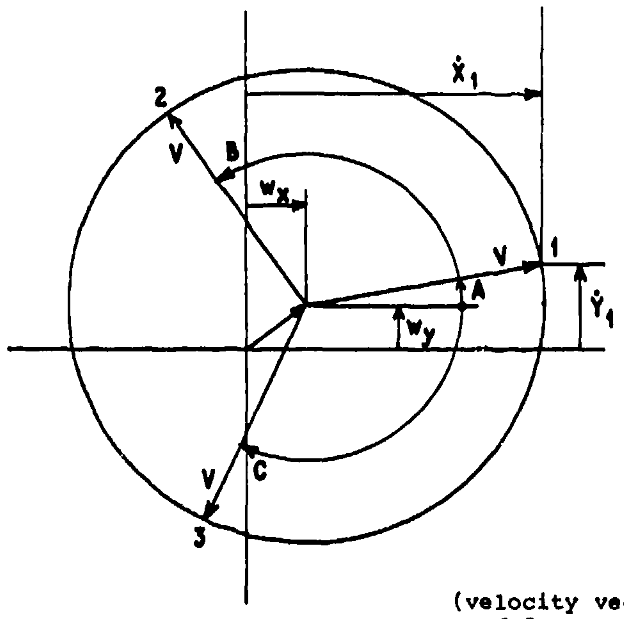
Advantage: Requires subject aircraft and radar only.

Limitation: Depends on some assumptions not independently confirmed (see above).

4.5 Radar Measurement of Aircraft Speed

It is evident that pressure error may be obtained by comparing the airspeed derived from the aircraft pressure system with a measurement of true airspeed, provided that the true airspeed measurement can be obtained with sufficient precision. This approach has been little used because true airspeed cannot be readily measured; it must be inferred from measurements of ground speed and wind-speed, and the accurate measurement of the latter, applicable at an aircraft in flight, presents substantial difficulty. Some use has been made in the past of the low-level "speed course", timing aircraft passage over surveyed points, but at ground level in particular the wind speed is liable to short-term unpredictable changes which make this approach unreliable.

At altitude the variation of wind speed within a short time interval is usually much less, because of the absence of ground-generated eddies. It is possible, particularly if favourable meteorological conditions are chosen for a trial, to assume that the wind speed is constant, though of unknown value, within the period of time required to perform three runs at steady airspeed, on tracks approximately 120° apart (ie 3-5 minutes). If it can be assumed that under conditions of constant engine setting and level flight the true airspeed is constant also, then if ground speed is known (eg measured by radar) in the three runs then, using an approach proposed by AFFTC at Edwards Air Force Base in the USA, the wind-speed and true airspeed can be obtained. In Figure 3 (below) we have the following:



$$\text{(i)}\quad \dot{X}_1 = w_x + V\cos A,\qquad \dot{Y}_1 = w_y + V\sin B$$

$$\text{(ii)}\quad \dot{X}_2 = w_x + V\cos B,\qquad \dot{Y}_2 = w_y + V\sin B \qquad (15)$$

$$\text{(iii)}\quad \dot{X}_3 = w_x + V\cos C,\qquad \dot{Y}_3 = w_y + V\sin C$$

(velocity vectors and components for points 2 and 3 are similar to those shown for point 1)

Figure 3 Velocity Vectors for Radar Speed Measurement

where $\dot{X}$, $\dot{Y}$ are the three pairs of values of the radar velocity components, V is the true airspeed, w_x and w_y are the windspeed components, and A, B and C are the aircraft heading angles whose values are known approximately to be 0°, 120°, and -120° relative to the X direction.

Hence $$\frac{\cos A - \cos B}{\sin A - \sin B} = \frac{\dot{X}_1 - \dot{X}_2}{\dot{Y}_1 - \dot{Y}_2} \quad \text{ie} \quad \tan\frac{A+B}{2} = -\frac{\dot{X}_1 - \dot{X}_2}{\dot{Y}_1 - \dot{Y}_2}$$

and similarly for tan((B+C)/2), tan((C+A)/2)

therefore $$A = \arctan\left(-\frac{\dot{X}_1 - \dot{X}_2}{\dot{Y}_1 - \dot{Y}_2}\right) - \arctan\left(-\frac{\dot{X}_2 - \dot{X}_3}{\dot{Y}_2 - \dot{Y}_3}\right) + \arctan\left(-\frac{\dot{X}_3 - \dot{X}_1}{\dot{Y}_3 - \dot{Y}_1}\right)$$

$$B = \text{ditto} + \text{ditto} - \text{ditto} \qquad (16)$$

$$C = -\ \text{ditto} + \text{ditto} + \text{ditto}$$

the arctans being taken with the knowledge that (A+B)/2, (B+C)/2, (C+A)/2 are approximately 60°, 0°, and -60° respectively.

And now $V = (\dot{X}_1 - \dot{X}_2)/(\cos A - \cos B)$, and w_x, w_y may be obtained if desired.

The assumption of constancy of wind velocity cannot be verified from the experimental data, but the assumption of constancy of V can be verified, or otherwise, from the air-data system; even for an uncalibrated system it will be possible to determine that, if V is the true airspeed on the first run, the airspeeds on the subsequent two are f_2V and f_3V respectively, where f_2 and f_3 are close to unity. In the ideal case where they are unity the solution above may be applied. If not, we have to find a point in Figure 3 which lies distant from the three points in the ratios 1, f_2, f_3 respectively.

The locus of a point whose distance from point 2 is in a fixed ratio f_2 to its distance from point 1 is a circle, whose equation is:

$$X^2 + Y^2 - 2\,\frac{f_2^2X_1 - X_2}{f_2^2-1}\,X - 2\,\frac{f_2^2Y_1 - Y_2}{f_2^2-1}\,Y + \frac{(f_2X_1)^2 - X_2^2 + (f_2Y_1)^2 - Y_2^2}{f_2^2-1} = 0 \qquad (17)$$

ie $\quad X^2 + Y^2 - CX - DY + E = 0$

and similarly for the circle which is the locus of the point whose distances from points 3 and 1 are in the ratio f_3.

$$X^2 + Y^2 - 2\,\frac{f_3^2X_1 - X_3}{f_3^2-1}\,X - 2\,\frac{f_3^2Y_1 - Y_3}{f_3^2-1}\,Y + \frac{(f_3X_1)^2 - X_3^2 + (f_3Y_1)^2 - Y_3^2}{f_3^2-1} = 0$$

ie $\quad X^2 + Y^2 - FX - GY + H = 0$

and a point of intersection of the two circles is the required point

The solution is $LY^2 + MY + N = 0$

where $L = \left(\frac{D-G}{C-F}\right)^2 + 1$, $M = C\,\frac{D-G}{C-F} - 2\left(\frac{D-G}{C-F}\right)\left(\frac{E-H}{C-F}\right) - D$, and $N = \left(\frac{E-H}{C-F}\right)^2 - C\,\frac{E-H}{C-F} + E$

ie $\quad Y = \left(-M \pm \sqrt{[M^2 - 4LN]}\right)/(2L)$

and $\quad X = \frac{E-H}{C-F} - \frac{D-G}{C-F}\,Y$

Thus, since circles which intersect must do so at two points, we have two solutions of which only one is realistic. The other lies well outside the circumscribing circle of points 1, 2, and 3, and we can see that a second point having the required distance relationships may occur there. The point within the circumscribing circle having been selected, the values of X and Y are those of w_x and w_y, and

$$V = \left[(\dot{X}_1 - w_x)^2 + (\dot{Y}_1 - w_y)^2\right]^{1/2} \qquad (18)$$

This solution fails if either f_2 or f_3 equals unity (the locus then being a straight line bisector). In that case the intersections of circle and straight line give solutions, or if V is attributed to the vector which is not equal to the other two, so that f_2, f_3 are equal but not unity, the above solution can be used.

The accuracy of determination of V depends upon that of measuring the components $\dot{X}$, $\dot{Y}$ of groundspeed and on the assumption of constancy of wind speed. Radar measurement of groundspeed can achieve very high accuracy; if wind speed components change between the three runs an error is introduced in the form of changes in $\dot{X}$, $\dot{Y}$ equal to the wind component changes, and consideration of the effect of such changes on the solution of Eq (15) leads to the conclusion that the resulting error in V is not more than one third of the greatest difference in wind velocity component between the three runs. If therefore steady meteorological conditions are chosen it may be assumed, perhaps pessimistically, that V may be obtained to within 1 kn of the true value.

V being known, the Mach number and stagnation pressure must be obtained in order to derive the required static pressure. For Mach number at known V the ambient temperature is required; if a radio-sonde survey is available, close in both time and geographic position to the trial, this may provide the temperature to within 1K; if, as is more probable given the difficulty of co-ordinating with a radio-sonde in time and space, such a survey is not available, the temperature must be obtained from on-board measurement using a probe on the aircraft. The stagnation pressure can be obtained by direct measurement of pitot pressure, with correction for pitot error if necessary. Then, from T and V, M (=V/a) can be obtained, and hence $\frac{p_p}{p}$, and then p. The sources of error in this determination are the errors in V, T and p_p.

If we consider first the error in V we have, at constant p_p (from Eq (7)):

$$\frac{dp}{dV} = \frac{\frac{d}{dV}\left(\frac{p_p}{p}\right)}{\frac{d}{dp}\left(\frac{p_p}{p}\right)} = \frac{3.5(1+M^2/5)^{2.5}(2V/5a^2)}{-p_p/p^2} = \frac{1.4(1+M^2/5)^{2.5}(M/a)}{-(1+M^2/5)^{3.5}/p}$$

$$= -\frac{1.4Mp}{a(1+M^2/5)}$$

and therefore $$\frac{\Delta p}{p} = -\frac{1.4M}{1+M^2/5}\left(\frac{\Delta V}{a}\right) \qquad (19)$$

Considering the error in T, we have that $\frac{T_p}{T} = 1 + kM^2/5 = \left[1+k\left(\frac{V}{a_{SL}}\right)^2 \frac{T_{SL}}{T} \Big/ 5\right]$

that is $$T = T_p - k\left(\frac{V}{a_{SL}}\right)^2 \frac{T_{SL}}{5} \qquad (20)$$

so the temperature increment $T_p - T$ is independent of T and M at given V, and T may be determined directly from T_p, V and k. The errors in this measurement of T are due to errors in T_p, k and V and we may write

$$|\Delta T| = \left|\frac{\partial T}{\partial T_p}\Delta T_p\right| + \left|\frac{\partial T}{\partial k}\Delta k\right| + \left|\frac{\partial T}{\partial V}\Delta V\right| \quad \text{(assuming that errors may be additive)}$$

$$= |\Delta T_p| + \left(\frac{V}{a_{SL}}\right)^2 \frac{T_{SL}}{5}|\Delta k| + 0.4kT_{SL}\frac{V}{a_{SL}^2}|\Delta V|$$

and assuming that k is close to unity and V <500 kn (it will be shown below that the method is applicable only at subsonic speed), and that $\Delta V < 1$ kn, we find that the third term is negligibly small (<0.13 K) compared with the other probable errors in measuring T.

therefore $$\Delta T = |\Delta T_p| + \left(\frac{V}{a_{SL}}\right)^2 \frac{T_{SL}}{5}|\Delta k|$$

$$= |\Delta T_p| + \frac{M^2}{5}T|\Delta k| \qquad (21)$$

$$\frac{dp}{dT} = \frac{\frac{d}{dT}\left(\frac{p_p}{p}\right)}{\frac{d}{dp}\left(\frac{p_p}{p}\right)} = \left[3.5(1+M^2/5)^{2.5}\left[-\left(\frac{V}{a_{SL}}\right)^2 \frac{T_{SL}}{5} /T^2\right]\right] \bigg/ \left[-(1+M^2/5)^{3.5}/p\right]$$

$$= \left[0.7\left(\frac{V}{a_{SL}}\right)^2 \frac{T_{SL}}{T^2}\, p\right] \bigg/ \left[1+M^2/5\right] = \frac{0.7M^2 p}{(1+M^2/5)T}$$

therefore $\quad \frac{\Delta p}{p} = \frac{0.7M^2}{(1+M^2/5)} \frac{\Delta T}{T}$

and from (21) $\quad \frac{\Delta p}{p} = \frac{0.7M^2}{(1+M^2/5)}\left(\left|\frac{\Delta T_p}{T}\right| + \frac{M^2}{5}\left|\Delta k\right|\right) \qquad (22)$

Adding errors from Eqs (19) and (22) we have

$$\frac{\Delta p}{p} = \frac{1.4M}{1+M^2/5}\left|\frac{\Delta V}{a}\right| + \frac{0.7M^2}{1+M^2/5}\left|\frac{\Delta T_p}{p}\right| + \frac{0.14M^4}{1+M^2/5}\left|\Delta k\right|$$

$$= \frac{1.4M}{1+M^2/5}\left[\left|\frac{\Delta V}{a}\right| + 0.5M\left|\frac{\Delta T_p}{T}\right| + 0.1M^3\left|\Delta k\right|\right] \qquad (23)$$

If we now assume that $\Delta V = 1$ kn, $\Delta T = 1K$, and $\Delta k = 0.02$, and that $T = 216$ K (corresponding to ISA stratosphere, ie the constant temperature band between 11 km and 20 km (36 089 to 65 616 feet) of altitude, we have

$$\frac{\Delta p}{p} = \frac{1.4M}{1+M^2/5}\left(\frac{1}{a} + \frac{M}{432} + 0.002M^3\right)$$

This function is plotted in Figure 4(a) below, which shows that the error in p increases rapidly with M, so that use of the method should be at the lowest available Mach number to establish values of p in relation to radar altitude, at two or more altitudes encompassing the required altitude range of the trial, and to use the relationship so established to determine values of p from measured radar altitude at other flight conditions. Because it is evident that accuracy is not maintained at supersonic conditions the above relationships have been calculated from the subsonic equations only. If the method is used at Mach numbers up to 0.6, we conclude that the errors in V and T_p are dominant, the effect of the assumed error in k being negligible except close to $M = 0.6$, and even then it is relatively small.

The assumption of additive errors is pessimistic; however for the small amount of data which will be generated on a trial the pessimistic case may occur, and statistical addition of RMS errors may be optimistic.

Figure 4(b) shows curves of Δp in millibars, derived on the above assumptions of $\Delta V = 1$ kn, $\Delta T = 1K$, and $\Delta k = 0.02$, using ISA values of T, for values of calibrated airspeed from 100 to 250 kn, plotted against pressure altitude. The increase of M as altitude is increased is more than balanced by the reduction in p, so that the total likely error decreases with altitude (as it does also for radio-sonde determination of pressure). Typically, for an aircraft of minimum calibrated airspeed 150 kn, p should be obtained to within 0.6 mb of the true value, which compares acceptably with the accuracy obtainable by radio-sonde unless a complex multi-sonde system of data aquisition and analysis is used. If better performance in data measurement can be achieved, particularly in measurement of T_p, total error may be consequently reduced. With good instrumentation errors in measured p_p should be small; slightly smaller errors in p will result.

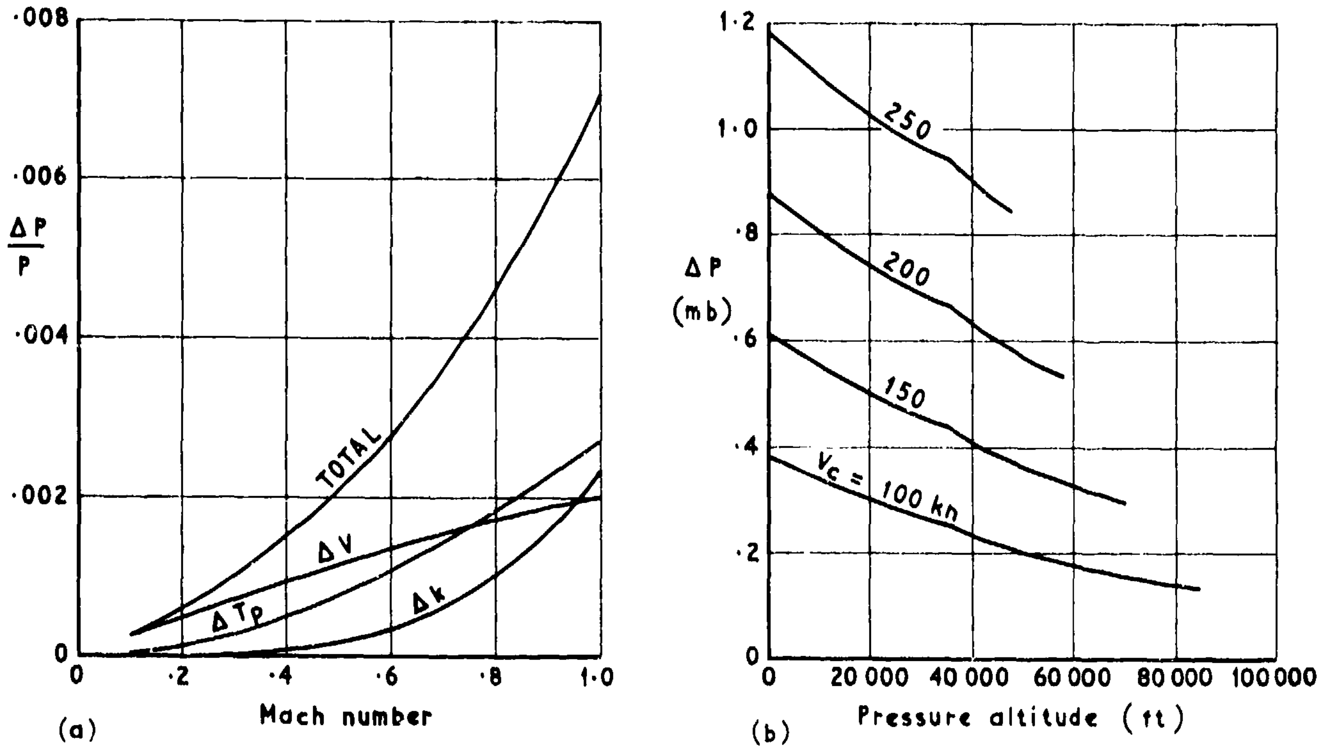


Figure 4 Estimated Errors in Measured Ambient Pressure for the Radar Speed Measurement Method

This method has not as yet been used to an extent where it can be described against a background of experience in its use. Because there is no published documentation elsewhere, and because its mathematics are a little more complex than for some of the other methods, it has been described here in some detail; this does not imply an intention to place particular emphasis on this compared with the other methods described.

Accuracy:	Measurement of ambient pressure at radar altitude	See Figure 4 above.
	Pressure measurement at test condition	±0.2 mb.
	Overall mean error	RMS of above errors.

Advantages: Fairly accurate method which may be superior to radio-sonde unless multi sondes are available.

Limitations: As yet a little tried method.
Requires steady meteorological conditions at test altitude.

4.6 Direct Measurement of True Airspeed by On-Board Instrumentation

Section 4.5 describes a method in which true airspeed is inferred from measurement of groundspeed. It is evident that if true airspeed can be directly measured, this method can be applied using that measurement. At the time of writing such direct measurement has not been achieved, but developments in laser anemometry are such that it may be possible in the near future. The analysis of Section 4.5 (apart from that of the groundspeed measurements) will apply, including the error analysis using appropriate assessment of the attainable accuracy of the true airspeed measurement

4.7 Calibration of Pitot Error by Reference Pitot

As is stated at the beginning of Section 3, the pitot system is less subject to error than is the static pressure system, and correct total head pressure p_p should be obtained for a well designed pitot except at extreme flight conditions. If calibration of the pitot system is required there is not of course, as for ambient static pressure, an aircraft-independent quantity which can be measured for comparative purposes, and an independent total-head source, free of error, is required for the aircraft. The pitot-in-venturi or fully swivelling pitot give full total head in subsonic conditions at extreme angles of flow; a number of designs, with their calibrations, are available (Ref (1)). The calibration method is therefore to mount a venturi or swivelling pitot on the aircraft in a position where it does not affect flow at the pitot system to be tested, and preferably, subject to the preceding condition, on a nose boom; the pressure difference between aircraft pitot and the reference pitot, which within the operating range of the reference pitot is a direct measure of the pitot pressure error, is then measured. The venturi pitot is subject to multiple shock patterns from the venturi in supersonic conditions and its use then is not recommended, but at such conditions a well designed pitot should give an error-free measurement of p_p.

4.8 Temperature Method at High Mach Number

The temperature T_{ps} measured by a sensor of recovery factor k is given by:

$$\frac{T_{ps}}{T} = 1 + kM^2/5 \qquad (24)$$

from which we obtain

$$M = \left[\frac{5}{k}\left(\frac{T_{ps}}{T} - 1\right)\right]^{1/2} \qquad (25)$$

so that if k, T_{ps}, T, p_p, and p_s are known by calibration or measurement M may be obtained (Eq (25)), and using Eqs (7) or (8) we obtain $\frac{p_p}{p}$ and hence p and the pressure error $p_s - p$. Temperature sensors with recovery factors of unity are available and are suited to this method. From Eq (25) we can obtain:

$$\frac{\partial M}{\partial T} = -\frac{2.5}{kTM}(1 + kM^2/5)$$

while from Eqs (7) and (8), with p_p constant, we obtain:

$$\frac{\partial M}{\partial T} = -\frac{1 + M^2/5}{1.4Mp} \quad M \leqslant 1, \qquad \text{or} \qquad \frac{\partial M}{\partial p} = -\frac{M(7M^2-1)}{7(2M^2-1)p} \quad M > 1$$

from which we obtain, assuming that k = 1

$$\frac{\Delta p}{p} = 3.5\frac{\Delta T}{T} \qquad M \leqslant 1$$

$$= 17.5\left(\frac{(1 + M^2/5)\ (2M^2 - 1)}{7M^2 - 1}\right)\left(\frac{\Delta T}{T}\right) \qquad M > 1 \qquad (26)$$

and assuming an accuracy in T of 1K (which is unlikely to be improved upon at the high Mach numbers and altitudes for which this method is applicable) we can plot $\frac{\Delta p}{p}$ against M in Figure 5, below. From this figure we can infer that, at M = 3, Δp in mb is 1.3 at 40 000 ft, and 0.2 at 80 000 ft, and conclude that this method may give some advantage over other methods at the highest Mach numbers and altitudes only, where other methods may present some difficulty. Note however (see Section 2) that in these conditions the assumption that $\gamma = 1.4$ may not be valid; instead of Eqs (25), (7) and (8), Eqs (25), (13) and (14) of Ref (1), using appropriate values of γ, may be required.

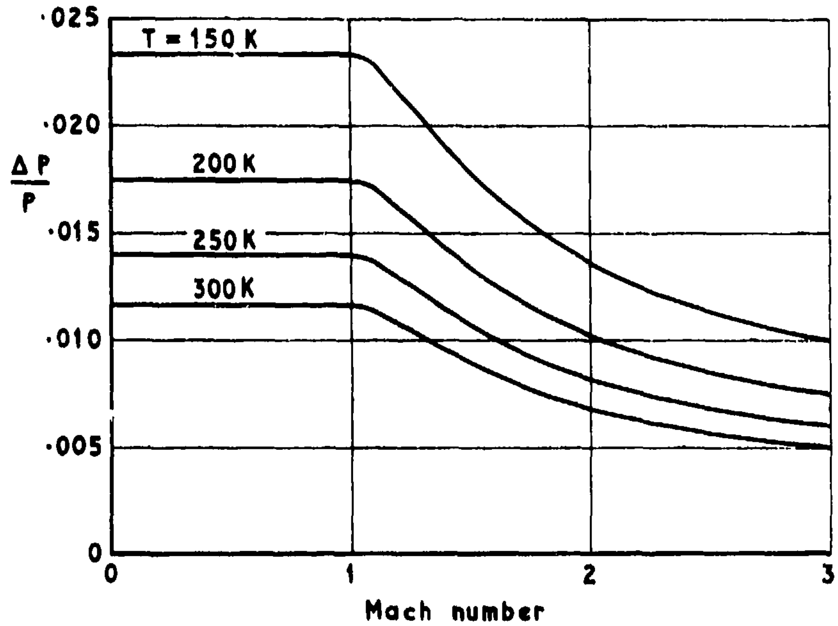


Figure 5   Estimated Error in p for Temperature Method

4.9 Calibration in Ground Effect

Pressure error in ground effect is in general relevant only for aircraft in the take-off or landing configuration, in which flight close to the ground may not be unduly hazardous, or possibly for terrain-following aircraft which may operate sufficiently close to the ground in other configurations for ground effect to be significant and which will require a control system which makes such operation acceptable for a pressure calibration trial. Calibration is made using the tower flypast method in the required conditions. The necessary accurate measurement of ground clearance may be obtained from the photographic record used in the usual height determination of the flypast method, since the height of the local surface relative to the observation point can be readily measured; alternatively a kinetheodolite tracking method may be used and this, if the surface height profile is included in the calculation program, can give height above surface throughout the length of a run which, if synchronised with the air data system, can give multiple data points from a single run; a high quality radio altimeter can be similarly used. The data required are the air data pressure, the reference point ambient pressure, the height of the aircraft relative to the surface, and the height relative to the reference pressure point; since ground effect trials will if at all possible be made above a level surface, the latter height will usually be obtainable with sufficient accuracy by inference from the former if, for example when a radio altimeter is used, it is not explicitly measured.

Pressure error calibration data may be acquired during take-offs and landings, and in particular during measured take-off and landing trials in which the aircraft path is recorded by kinetheodolites or other method, so that height relative to an ambient pressure datum position may be obtained; the data can then be processed as for a fly-past calibration. If aircraft groundspeed is obtained, as usually it will be during such trials, and if the windspeed is accurately recorded, a true airspeed is obtained and hence a stagnation pressure which can be used to check the calibration of the pitot system, and a calibrated airspeed may be obtained for comparison with that of the air data system; the accuracy of knowledge of windspeed at the aircraft will not usually be sufficient for this true airspeed data to be used as a primary calibration method.

Calibration of the pitot system by venturi or swivelling pitot is a trial self-contained within the aircraft which can equally well be made in ground effect if necessary; in the flight conditions occurring in ground effect a pitot calibration error should not exist in any well designed system, and a ground effect on such error should be improbable.

4.10 Calibration Under Normal Acceleration

The pressure error coefficients ΔC_p, for pitot and static, are generally functions of C_L and M in level flight (and they may also be dependent on Reynolds number but this effect should be small). Level flight relationships between ΔC_p, C_L and M should be valid under normal acceleration also provided that the acceleration itself does not change the relationship by distortion of the airframe and/or the pitot-static installation; whether this applies in a particular case is a matter of judgement with input from structural engineering and aerodynamics. Since $C_L = \frac{nW}{0.7pM^2S}$ a given combination of C_L and M can be obtained at all combinations of weight, normal acceleration and pressure altitude which give the appropriate value of $\frac{nW}{pM^2}$; some comparative trials should be made before reliance is placed on the invariability of pressure error coefficient with normal acceleration per se at constant C_L and M.

Measurement of pressure error under normal acceleration can be made using those methods already described which permit height determination during manoeuvring flight. Tower flypast and pacer formation methods are therefore excluded, while radar and radio altimeter methods may be used (subject to confirmation of the calibration of the latter for non-level altitudes). A level-flight calibration is established by methods already described and a reference point of known ambient pressure is obtained in level flight from this calibration, together with a height measurement by the method to be used. Normal acceleration is then applied (positive or negative) by appropriate manoeuvres such as turns or roller coasters, and true ambient pressure is derived from the reference pressure corrected by the relations $\Delta p = -\rho g_0 \Delta H$, or $H = H_{ref} + \Delta H \frac{T_{ISA}}{T}$

If practicable, an inverted flight point may be obtained to give n=-1. Height changes need not be rapid but correction should be made if necessary for lag effects (see Section 5).

5 LAG IN THE AIRCRAFT PRESSURE SYSTEM

5.1 Assessment of Lag in the Air Data System

Our consideration to this stage has been of a "steady" system in which the pressure at the sensor has been measured at the aircraft instrument on the assumption that, apart from the self compensating and usually trivial effects due to height difference between sensor and instrument, these pressures are the same. However if the pressure at the sensor is changing with time, there may be a lag in transmission of pressure to the instrument such that the pressure at the instrument is not equal to that at the sensor. This may cause an error, referred to as "lag error", in the measured pressure altitude, airspeed, or both. This lag can be minimised if the transducers are placed close to the sensor (and improvement in instrumentation techniques may make this increasingly possible).

The lag in pressure transmission may be due to the sum of both acoustic lag and pressure lag. The former, due to the time required for transmission at local speed of sound, is given by

$$\tau = L/a \tag{27}$$

where L is length of pipe. For usual pipe lengths τ is negligibly small.

Pressure lag arises because air must flow along the piping to cause pressure change at the instrument, and this flow requires a pressure difference to overcome viscous effects. This can be expressed by

$$\lambda \dot{p}_R = p_s - p_R \tag{28}$$

where λ is a time constant, equal to the time required for a pressure difference $p_s - p_R$ to decay to 1/e times its initial value, if p_s is maintained constant. The relevant relationships for the ideal case of laminar flow in straight pipes are developed in Chapter 5 of Ref (1) where it is shown that λ (designated τ in Ref (1)) is given by

$$\lambda = \frac{128\mu LV}{\lambda d^4 p}$$

where μ is dynamic viscosity, L the pipe length, V the system volume, d the pipe diameter and p the system mean pressure. This expression indicates the important effects of pipe diameter and system volume; it may be used to give first estimates of the lag constant but it leaves out of account important factors such as bends in piping, inflow or outflow effects at sensor orifices, and local turbulence in piping. Testing of the actual system to determine lag effects is essential. However, the linear differential equation model of Eq (28) is an adequate representation of the lag characteristics of air data systems in which flow rates and pressure differences are small, if λ can be determined.

Eq (28) suggests a means of evaluating λ by a ground test, since imposition of a rate of change on p_s, and measurement of the time histories of p_s and p_R, can give the parameters required to solve the equation for λ. Since time scales are short and high accuracy is necessary, high quality instrumentation with short response time is required. Three methods may be used as follows:

(i) Application of a step function at the sensor. With the sensor closed to atmosphere, the system pressure is raised or lowered by a small amount. By puncturing a membrane (or other method for rapid opening) the sensor is exposed to atmospheric pressure. In principle, the transfer function can then be calculated; in practice, unless λ is large the time scales are very short, making heavy demands on instrumentation performance, and the impulsive nature of flow initiation may cause departure from the behaviour implied by the linear equation model. Consequently requisite accuracy is very difficult to attain and the method is not usually used.

(ii) Application of a constant pressure rate at the sensor. This requires a specialised type of pressure generator but it does simulate closely what occurs in flight (apart from external flow effects referred to below) and gives accurate results in the ground test mode.

(iii) Application of sinusoidal pressure rate at the sensor. This is a convenient method if an appropriate generator is available. In this case, for the linear model considered, p_R will lag on p_s by a phase angle Ψ, and if ω is the applied frequency in radians per second, we have

$$\lambda = \tan \Psi/\omega \qquad (29)$$

and amplitude of p_R is attenuated, relative to p_s, by $\cos \Psi$. See ref (19)

Unfortunately these methods of assessing lag constants do not always work well in practice for the static pressure system where lag times much greater than those obtained by ground test have sometimes been observed. Two possible causes of this discrepancy have been proposed, and either or both may be applicable in particular cases. The external flow may affect the inflow or outflow at the sensor orifices, changing their effective area and thus the lag constant, or the inflow or outflow at the orifices may modify the external flow sufficiently to cause a change of local pressure; this latter effect would not strictly be a lag but a change of pressure error, but in practice there is no way of distinguishing them and both are conveniently included within the lag error. In view of the configuration-dependent nature of these factors it is not surprising that in some instances large differences between ground and flight results (up to a factor of 3, at NATC in USA Ref (6)) have been found while in other cases there has been good agreement (at NLR in Holland). The conclusion, however, must be that flight test calibration is necessary if consideration of system dimensions, or an exploratory ground test, show that lag may be significant.

Lag effects in the pitot system tend to be smaller than in the static system because pipe lengths and system volumes tend to be smaller; neither of the causes postulated above for discrepancy between ground test and flight test are likely to apply to pitot pressure measurement, while the accurate value of true pitot pressure, free of lag effects, during a climb or descent cannot be related directly to radar altitude as for the static system, so its measurement constitutes a problem which at present is difficult to solve accurately; consequently determination of pitot system lag has not yet been addressed by flight test techniques. If lag effects in a pitot system are suspected, a ground test should be used to evaluate it.

In a flight test of the static system, the aircraft is flown at known rates of climb or descent through a point at which the static pressure is known. First, static pressure is determined in terms of radar altitude by flying level at a range of altitudes covering the test range; then the aircraft is climbed or dived under radar observation and static pressure at the instrument is observed against radar altitude, from which true pressure and rate of pressure change can be determined. The following are the essential requirements for this method:

(i) A high-accuracy low-volume low-lag pressure transducer to measure pressure at the instrument station.

(ii) Accurate synchronisation of radar altitude and pressure measurements on board, by recording on a common base or by synchronising signals.

The necessary parameters are the pressures at the sensor and the instrument, and their rate of change; therefore static pressure error correction should not be applied and the initial correlation of radar measured altitude with pressure should be in terms of the sensed pressure in level flight. A steady climb or descent should be made through the reference altitude and this steady altitude change should be maintained, before passing through the reference altitude, for a length of time, in terms of anticipated lag, which would correspond to at least a 99% attenuation of pressure difference derived from Eq (29) that is:

$$t_2 - t_1 = -\lambda \ln 0.01 \simeq 4.6\lambda$$

The method of lag determination described above for the static system may be applied over a range of altitudes; it does not require a radar altitude vs pressure survey ,only the "steady-level" values of radar altitude and sensed static pressure, followed by values measured during climb or descent. A relationship for the effect on λ of change of pressure or temperature (the latter having its effect through change of viscosity) is given in Ref (7) and reproduced here:

$$\frac{\lambda_2}{\lambda_1} = \frac{p_1}{p_2}\,\frac{\mu_2}{\mu_1} \qquad (30)$$

where subscripts 1 and 2 refer to conditions to be compared, p is pressure at the sensor inlet, and μ the viscosity.

Applied to the static system, and assuming λ_1 is at sea-level:

$$\frac{\lambda}{\lambda_{SL}} = \frac{\mu}{\mu_{SL}} / \delta \qquad (31)$$

and applied to the pitot system, assuming λ_1 is obtained in a ground test at sea-level pressure

$$\frac{\lambda}{\lambda_{G\ TEST}} = \frac{\mu}{\mu_{SL}} / \frac{p_p}{p_{SL}} \qquad (32)$$

and applied to the pitot system, assuming λ_1 is at a comparable flight condition at sea-level

$$\frac{\lambda}{\lambda_{SL}} = \frac{\mu}{\mu_{SL}} / \frac{p_p}{p_{pSL}} \qquad (33)$$

and

$$\frac{\mu}{\mu_{SL}} = \left(\frac{T}{T_{SL}}\right)^{1.5} \frac{T_{SL} + 110.4}{T + 110.4} = \theta^{1.5} \frac{1.38313}{\theta + 0.38313} \qquad \text{Ref (8)} \qquad (34)$$

and the effect of a 20 K change of temperature is to cause a change in μ of 5% to 8.5% at 300 K and 200 K respectively, so that corrections for small changes in temperature are not necessary and in general a value of μ calculated for ISA values of temperature will be sufficiently accurate.

For the static system we can use Eq (31), and making the calculation for the International Standard Atmosphere, we obtain Figure 6 below, $\frac{\lambda}{\lambda_{SL}}$ being applicable as the ratio relative to that in flight at sea-level, or relative to that obtained in a ground test in principle, but subject in that case to the qualification that, as noted above, ground test results may not be representative of those in flight.

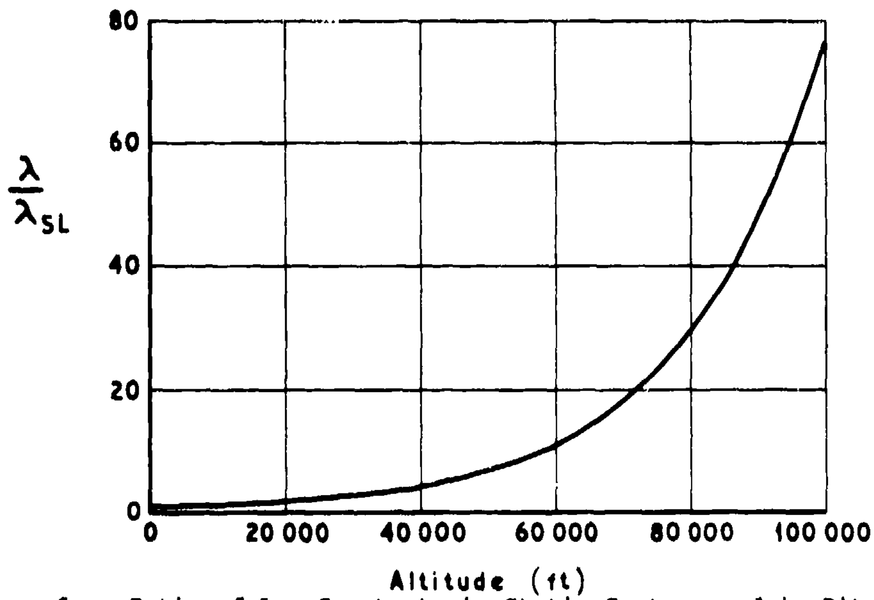


Figure 6 Ratio of Lag Constants in Static System, and in Pitot System at Constant Mach Number

For the pitot system a flight measurement of lag would require knowledge, in passing through a reference altitude, of the true pitot pressure, for comparison with that measured at the instrument. If it is possible (which usually it is not) to connect separate, low-volume, low-lag transducers at the pitot sensor and at the instrument, pressure and pressure rate data at these positions allow λ to be calculated. Otherwise, there is no convenient way of obtaining true pitot pressure since that requires both ambient pressure (known via the static system) and true lag-free dynamic pressure (not known). However the lesser sensitivity of pitot systems to lag, and to the probable reasons for discrepancy between ground-test and flight values of lag, make it valid to use a ground test in this case, and to use Eq (32) to correct to the value applicable in flight. Curves of $\frac{\lambda}{\lambda_{G\ TEST}}$ for constant values of M and V_R are plotted in Figure 7 below where $\lambda_{G\ TEST}$ is the value obtained with p_{ps} equal to the sea level pressure - see Eq (32).

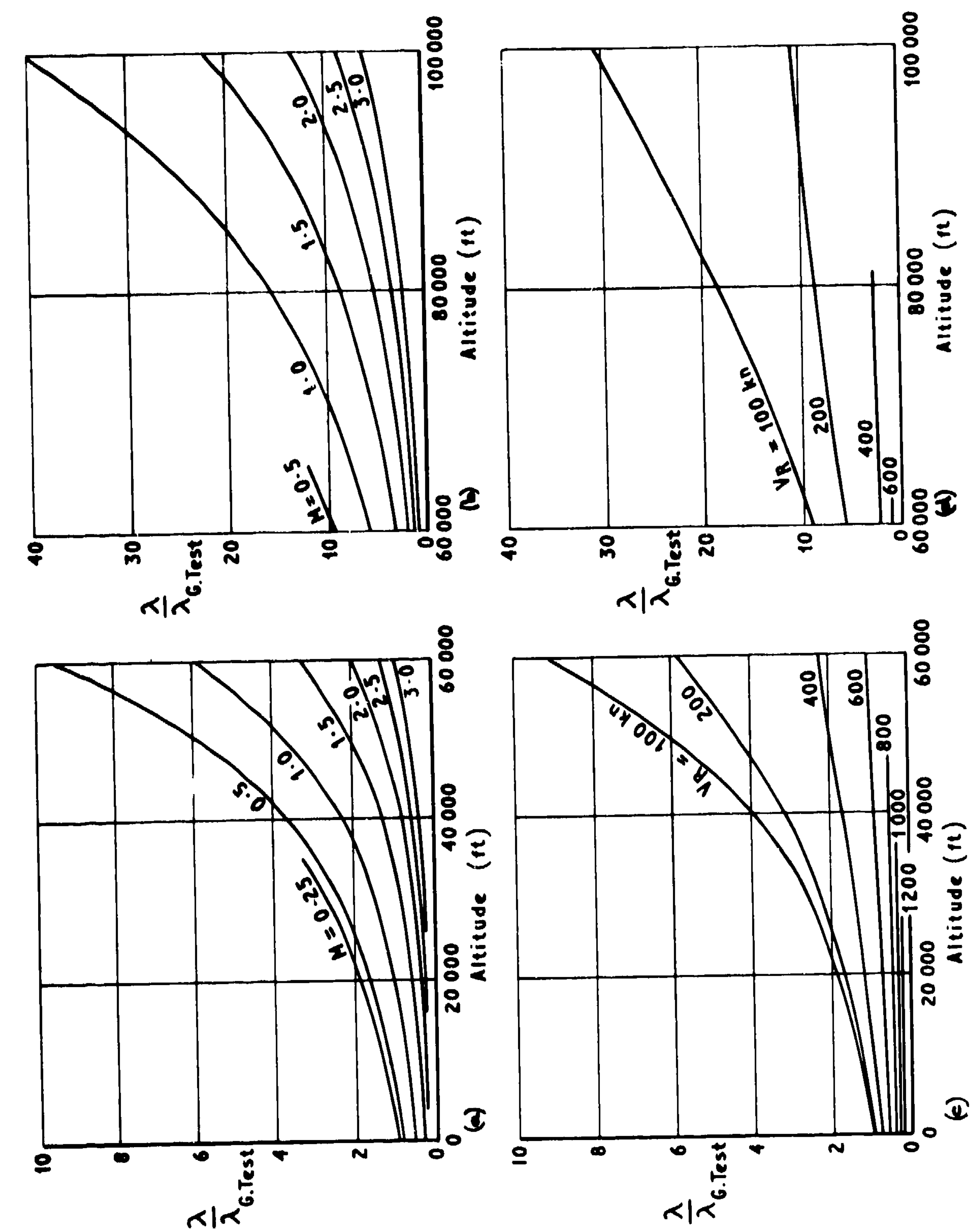


Figure 7 Ratio of Lag Constants in Pitot System, Relative to Ground Test Value

To illustrate the effect of altitude on pitot systems, in relation to that in flight at sea-level, a curve of $\frac{\lambda}{\lambda_{SL}}$ is plotted in Figure 8 below for constant values of indicated airspeed: the value of p_{ps} is that corresponding to the flight condition - see Eq (33).

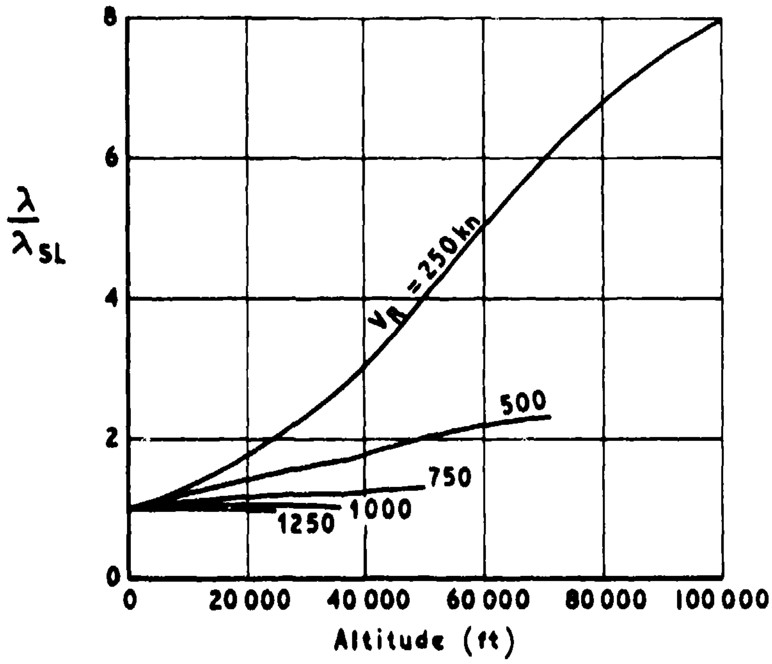


Figure 8 Ratio of Lag Constants in Pitot System at Constant Airspeed

At constant Mach number, the relationship is (as for constant airspeed)

$$\frac{\lambda}{\lambda_{SL}} = \frac{\mu}{\mu_{SL}} \Big/ \frac{p_p}{p_{pSL}} = \frac{\mu}{\mu_{SL}} \Big/ \left[\left(\frac{p_p}{p} p\right) \Big/ \left(\frac{p_{pSL}}{p_{SL}} p_{SL}\right)\right]$$

$$= \frac{\mu}{\mu_{SL}} \Big/ \frac{p}{p_{SL}} \qquad \left(\text{since } \frac{p_p}{p} \text{ is constant if M is constant}\right)$$

so the relationship, independent of Mach number, is the same as that plotted for the static system in Figure 6.

Since at any given altitude $\frac{dp}{dH} = -\rho g_o$, we have

$$\lambda \frac{dp_R}{dt} = p_s - p_R$$

$$-\rho g_o \lambda \frac{dH_R}{dt} = -\rho g_o (H_s - H_R)$$

$$\lambda \frac{dH_R}{dt} = H_s - H_R$$

and therefore the calibration for lag may be made, and applied, depending on the instrumentation in use, in terms of pressure or of pressure altitude.

5.2 Application of Correction for Lag Errors to In-Flight Air Data

For flight data to be corrected for lag errors, λ for the static system, and for the pitot system if applicable, must be know from measurement as described above. Then for the static system:

$$p_s = p_R + \lambda \dot{p}_R, \text{ or } H_s = H_R + \lambda \dot{H}_R \tag{35}$$

for a steady climb or descent, where "steady" means that $\dot{p}_R$ or $\dot{H}_R$ has been sensibly constant for a period of not less than 2λ (corresponding to attenuation of difference, at the beginning of the period, from the value in a prolonged steady descent, of 85%).

If we know λ and λ_p, and also p_R, $\dot{p}_R$, V_R, $\dot{V}_R$ (p_R and $\dot{p}_R$ being derived from pressure altitudes if necessary), then:

$$p_s = p_R + \lambda \dot{p}_R \qquad \text{(Eq (28))}$$

If there is lag error in the pitot system its effect is evaluated as follows:

From differentiation of Eqs (9) and (10), using p_{pR}, p_R as the pressures which at any instant correspond to V_R,

$$\frac{d}{dt}\,\frac{p_{pR}-p_R}{p_{SL}} = 1.4\left(\frac{V_R}{a_{SL}}\right)^2\left[1+\left(\frac{V_R}{a_{SL}}\right)^2 \Big/ 5\right]^{2.5}\frac{\dot{V}_R}{V_R} \qquad V_R < a_{SL}$$

$$\frac{d}{dt}\,\frac{p_{pR}-p_R}{p_{SL}} = 1.2^{3.5}\,(6/7)^{2.5}\left[2\left(\frac{V_R}{a_{SL}}\right)^2-1\right]\Big/\left[1-1\Big/\left[7\left(\frac{V_R}{a_{SL}}\right)^2\right]\right]^{3.5}\frac{\dot{V}_R}{V_R} \tag{36}$$

for $V_R > a_{SL}$

and we may therefore plot $(\dot{p}_{pR}-\dot{p}_R)/(p_{SL}\dot{V}_R)$ against V_R as in Figure 9 below:

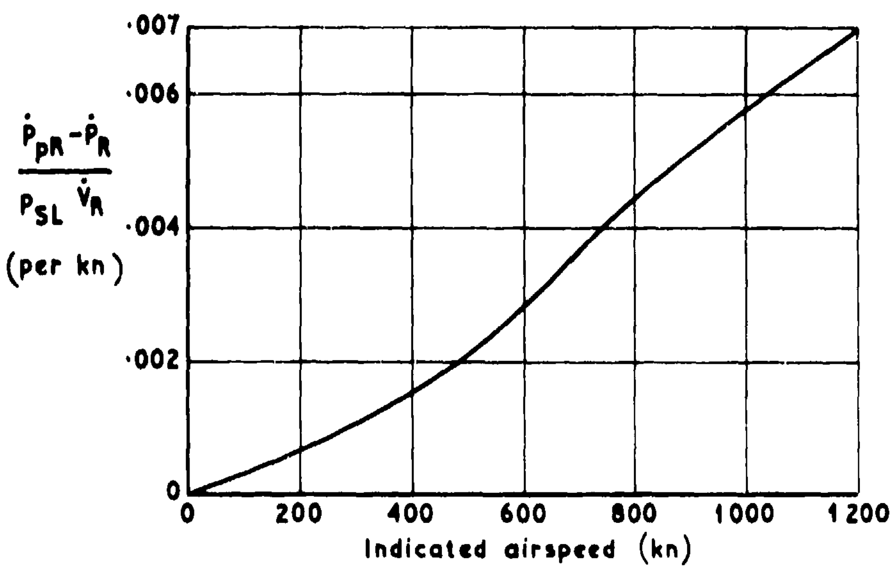


Figure 9 Values for Application of Lag Correction in Pitot System

so that, at known V_R and $\dot{V}_R$ we may obtain $\dot{p}_{pR} - \dot{p}_R$, and $\dot{p}_R$ being known for the static system, we obtain $\dot{p}_{pR}$.

(Note that although $\dot{V}_R$ may be zero, this means that $\dot{p}_{pR} = \dot{p}_R$ and there may be a pitot lag correction).

Then $$p_p = p_{pR} + \lambda_p \dot{p}_{pR}$$

and from the corrected values of p_p and p_s thus obtained, corrected values of V_i, M_i, and H_s (which are subject to pressure error correction in the usual way) can be obtained using Eqs (1) to (12).

6 TEMPERATURE CALIBRATION

The temperature recovery factor k is defined in Eq (24), ie:

$$\frac{T_{ps}}{T} = 1 + kM^2/5$$

where T_{ps} is the sensed temperature. The full stagnation temperature T_{pp} corresponds to k = 1, ie:

$$\frac{T_{pp}}{T} = 1 + M^2/5$$

The measurement of the recovery factor k can be made if T is known (for example during a flypast trial where T can be measured), by measuring T_{ps} at each of a range of values of M. Then at each test condition we have

$$k = \frac{5}{M^2}\left(\frac{T_{ps}}{T} - 1\right)$$

and if Eq (24) is valid k should have a constant value independent of M. In some instances k may be found to be weakly dependent on M.

In using this method care is necessary in the measurement of T, which must be ambient temperature at (or near) the aircraft. The measurement must be taken at a station free of radiation and convection effects from adjacent buildings; ground radiation and convection should be only that which has the equivalent effect on the air through which the aircraft flies. This means that a suitable meteorological thermometer is required, at a height similar to that of the aircraft and above a surface (eg runway or grass) which is similar to that below the aircraft flight-path.

At higher altitude, which may be necessary in order to obtain requisite higher values of M, the calibration may be made by measuring T_{ps} at a range of values of M and plotting T_{ps} against M^2. T is then the intercept at $M^2 = 0$ and from Eq (24).

$$k = \frac{5}{T}\frac{dT_{ps}}{dM^2}$$

This trial should be conducted so that stabilised conditions are achieved, and readings taken, in the same geographic position as closely as possible for each reading, to avoid change of T during the trial; for the same reason, as for all pressure error work, the atmospheric conditions should be stable. The range of M should cover that required for the calibration, and inclusion of the lowest practicable value of M, and a range of M from there to the highest required, will give good definition of T and of $\frac{dT_{ps}}{dM^2}$

With these precautions, k should be measurable to within 0.01 of its correct value.

Since the recovery factor is primarily a function of the temperature probe itself rather than of its environment on the aircraft, it may be calibrated in a wind-tunnel. In this case the ambient temperature in the tunnel itself must be known from the tunnel calibration; measurement of ambient temperature at low velocity in the tunnel settling chamber and application of the equation

$$\frac{T_1}{T_2} = \frac{1+M_2^2/5}{1+M_1^2/5}$$ may be the method used.

7 CALCULATION AND PRESENTATION OF PRESSURE ERROR RESULTS

7.1 Methods of Calculation and Presentation

In the pressure error trials described in this volume, the output data will consist of V_i, p_s, p, p_p, W, n, (p_s and p being converted from H_s, H by Eqs (1) to (3) if necessary), and results may be presented in terms of ΔC_p, ΔV_i, ΔH as functions of V_i, H_s and nW, or of M and C_L, or $C_{p\beta}$ as a function of M, $C_{L\beta}$ (section 4.4.3). Presentation in terms of ΔC_p, M, and C_L is consistent with what one would expect on aerodynamic grounds and has much to commend it; however applying a correction presented in this form requires an iteration since until the correction has been applied exact values of M and C_L are not known; if the calibration has been characterised in a form which can be programmed on a calculator or computer the iteration, as described below, is simply performed. However it is sometimes convenient to express the calibration in terms of directly measurable quantities, avoiding the need for iteration. For each of the "fundamental" parameters ΔC_p, M, C_L, one can write a corresponding "empirical" parameter, denoted by a dash (').

$$\Delta C_p = \frac{p - p_s}{\frac{1}{2}\rho V^2} = \frac{p - p_s}{0.7pM^2} = \frac{1}{0.7M^2}\left(1 - \frac{p_s}{p}\right) \qquad (37)$$

$$\Delta C_p' = \frac{p - p_s}{\frac{1}{2}\rho_{SL}V_i^2} = \frac{p - p_s}{0.7p_{SL}(V_i/a_{SL})^2} \qquad (38)$$

$$M = \frac{V}{a} = \frac{V_e}{a_{SL}\sqrt{\delta}} \qquad (39)$$

$$M' = \frac{V_i}{a_{SL}\sqrt{\delta_s}} \qquad (40)$$

(The effect of using V_i instead of V_e is to exclude the pressure error and compressibility corrections, while using δ_s instead of δ also omits pressure error correction, but both the omitted corrections are themselves functions of M and δ).

$$C_L = \frac{nW}{\frac{1}{2}\rho V^2 S} = \frac{nW}{0.7p_{SL}\delta M^2 S} \qquad (41)$$

$$C_L' = \frac{nW}{\frac{1}{2}\rho_{SL}V_i^2 S} = \frac{nW}{0.7p_{SL}(V_i/a_{SL})^2 S} \qquad (42)$$

Each of these parameters may be calculated from the known data (M being obtained from V_i, $\frac{q_c}{p_{SL}}$ (Eq (9) or (10)), $\frac{q_c}{p}$, and Eq (7) or (11). Results may then be presented as required, the following being commonly used forms:

(i) ΔC_p vs M for various constant C_L. A multiple regression to extract the (usually smaller) effect of C_L may be used to derive the constant C_L values, or they may be obtained by cross-plotting against C_L at required values of M.

(ii) $\Delta C_p'$ vs M' for various constant C_L'.

(iii) $H - H_s$ and $V_c - V_i$ against V_i and H_s; if appropriate, separate plots for values of nW. It may be convenient, since the compressibility correction $V_e - V_c$ is also a function of V_c and H, to include this in a single correction $V_e - V_i$ instead of $V_c - V_i$. If the complete set of results can be analysed and characterised as in presentation (i) or (ii) in the form

$$\Delta C_p = f(M, C_L) \text{ or } \Delta C_p' = f(M', C_L')$$

this should be done, and presentation (iii) can then be developed from a set of self-consistent data derived from all the calibration results. Given values of V_i, H_s and nW permit calculation of M', C_L', ΔC_p, $p-p_s$ p, p_p-p_s (Eq (7) or (8)) p_p-p, V_c (Eq (7) or (12)), H (Eqs (4) to (6) and V_e (Eq (13)). If the calibration is initially characterised in terms of the "pure" values ΔC_p, M, and C_L then an iteration is required as follows:

(a) Obtain $\dfrac{p_p-p_s}{p_{SL}}$, δ_s and calculate $\dfrac{p_p-p_s}{p_s} = \dfrac{p_p-p_s}{p_{SL}} \Big/ \delta_s$, and $\dfrac{p_p}{p_s} = \dfrac{p_p-p_s}{p_s} + 1$

(b) From $\dfrac{p_p-p_s}{p_{SL}}$ calculate M_i (Eq (7) or (11)) and using this as an approximate M, and δ_s as δ, calculate an approximate C_L (Eq (41)). Use these values to obtain an approximation to ΔC_p.

(c) Calculate $\dfrac{p_s}{p} = 1-0.7M^2 \, \Delta C_p$ (from Eq (37)).

(d) Calculate $\dfrac{p_p-p}{p} = \dfrac{p_p}{p_s} \dfrac{p_s}{p} - 1$ and hence M (Eq (7) or (11)).

(e) Calculate $\delta = \delta_s \Big/ \dfrac{p_s}{p}$, and C_L (Eq (41)) and obtain a new value of ΔC_p. Return to (c) until convergence is obtained.

(f) After convergence, obtain $p-p_s$ from Eq (37) and proceed as above.

If there is also a pitot error, so that instead of correct pitot pressure p_p, a reduced value p_{ps} is sensed, and the error has been calibrated and expressed as $\Delta C_{pp} = \dfrac{p_p-p_{ps}}{0.7pM^2}$, then the iteration is similar, but modified as follows:

(a) Obtain $\dfrac{p_{ps}-p_s}{p_{SL}}$ from V_i, δ_s, and calculate $\dfrac{p_{ps}-p_s}{p_s} = \dfrac{p_{ps}-p_s}{p_{SL}} \Big/ \delta_s$, and $\dfrac{p_{ps}}{p_s}$.

(b) From $\dfrac{p_{ps}-p_s}{p_s}$ calculate M_i and using this as an approximation to M, and δ_s as δ, calculate approximate C_L. Use these values to obtain approximations to ΔC_p and ΔC_{pp}.

(c) Calculate $\dfrac{p_s}{p} = 1 - 0.7\, M^2\, \Delta C_p$.

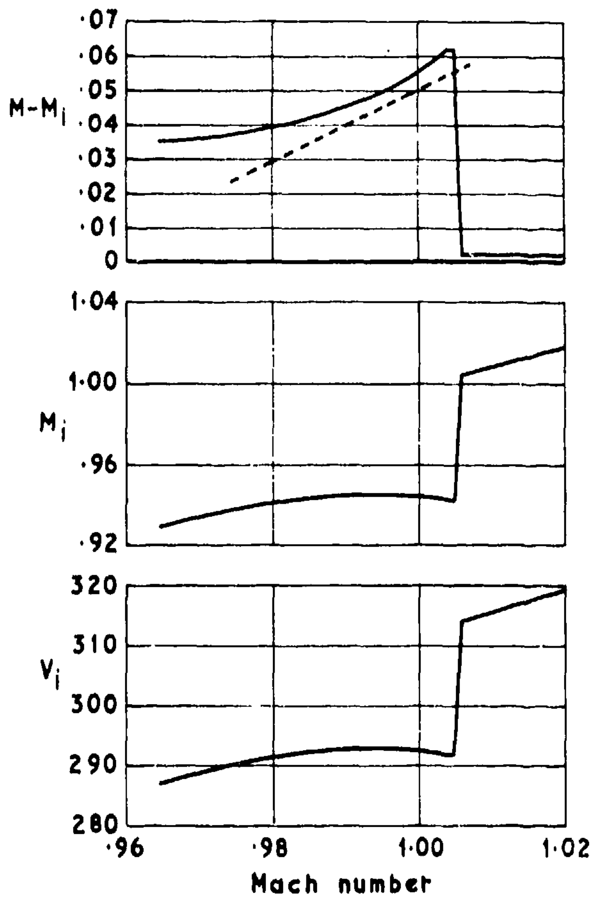
(d) Calculate $\dfrac{p_p-p}{p} = \dfrac{p_p}{p_s}\dfrac{p_s}{p} - 1 = \left(\dfrac{p_{ps}}{p_s} + \dfrac{p_p-p_{ps}}{p_s}\right)\dfrac{p_s}{p} - 1 = \dfrac{p_{ps}}{p_s}\dfrac{p_s}{p} + 0.7\, M^2 \Delta C_{pp} - 1$, and hence M.

(e) Calculate δ, C_L, and obtain new values of ΔC_p, ΔC_{pp}. Return to (c) until convergence is obtained.

When the iteration is complete we have values of M and $\dfrac{p_p-p}{p}$, δ from which $\dfrac{p_p-p}{p_{SL}}$ and V_c may be obtained.

7.2 Ambiguity of Calibration in the Transonic Region

The measurement of the calibration of an aircraft pressure system in the transonic region presents no special difficulty provided that the aircraft can be flown in a stabilised condition in the required speed range. The tower flypast method is impracticable and trailing sensors (bomb or cone) are unlikely to fly acceptably and in any case their own calibrations will be unreliable, but the radar methods here described, in relation to radio- sonde, pacer aircraft, or the subject aircraft itself at other conditions, are available and effective. However the pressure error may change very rapidly with Mach number in the transonic region, usually in the sense that $M-M_i$ becomes increasingly positive and then, close to M = 1, falls rapidly to a value close to zero. If the slope $\frac{d}{dM}(M - M_i)$ becomes greater than 1, M_i will decrease as M increases. An example is plotted in Figure 10, where V_i (taking H = 40 000 ft) is also shown. It is apparent that there is an ambiguity since a given value of M_i or of V_i may imply two different values of M or of V_c, and within a range in this case of 0.02 in M a unique value of M or of V_c is not obtainable from air-data readings; in this region the calculation methods described above may yield values of M and V_c but care should be taken in their interpretation.



The dotted line indicates the slope $\frac{d}{dM}(M-Mi) = 1$

Figure 10 Ambiguity of Calibration in Transonic Region

8 INTERRELATIONSHIP OF THE FORMS IN WHICH STATIC PRESSURE ERROR IS EXPRESSED

Errors in sensed static pressure cause errors in indicated altitude, airspeed and Mach number as has been described, and the error may be expressed as errors in these quantities or as a pressure error coefficient ΔC_p. It is often necessary, knowing the error expressed in one form, to find corresponding values in other forms. In the following figures (Figures 11(a) to 11(m)) the interrelationships are plotted as values of the ratios of the different forms of expressing the error, as functions of V_c, M and H. These ratios are obtained by differentiation of Eqs (1) to (12) as appropriate and are therefore applicable only for small errors, but they are adequate for most purposes within the range of acceptable magnitude of pressure error.

For example, for H_s = 20 000, V_i = 400, ΔH = 1000 we have that M_i = 0.8536, $\frac{\Delta H}{\Delta V}$ = 79, $\frac{\Delta H}{\Delta M}$ = 25 000, and therefore the values of ΔV and ΔM are 12.7 and 0.040 respectively, so that H = 21 000, V_c = 412.7, and M = 0.8936. A detailed calculation gives V_c = 412.2, M = 0.8932. Thus in a case where pressure error is substantial, acceptably accurate determination of equivalent error is obtained.

To enable conversion of presure errors expressed in millibars into the other forms above, Figure 12 shows $\frac{\Delta H}{\Delta p}$ (ft per mb) for the International Standard Atmosphere.

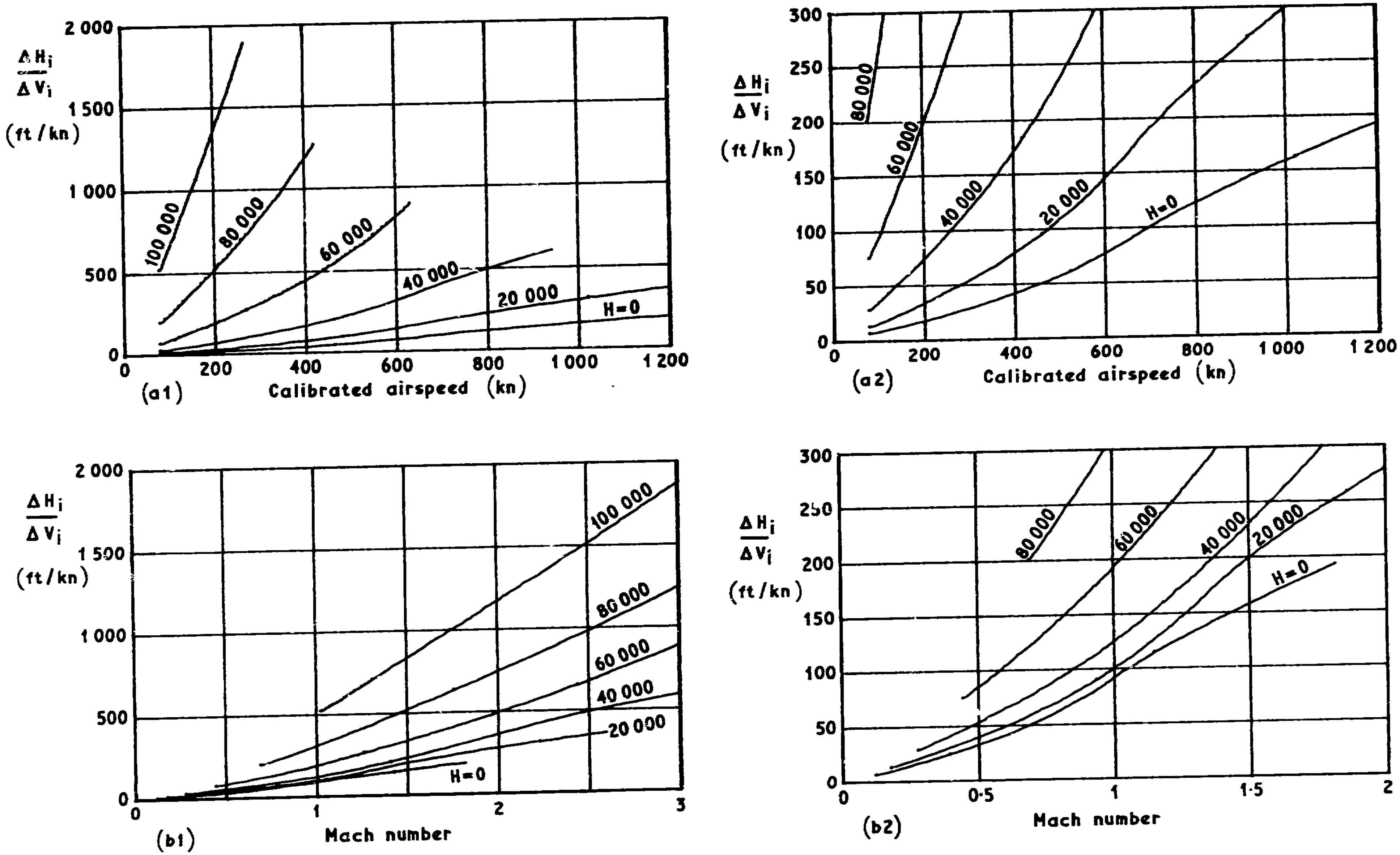


Figure 11 Relationships between forms of pressure error

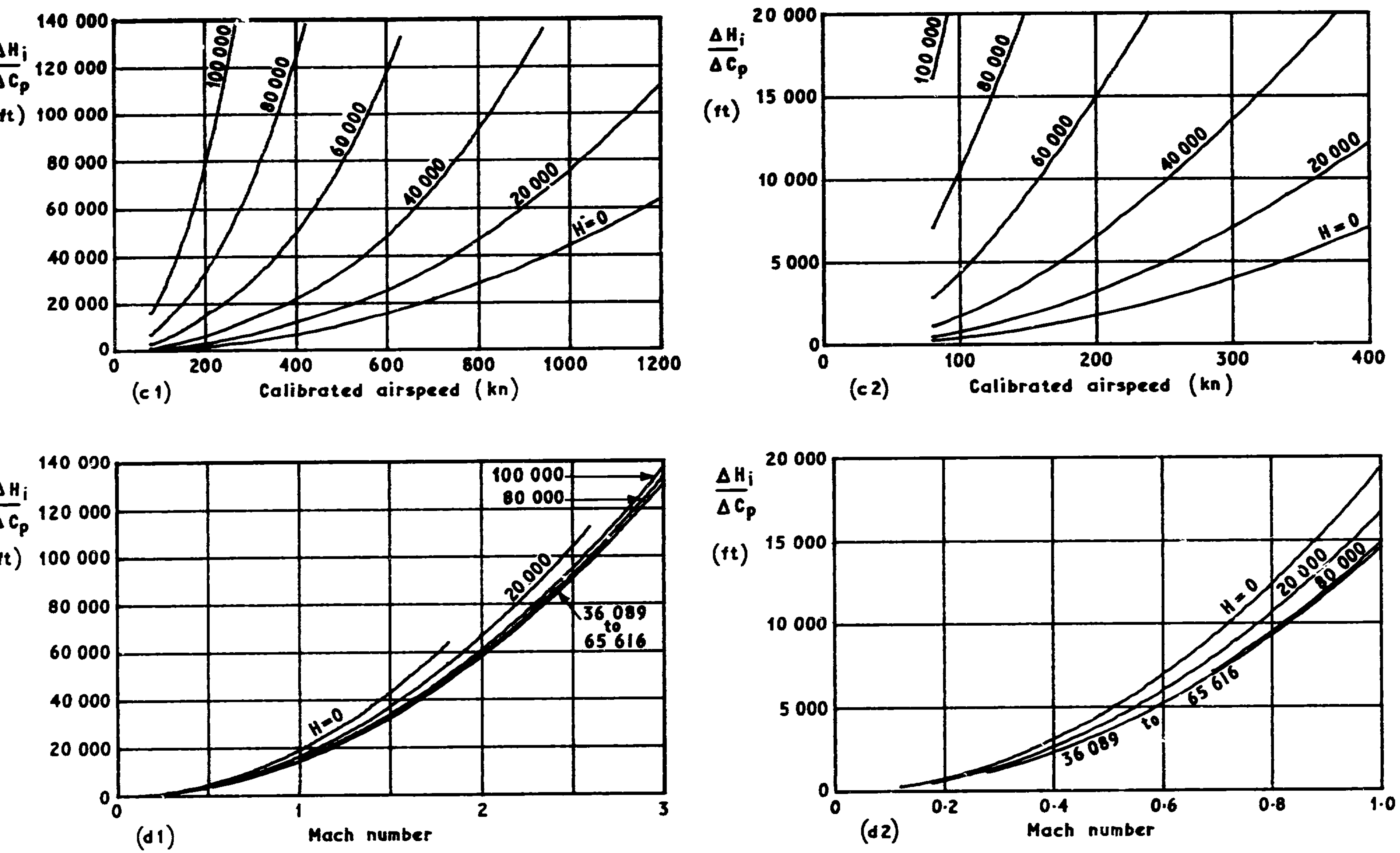


Figure 11 (continued)

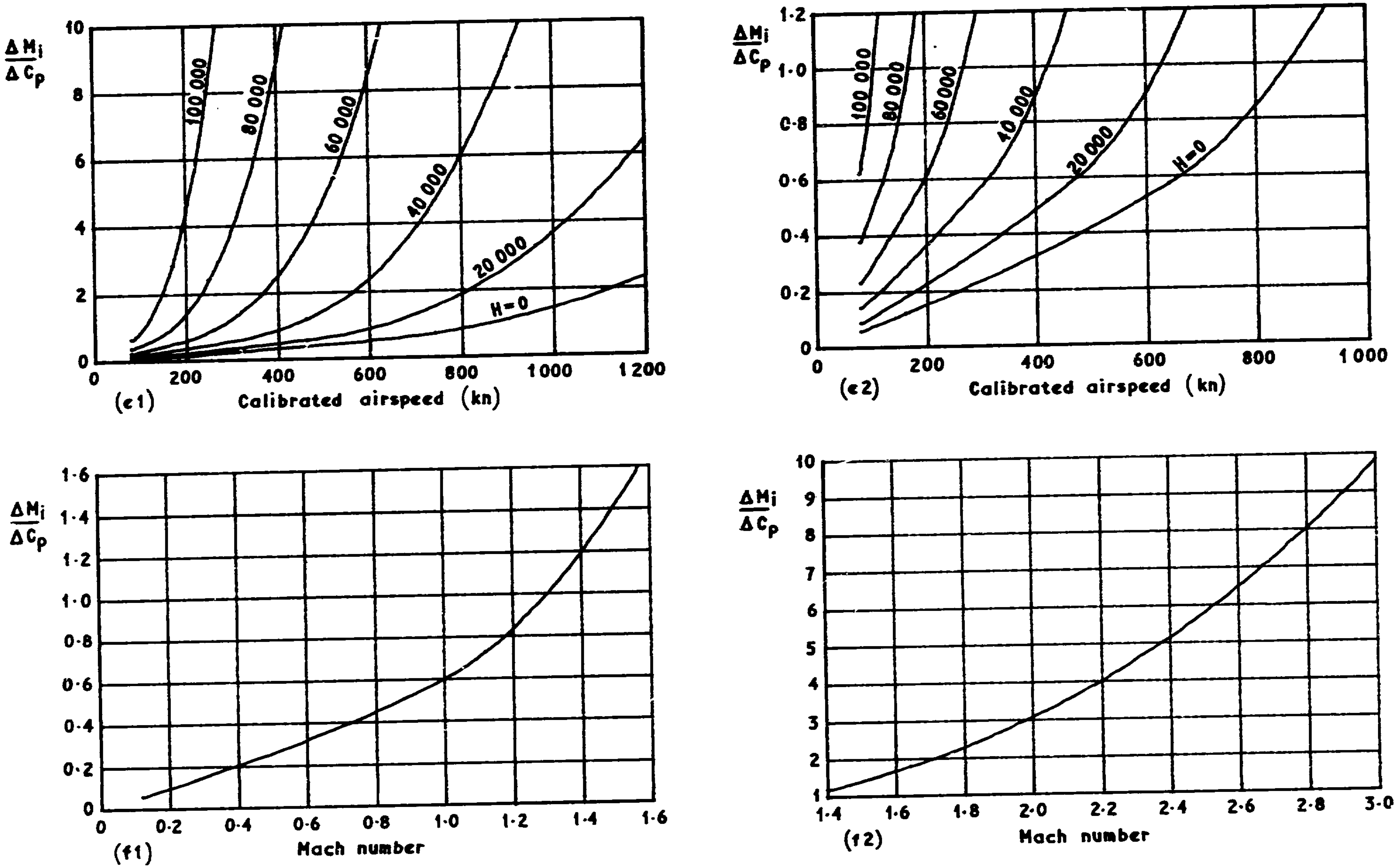
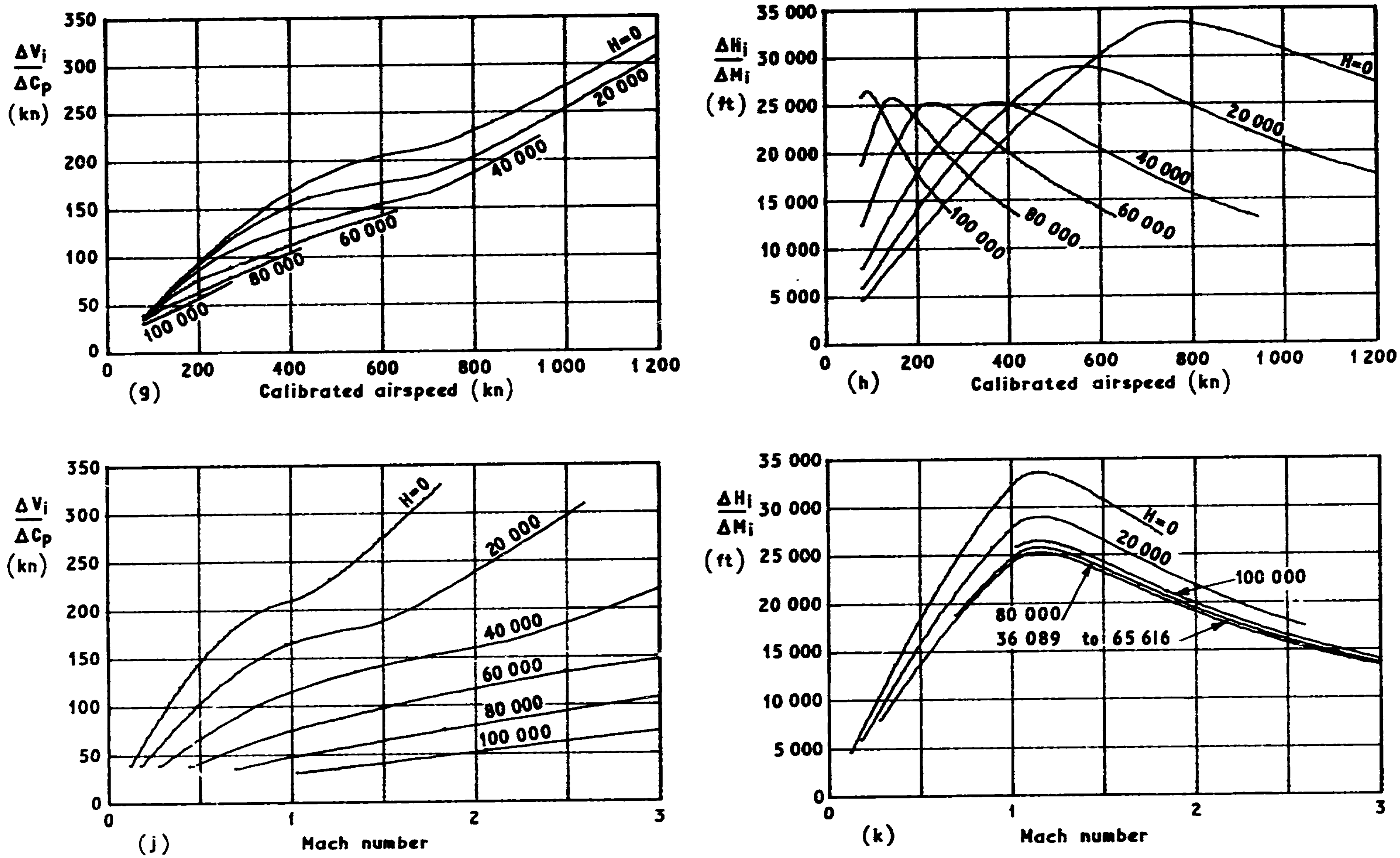


Figure 11 (continued)



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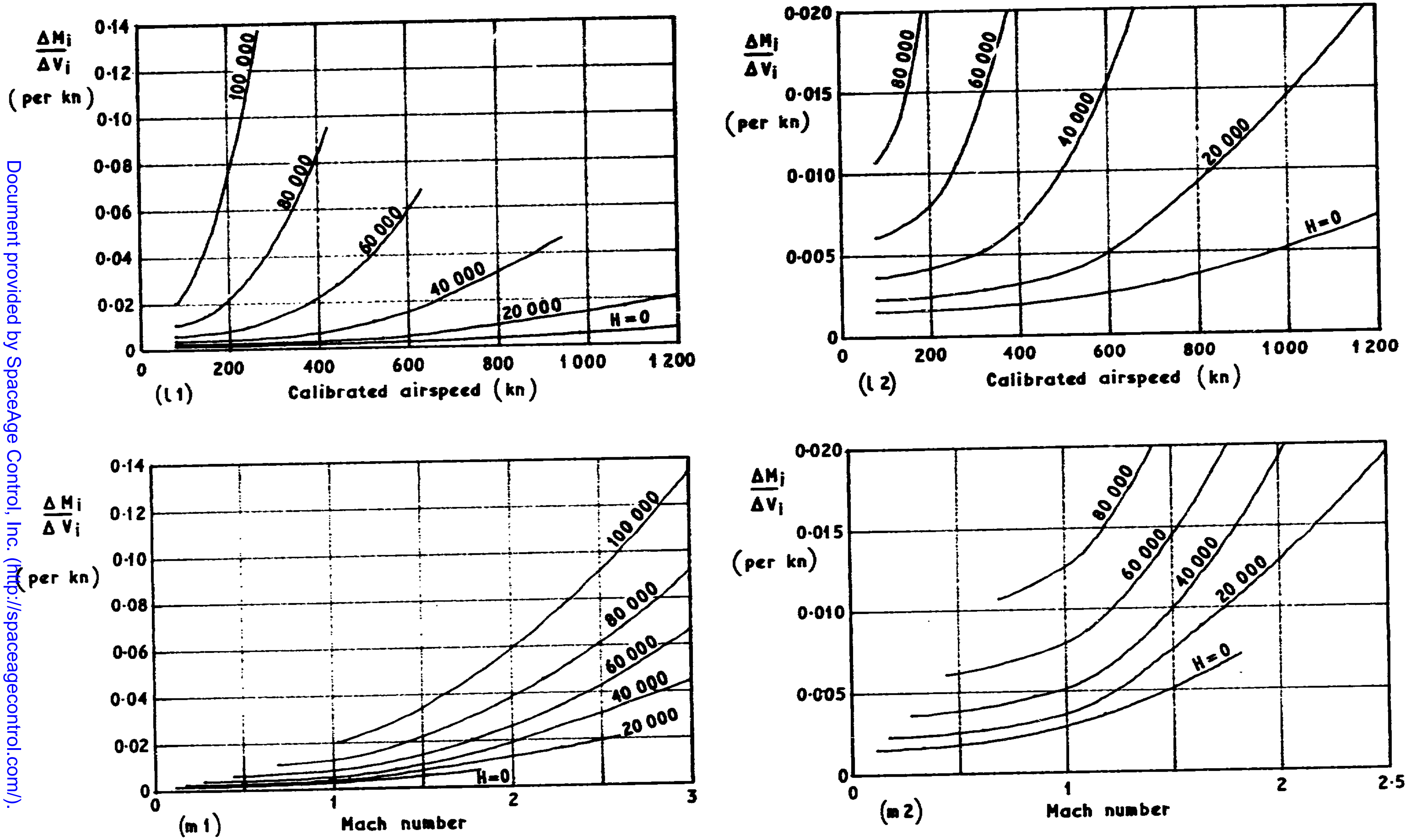

Figure 11 (continued)

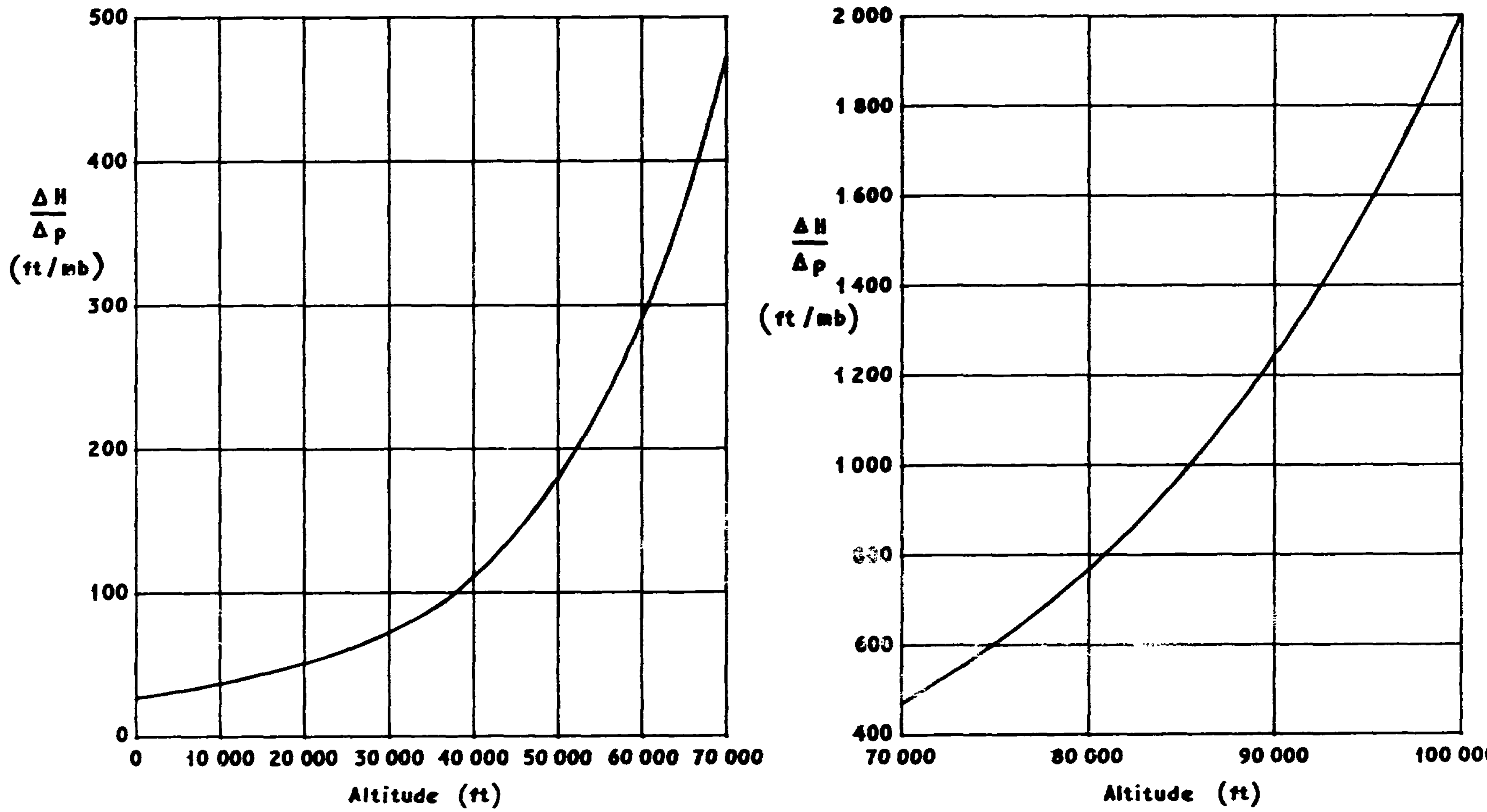

Figure 12 Altitude-Pressure gradient in International Standard Atmosphere

PART 2 FLOW DIRECTION SENSOR CALIBRATION

9 SENSOR TYPES AND CALIBRATION METHODS

There are three basic types of flow direction sensors available Ref(1). These are:

1. Vanes
2. Null-seeking servoed differential pressure sensors such as "Airstream Direction Detector" probes.
3. Differential pressure probes.

All of these sensors measure flow direction at their mounting position. Since they must be positioned on, or if a nose boom installation is used, close to the airframe, the measured flow direction will contain components of upwash and sidewash induced by the airframe at the sensor location. The relationship between sensed angle of attack or sideslip angle and corresponding free-stream values must therefore be established. There are three methods available for establishing a flow direction calibration.

1. Calculate the upwash and sidewash angles induced by the fuselage/wing combination using a theoretical method.
2. Measure the flow at the sensor position on a model of the aeroplane mounted in a wind tunnel. If a noseboom installation is used it may be necessary to measure the local flow effects around the noseboom by mounting this in a wind tunnel.
3. Calibrate the sensor in flight.

The theoretical, wind tunnel, and flight test techniques for obtaining angle of attack and sideslip angle calibration data will be discussed in this section. More attention will be paid to techniques for calibrating the sensors in flight than to other means, primarily because there are factors which limit the applicability of wind tunnel and theoretical data to flight test.

10 THEORETICAL METHOD

A theoretically based method of deriving upwash angle at a noseboom-mounted vane has been used for a number of years at the NASA Dryden Flight Research Facility and the Air Force Flight Test Center, Edwards Air Force Base, USA, and is described in Ref (9). The procedure is derived from two papers by Yaggy Ref (10) and Rogallo Ref (11), which addressed the problem of predicting the upwash angle at the plane of a rotating propeller on a wing-mounted engine nacelle. In Ref (10) the upwash generated by a cylindrical body of revolution was computed using the theoretical flow about a series of doublets in a uniform incompressible flow field. The noseboom and fuselage effects can be added to the wing effect to produce the upwash or sidewash generated at the angle of attack or sideslip vane positions. The upwash angle due to the fuselage-noseboom combination is computed using the expression for the upwash angle in the plane of symmetry of a cylinder of circular cross section, that is

$$\frac{\epsilon}{\alpha} = \frac{\sin^2\Omega - \cos^2\Omega}{2r^2} \int_{\delta_{LE}}^{\delta_{TE}} R^2 \sin\delta \, d\delta \qquad (43)$$

Where ϵ is the induced upwash angle, Ω is the angle between the axis of rotation of the angle of attack sensor and the vertical plane of symmetry of the aircraft, and r is the distance of the sensor centre of pressure, measured outboard from the centre line of the boom or fuselage. R is the equivalent radius of the boom or fuselage, such that

$$R = \sqrt{\text{Area}/\pi}$$

The ordinate δ is defined by $\delta = \text{arc cot} \dfrac{d}{|r|}$

Where d is the distance from the centre of pressure of the sensor to the chosen point on the boom or fuselage, positive forward of the sensor.

The effect of compressibility is effectively to increase the distance aft of the sensor and decrease the distance forward of the sensor by a factor

$$\beta = \sqrt{(1-M^2)}$$

such that $d = d \times \beta$ for forward locations

$d = d/\beta$ for aft locations

It should be noted that the method is applicable only in the subsonic regime, since β becomes undefined for Mach numbers greater than unity. In theory a noseboom-mounted sensor should not be affected by induced upwash in supersonic flight, since pressure propagation should not occur in supersonic flow. In practice however it has been found Ref (9) that some small effects due to shock wave interaction still exist; these cannot be predicted by this method.

Calculation of the noseboom/fuselage upwash angle is performed by evaluating the integral given in Eq (43), numerically, along the noseboom and fuselage. The recommendation given in Ref (9) is to select points on the fuselage area versus fuselage station curve as far apart as practicable, whilst still adequately representing the curve between selected points as a straight line.

The upwash at the vane due to the presence of the wing must next be computed. This upwash will depend on the spanwise load distribution on the wing. A detailed explanation of this upwash calculation is given in Ref (9), which includes the description of a FORTRAN program which will perform the necessary upwash calculations for both the wing and fuselage. The results indicate, however, that the wing-fuselage upwash can generally be regarded as a linear function of the true angle of attack (provided the aircraft lift curve is linear) and so the relationship between sensed and true angle of attack can be assumed to be

$$\alpha = \alpha_0 + K\alpha_R \qquad (44)$$

where the value of the calibration factor, K, will be less than unity and will vary with Mach number. A similar relationship will also hold between true and sensed sideslip angles.

The Yaggy-Rogallo technique has so far been used only to compute the upwash induced at a noseboom-mounted vane Ref (9). It may however be used to compute the upwash and sidewash angles for fuselage-mounted installations. Experience with noseboom-mounted vanes has indicated Ref (9) that the technique does not always predict the correct value although the variation with subsonic Mach number is as predicted. It should also be noted that the technique assumes attached flow round the wing-fuselage-nose-probe combination and is therefore valid only at small or moderate (<15°) angles of attack or sideslip.

The effects of structural distortion under load must also be added to the upwash effects in order to provide a complete calibration of the angle of attack or sideslip sensor, and these effects may be difficult to assess without flight testing. In view of the errors likely to be incurred when using theoretical techniques in conjunction with estimates of fuselage/noseboom deflection under load, the best approach is to use the theoretical data as a basis for planning the flight tests and interpreting the flight data.

11 WIND TUNNEL TESTING

The relationship between local flow direction and true angle of attack and sideslip angle may be investigated for all likely flight conditions by testing a model in a wind tunnel. There may be differences between the wind tunnel calibration and the calibration obtained in flight, because of effects such as wind tunnel wall interference, and possible engine effects if the sensor is mounted on the fuselage.

The wind tunnel tests will, however, indicate any likely changes of sensor calibration with Mach number, due to shock wave interaction, and any cross coupling effects between sensed angle of attack and sideslip angle, and so can be used as a basis for planning the flight tests.

12 IN-FLIGHT CALIBRATION OF ANGLE OF ATTACK SENSORS

12.1 General Considerations

The previous paragraphs have dealt with theoretical and wind tunnel methods of calibrating flow-direction sensors, there being no need to distinguish between angles of attack and sideslip. However when considering the problem of in-flight calibration the flight manoeuvres and subsequent analysis for angle of attack and for sideslip angle differ sufficiently to require separate treatment for each. Because of the importance of angle of attack in dynamic performance and in stability and control derivative estimation, its calibration has received more attention than that of sideslip sensors, and will be treated first.

The test manoeuvres and data analysis techniques depend on whether the data are acquired during "steady state" or quasi-steady or dynamic manoeuvres. Calibration data may be obtained from steady-state manoeuvres with very little analysis effort, but such manoeuvres are relatively expensive in flight time, and the calibration laws obtained apply only at or near the 1 'g' level-flight condition; accuracy may be limited also by the precision with which the pilot can fly the manoeuvre. The instrumentation requirement is low however and may sometimes be satisfied using ground-based equipment rather than instrumentation installed in the aircraft. Determination of calibration data from quasi-steady manoeuvres such as wind-up turns requires an accurate data source such as an inertial navigation system.

Calibration data may be obtained from dynamic manoeuvres only with substantial analysis effort; digitally recorded data and a digital computer are indeed essential if the effort required is not to become prohibitive. Calibration laws applicable throughout the flight envelope may however be obtained, and dynamic manoeuvres are very economical in terms of flight time. Dynamic analysis techniques may be the only ones capable of obtaining calibration data at or near the manoeuvre limitations of the aeroplane.

12.2 "Steady-Flight" Calibrations

In constant-speed, zero-sideslip flight, the angle of attack is equal to the difference between the aircraft pitch angle and the climb angle (descent being represented by negative climb angle). That is:

$$\alpha = \theta - \gamma \tag{45}$$

For small angles of climb (or descent) the climb angle is proportional to rate of climb so that:

$$\alpha = \theta - \dot{h}/_V \tag{46}$$

If the aircraft is flown in steady level flight it is necessary to measure only the pitch angle θ to obtain the true aircraft angle of attack. Pitch angle may be measured in three ways.

The first method is to photograph the aircraft from a tower as it flies past in steady level flight; this "tower flypast" method may therefore be used on non-instrumented aircraft. The camera must be capable of being accurately levelled, and it is desirable to have a grid on the film to define the horizontal and simplify the measurement of pitch angle. Identifying marks should be painted on the aircraft datum line at its nose and rear. The aircraft should be flown past the camera at speeds covering the range from minimum to maximum. The advantage of this technique is its simplicity; in the author's experience there are four main disadvantages:

(i) The resolution of the film image usually limits the accuracy in obtaining pitch angle to ±0.25°.

(ii) Because the aircraft is flown in steady level flight at low altitude, the angle of attack range is limited by the aircraft's maximum speed and minimum weight; this range can therefore be quite small.

(iii) The method is expensive in terms of flight time.

(iv) Over some parts of the speed range, the aircraft's handling characteristics may limit the precision with which steady level flight may be obtained. In these cases "scatter" of approximately ±1° is to be expected on the final results.

The effects of small rates of climb or descent can be reduced if airspeed and altitude instrumentation is available. The scatter on angle of attack is then expected to be approximately ±0.5°. Care must be taken to ensure that the instrumentation data are properly synchronised with the photographic data.

A variation on this technique, the 'mountain fly at' method, may be used when the aircraft is fitted with a gun sight. During calibration of the gun sight the aircraft is precisely levelled and the depression angle for zero sight line angle is determined. Thus, whenever the pilot looks through the aiming reticle with the zero sight line depression angle set, he is looking parallel to the aircraft datum line. The aircraft is stabilized in level flight some 30 nautical miles away from a prominent landmark, such as a mountain peak, (hence the name "mountain fly at" method), at the tapeline height of the landmark. The gun sight depression angle is then increased until the aiming mark (or pipper) is on the top of the land mark, and is then kept there. The difference between the reference setting and the final setting is the aircraft pitch angle. The disadvantage of this method is that it is expensive in terms of flight time, because of the need to stabilise at each angle of attack.

It is clearly advantageous to eliminate the need for a visual reference by measuring pitch angle with an on-board transducer. There are three methods of doing this:

(i) Position Gyroscope.

(ii) Pitch pendulum or sensitive longitudinal accelerometer.

(iii) Use of an inertial platform.

Use of a position gyroscope of the type normally fitted for stability and control work is not recommended, because the mechanism which controls drift from the vertical (such as mercury switches) will cause the gyroscope to tend to align itself with the direction of the local resultant of gravitational and inertial forces. Stieler and Winter Ref (12) describe in some detail the differing types of gyroscopes currently available.

A pitch pendulum or sensitive accelerometer is a most convenient device for measuring pitch angle. The accelerometer is becoming accepted as the standard transducer in the United Kingdom, since the advent of moderately priced servo (force balance) accelerometers. A typical transducer will have an accuracy of 0.002 'g', which is equivalent to an angle error of approximately 0.1°. The accelerometer reading is related to pitch angle by:

$$\theta = \arcsin a_x \qquad (47)$$

in steady level flight.

It is important to correct for any small rates of change of forward speed, if accelerometer data are to give accurate values of pitch angle. The following relationship may be used:

$$\theta = a_x - \dot{V}/g_o \qquad (48)$$

It can be seen from Eq (48) that a rate of change of forward speed of 0.1 kn/s will give an error in pitch angle of approximately 0.3°, which is significant.

The accelerometer must be mounted rigidly to the aircraft structure to minimise changes in bias errors during flight, and should preferably be mounted near the aircraft centre of gravity to eliminate any effects due to airframe distortion or small pitch accelerations. Any bias due to possible misalignment can be removed by reading, prior to flight, the accelerometer reading and the inclination of the aircraft datum line. With precautions to eliminate or minimise errors as given above, the accuracy of the calibration can approach the accuracy of the angle of attack transducer. However a "scatter band" of 0.5° is more usual. An inertial navigation system can be used to provide accurate pitch angle reference, the accuracy obtainable being approximately 0.1°.

12.3 Calibration from Dynamic Manoeuvres

As shown in Section 12.2, an angle of attack calibration can be obtained in steady level flight with little analysis effort. However this calibration should strictly apply only at or close to the level-flight condition. In the author's experience, the level-flight calibration will apply also in manoeuvring flight for a noseboom mounted vane at low to moderate Mach numbers (<0.8), provided that the effects of fuselage and boom distortion under load are accounted for. At higher Mach numbers there is a tendency for the vane calibration to change with Mach number, as illustrated in Ref (9). Null-seeking sensors, such as Airstream Direction Detector probes, do not have this tendency but are generally mounted on the side of the fuselage, so that changes in power setting can slightly affect the calibration. It is therefore desirable to calibrate the angle of attack sensor in manoeuvring flight. This is a more complex process than for steady-state data because the "true" angle of attack time history must be synthesised from other measurements and the effects of rate of rotation and boom bending must be allowed for. Whereas steady-state calibration data can be obtained from aircraft fitted with paper-trace recorders, with comparatively little effort, a digital recording system should be employed if dynamic calibrations are to be undertaken (otherwise a considerable effort in digitising from the trace will be required). A digital computer is required for the analysis.

12.3.1 Calibration using an Inertial Navigation System

It is possible to use an inertial navigation system to derive an angle of attack calibration from both static and dynamic manoeuvres. Inertial navigation systems, whether strapped down or gimballed, generally provide more accurate data than that obtainable from the accelerometers and gyroscopes fitted for stability and control work, but may be too costly to justify fitting purely as an instrumentation system, in addition to requiring more maintenance.

The design of such systems varies, as does the data available to the instrumentation system, so the analysis will depend on the data available.

The analysis techniques described herein may be used when aircraft pitch angle, roll angle and yaw angle are available, together with north velocity, east-west velocity and down velocity, and wind velocity components. Because the calibration manoeuvres should be performed in calm air conditions, it may be assumed that vertical components of wind velocity are negligible.

The three manoeuvres which have been found to be most suitable for calibration work are wind up turns, roller coasters and split 'S' manoeuvres. Each of these manoeuvres allows a calibration to be obtained at more or less constant Mach number.

The analysis assumes that the wind velocity remains constant throughout the manoeuvre. The reason for not using the values of wind velocity is that these are derived from the inertial navigation accelerometer data and true airspeed obtained from the pilot static system. These pneumatic data will be subject to lag in dynamic manoeuvres.

The air mass velocity components are defined in terms of the North, East and Down velocities V_N, V_E, V_D, as follows:

$$\begin{aligned} V_f &= V_N - V_{WN} \\ V_l &= V_E - V_{WE} \\ V_d &= V_D \end{aligned} \qquad (49)$$

Where V_{WN} and V_{WE} are the North and East components of wind velocity, and V_f, V_l, V_d are the forward, lateral and down components of velocity.

In the technique developed by Duxbury of BAe Warton (UK) the aircraft is held in steady level flight prior to the manoeuvre, in order to estimate the wind velocity components.

The technique employed by Olsen at the AFFTC, Edwards AFB is different in that the wind velocity components are estimated during the manoeuvre by the following process:

The relationship between the true airspeed and the North, East and down velocity components is:

$$V^2 = (V_N - V_{WN})^2 + (V_E - V_{WE})^2 + V_D^2 \qquad (50)$$

The sum of the squares of the residual errors in true airspeed, over the whole manoeuvre, is therefore:

$$ERRSUM = \sum_{j=1}^{N} \left[V_j^2 - (V_{N_j} - V_{WN})^2 - (V_{E_j} - V_{WE})^2 - V_{D_j}^2 \right]^2 \qquad (51)$$

Values of the (unknown) wind velocity components V_{WE} and V_{WN} need to be chosen such that this error sum is a minimum. At this minimum, the partial derivative of the error sum with respect to each of the wind velocity components will be zero, that is:

$$\frac{\partial ERRSUM}{\partial V_{WN}} = \sum_{j=1}^{N} \left[V_j^2 - (V_{N_j} - V_{W_N})^2 - (V_{E_j} - V_{W_E})^2 - V_{D_j} \right] (V_{N_j} - V_{W_N}) = 0 \qquad (52)$$

and

$$\frac{\partial ERRSUM}{\partial V_{WN}} = \sum_{j=1}^{N} \left[V_j^2 - (V_{N_j} - V_{W_N})^2 - (V_{E_j} - V_{W_E})^2 - V_{D_j} \right] (V_{E_j} - V_{W_E}) = 0 \qquad (53)$$

Eqs (52) and (53) may be solved by a two dimensional Newton Raphson iteration procedure.

The air mass velocities are then transformd to body axes using the transformation matrix.

$$\begin{vmatrix} u \\ v \\ w \end{vmatrix} = \begin{matrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\cos\theta\sin\psi+\sin\theta\sin\phi\cos\psi & \sin\theta\sin\phi\sin\psi+\cos\theta\cos\psi & \cos\theta\sin\phi \\ \sin\theta\cos\phi\cos\psi+\sin\phi\sin\psi & \sin\theta\cos\phi\sin\psi-\sin\theta\cos\psi & \cos\theta\cos\phi \end{matrix} \begin{Vmatrix} V_f \\ V_l \\ V_d \end{Vmatrix} \qquad (54)$$

The following correction for aircraft rotation are required:

$$\begin{aligned} \Delta u &= -qL_y + rL_p \\ \Delta v &= -rL_r + pL_y \\ \Delta w &= -pL_p + qL_r \end{aligned} \qquad (55)$$

Where L_r, L_p, L_y are the longitudinal, lateral, and vertical distances from the aircraft centre of gravity to the inertial platform. It should be noted that, although the inertial platform values of pitch angle, roll angle and yaw angle may be differentiated to give the rates of rotation, more accurate data will be obtained by measuring the rates directly with rate gyroscopes. The corrections are added to the sensed body-axis velocity components.

The resultant velocity vector V may be obtained by addition of the component velocities. Angle of attack may then be obtained from:

$$\alpha = \arcsin\left(\frac{w}{\sqrt{u^2 + w^2}}\right) = \arctan \frac{w}{u} \qquad (56)$$

The true angle of attack may then be compared with the corresponding measured angle to give a calibration. Corrections must be made to the measured angle of attack to allow for the effects of aircraft rotation and boom bending, as outlined in Eqs (66) and (67) of section 12.3.3.

12.3.2 Calibration using Accelerometers and Rate Gyroscopes

The following technique has been used at the Aeroplane and Armament Experimental Establishment in UK to provide a calibration on aircraft equipped with the type of instrumentation employed for 'flying qualities' flight tests. The calibration provided is of the form

$$\alpha_{true} = \alpha_o + K \alpha_R \qquad (57)$$

where the values of α_o and K may vary with Mach number. The preferred manoeuvre is the roller coaster, as this will allow angle of attack to be varied throughout the usable range at approximately constant Mach number.

12.3.2.1 Basic Equations

The rates of change of the velocity components along and normal to the body axis may be written in terms of the other state variables as follows:

$$\dot{u} = g_o a_x - g_o \sin\theta - qw \qquad (58)$$

$$\dot{w} = g_o a_z + g_o \cos\theta + qu \qquad (59)$$

The value of angle of attack during the steady portion of the manoeuvre is assumed to be constant and is obtained from Eqs (46) and (48) using mean values of a_x to give θ. Then:

$$u_o = V \cos\alpha_o$$
$$w_o = V \sin\alpha_o \qquad (60)$$

Given sufficiently accurate instrumentation it is necessary only to integrate Eqs (58) and (59) step by step in order to arrive at time histories of u and w; the total velocity component V can then be obtained by vector addition, and the angle of attack from Eq (56).

In the case of accurate instrumentation, the true angle of attack time history may be compared with the sensor time history to yield a calibration.

12.3.2.2 Errors

Unfortunately all instrumentation systems mounted in aircraft are subject to errors, which fall into two broad categories:

(i) Errors which arise because of the positioning of the transducer in the aircraft; for example accelerometers are affected by rotational rates and accelerations when not mounted on the aircraft centre of gravity, and angle of attack sensors are affected by pitch rate, and by deformation under load of the fuselage and noseboom. These errors may be accounted for as shown in section 12.3.3, whereas those of the following set must be estimated for each manoeuvre.

(ii) Errors inherent in the transducer and recording system, such as hysteresis, inaccuracy and/or resolution of the recording system, drift of the transducer due to environmental change such as in temperature. When digital recording systems are used then aliasing errors may also contribute to the instrumentation error. (Aliasing is discussed in some detail in Ref (13)). These errors may be represented by a fixed (bias) component and a random component shaving a mean of zero. The bias error is assumed to remain constant throughout the manoeuvre.

12.3.2.3 Effect of Random Errors

It can be shown that the variance $\sigma_2{}^2$ of the integral of a digitally sampled signal having variance $\sigma_1{}^2$ is equal to:

$$\sigma_2{}^2 = \sigma_1{}^2\ {}^t\!/_f \qquad (61)$$

where t is the time from the start of the manoeuvre and f is the sampling frequency. It may be shown from the above that if the manoeuvre is of short duration and the sampling frequency is high, then it is often possible to neglect the effects of random errors when data are produced by integrating a noisy signal. For example, assume that a roller-coaster manoeuvre is performed at 200 m/s true airspeed and the recorded data have the following characteristics:

$$\sigma_{a_z} = 0.01\text{'g'}$$

$$\sigma_q = 0.1^\circ \text{ per sec}$$

Duration of manoeuvre = 20 s; sample frequency 20/sec.

The variance on the rate of change of angle of attack can be expressed as:

$$\sigma_{\dot{\alpha}}^2 = \left[0.1^2 + \left(\frac{9.807\text{x}57.30\text{x}0.01}{200}\right)^2\right] \qquad (62)$$

Which gives, at the end of the manoeuvre, a maximum error of 0.1° in angle of attack. It may therefore be concluded that errors in the integrated data are negligible.

12.3.2.4 Effects of Bias Errors

As stated previously, if instrumentation is fitted of such accuracy that both random and bias errors can be neglected, then the true angle of attack may be obtained by integrating Eqs (58) and (59). This is however rarely the case, so the effect of errors in the rate and accelerometer data must be considered.

The relationship between rate of change of angle of attack and the aircraft state is given by the following equation:

$$\dot{w} = (V\dot{\alpha}) = qV\cos\alpha - g_o\ (a_z - \cos\theta) \qquad (63)$$

(It is assumed that any errors in speed can be neglected)

If the observed values of the state variables contained no errors, the calibration could be obtained by comparing the integral of Eq (63) with the w velocity perturbation Vα recorded by the vane and the airspeed sensor, the lg value being obtained from a period of "steady" flight before the dynamic manoeuvre. The observations will, however, be corrupted by random and bias errors. (In this context a bias error is defined as an error which remains constant throughout the manoeuvre, although its value may change between manoeuvres). It is assumed that the integration process will so attenuate the random errors that they become negligible, in which case:

$$(V\alpha)_{nom} - V\alpha = q_b\int V\cos\alpha dt - g_o\ a_{z_b}\int dt$$

For small angles of attack (α<15°) Vcosα can be replaced by V so that:

$$(V\alpha)_{nom} - V\alpha = q_b\int V dt - g_o\ a_{z_b}\int dt \qquad (64)$$

It can be seen from Eq (64) that the bias error components will cause the computed value of w value to drift away from the true value as time increases. The angle of attack vane, however, will measure a linear function of the true angle of attack, although corrupted by random noise, so that:

$$(V\alpha)_{nom} = V(\alpha_o) + KV\ (\alpha_R) + C_1\ Vdt + C_2 t \qquad (65)$$

where C_1 and C_2 are constants. The value of the calibration factor, K, may therefore be extracted from Eq (65) using multiple linear regression analysis.

Although the regression technique outlined above has been developed assuming a linear relationship between the angle of attack vane and the true angle of attack, it may, in principle, be used for non-linear calibrations by a suitable substitution for $K(V\alpha)$ in Eq (65).

In practice however, it may be found that the calibration accuracy decreases sharply with increasing complexity of the calibration law, so the simplest possible law should be used.

The analysis described above is the simplest method for obtaining an angle of attack calibration using least squares techniques. If an accurate angle of attack is required for performance purposes then the flight-path reconstruction techniques described in Ref (14) will produce the optimum estimate. For stability and control estimation it may be more convenient to include the angle of attack calibration factor as one of the coefficients of the mathematical model relating the aircraft dynamics to the measured data, in the manner described in Ref (15).

12.3.3 Corrections to Recorded Data

When analysing dynamic manoeuvres corrections must be made for the effects of:

(i) Aircraft rotation on transducers not mounted on the aircraft centre of gravity.

(ii) Structural deformation of the aircraft nose boom and front fuselage.

(iii) Response characteristics of the angle of attack sensors.

otherwise significant errors may be introduced in the calibration data. The corrections required are detailed in this section.

12.3.3.1 Pitch Rate Correction

The true angle of attack is related to its apparent value by:

$$\alpha = \alpha_o + K\left[\alpha_R + q\ {Lr}/{V}\right] \qquad (66)$$

12.3.3.2 Accelerometer Offset Correction

The accelerometer data may be corrected to the centre of gravity using the following relationships:

$$\begin{aligned} a_x &= a_{x_r} - \dot{q}Lp/g_o + q^2Lr/g_o \\ a_z &= a_{z_R} + \dot{q}Lr/g_o - q^2Lp/g_o \end{aligned} \qquad (67)$$

12.3.3.3 Boom Bending

The effects of fuselage and boom bending are typically small and may generally be assumed to be linearly proportional to applied normal acceleration. A representative value of deflection, for an aircraft fitted with a long noseboom on which a vane assembly is mounted, is 0.1° per applied 'g'.

If it can be assumed that the aerodynamic loads on the fuselage and noseboom are small compared with the inertia loads in manoeuvring flight the deflection can be obtained from ground tests. For this the aircraft is supported on jacks and the inertia loads are applied (for example using sandbags). A sensitive inclinometer is mounted on the aircraft rigging position (near the centre of gravity) and another on the noseboom at the vane position. The deflection is obtained from the difference in the two inclinometer readings.

If the aerodynamic loading cannot be discounted it may be possible to assess its affect with sufficient accuracy from wind tunnel pressure data or by theoretical calculations.

One flight technique which has been documented in the USA Ref (16) is to mount one sensitive longitudinal accelerometer at the aircraft centre of gravity and another on the noseboom at the vane position. In theory the difference between the accelerometer readings at any flight condition (wings level) will give an estimate of boom bending. The drawbacks with this technique are mainly due to the fact that the noseboom accelerometer is positioned an appreciable distance from the centre of gravity and is therefore affected by pitch rate effects, which implies that a sensitive pitch rate gyroscope must be fitted to allow accurate corrections for pitch rate. Also this method can be used only at low to moderate bank angles because the difference in accelerometer readings depends on the value of $\cos\phi\sin\theta$.

Since the boom deflection is usually proportional to normal acceleration, which in turn is proportional to angle of attack at constant Mach number, it is common practice in UK to include the bending effects in the calibration coefficient K, and to express the variation of α_0 and K with Mach number and altitude. Experience has shown that for noseboom-mounted systems the variations of K with altitude can generally be neglected.

12.3.3.4 Dynamic Response of Angle of Attack Sensors

The methods documented at AFFTC Edwards Ref (16) and used at the Aeroplane and Armament Establishment in UK are based on the assumption that the sensor is a second-order system having a transfer function of the form

$$\frac{\alpha_R}{\alpha} = \frac{k\omega_n^2}{S^2+2\omega_n S+\omega_n^2} \qquad (68)$$

The equivalent expression in the time domain becomes:

$$(\alpha)_t = \frac{1}{k\omega_n^2}\left[\alpha_R(t)+2\zeta\omega_n\dot{\alpha}_R(t)+\omega_n^2\ddot{\alpha}_R(t)\right] \qquad (69)$$

The values of natural frequency ω_n and damping ratio ζ may be obtained from manufacturers' literature or from wind tunnel tests. It can be seen from equation (69) that it is necessary to know the first and second derivatives of the measured angle of attack. This differentiation would usually be performed on a digital computer probably using digitally recorded data. The problem with differentiating such data is that errors are amplified. At the Aeroplane and Armament Experimental Establishment in UK it has been found necessary to filter the data, using a digital low-pass filter such as that described in Ref (17) both before and after differentiation. Use of on-board filtering can introduce unacceptable phase shift into the recorded data, whereas digital filters can be designed to give zero phase-shift.

12.4 Test Techniques and Instrumentation

Previous sections have dealt with the analysis techniques required for extracting angle of attack calibrations from dynamic manoeuvres. In order to provide information for planning the flight trials, this section will describe the test manoeuvres and attempt to outline the required instrumentation.

12.4.1 Test Manoeuvres

The test manoeuvres most commonly employed are the wind-up turn, the split 'S', and the roller coaster. The roller coaster manoeuvre may be employed when either intertial platform or 'stability and control' instrumentation is fitted. The other manoeuvres are suitable only when an inertial navigation system is available. Calm air conditions and VMC are required for all calibration manoeuvres

12.4.1.1 Wind-Up Turn

The aircraft should be stabilised in steady level flight for approximately 10 seconds. The normal acceleration should then be gradually increased by banking the aircraft, whilst descending so that Mach number is kept approximately constant. Throttle setting should preferably be kept constant throughout the manoeuvre. The test should be discontinued at buffet onset, the aircraft normal acceleration limit, or the onset of lateral instability. The manoeuvre should be performed in a smooth and progressive manner so avoiding abrupt changes in bank angle and keeping sideslip deviations to a minimum. The altitude at the start of the manoeuvre should be sufficient to ensure that recovery from the manoeuvre can be safely accomplished.

12.4.1.2 Split 'S' Manoeuvres

In this manoeuvre the aircraft is pulled through from inverted flight to level flight at lower altitude.

12.4.1.3 Roller Coaster Manoeuvres

The aircraft is stabilised in steady level flight for 10 to 20 seconds, then pulled up to buffet onset, the normal acceleration limit, or the onset of lateral instability. It is then allowed to return to 1'g' flight and then pushed forward to the negative 'g' limit or buffet onset, finally being allowed to return to level flight, which is maintained for 10 seconds. The rate of application of normal acceleration should be kept below 0.5 g/second, otherwise dynamic effects become significant.

12.4.2 Instrumentation

The instrumentation details given in the following sub-paragraphs are intended only as a guide. They should not be taken to represent the most accurate instrumentation currently available. Examples of modern instrumentation systems are given in Ref (12).

12.4.2.1 Inertial Navigation Systems

The information given in the Table 1 is reprinted from Ref (12).

Table 1 Accuracy of a Typical Inertial Navigation System

Parameter	Typical Accuracy
Baro-inertial altitude	depending on air data accuracy
True airspeed	depending on air data accuracy
Ground speed	3 m/s
Latitude	1 nm per hour of flight time
Longitude	1 nm per hour of flight time
Wind direction	depending on air data accuracy
Wind speed	depending on air data accuracy
Track angle	0.5°
Drift angle	0.5°
North velocity	1 m/s
East velocity	1 m/s
Vertical velocity	1 m/s
North acceleration	10^{-3}g
East acceleration	10^{-3}g
Vertical acceleration	10^{-3}g
True heading	20 arc min
Pitch	6 arc min
Roll	6 arc min
Yaw rate	0.1% of full scale
Pitch rate	0.1% of full scale
Roll rate	0.1% of full scale

12.4.2.2 'Stability and Control' Instrumentation

Instrumentation of the type usually fitted for performance and handling or stability and control work is generally of acceptable accuracy for defining an angle of attack calibration. Table 2 presents details of a typical system.

Table 2 Accuracy of Typical 'Stability' Instrumentation

Parameter	Range	Typical Accuracy (1 Standard Deviation)
Indicated Airspeed	0 to 700 kn	0.5 kn
Altitude	0 to 50 000 ft	80 ft
Total Temperature	-60 to +150K	0.5K
Angle of Attack	-10° to +30°	0.15°
Pitch Angle	±60°	1°
Bank Angle	±180°	2°
Pitch Rate	±30°/sec	0.5°
Roll Rate	±30°/sec	0.5°
Longitudinal Acceleration	±1 'g'	0.005 'g'
Normal Acceleration	-3 to +7 'g'	0.05 'g'
Lateral Acceleration	±1 'g'	0.005 'g'

13 IN-FLIGHT SIDESLIP ANGLE CALIBRATIONS

13.1 Steady State Calibrations

In principle sideslip angle may be obtained from the difference between track and heading, to about the same accuracy as angle of attack can be obtained from the difference between pitch angle and climb angle. However whereas it is possible to neglect the vertical wind component when computing angle of attack (provided the manoeuvre is flown in calm air conditions), the analagous assumption in respect of horizontal component when computing angle of sideslip cannot be made. The horizontal wind component may be appreciable, and may change appreciably, during a set of steady sideslip calibration runs.

The use of steady state methods for sideslip angle calibrations is therefore not recommended as the accuracy of the calibration data is not expected to be good.

13.2 Quasi-Steady Calibrations

If an inertial navigation system is fitted to the aircraft, the sideslip calibration may be obtained from 'steady heading sideslip' manoeuvres. In this manoeuvre the aircraft is maintained in steady level zero (indicated) sideslip flight to enable the wind velocity to be determined. This velocity is assumed constant over the rest of the manoeuvre which is a reasonable assumption if the manoeuvre is performed in non turbulent conditions. The sideslip angle is increased by banking the aircraft, keeping heading approximately constant, in increments until the maximum sideslip angle is reached. The aircraft is returned to level flight and the manoeuvre repeated, increasing the sideslip angle in the opposite sense.

In order that sideslip angle limitations are not exceeded, the sideslip angle should be displayed to the pilot during the course of the manoeuvre. Even though the calibration of the sideslip sensor is not known initially, the effect of the aircraft's flow field is to cause the sensor to over-read, so that taking the maximum sideslip angle to be the maximum indicated value will always be conservative.

The analysis procedure requires that the inertial navigation system North, East, Down velocity components are transferred to the body axis co-ordinate system using the relationships given in Section 12.3. The true sideslip angle is then obtained from:

$$\beta = \arcsin v/V \qquad (70)$$

The measured values may then be compared with the true values to yield a calibration.

13.3 Calibration from Dynamic Manoeuvres

The following technique has been successfully used in the UK Ref (18) to obtain sideslip calibration data from dynamic manoeuvres such as Dutch rolls or flat turns.

13.3.1 Analysis Technique

The calibration produced is of the form:

$$\beta_R = \beta_O + K\beta \qquad (71)$$

where the value of K may vary with Mach number and altitude.

In the analysis the true sideslip angle is synthesised from accelerometer and rate gyroscope data, and this is compared with the values from the sideslip sensor to yield a calibration, as follows:

The relationship between the rate of change of sideslip and the aircraft state is as follows:

$$\dot{\beta} = g_o/V\ a_y + p\sin\alpha - r\cos\alpha + g_o/V\cos\theta\sin\phi \qquad (72)$$

Measurements of the aircraft state (observations) are assumed to contain bias errors (constant throughout the manoeuvre) and random errors. When Eq (72) is integrated with respect to time the random errors will be reduced considerably. It is assumed in the following that they become negligible. Then:

$$\beta_{nom} = \int g_o/V(a_y + a_{y_b}) + (p+p_b)\sin(\alpha+\alpha_b)$$
$$-(r+r_b)\cos(\alpha+\alpha_b)\, g_o/V\cos(\theta+\theta_b)\sin(\phi+\phi_b)]dt \qquad (73)$$

(It is reasonable to suppose that errors in airspeed measurement are insignificant). If speed changes can be neglected, then after expansion and omission of second order terms, Eq (73).

$$\beta_{nom} = \int g_o/V) a_y + \cos\theta\sin\phi) + p\sin\alpha - r\cos\alpha]\ dt$$
$$+ \int g_o/V(a\dot{y}_p + \phi_b\cos\phi\cos\theta - \theta_b\sin\phi\sin\theta)dt \qquad (74)$$
$$+ \int [(p\alpha_b - r_b)\cos\alpha + (r\alpha_b + p_b)\sin\alpha]dt$$

Where the first integral in Eq (74) represents the true sideslip angle, and the second integral contains the error terms.

If airspeed remains approximately constant, then changes in α and θ should also be small so that α and θ in the second integral of Eq (74) may be replaced by their mean values $\tilde{\alpha}$ and $\tilde{\theta}$, giving:

$$\beta_{nom} = \beta + [g_o/V a_{y_b} + p_b\sin\tilde{\alpha} - r_b\cos\tilde{\alpha}]\int dt + \alpha_b\cos\tilde{\alpha}\int p dt$$
$$+\alpha_b\sin\tilde{\alpha}\int r dt + g_o/V\cos\tilde{\theta}\int\cos\phi dt - \theta_b\sin\tilde{\theta}\int\sin\phi dt \qquad (75)$$

In general α and θ will be small (<10°) so that terms involving $\theta_b\ \sin\theta$ and $\alpha_b\sin\alpha$ may be neglected, giving:

$$\beta_{nom} = \beta + [g_o/V a_{y_b} + p_b\sin\tilde{\alpha} - r_b\cos\tilde{\alpha}]\int dt + \alpha_b\cos\tilde{\alpha}\int p dt + g_o/V\phi_b\cos\tilde{\theta}\int\cos\phi\, dt \qquad (76)$$

The second term on the right hand side of Eq (76) represents the linear 'drift', with time, of the calculated sideslip angle away from the true sideslip angle. The third term represents an error in the sideslip angle due to an error in angle of attack, the error being dependent upon the magnitude of the bank angle changes during the manoeuvre. The fourth term represents an error due to bias on bank angle, which is weakly dependent upon bank angle changes. If the bank angle changes are moderate, say less than 30°, then this fourth term may be included as part of the first term and Eq (76) may be re-written as:

$$\beta_{nom} = \beta + C_1 t + C_2\int p dt \qquad (77)$$

Since the sideslip vane will measure a linear function of the true sideslip angle, albeit corrupted by random noise, β_R may be substituted for β in Eq (77), giving the following:

$$\beta_R = \beta_o + K\beta_{nom} - C_1 t - C_2 \int \dot{p} dt \qquad (78)$$

The value of the sideslip calibration factor, K, may therefore be extracted by multiple linear regression analysis using Eq (78).

For a jet engined aircraft it may usually be assumed that β_o is equal to zero if the aircraft is symmetrically loaded. The sidewash generated by propellors may, however, cause the vane reading to be non-zero at zero true sideslip. For most practical purposes, though, the true sideslip angle may be assumed to be zero when the aircraft is in steady wings level flight, and the value of β_o due to propeller sidewash may be obtained from this portion of the manoeuvre.

There is a potential problem in that, if α remains constant during the manoeuvre, and rolling motion dominates, the third term in Eq (78) becomes linearly dependent upon β_{nom} and so incapable of separation from it by regression techniques, therefore it should be excluded from the analysis. For this reason this technique may yield good results only when bank angle changes are small.

In the preceding equations it is assumed that the sideslip vane, or probe, is mounted at the aircraft CG. Since this is in general not true β_{nom} must be corrected to the vane location before proceeding with the regression analysis; that is:

$$\beta_{nom} = \beta_{nom} + r\frac{Lr}{V} - \frac{pLp}{V} \qquad (79)$$

(The correction terms in Eq (79) are second order so that noise on the rates may be neglected. Bias errors will produce a constant shift in β_{nom}, so will not affect the regressions).

13.3.2 Test Manoeuvres

Although adequate sideslip angle calibration data have been obtained from Dutch roll manoeuvres the preferred manoeuvre is the 'flat turn', which should be performed as follows:

The aircraft should be stabilised in level, zero sideslip flight for approximately 10 seconds and then sideslip should be progressively increased to the maximum indicated value, keeping wings level as far as possible but avoiding abrupt changes in bank angle. The sideslip should then be reduced to zero and then increased to the maximum value in the opposite sense, whilst again attempting to keep bank angle zero. The aircraft should finally be returned to zero sideslip flight and this held for approximately 10 seconds.

The rate of application of sideslip angle should be relatively low, provided that speed changes in the manoeuvre are kept to within approximately ±15 percent of the initial speed. Changes of throttle should be avoided during the manoeuvre, and the use of asymmetric thrust to control sideslip is not recommended.

On propellor driven aircraft, it may be difficult to achieve the maximum sideslip angle in one direction due to propellor effects and so the manoeuvre should be practiced before attempting to obtain calibration data.

A sideslip angle gauge in the cockpit is essential to ensure that sideslip angles achieved in the manoeuvre do not exceed limit values (dictated by fin strength or rudder stall characteristics).

The test manoeuvre should be performed at both high and low altitude and at various Mach numbers, since experience in the UK has shown that sideslip sensors, especially when mounted beneath the aircraft fuselage, may change calibration with both altitude and Mach number.

13.3.3 Instrumentation

Instrumentation of the standard required for performance and handling trials is usually adequate. Typical ranges and accuracies of the transducers are given in Table 3 as an aid to flight planning.

Table 3 Accuracy of Typical 'Stability' Instrumentation

Parameter	Range	Accuracy (1 Standard Deviation)
Sideslip Angle	±25°	0.15°
Angle of Attack	±25°	0.15°
Airspeed	0 to 700 kn	0.5 kn
Altitude	0 to 30,000 ft	100 ft
Lateral Acceleration	±1 'g'	0.005 'g'
Yaw Rate	±30°/sec	0.15°
Roll Rate	±30°/sec	0.15°
Pitch Angle	0 to 30°	1°

14 REFERENCES

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4	C S Barnes	Flight Assessment of a Douglas Trailing Cone Static Pressure Probe at Subsonic Speeds, Royal Aircraft Establishment (UK) Technical Report 69139 - July 1969
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9	K Rawlings III	A Method of Estimating Upwash at Noseboom-Mounted Vanes, USA Air Force Flight Test Center AFFTC-TIM-81-1 - June 1981
10	P F Yaggy	A Method for Predicting the Upwash Angles Induced at the Propeller Plane of a Combinatin of Bodies with an Unswept Wing USA NACA TN 2528 - October 1951
11	V L Rogallo	Effects of Wing Sweep on the Upwash at the Propeller Planes of Multi-engined Airplanes, USA NACA TN 2795 - September 1952
12	B Stieler H Winter	AGARD Flight Test Instrumentation Series, Vol 15: Gyroscopic Instruments and their Application to Flight Testing. AGARDograph 160, - September 1982.
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18		Aeroplane and Armament Experimental Establishment (UK), Handbook of Test Methods for Assessing the Flying Qualities and Performance of Military Aircraft Volume 1 Aeroplanes
19	J H Breeman	Method for Determining the Delay of the Pitot-Static Tubing System of Aircraft. NLR Report TR 83075U - June 1983.

Annex 1

AGARD FLIGHT TEST INSTRUMENTATION AND FLIGHT TEST TECHNIQUES SERIES

1. Volumes in the AGARD Flight Test Instrumentation Series, AGARDograph 160

Volume Number	*Title*	*Publication Date*
1.	Basic Principles of Flight Test Instrumentation Engineering by A.Pool and D.Bosman	1974
2.	In-Flight Temperature Measurements by F.Trenkle and M.Reinhardt	1973
3.	The Measurement of Fuel Flow by J.T.France	1972
4.	The Measurement of Engine Rotation Speed by M.Vedrunes	1973
5.	Magnetic Recording of Flight Test Data by G.E.Bennett	1974
6.	Open and Closed Loop Accelerometers by I.McLaren	1974
7.	Strain Gauge Measurements on Aircraft by E.Kottkamp, H.Wilhelm and D.Kohl	1976
8.	Linear and Angular Position Measurement of Aircraft Components by J.C. van der Linden and H.A.Mensink	1977
9.	Aeroelastic Flight Test Techniques and Instrumentation by J.W.G. van Nunen and G.Piazzoli	1979
10.	Helicopter Flight Test Instrumentation by K.R.Ferrell	1980
11.	Pressure and Flow Measurement by W.Wuest	1980
12.	Aircraft Flight Test Data Processing – A Review of the State of the Art by L.J.Smith and N.O.Matthews	1980
13.	Practical Aspects of Instrumentation System Installation by R.W.Borek	1981
14.	The Analysis of Random Data by D.A.Williams	1981
15.	Gyroscopic Instruments and their Application to Flight Testing by B.Stieler and H.Winter	1982

At the time of publication of the present volume the following volumes were in preparation:

Flight Test Instrumentation Signal Conditioning
by D.W.Veatch

Trajectory Measurements for Take Off and Landing and Other Short Range Applications
by P. de Benque d'Agut, H.Riebeek and A.Pool

Microprocessor Applications in Airborne Flight Test Instrumentation
by M.Prickett

2. **Volumes in the AGARD Flight Test Techniques Series**

	Title	*Publication Date*
AG 237	Guide to In-Flight Thrust Measurement of Turbojets and Fan Engines by the MIDAP Study Group (UK)	1979

The remaining volumes will be published as a sequence of Volume Numbers of AGARDograph 300:

Volume Number	*Title*	*Publication Date*
1.	Calibration of Air-Data Systems and Flow Direction Sensors by J.A.Lawford and K.R.Nippress	1983

At the time of publication of the present volume the following volumes were in preparation:

Identification of Dynamic Systems: Theory and Application to Aircraft Stability and Control
by R.E.Maine and K.W.Iliff

Flight Testing of Digital Navigation and Flight Control Systems
by F.J.Abbink and H.A.Timmers

Determination of Antenna Pattern and Radar Reflection Characteristics of Aircraft
by H.Bothe and D.Macdonald

Stores Separation Flight Testing
by R.J.Arnold and C.S.Epstein

Annex 2

AVAILABLE FLIGHT TEST HANDBOOKS

This annex is presented to make readers aware of handbooks that are available on a variety of flight test subjects not necessarily related to the contents of this volume.

Requests for A&AEE documents should be addressed to the Technical Information Library, St Mary Cray. Requests for US documents should be addressed to the DOD Document Centre (or in one case, the Library of Congress).

Number	*Author*	*Title*	*Date*
NATC-TM76-ISA	Simpson, W.R.	Development of a Time-Variant Figure-of-Merit for Use in Analysis of Air Combat Maneuvering Engagements	1976
NATC-TM76-3SA	Simpson, W.R.	The Development of Primary Equations for the Use of On-Board Accelerometers in Determining Aircraft Performance	1977
NATC-TM77-IRW	Woomer, C. Carico, D.	A Program for Increased Flight Fidelity in Helicopter Simulation	1977
NATC-TM77-2SA	Simpson, W.R. Oberle, R.A.	The Numerical Analysis of Air Combat Engagements Dominated by Maneuvering Performance	1977
NATC-TM77-1SY	Gregoire, H.G.	Analysis of Flight Clothing Effects on Aircrew Station Geometry	1977
NATC-TM78-2RW	Woomer, G.W. Williams, R.L.	Environmental Requirements for Simulated Helicopter/VTOL Operations from Small Ships and Carriers	1978
NATC-TM78-1RW	Yeend, R. Carico, D.	A Program for Determining Flight Simulator Field-of-View Requirements	1978
NATC-TM79-3SA	Chapin, P.W.	A Comprehensive Approach to In-Flight Thrust Determination	1980
NATC-TM79-3SY	Schiflett, S.G. Loikith, G.J.	Voice Stress Analysis as a Measure of Operator Workload	1980
NWC-TM-3485	Rogers, R.M.	Six-Degree-of-Freedom Store Program	1978
WSAMC-AMCP 706-204	–	Engineering Design Handbook, Helicopter Performance Testing	1974
NASA-CR-3406	Bennett, R.L. and Pearsons, K.S.	Handbook on Aircraft Noise Metrics	1981
–	–	Pilot's Handbook for Critical and Exploratory Flight Testing. (Sponsored by AIAA & SETP – Library of Congress Card No.76-189165)	1972
–	–	A&AEE Performance Division Handbook of Test Methods for Assessing the Flying Qualities and Performance of Military Aircraft. Vol.1 Airplanes	1979
A&AEE Note 2111	Appleford, J.K.	Performance Division: Clearance Philosophies for Fixed Wing Aircraft	1978

Number	Author	Title	Date
A&AEE Note 2113 (Issue 2)	Norris, E.J.	Test Methods and Flight Safety Procedures for Aircraft Trials Which May Lead to Departures from Controlled Flight	1980
AFFTC-TD-75-3	Mahlum, R.	Flight Measurements of Aircraft Antenna Patterns	1973
AFFTC-TIH-76-1	Reeser, K. Brinkley, C. and Plews, L.	Inertial Navigation Systems Testing Handbook	1976
AFFTC-TIH-79-1	–	USAF Test Pilot School (USAFTPS) Flight Test Handbook. Performance: Theory and Flight Techniques	1979
AFFTC-TIH-79-2	–	USAFTPS Flight Test Handbook. Flying Qualities: Theory (Vol.1) and Flight Test Techniques (Vol.2)	1979
AFFTC-TIM-81-1	Rawlings, K., III	A Method of Estimating Upwash Angle at Noseboom-Mounted Vanes	1981
AFFTC-TIH-81-1	Plews, L. and Mandt, G.	Aircraft Brake Systems Testing Handbook	1981
AFFTC-TIH-81-5	DeAnda, A.G.	AFFTC Standard Airspeed Calibration Procedures	1981
AFFTC-TIH-81-6	Lush, K.	Fuel Subsystems Flight Test Handbook	1981
AFEWC-DR 1-81	–	Radar Cross Section Handbook	1981
NATC-TM71-ISA226	Hewett, M.D. Galloway, R.T.	On Improving the Flight Fidelity of Operational Flight/ Weapon System Trainers	1975
NATC-TM-TPS76-1	Bowes, W.C. Miller, R.V.	Inertially Derived Flying Qualities and Performance Parameters	1976
NASA Ref. Publ. 1008	Fisher, F.A. Plumer, J.A.	Lightning Protection of Aircraft	1977
NASA Ref. Publ. 1046	Gracey, W.	Measurement of Aircraft Speed and Altitude	1980
NASA Ref. Publ. 1075	Kalil, F.	Magnetic Tape Recording for the Eighties (Sponsored by: Tape Head Interface Committee)	1982

The following handbooks are written in French and are edited by the French Test Pilot School (EPNER Ecole du Personnel Navigant d'Essais et de Réception ISTRES – FRANCE), to which requests should be addressed.

Number EPNER Reference	Author	Title	Price (1983) French Francs	Notes
2	G.Leblanc	L'analyse dimensionnelle	20	Réédition 1977
7	EPNER	Manuel d'exploitation des enregistrements d'Essais en vol	60	6ème Edition 1970
8	M.Durand	La mécanique du vol de l'hélicoptère	155	1ère Edition 1981
12	C.Laburthe	Mécanique du vol de l'avion appliquée aux essais en vol	160	Réédition en cours
15	A.Hisler	La prise en main d'un avion nouveau	50	1ère Edition 1964

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16	Candau	Programme d'essais pour l'évaluation d'un hélicoptère et d'un pilote automatique d'hélicoptère	20	2ème Edition 1970
22	Cattaneo	Cours de métrologie	45	Réédition 1982
24	G.Fraysse F.Cousson	Pratique des essais en vol (en 3 Tomes)	T 1 = 160 T 2 = 160 T 3 = 120	1ère Edition 1973
25	EPNER	Pratique des essais en vol hélicoptère (en 2 Tomes)	T 1 = 150 T 2 = 150	Edition 1981
26	J.C. Wanner	Bang sonique	60	
31	Tarnowski	Inertie-verticale-sécurité	50	1ère Edition 1981
32	B.Pennacchioni	Aéroélasticité – le flottement des avions	40	1ère Edition 1980
33	C.Lelaie	Les vrilles et leurs essais	110	Edition 1981
37	S.Allenic	Electricité à bord des aéronefs	100	Edition 1978
53	J.C.Wanner	Le moteur d'avion (en 2 Tomes) T 1 Le réacteur .... T 2 Le turbopropulseur	 85 85	Réédition 1982
55	De Cennival	Installation des turbomoteurs sur hélicoptères	60	2ème Edition 1980
63	Gremont	Aperçu sur les pneumatiques et leurs propriétés	25	3ème Edition 1972
77	Gremont	L'atterrissage et le problème du freinage	40	2ème Edition 1978
82	Auffret	Manuel de médecine aéronautique	55	Edition 1979
85	Monnier	Conditions de calcul des structures d'avions	25	1ère Edition 1964
88	Richard	Technologie hélicoptère	95	Réédition 1971