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BLADE VORTEX INTERACTION PROBLEM AT HELICOPTER ROTORS

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ABSTRACT

The helicopter rotor blade excites a vortex system, which consists of bound, trailed and shed vortices. The trailed and shed vortices rolled up to the blade tip vortex. This strong tip vortex has a remarkable induced velocity field, which can cause pregnant changes in the aerodynamic loads of the rotor blade.

This tip vortex moves together with the ambient air masses. Arises a "Blade Vortex Interaction" (BVI) problem when this tip vortex moves closely to a rotor blade. In order to get physically real induced velocity field a real vortex model with finite core must be apply.

Key Words: finite core, real vortex, vortex segment

NOMENCLATURE

The symbols for physical quantities and also their dimensions are the follows:

e	$[m]$	flapping hinge offset
$\underline{ds}$	$[m]$	tangential vector
J	$[m^4/s]$	second moment of vorticity
K	$[m^3/s]$	first moment of vorticity
L	$[m]$	rotor blade length
m	$[kg]$	rotor blade mass
r	$[m]$	place vector or distance
rc	$[m]$	vortex core radius
t	$[s]$	time
x_l	$[m]$	coordinate along the rotor blade
x_s	$[m]$	centre of gravity of the blade
$v(r)$	$[m/s]$	swirl (induced) velocity
z	$[m]$	local coordinate
w	$[m/s]$	induced velocity
$\underline{\rho}$	$[m]$	vector along the vortex line
β	$[deg]$	angle between the vortex and the blade
β_l	$[rad]$	rotor blade flapping angle
δ	[-]	eddy viscosity coefficient
$\varphi(r, rc)$	[-]	core function
ν	$[m^2/s]$	kinematical viscosity
ρ	$[kg/m^3]$	air density
Γ	$[m^2/s]$	circulation
$\gamma(x_l)$	$[m/s]$	distributed circulation
ζ	$[rad]$	vortex age
Θ_y	$[kgm^2]$	inertial moment of the blade
Ω	$[1/s]$	angular velocity of the rotor

1. INTRODUCTION

The most characteristic part of the helicopter is the main rotor. In flight the helicopter main rotor provides three basic functions: lift generation, propulsive force generation for forward flight and means to generating forces and moments to the control of the attitude and position of the helicopter. The rotor has rotor blades, which generate aerodynamic forces in order to produce lift, propulsive force and control moments. The rotor blade is a rotating wing, which works under complicated circumstances. The rotor blade motion determines the aerodynamic forces acting on the blade. But on the other hand the aerodynamic forces determine the rotor blade motion. This means that the forces and the motion of the blades are connected together. In the aerodynamic calculation of the helicopter rotor blades must be included the blade dynamic and control to. In order to show the blade vortex interaction problem the most simplest case was chosen: horizontal flight with constant flying velocity.

The aerodynamic force generation means that the rotor blade produced a vortex-system (see Fig. 6). Connected to the blade exists the bound vortex. From the changing of this bound vortex arise the trailed and the shed vortices. The trailed and the shed vortices quickly rolled up and from this precede the tip vortex of the rotor blade. The intensity of the tip vortex is high and this vortex has a long lifetime. This tip vortex moves together with the ambient air masses and causes blade vortex interaction (BVI) problems if coincide with a rotor blade of the helicopter.

For the BVI problems the distance between the rotor blade and the vortex is very important. If this distance small (for example smaller than the twice vortex core radius) then an ideal vortex produces physically unrealistic induced velocities. In order to avoid this the real vortices with finite core was introduced. This finite vortex core gives the possibility to take account the vortex age too.

2. VORTEX BEHAVIOUR

The rotor flow field is laden with vortical structures. Experimentally, blade tip vortices have been found to be the most dominant structures in the flow field. Tip vortices form quickly behind the rotor blades as they rotate. Physically the tip vortices in the rotor wake are convected downstream of the rotor at the local flow field velocity. Unlike wing-tip vortices of fixed wing aircraft blade vortices remain close to the plane of the rotor, and have a powerful influence on the spatial and temporal variations in the aerodynamic loading on all the blades. The strengths and positions of these tip vortices are affected by many interrelated geometric parameters (e.g., number of blades, blade twist, blade plan form) and operational conditions (e.g., rotor thrust, advance ratio, climb velocity and tip path plane (TPP) angle of attack).

Betz's theory ([2],[12]) uses three conservation equations for ideal vortex systems to relate the structure of the vortex sheet behind an isolated half span wing or a rotor blade to the structure of a single, fully developed vortex. The three conservation laws that relate the circulation on the wing to that in the fully developed vortex (Γ) are then:

I. The circulation is conserved:

$$\Gamma = \int_0^L \gamma\,(x_l)\; dx_l \; = const \qquad (1)$$

II. The first moment of vorticity is conserved (the centroid of vorticity remains at a fixed span-wise location)

$$K = \int_0^L \gamma(x_l)\, x_l \; dx_l \qquad (2)$$

III. The second moment of vorticity is conserved:

$$J = \int_0^L \gamma(x_l)\, x_l^2 \; dx_l \qquad (3)$$

Applying these conservation laws the uprolled vortex position can be calculated at the end of the near wake. Since the rolling up process at helicopter rotor blades is very fast the effect of viscosity - in this process - can be neglected.

Very high swirl velocities in the tip vortices can produce large spatial variations in induced velocities at each rotor blade, which can contribute significantly to the overall unsteadiness of the rotor flow field. In order to avoid the physically unrealistic high swirl velocities the viscous core structure of the vortices would be introduced. Modelling this core structure of lift-generated trailing vortices has been a continuing challenge in rotating wing aerodynamic problems. There are more vortex models, which include the effect of viscosity. The simplest model is the Rankine-model; the Kaufmann (Scully) model uses an algebraic velocity profile. Vatistas also proposed a family of desingularized algebraic swirl-velocity profile for stationary vortices. Another vortex model was given by Lamb and also by Oseen, this model is a solution to the one-dimensional simplified Navier-Stokes equations. This is an axisymmetric solution for the swirl velocity with the assumption that the axial and radial velocities are zero. The Lamb-Oseen model for the swirl velocity is:

$$v(r) = \frac{\Gamma}{4\pi}\left[1 - \exp\left(-\frac{r^2}{4\nu t}\right)\right] \qquad (4)$$

The viscous core radius is the radial location where the swirl velocity is the maximum. Differentiating the above equation by the "r" and setting the derivative to zero, the viscous core radius (growing in the time) can be calculated as:

$$rc = \sqrt{-4 \nu t \, LambertW\left(-1, -\frac{1}{2} e^{-\frac{1}{2}}\right) - 2 \nu t} \tag{5}$$

Where *LambertW*(*k*,*x*) is the Lambert's W function. The actual value of this function in this case: - 1.75643. Substituting this value into the equation above we get the well-known expression for the vortex core radius in laminar case:

$$rc = \sqrt{4 \alpha \nu t} \qquad (\alpha = 1.2564312.) \tag{6}$$

The Lamb-Oseen model can be applied only for laminar flow. To generalise such a formula to the turbulent flow Squire proposed to introduce an "eddy" viscosity: $\delta\nu$. Therefore for turbulent flow the core radius becomes:

$$rc = \sqrt{4 \alpha (\delta \nu) t} \tag{7}$$

For the calculation of the eddy viscosity coefficient the vortex Reynolds number can be introduced:

$$\mathrm{Re}_\Gamma = \Gamma/\nu \qquad \text{and:} \qquad \delta = 1 + a \, \mathrm{Re}_\Gamma$$

The " a " is an empirical parameter. In our calculation (referring to [3]) its value is 0.0002. By using of this eddy viscosity and the vortex core radius the Lamb-Oseen model can be rewritten as:

$$v(r) = \frac{\Gamma}{4\pi}\left[1 - \exp\left(-\alpha\delta \frac{r^2}{rc^2}\right)\right] \tag{8}$$

This modified Lamb-Oseen model is suitable for the swirl velocity calculation in the whole region around a vortex line and can take into account the vortex aging process too. This model has not a singular point, at the zero distance gives zero swirl velocity (see Fig. 4.).

3. IDEAL VORTICES IN THREE DIMENSIONS

The results of the Lamb-Oseen model including the "eddy" viscosity are applicable only for one-dimensional flow, but we should turn to the three-dimensional model. The induced velocity such a vortex can be calculated by Biot-Savart law:

$$\underline{w} = \frac{\Gamma}{4\pi} \int \frac{\underline{ds} \times \underline{r}}{|\underline{r}|^3} \tag{9}$$

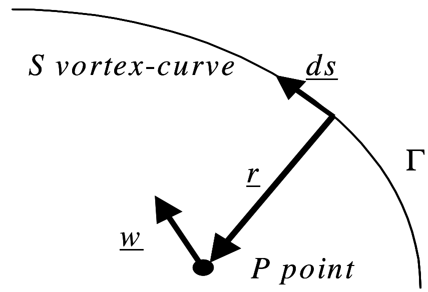

Figure 1. The induced velocity at point *P*

In the simplest case if the vortex curve is a straight line then in plane perpendicular to the vortex line the problem reduces to one dimension and the induced velocity – the so called swirl velocity – can be calculated as:

$$v = \Gamma / (2 \pi r) \tag{10}$$

In case of ideal fluid when the kinematical viscosity is zero the result of Eqn. 4 is identical to Eqn. 10. This identity means that like Eqn. 10 in case of the real fluid the induced velocity can be calculated as the product of the induced velocity in inviscid flow and function of distance *r*. (This function is introduced later, in Eqn. 13.)

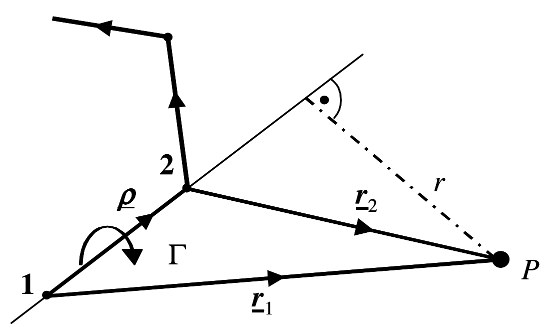

Figure 2. Geometrical relations of a vortex segment

In the practice the vortex-curve "S" can be divided into straight-line segments (Fig.6). In ideal case according to [11] the induced velocity of the vortex lying along the straight-line interval can be calculated from Eqn.11:

$$\underline{w} = \frac{\Gamma}{4\pi} \frac{\underline{r}_1 \times \underline{r}_2}{|\underline{r}_1 \times \underline{r}_2|^2} \left\{ \underline{\rho} \left[\frac{\underline{r}_1}{|\underline{r}_1|} - \frac{\underline{r}_2}{|\underline{r}_2|} \right] \right\} \tag{11}$$

The viscous core of the real vortices has a limited influence domain. In order to find this domain a constant induced velocity surface of a vortex segment (a length of "*h*") can be defined. In the Fig. 3. a local coordinate system is defined.

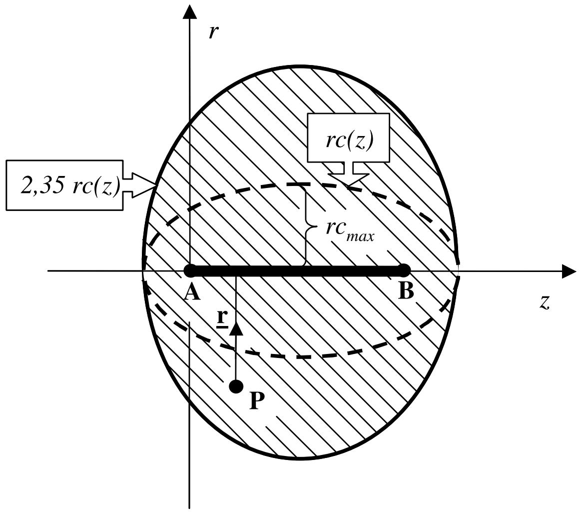

Figure 3. Surfaces with constant swirl velocity

The equation of the constant induced velocity surface can be find in this *(r – z)* local coordinate system. The swirl velocity component is readily found by substituting the geometrical parameters into Eqn. 11, that is:

$$w_{\varphi} = \frac{\Gamma}{4\pi}\frac{1}{r}\left[\frac{z}{\sqrt{z^2+r^2}} - \frac{z-h}{\sqrt{(z-h)^2+r^2}}\right] = Const \tag{12}$$

The swirl velocity achieves its maximum value at the vortex core radius. Choosing $z=h/2$ and $r = rc = rc_{\max}$ the *"Const"* can easily calculate – so the curve of $rc = rc(z)$ as the place of the maximum swirl velocities (Fig. 3.) can be drawn. This curve is the meridian segment of the maximum swirl velocity surface. In Fig. 3. this body of revolution is illustrated by the dotted line.

4. VORTICES WITH VISCOUS CORE

The induced velocities of a vortex segment with viscous core (Fig. 3. line **AB**) can be calculated by the combination of the equations 8 and 11:

$$\underline{w} = \frac{\Gamma}{4\pi}\frac{\underline{r}_1\times\underline{r}_2}{\left|\underline{r}_1\times\underline{r}_2\right|^2}\left\{\underline{\rho}\left[\frac{\underline{r}_1}{\left|\underline{r}_1\right|} - \frac{\underline{r}_2}{\left|\underline{r}_2\right|}\right]\right\}\varphi\left(\frac{\left|\underline{r}\right|}{rc(z)}\right) \tag{12}$$

Where:

$$\varphi\left(\frac{\left|\underline{r}\right|}{rc(z)}\right) = 1 - \exp\left(-\alpha\delta\frac{\left|\underline{r}\right|^2}{rc^2}\right) \tag{13}$$

This core function φ was applied originally in the local coordinate system. In this system the induced velocity vector has only tangential component and the other components are zero. This means, that the whole induced velocity vector must be reduced. Applying function φ in Eqn. 12 means that the components of the induced velocity are reduced. In this way the whole induced velocity is suitably reduced too.

The effect of viscous core suddenly reduces to zero. We can also choose an outer surface, where the distance of point **P** is equal to *2.35 rc(z)*. When the point **P** is chosen outside of this influence region then the influence of the core effect is smaller then 0.1 % - in this case we can use Eqn. 11. Further if the point **P** is inside of this region for the calculation we must use the Eqn. 12.

In order to show the effect of the viscous core a numerical example is presented too. In this example a vortex segment of unit length moves closely to a helicopter rotor blade. The rotor blade belongs to a MD 500 type helicopter and the other data of the example refers to the real dataset of this type of helicopter rotor blade. In the Fig. 4 the lift coefficient changing is shown.

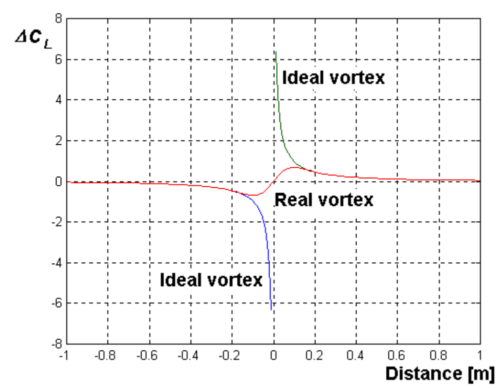

Figure 4. Lift coefficient changing

In the case shown in Fig. 4. the vortex angle *(β)* is zero and the vortex core radius is approximately equal to 0.1 m. The lift coefficient changing outside of the core influence region is the same in the ideal and real case too. In the core influence region the using of core function φ gives physically good results. The values of the lift coefficient changing in the ideal case are physically unrealistic. This means, that we have to apply a vortex model of viscous core.

If the vortex segment has a non-zero β angle, namely the segment is not parallel to the length axis of the blade, then the effect of the vortex segment decreases.

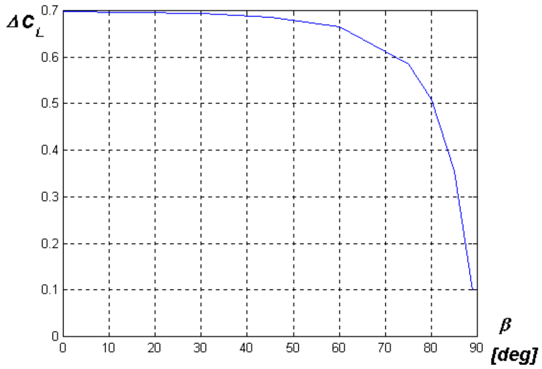

Figure 5. Effect of the vortex segment position

In the Fig. 5 can be seen, that the lift coefficient changing is maximal, if the angle between the blade and vortex segment is equal to zero. The changing go to zero, if the angle going to 90 degrees but at smaller angles the changing of the lift coefficient is small.

We can state that in the blade vortex interaction (BVI) problems not only the distance between the blade and the vortex, but the vortex position related to the blade has a significant role.

5. THE FLAPPING MOTION OF THE ROTOR BLADES

For the simplicity the lagging, feathering and elastic motions are neglected, only the flapping motion is taken into account. In the calculation we apply the classical linear form of the flapping equation.

$$\beta_l'' + (1+\varepsilon)\,\beta_l = M_y \big/ \left(\Theta_y\,\Omega^2\right) \qquad (14)$$

where: $\varepsilon = (m\,x_s\,e)/\Theta_y$; the Lock number

The flapping motion is determined by the M_y aerodynamic moment but this moment depends on the flapping velocity. The Eqn. 14 is an ordinary differential equation which can be solved together with the induced velocity calculation.

The helicopter rotor blades are controlled by cyclic and collective way. The values of the control parameters for the case of example are known from other calculations of this helicopter ([6], [7]).

6. INVESTIGATION OF THE ROTOR OF A MD 500 HELICOPTER

The vortex model of the helicopter rotor plays very important role because the local interactions of the tip vortices with the blades have appearance in unsteady blade loads, vibration and noise. Interactions with the empennage and fuselage lead to the further complication in the flow environment and contributes to the vibratory air loads on the helicopter fuselage.

In the working of helicopter rotor blades the distribution of the bound vortex is special: going to the tip of the rotor blade the strength of the bound vortex strongly increases and only near to the blade tip rapidly goes to zero. Follows that the free vortices in a short distance behind the rotor blade rolled up - forms the tip vortex (see Figure 6). In this article is assumed that at the vortex age of 72 degrees the rolling up process is finished. The initial position of the vortex filament is given by three independent coordinates. These determined by the Betz's vortex laws (Equations 1, 2 and 3).

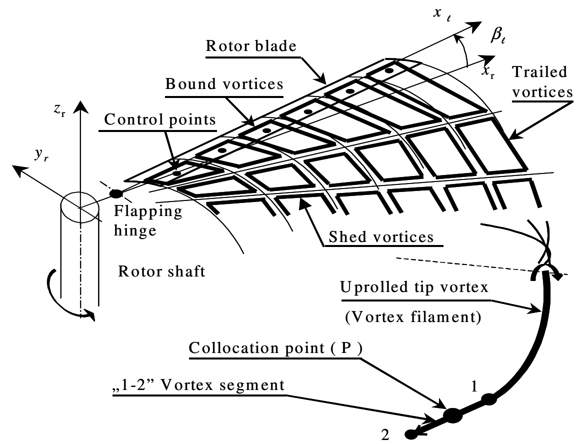

Figure 6. Vortex system of a rotor blade

In this example a prescribed wake and a free wake model is applied. The free wake and the prescribed wake methods determine the rotor induced velocity field by the combination of the induced velocity contributions of all free vortices (trailed and shed vortices) in the wake and of the bound vortices represented the lifting blades at a given point in the flow field. It is accomplished with the evaluation of the Biot-Savart integral along the length of the vortex filaments. In the calculation was used the non-rotating coordinate system connected to the rotor hub.

In our calculation the MD 500 helicopter flies horizontally with advance velocity of 144 [km/h]. During the calculation evolved the complex vortex system around the rotor and the rotor blade reaches an asymptotic dynamic equilibrium state. From the calculation the path of the blades and of the tip vortex are known. We can find the places, where the blade vortex interaction becomes.

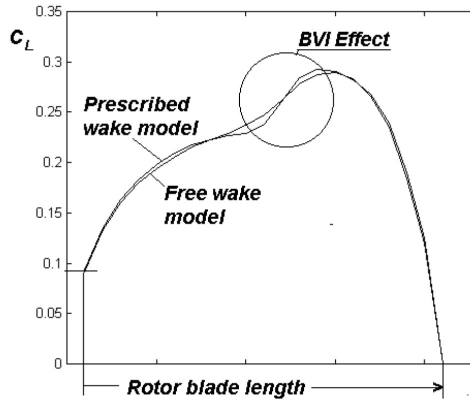

Figure 7. Typical blade vortex interaction

Figure 7. illustrates the lift coefficient distribution along a rotor blade. In this figure the effect of blade vortex interaction is shown. In the calculated flying case the distance between the blade and the tip vortex is not to small, so the BVI effect is also restrained.

7. SUMMARY

The “real” vortex theory is an advanced method for the aerodynamic calculation of the helicopter rotor blades. By this method the unsteady effects, the BVI effects and the vortex interactions of other helicopter parts can be calculated. In the calculation with the induced velocities the distance measured from the vortex line has a very important role. A small inaccuracy of the vortex position can lead to a great error in the induced velocity value.

Because the required accuracy the time demand of the calculation is quite big, so this method for real time simulations for the present is not suitable.

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