Following the Polarization in a Martin-Puplett Interferometer

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1 Introduction

This document will explain how the polarization of light entering a Martin-Puplett interferometer is affected by the presence of each optical element in the interferometer. A Martin-Puplett interferometer is similar to a Michelson interferometer, however in the Martin-Puplett interferometer the beam splitter is a wire grid polarizer, not a half-silvered mirror, and the mirrors which redirect the beam on its round trip path are rooftop mirrors, not simple plane mirrors. In order to track the polarization of light waves as they pass through the interferometer, one must understand how a wire grid polarizer and a rooftop mirror each affect the electromagnetic wave. After discussing how these two optical elements affect the light, an explanation of what happens to the light at each element in the interferometer will be provided.

2 Background

2.1 Wire Grid Polarizer

A wire grid polarizer consists of many thin wires separated by a small distance which are strung parallel to each other in a plane. The Martin-Puplett interferometer makes use of wire grid polarizers as beam splitters. When light enters the polarizer, the component of the electric field vector which oscillates parallel to the direction in which the wires are strung will induce a current along the wires. Thus, to this component of the incoming E field, the collection of wires in the polarizer will act as a plane metal surface, and thus reflect the light (with the usual 180 degree phase shift from reflection by a plane mirror). By contrast, the component of the electric field vector which oscillates perpendicular to the direction of the wires does not induce a significant current in the transverse direction, and therefore this component of the field is unaffected and light with this polarization simply transmits through the polarizer. Thus, if light which is polarized along a particular axis, say horizontally, is incident on a polarizer oriented at an angle of 45 degrees to the initial polarization axis, half of the light will transmit, and half of the light will reflect.

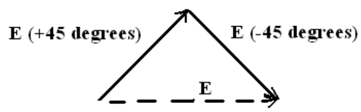

Figure 1: Horizontal polarization (dotted line) as the sum of two polarization vectors at 45 degrees

This is because horizontally polarized light can be seen as a superposition of light with an electric field vector at +45 degrees, and an electric field vector of equal magnitude oriented at -45 degrees; see Figure 1. Since the plane mirror reflection results in a phase shift of 180 degrees and the transmitted wave is unaffected by the polarizer, and thus undergoes no phase shift, the reflected and transmitted waves are 180 degrees out of phase.

2.2 Rooftop Mirror

A rooftop mirror is a pair of plane metal surfaces which are placed at right angles to one another. The line of intersection of the two planes is called the "roof line". The net effect of a rooftop mirror is to reflect a beam which is incident perpendicular to the roof line back in the direction of incidence while it also rotates the polarization by an angle of 2θ, where θ is the angle between the roof line and the incident polarization direction (see Figure 2). If θ is 45 degrees,

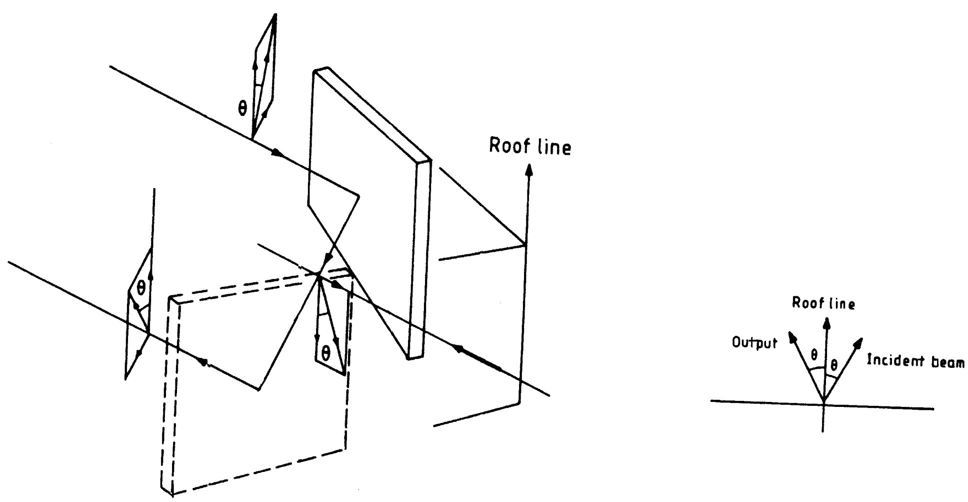

Figure 2: Transformation of the electric field vector as it bounces off a rooftop mirror. The net polarization shift is shown in the figure to the right. (From LeSurf 1990)

then the shift in polarization is 90 degrees. One can see how the rooftop mirror achieves this polarization shift of 2θ by looking at the boundary conditions of the electric field at the conducting surface. If the reader would like to bypass this analysis, he or she can skip down to the next section entitled 'Polarization in the Martin-Puplett Interferometer'.

The problem of understanding the phase shift produced by a rooftop mirror is greatly simplified by separately considering the effect of the mirror on the components of an incident E field parallel and perpendicular to the roof line; the effect of the rooftop mirror on an arbitrarily oriented E field is found by summing the effect of the mirror on these two components. Analysis of an incident E field parallel to the roof line shows that we can ignore the effect of the rooftop mirror on this component. From boundary conditions on the E field (continuity of the component of the E field tangent to the surface at a medium change) we know that the tangential component of the field outside the surface must equal the tangential component inside the material. Since the E field inside the reflecting conducting surface is zero, the tangential components of the incident and reflected fields outside the surface at the point of reflection must sum to zero. This allows us to conclude that when incident light with an

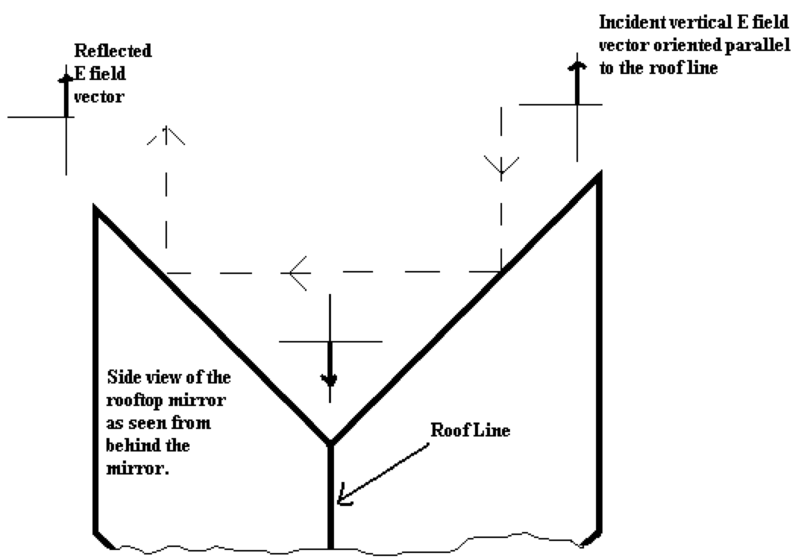

Figure 3: Transformation of the electric field component parallel to the roof line as it bounces off the rooftop mirror.

E field vector parallel to the roof line (see Figure 3) strikes the first mirror, it will reflect with an E field vector in the opposite direction but with the same magnitude, thus undergoing a phase shift of 180 degrees. Since the light incident on a rooftop mirror undergoes two reflections, the net phase shift for any E field component parallel to the roof line will be 180+180=360 degrees, or no net effect at all.

The effect of the rooftop mirror on an E field vector which is perpendicular to the roof line is a bit more complicated. As shown in Figure 4, the polarization vector which is perpendicular to the roof line, E, has one component tangential to and the other component perpendicular to Rooftop Mirror Surface A. The component tangential to the surface, E1, must undergo a rotation of 180 degrees, shown as E1' in Figure 4. This ensures that the sum of the incident

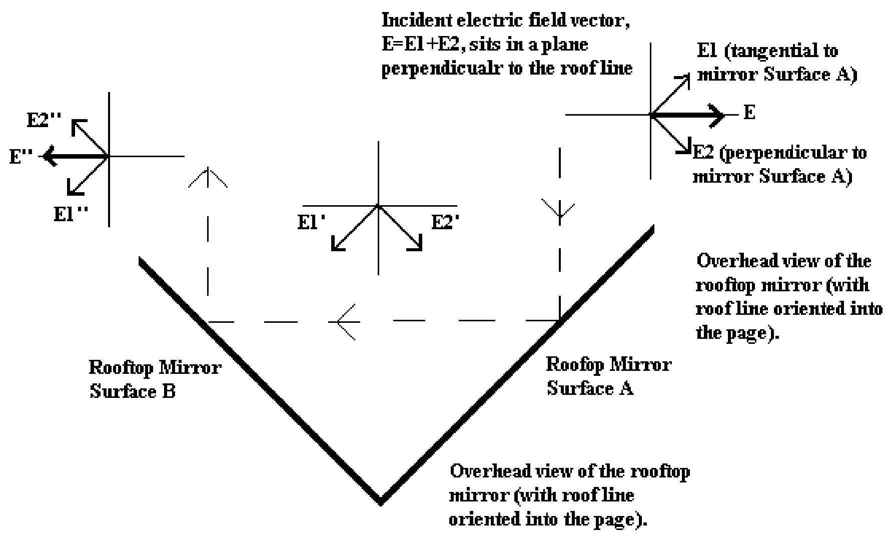

Figure 4: Transformation of the electric field component perpendicular to the roof line as it bounces off the rooftop mirror.

and reflected tangential components of the E field equals zero, thus obeying the boundary condition at the conducting surface. The component which is initially perpendicular to the surface, E2, is constrained by the fact that the total E field vector must be perpendicular to the direction of propagation of the wave. For E to be perpendicular to the direction of propagation after the first reflection, the component which was initially perpendicular to the surface is unchanged, as shown by the E2' component in Figure 4. When the wave encounters Rooftop Mirror Surface B, E1' is now perpendicular to Surface B, and thus will remain unchanged after the reflection. But E2' is now tangential to surface B, and thus will shift by 180 degrees. The result of the reflection at surface B is E" which is a sum of E1" and E2". Figure 2 shows the net effect of the rooftop mirror on an incoming light wave with arbitrary polarization. The component initially parallel to the roof line remains unchanged and the component initially perpendicular to the roof line shifts by 180 degrees. Overall, there is a polarization shift of 2θ and a reversal in direction of the wave.

3 Following the Polarization through the Interferometer

We can now understand how the polarization of light traveling through a Martin-Puplett interferometer is affected by the presence of the polarizers and mirrors in the interferometer. A schematic of a Martin Puplett interferometer is show in Figure 5. Polarizer A will be oriented at some angle, say horizontally for

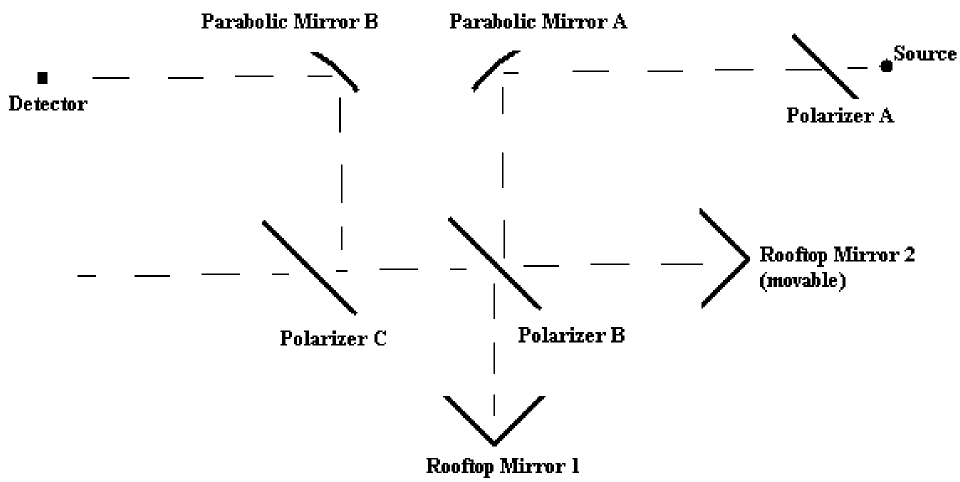

Figure 5: A Martin-Puplett interferometer

this analysis, (which corresponds to vertically strung wires). Polarizer B is then oriented at a 45 degree angle to Polarizer A. For example, if Polarizer A is oriented in the **x** direction (where **x** indicates horizontal polarization in the positive **x** direction), we choose Polarizer B to be oriented in the **(x+y)** direction; we could also choose to orient Polarizer B in the **(x-y)** direction and still obtain a beam splitting effect. When unpolarized light is incident upon Polarizer A from the source at the right, half the light passes through the polarizer with a horizontal polarization and the other half of the light is reflected upwards and lost from the system. The light which transmits through Polarizer A has an electric field vector

$$E_i = A \sin(\frac{2\pi ct}{\lambda})\mathbf{x} \tag{1}$$

where A is the amplitude of the field at this point and λ is the wavelength of the light. Light which is incident from below Polarizer A will also enter the system by reflecting off Polarizer A. If the light from below is vertically polarized it will enter the system by reflecting off the horizontally oriented polarizer and propagate to the left. However, light from below which is horizontally polarized will transmit upwards through the polarizer and leave the system. Note that the Martin-Puplett interferometer is a 4-port device, with two ports at the input and two ports at the output. (The input and output ports are actually interchangeable, as the device is symmetrical). In the analysis below I will follow only the light which is incident on Polarizer A from the right with a horizontal polarization for simplicity, however the principles can be generalized.

After traveling through Polarizer A the light strikes Parabolic Mirror A, which collimates and redirects the beam. The light from the source that passed through the focus of Parabolic Mirror A before striking the mirror will leave the mirror parallel to the plane of the interferometer towards Polarizer B. The effect of the parabolic mirror is only to phase shift the light wave, leaving the polarization unaltered; since the light from the source is, in general, incoherent, the effect of Parabolic Mirror A is insignificant to following the polarization in the interferometer.

When the light strikes Polarizer B, half the light is transmitted through the polarizer and half of it is reflected since Polarizer B is oriented at an angle of 45 degrees to Polarizer A. This transmitted light has an electric field vector of

$$E_t = \frac{1}{2} A \sin\left(\frac{2\pi ct}{\lambda}\right)(\mathbf{x}+\mathbf{y}) \tag{2}$$

(assuming the polarizer is oriented at +45 degrees, or $(\mathbf{x}+\mathbf{y})$. The transmitted light is incident on Rooftop Mirror 1 with a polarization oriented at 45 degrees to the roof line. Thus, at Rooftop Mirror 1 the light reflects and undergoes a polarization shift of 90 degrees (leaving it with an **x-y** polarization). When this light has returned to Polarizer B, it has an electric field vector of

$$E_t' = \frac{1}{2} A \sin\left(\frac{2\pi(ct - Z_1)}{\lambda}\right)(\mathbf{x}-\mathbf{y}) \tag{3}$$

where Z_1 is the round trip distance the light traveled from Polarizer B to Rooftop Mirror 1 and back again.

The light which initially reflected off Polarizer B has an electric field of

$$E_r = \frac{1}{2} A \sin\left(\frac{2\pi ct}{\lambda} + \pi\right)(\mathbf{x}-\mathbf{y}) \tag{4}$$

where the polarization is oriented at -45 degrees (or **x-y**), and a 180 degree phase shift is introduced by the effective plane mirror reflection. This light travels towards Rooftop Mirror 2 with a polarization oriented at 45 degrees to the roof line. Thus, this light also undergoes a polarization shift of 90 degrees (leaving it with an **x+y** polarization), and a phase shift due to this light traveling a round trip distance of Z_2. This light has an electric field vector of

$$E_r' = \frac{1}{2} A \sin\left(\frac{2\pi(ct - Z_2)}{\lambda} + \pi\right)(\mathbf{x}+\mathbf{y}) \tag{5}$$

when it returns to Polarizer B.

The two light beams then are again incident on Polarizer B. The beam that originally transmitted through Polarizer B will now reflect due to the 90 degree polarization shift that Rooftop Mirror 1 caused. The reflection will result in a 180 degree phase shift, leaving this beam with an electric field of

$$E_{tr} = \frac{1}{2} A \sin\Big(\frac{2\pi(ct - Z_1)}{\lambda} + \pi\Big)(\mathbf{x} - \mathbf{y}). \tag{6}$$

And the light beam that originally reflected off Polarizer B will now transmit through the polarizer since it also picked up a 90 degree polarization rotation at Rooftop Mirror 2. During transmission the E field of this beam is unaltered, so we can now write the electric field of the initially reflected beam as

$$E_{rt} = \frac{1}{2} A \sin\Big(\frac{2\pi(ct - Z_2)}{\lambda} + \pi\Big)(\mathbf{x} + \mathbf{y}). \tag{7}$$

The two beams emerge from Polarizer B in parallel, where the total E field is just the sum of E_{tr} and E_{rt}:

$$E_{tr}+E_{rt} = \frac{1}{2} A \sin\Big(\frac{2\pi(ct - Z_1)}{\lambda}+\pi\Big)(\mathbf{x} - \mathbf{y})+\frac{1}{2} A \sin\Big(\frac{2\pi(ct - Z_2)}{\lambda}+\pi\Big)(\mathbf{x} + \mathbf{y}). \tag{8}$$

One can use trigomometry identities to manipulate this expression and obtain the total E field exiting Polarizer B, E_{TOT}

$$\begin{aligned} E_{TOT} = {} & A \sin\Big(\frac{\pi(2ct - Z_1 - Z_2)}{\lambda} + \pi\Big) \cos\Big(\frac{\pi(Z_1 - Z_2)}{\lambda}\Big)\mathbf{x} + \\ & + A \cos\Big(\frac{\pi(2ct - Z_1 - Z_2)}{\lambda} + \pi\Big) \sin\Big(\frac{\pi(Z_1 - Z_2)}{\lambda}\Big)\mathbf{y}. \end{aligned} \tag{9}$$

Note that both the $\mathbf{x}$ and $\mathbf{y}$ components of the E field contain the product of a sine and cosine term. The first term in both products contains a sine or cosine with an argument of $(\frac{\pi(2ct-Z_1-Z_2)}{\lambda} + \pi)$; the sine term will vanish at the times when $2ct - Z_1 - Z_2$ is an integral number of wavelengths and the cosine term will vanish when $2ct - Z_1 - Z_2$ is a half number of wavelengths. The second term in both products contains a sine or cosine term with an argument of $(\frac{\pi(Z_1-Z_2)}{\lambda})$; the sine term will vanish when $Z_1 - Z_2$ is an integral number of wavelengths and the cosine term will vanish when $Z_1 - Z_2$ is a half number of wavelengths.

3.1 Case 1: When $Z_1 - Z_2$ is an Integral Number of Wavelengths

The simplest case to consider is one in which the path difference, $Z_1 - Z_2$ is an integral number of wavelengths. In this case the $\sin(\frac{\pi(Z_1-Z_2)}{\lambda})$ term in E_{TOT} is zero, and the net electric field is again horizontally polarized. This is important because it tells us that if the phase shift introduced by Z_1 and Z_2 can be ignored, then the E field vectors emerging from Polarizer B will add constructively, producing a maximum in intensity at the detector as long as Polarizer C is chosen appropriately. To see how this occurs vectorally, see Figure 6; it shows how horizontally polarized light which enters the Polarizer B/Rooftop

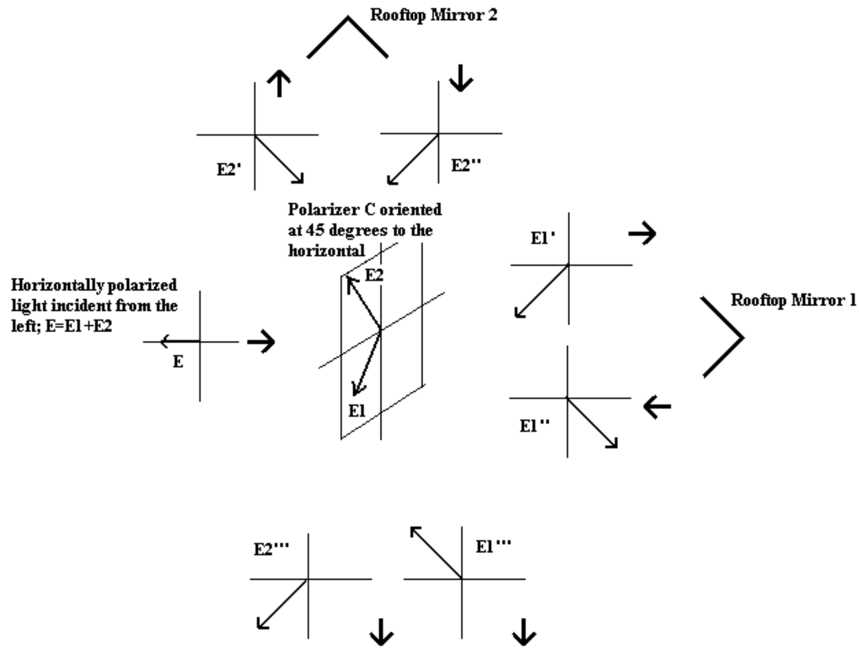

Figure 6: Transformation of the polarization vector when $Z_1 - Z_2$ is an integral number of wavelengths.

Mirror1/Rooftop Mirror 2 configuration will emerge from the configuration with a horizontal polarization if $Z_1 - Z_2$ is a full integral number of wavelengths. In Figure 6, the initial E field vector, a sum of E1 and E2, points in the negative **x** direction. The transmitted wave has an E field of E1', and the reflected wave has an E field of E2'. After interaction with Rooftop Mirrors 1 and 2, the E field vectors will be E1" and E2", respectively. Finally, after interaction with Polarizer C the E field vectors are E1"' and E2"'. The sum of E1"' and E2"' is just the E field from the source which was horizontally polarized. Finally, the light encounters Polarizer C, which can be oriented vertically such that it reflects the entire desired signal towards Parabolic Mirror B, and then Detector A. Note that if the path difference $Z_1 - Z_2$ is on the order of a half wavelength then the $\cos(\frac{\pi(Z_1-Z_2)}{\lambda})$ term in E_{TOT} will vanish and the net electric field will be vertically, not horizontally polarized. By analogy we can make some conclusions about the light which entered the system by reflecting off Polarizer A from below with a vertical polarization. This light will be incident on Polarizer C with a vertical polarization in the case of a full wavelength path difference, and a horizontal polarization in the case of a half wavelength path difference.

3.1.1 Testing for Polarization

In order to test the polarization of an optical element one can place the element in front of the detector and measure the transmitted intensity as the optical element is rotated through 90 degrees. If the test element has no affect on the polarization of light, the measured intensity will be independent of the angle. If the test element has polarizing properties, one will observe a maximum of intensity when the polarizing axis of the test element is aligned with the polarization of the light, and a minimum of intensity when the polarizing axis of the test element is orthogonal to the polarization axis of the transmitted light. When testing for polarization, the movable Rooftop Mirror 2 needn't be moved at all, but the position of the movable mirror will determine which wavelength of light will be incident on the test element since the path difference will result in a particular wavelength of light being at a maximum.

3.2 Case 2: When $Z_1 - Z_2$ is not an Integral Number of Wavelengths

The situation is less simplified when $Z_1 - Z_2$ is not an integral number of wavelengths. In this case, we must consider E_{TOT} as defined in Equation (9) in its general form since neither the sine nor cosine envelope term vanishes. This means we have both a horizontal and vertical component in our E field even if we started with horizontally (or vertically) polarized light. If Polarizer C is oriented at some fixed polarization, then only part of our original signal will reflect at Polarizer C towards Parabolic Mirror 2 and the detector. For example, if Polarizer C is oriented vertically, then only the horizontally polarized E field component will reach the detector. The rest of the signal will transmit through Polarizer C. As the movable rooftop mirror moves, the path difference $Z_1 - Z_2$ changes from an integral number of wavelengths (fully horizontally polarized light which is incident on Polarizer C, in our example), to a half number of wavelengths (fully vertically polarized light which is incident on Polarizer C, in our example), and then back to an integral number of wavelengths (again, fully horizontally polarized light in our example). This will result in a variation of intensity observed at the detector from a maximum to a minimum and back to a maximum in the case that Polarizer C is oriented vertically.

3.3 Presence of a Load in the System

If there is a load present in the system in front of both the input ports (say a 300K blackbody) then light will enter our system through transmission and reflection at Polarizer A. Regardless of the path difference, an equal amount of intensity from an isotropic and unpolarized load will always strike the detector since the load is entering the system through both input ports. In our example where Polarizer A is oriented vertically, radiation from a load will be both transmitted from the right and reflected from below by Polarizer A. The transmitted component will only have a horizontal polarization and the reflected component will only have a vertical polarization. Thus, the two components of radiation from the load will sum to unpolarized light after interacting with Polarizer A as long as the load is isotropic and unpolarized. When this unpolarized light is incident on Polarizer B, it will split and recombine such that it will be unpolarized when it is incident on Polarizer C. This can be seen if one thinks of the unpolarized light as a sum of horizontally and vertically polarized light. The component of load radiation incident on Polarizer B which is horizontally polarized will have the form of E_{TOT} as defined in Equation (9) when it is incident on Polarizer C. By analogy, the component of load radiation incident on Polarizer B which is vertically polarized will have the form of

$$E'_{TOT} = A\cos\left(\frac{\pi(2ct - Z_1 - Z_2)}{\lambda}\right)\sin\left(\frac{\pi(Z_1 - Z_2)}{\lambda}\right)\mathbf{x} + A\sin\left(\frac{\pi(2ct - Z_1 - Z_2)}{\lambda}\right)\cos\left(\frac{\pi(Z_1 - Z_2)}{\lambda}\right)\mathbf{y}. \tag{10}$$

Then the sum of the contributions of both the horizontally and vertically polarized components after they emerge from Polarizer B will be $E_{TOT} + E'_{TOT}$

$$E_{TOT} + E'_{TOT} = A\Big\{\sin\left(\frac{\pi(2ct - Z_1 - Z_2)}{\lambda}\right)\cos\left(\frac{\pi(Z_1 - Z_2)}{\lambda}\right) + \cos\left(\frac{\pi(2ct - Z_1 - Z_2)}{\lambda}\right)\sin\left(\frac{\pi(Z_1 - Z_2)}{\lambda}\right)\Big\}(\mathbf{x} + \mathbf{y}). \tag{11}$$

Equation (11) describes unpolarized light. Therefore, regardless of the orientation of Polarizer C, half the light will reflect off the polarizer and reach the detector, and the other half of the light will transmit through the polarizer. Thus, the light from the load will affect the absolute measurement of the intensity in a constant way as long as the load is unpolarized, isotropic, and uniform in time.

4 Conclusion

We plan to use the FTS in our lab to test optics for the telescopes we collaborate on; the FTS will not be used to collect CMB data directly by looking at the sky. We will either use a strong source of a particular frequency or range of frequencies or we will use a source of a blackbody temperature load such as a 77K load in the form of a bucket of liquid nitrogen; the location of the source will be as labeled in Figure 5. In either case, a 300K blackbody will sit in front of the second input port of the FTS which is below Polarizer A, as shown in Figure 5.

References

[1] LeSurf, James, *Millimetre-Wave Optics, Devices and Systems.* 1990, page 136–page 155.