Growing evanescent waves in negative-refractive-index transmission-line media

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(Received 15 November 2002; accepted 23 January 2003)

We show the enhancement of evanescent waves by a realizable negative-refractive-index (NRI) medium consisting of a periodic 2-D L,C loaded transmission-line (TL) network. This network is referred to as a dual TL structure. Growing evanescent waves within the dual TL structure are predicted analytically and demonstrated through simulation. These findings confirm that the dual TL structure is not simply a phase compensator that corrects the phase of propagating waves, but is in fact a NRI medium, since it also enhances the amplitudes of evanescent waves. This structure is a likely candidate for microwave subwavelength focusing and imaging applications. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1561167]

The possible subwavelength resolving property of a slab of negative refractive index (NRI) metamaterial predicted by Pendry[1] has renewed interest in media possessing negative material parameters (ϵ,μ), which were first investigated by Veselago in the 1960s.[2] Pendry reveals that such "perfect lenses" not only apply a phase correction to the propagating Fourier components, which decay in phase away from their source (as do conventional lenses), but also enhance the evanescent waves, which normally decay in amplitude. In other words, these perfect lenses achieve diffraction-free, near-field focusing. The NRI metamaterial reported in Ref. 3 combined an array of metallic wires to attain negative permittivity and an array of split-ring resonators to achieve negative permeability.[3] More recently, the periodic 2-D L,C loaded transmission-line (TL) network shown in Figure 1(a) was shown to exhibit NRI properties over a broad frequency range.[4] This network will be referred to as a dual TL structure since it is of a high-pass configuration, as opposed to the low-pass representation of a conventional TL mesh [see Fig. 1(b)]. Dual TL structures have been used to experimentally demonstrate backward-wave radiation and focusing at microwave frequencies.[5–7] In this letter, we present analytic expressions and microwave circuit simulation results using Agilent's Advanced Design System (ADS)™, which demonstrate the enhancement of evanescent waves by a realizable NRI metamaterial consisting of a dual TL structure. The enhancement of evanescent waves has to date only been treated theoretically and has not been demonstrated in any realizable structure. We show that evanescent waves actually grow within the dual TL structure. These findings confirm that the dual TL structure is not simply a phase compensator that only corrects the phase of propagating waves, but is in fact an NRI medium, since it also enhances the amplitude of evanescent waves.

The transmission of an evanescent voltage plane wave through a finite width NRI slab is depicted in Fig. 2. The plane wave voltage V_1 is y-directed and incident from a positive refractive index (PRI) medium. The incident plane wave possesses a large transverse wave number (k_{xp}) that exceeds the intrinsic wave number (k_p) of the PRI medium. This results in an inhomogeneous wave with an imaginary longitudinal wave number $k_{zp} = -j\alpha$. The NRI medium is designed to have a relative refractive index of -1 compared to the PRI medium. In addition, both media are impedance matched to eliminate reflections at the two interfaces.

The PRI medium utilized is a TL mesh composed of the unit cells shown in Fig. 1(b) while the NRI medium is made up of the dual TL unit cells depicted in Fig. 1(a). The dual TL structure exhibits isotropic and homogeneous backward-wave propagation characteristics when the interconnecting transmission line sections are electrically short ($\beta_n d \ll 1$) and the per-unit-cell phase delays in the z and x directions are small: $k_{zn}d \ll 1$, $k_{xn}d \ll 1$, where d is the unit cell dimension. Therefore, it can be considered a homogeneous NRI medium. For frequencies where there is nearly isotropic backward-wave propagation, the dispersion relation becomes

$$k_n d = -\sqrt{\left(\frac{Z_{on}}{\omega L} - 2\beta_n d\right)\left(\frac{1}{Z_{on}\omega C} - \beta_n d\right)}, \qquad (1)$$

where k_n is the intrinsic wave number in the NRI TL medium, ω is the radial frequency, $\beta_n d$ is the phase delay, and Z_{on} is the characteristic impedance of the interconnecting transmission line sections, C is the loading series capacitance, and L is the loading shunt inductance. The voltages

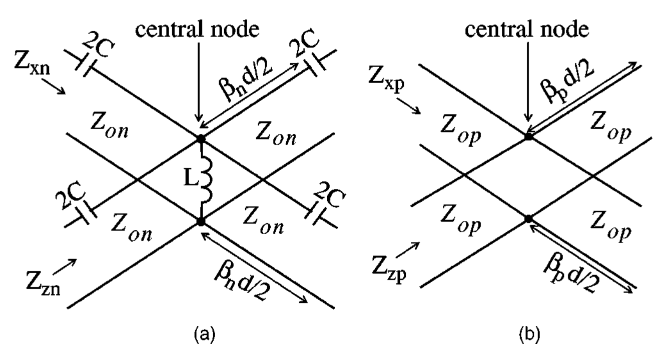

FIG. 1. TL unit cells. (a) Unit cell of dual TL structure (NRI medium) and (b) unit cell of TL mesh (PRI medium).

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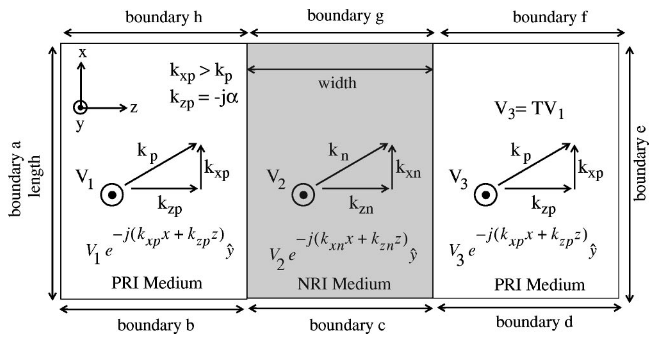

FIG. 2. Transmission through a NRI medium consisting of a dual TL structure (reflected voltages not shown for clarity).

and currents at the terminals of the dual TL unit cells are related by the structure's Bloch impedances. The Bloch impedances, Z_{zn} and Z_{xn}, looking into the unit cell terminals in the z and x directions, respectively, are

$$Z_{zn}=\frac{Z_n k_n d}{k_{zn} d},\quad Z_{xn}=\frac{Z_n k_n d}{k_{xn} d},$$
$$Z_n = Z_{on}\sqrt{\frac{\dfrac{1}{Z_{on}\omega C}-\beta_n d}{\dfrac{Z_{on}}{\omega L}-2\beta_n d}}. \tag{2}$$

Similarly, a dispersion relation and Bloch impedances for the PRI TL structure shown in Fig. 1(b) can be derived. This structure exhibits isotropic and homogeneous forward-wave propagation characteristics when the interconnecting TL sections are electrically short ($\beta_p d \ll 1$) and can therefore be considered a homogeneous PRI medium. Its dispersion relation is

$$k_p d = \sqrt{(k_{xp}d)^2+(k_{zp}d)^2} = \sqrt{2}\beta_p d, \tag{3}$$

where d is the dimension of the TL mesh unit cell, k_p is the intrinsic wave number, k_{zp} is the wave number in the z direction, k_{xp} is the wave number in the x direction, $\beta_p d$ is the phase delay, and Z_{op} is the characteristic impedance of the interconnecting transmission line sections.

The Bloch impedances Z_{zp} and Z_{xp}, looking into the TL mesh unit cell terminals in the z and x directions, are

$$Z_{zp}=\frac{Z_p k_p d}{k_{zp} d},\quad Z_{xp}=\frac{Z_p k_p d}{k_{xp} d},\quad Z_p=\frac{Z_{\mathrm{op}}}{\sqrt{2}}. \tag{4}$$

To ensure a relative refractive index of -1 between the PRI and NRI media, the wave numbers in each medium are set equal in magnitude but opposite in sign: $kd=-k_n d = k_p d$. Additionally, reflections at the two interfaces are eliminated by matching the two media: $Z=Z_n=Z_p$. The electrical parameters of two such PRI and NRI media that utilize the same interconnecting TL sections ($Z_o=Z_{on}=Z_{\mathrm{op}}$, $\beta d=\beta_n d=\beta_p d$) are shown in Table I for a 1-GHz frequency of operation.

TABLE I. Electrical parameters for the PRI and NRI media.

kd	Z	Z_0	βd	L	C
0.4363 rad	50 Ω	71.574 Ω	0.3073 rad	9.1943 nH	3.5895 pF

Using the Bloch impedance expressions and dispersion relations for the PRI and NRI media, the following transmission coefficient T can be derived for a voltage plane wave incident on a NRI (dual TL) medium that is infinite in the x direction and m cells wide in the z direction:

$$T=\frac{V_3}{V_1}=\frac{tt' e^{-jk_{zn}md}}{1-r'^2 e^{-jk_{zn}md}}, \tag{5}$$

where an $e^{j\omega t}$ time variation is assumed, and t and r are the transmission and reflection coefficients, respectively, initially seen by the incident voltage wave at the first interface:

$$t=\frac{2k_{zp}k_n Z_n}{k_{zp}k_n Z_n + k_{zn}k_p Z_p},\quad r=\frac{k_{zp}k_n Z_n - k_{zn}k_p Z_p}{k_{zp}k_n Z_n + k_{zn}k_p Z_p}, \tag{6}$$

while t' and r' are the transmission and reflection coefficients, respectively, initially seen by the voltage wave at the second interface:

$$t'=\frac{2k_p Z_p k_{zn}}{k_p Z_p k_{zn} + k_{zp}k_n Z_n},\quad r'=\frac{k_p Z_p k_{zn} - k_{zp}k_n Z_n}{k_p Z_p k_{zn} + k_{zp}k_n Z_n}. \tag{7}$$

Assuming both TL media are matched and their relative refractive index is -1 ($k=k_p=-k_n$, $k_z=k_{zp}=-k_{zn}$, $Z=Z_n=Z_p$) yields

$$T=e^{jk_z md}. \tag{8}$$

In the case of the evanescent voltage wave shown in Fig. 2, k_z is replaced by an attenuation constant $-j\alpha$. The transmission constant through the dual TL structure becomes $T=e^{\alpha md}$, which suggests a growing wave. The amplification per unit cell is simply $e^{jk_z d}=e^{\alpha d}$.

In the circuit simulations, the inhomogeneous plane wave was excited using an array of voltage generators along boundaries (a) and (b) (see Fig. 2). The transverse wave number of the plane-wave was chosen slightly larger than the intrinsic wave number k. For convenience, the attenuation per unit cell in the z direction $e^{-jk_z d}$ was set to 0.8. Therefore, $k_z d=-0.2231j$ radians per cell. Substituting this value for $k_z d$ into Eq. (3) yields $k_x d=0.4920$ radians, which exceeds $kd=0.4363$ of the two media. Adjacent generator voltages along boundary (a) were related by $k_x d$ and those along boundary (b) by $k_z d$. The Bloch impedances for this particular evanescent plane wave in both media can be found using Eqs. (2), and (4)

$$Z_z=99.7823j\Omega,\quad Z_x=44.1474\Omega. \tag{9}$$

The Bloch impedance in the z direction is reactive due to the fact that the evanescent plane wave does not transport energy in the z direction. In the simulations, the boundaries parallel to the z and x axes were terminated in Z_z and Z_x, respectively.

In Fig. 3, Agilent ADS circuit simulation results are shown for the transmission of the aforementioned evanescent plane wave through an infinitely long (in the x direction) NRI medium that is four cells wide (in the z direction). Figure 3 shows the voltage magnitudes at the central nodes of the unit cells for a section 10 cells wide and 15 cells long. The plane wave decays in both PRI media (TL mesh), but grows within the NRI medium (dual TL structure). As anticipated, the attenuation per unit cell in the PRI medium is 0.8, while the amplification per unit cell in the NRI medium is

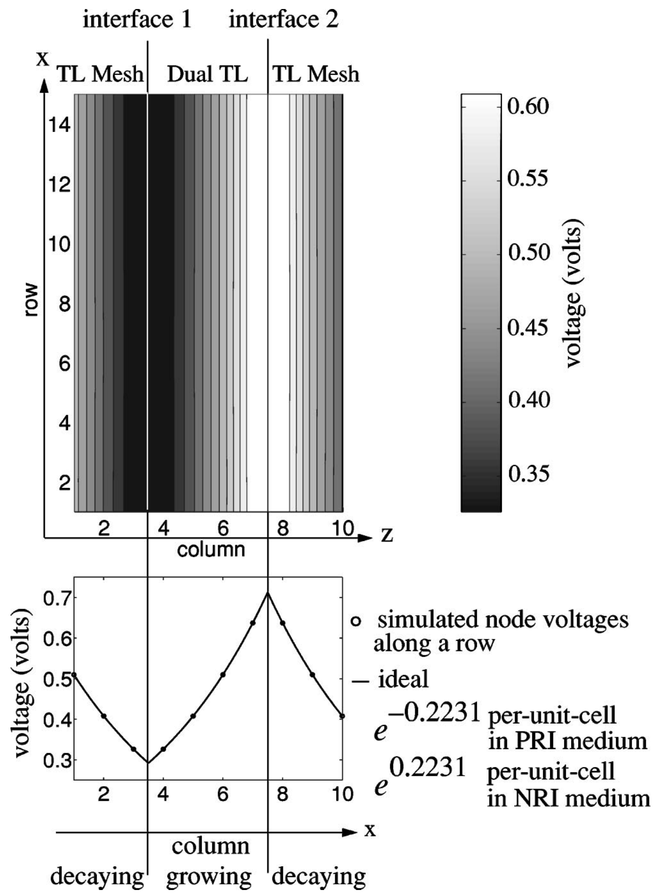

FIG. 3. Simulated voltage magnitudes for infinitely long interfaces (in the x direction).

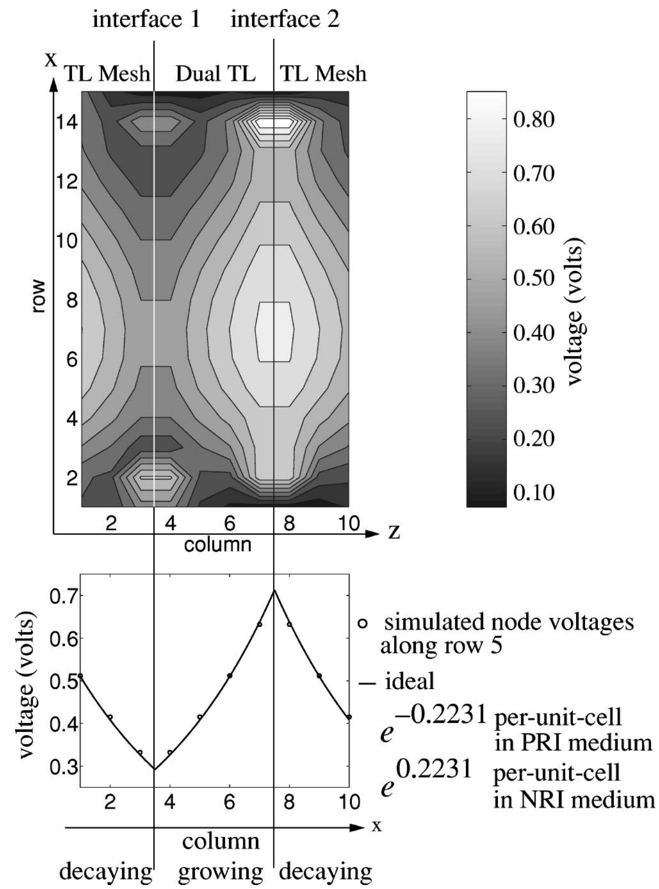

FIG. 4. Simulated voltage magnitudes for finite interfaces (in the x direction).

$0.8^{-1}=1.25$, exactly canceling the decay of the evanescent wave. The phase progression in the x direction, which is not shown, is linear ($k_x d = 0.4920$ radians per unit cell) in all three media. In order to simulate these infinitely long interfaces, additional voltage generators were placed along boundaries (d) and (g). The voltage generators along boundary (g) compensate for the power absorbed by boundary (h), and the generators along boundary (d) compensate for the power absorbed by boundary (c).

In Fig. 4, the voltage magnitudes are plotted for a finite sized structure that is only 10 cells wide and 15 cells in length. Voltage generators were placed only along boundaries (a) and (b) to excite an incident plane wave, while all other boundaries were terminated in their respective Bloch impedances. Edge effects are apparent in these simulations due to the structure's finite size. Nevertheless, the growing evanescent wave within this finite practical structure is still quite evident.

The enhancement of evanescent waves by a realizable NRI medium consisting of a periodic 2-D L,C loaded TL network has been predicted analytically as well as demonstrated through circuit simulation. Evidence of growing evanescent waves within the dual TL medium is shown for both infinite and finite length structures. The dual TL medium is a likely candidate for microwave subwavelength focusing and imaging applications.

[1] J. B. Pendry, Phys. Rev. Lett. **85**, 3966 (2000).
[2] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[3] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[4] A. K. Iyer and G. V. Eleftheriades, in *IEEE MTT-S International Microwave Symposium Digest*, *Seattle, WA, 2002* (IEEE, New York, 2002), Vol. 2, pp. 1067–1070.
[5] A. Grbic and G. V. Eleftheriades, in *IEEE International Symposium on Antennas and Propagation, San Antonio, TX, 2002* (IEEE, New York, 2002), Vol. 4, pp. 340–343.
[6] A. Grbic and G. V. Eleftheriades, J. Appl. Phys. **92**, 5930 (2002).
[7] G. V. Eleftheriades, A. K. Iyer, and P. C. Kremer, IEEE Trans. Microwave Theory Tech. **50**, 2702 (2002).