

# List of abstracts submitted to the 7th DEDS conference<sup>†</sup>

15-18 December, 2010  
University of South Florida, Tampa, Florida, USA

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# 1 Plenary speeches

## Nonlinear waves: from beaches to lasers

**Mark J. Ablowitz\***, University of Colorado at Boulder, USA [mark.ablowitz@colorado.edu]

**Abstract:** The study of localized waves has a long history dating back to the discoveries in the 1800s by Russell, Boussinesq and Korteweg-de Vries (KdV) describing water waves in shallow water. In the 1960s research on the KdV equation led to the concept of solitons which are solitary waves which interact “elastically”. More recently both in fluid dynamics and nonlinear optics there has been considerable interest in various aspects of localized waves. Some of the topics that will be discussed include: water waves, ultra-short pulses in lasers and nonlinear waves in optical lattices. The talk will be in a colloquium style—aimed at a broad audience.

## A brief survey of recent developments on control and filter theory on infinite dimensional Banach spaces

**Nasir Uddin Ahmed\***, University of Ottawa, Canada [ahmed@site.uottawa.ca]

**Abstract:** In this talk we present a brief account of authors recent work in the area of dynamics, control and filter theory on infinite dimensional Banach spaces. First we present a basic introduction to differential equations and inclusions on Banach spaces with regular vector fields admitting mild solutions and discuss classical control problems. Relaxed controls are mentioned briefly. Next we consider impulsive systems driven by vector measures as controls. We consider also stochastic differential equations and inclusions and their control invoking HJB equations on a Hilbert space like  $L_2(\mu, H)$  where  $\mu$  is an invariant measure on  $H$  induced by Ornstein-Uhlenbeck process. Next we consider differential equations with discontinuous and possibly unbounded vector fields and present measure valued solutions and their control. We present an application to stochastic Navier-Stokes equations considering measure solutions (as turbulent solutions) and present existence of optimal controls minimizing a certain measure of turbulence. Finally we present some results on linear and nonlinear filter theory after a brief introduction. We consider Kalman type linear filtering of stochastic systems where both the state and observation dynamics are perturbed by signed measures. Relatively optimal filters are derived which reduce to Kalman filter in the absence of perturbing measures. For nonlinear filters, we consider the observer or measurement dynamics as a design or control variable and prove existence of optimal observers from a suitable class of operators.

## Theory and application of model equations for water waves

Jerry Bona\*, University of Illinois at Chicago, USA [bona@math.uic.edu]

**Abstract:** The lecture will begin with a little history to set the stage for the derivation of classes of model equations for surface water waves. A sketch of the theory pertaining to these models will be followed by examples of their application in problems arising in oceanography.

## Fokker-Planck equations for a free energy functional or Markov process on a graph

Shui-Nee Chow\*, Georgia Institute of Technology, USA [chow@math.gatech.edu]

**Abstract:** The classical Fokker-Planck equation is a linear parabolic equation which describes the time evolution of probability distribution of a stochastic process defined on a Euclidean space. Corresponding to the stochastic process, there often exists a free energy functional which is defined on the space of probability distributions and is a linear combination of a potential and an entropy. In recent years, it has been shown that Fokker-Planck equation is the gradient flow of the free energy functional defined on the Riemannian manifold of probability distributions whose inner product is generated by a 2-Wasserstein distance. In this talk, we consider similar matters for a free energy functional or Markov process defined on a graph with a finite number of vertices and edges. If  $N \geq 2$  is the number of vertices of the graph, we show that the corresponding Fokker-Planck equation is a system of  $N$  nonlinear ordinary differential equations defined on a Riemannian manifold of probability distributions. However, in contrast to the case of stochastic processes defined on Euclidean spaces, situation is more subtle for discrete spaces. We have different choices for inner products on the space of probability distributions resulting in different Fokker-Planck equations for the same process. It is shown that there is a strong connection but also substantial differences between the systems of ordinary differential equations and the classical Fokker-Planck equation on Euclidean spaces. Furthermore, each of these systems of ordinary differential equations is a gradient flow for the free energy functional defined on a Riemannian manifold whose metric is closely related to certain Wasserstein metrics. Some examples will also be discussed.

## Inverse problem of KP solitons and applications to solitary wave interactions in shallow water

Yuji Kodama\*, Ohio State University, USA [kodama@math.ohio-state.edu]

**Abstract:** I start to explain briefly a classification theorem for exact soliton solutions of the KP equation. The classification theorem is closely related to the theory of the totally non-negative Grassmann manifold,  $\text{Gr}_+(N; M)$ . Each of the exact solutions is then identified as an  $N \times M$  matrix  $A$ , and is parametrized by a unique chord diagram representing the derangement of the permutation group  $S_M$ . The corresponding chord diagram also describes the asymptotic structure of the solution which consists of arbitrary numbers of line-solitons in both  $y \rightarrow \infty$ . The chord diagram determines a structure of the  $N \times M$  matrix  $A$ , and the entries of the matrix determine the pattern of the solution in the  $xy$ -plane. The inverse problem is to reconstruct the matrix  $A$  from the wave pattern given by the interaction of the KP line-solitons. We propose a method to solve the inverse problem, in which the (scattering) data consists of the locations of the line-solitons in the wave pattern. We apply the method to real experimental data of Mach refection phenomena in shallow water waves.

## The Grunsky operator, Ahlfors' question and geometry of the universal Teichmuller space

Samuel Krushkal\*, Bar-Ilan University, Israel; University of Virginia, USA  
[slk6z@virginia.edu, krushkal@math.biu.ac.il]

**Abstract:** We prove the Grinshpan conjecture on the norm of the Grunsky operator generated by univalent functions in the disk. This conjecture is deeply connected with the theory of renormalizations and measurable foliations of dynamical systems, which forces some essential restrictions. The result implies fundamental consequences for the Ahlfors problem concerning the quasiconformal (subject to the generalized Beltrami differential equation) extensions of holomorphic maps and for geometric, plurisubharmonic and pluripotential features of the universal Teichmuller space.

## The symmetry studies of partial differential equations and its applications in atmospheric and oceanic dynamics

Sen-Yue Lou\*, Ningbo University, P.R. China [lousenyue@nbu.edu.cn]

**Abstract:** In this report, we simply review the basic methods on the symmetry studies of the nonlinear partial differential equations (PDEs), especially on the direct methods: the direct method on the symmetry reductions, the direct methods on the finite symmetry groups, the direct classification methods on the PDEs with same Lie point symmetry algebra and the direct method on the conservation laws with the same symmetries.

## 2 Invited and contributed talks

### Mathematical analysis of the transmission dynamics of bovine tuberculosis model

**F. B. Augusto\***, **S. Lenhart**, University of Tennessee, USA [fbagusto@gmail.com]

**A. B. Gumel**, University of Manitoba, Canada

**A. Odoi**, University of Tennessee, USA

**Abstract:** A deterministic model for studying the transmission dynamics of bovine tuberculosis in a single cattle herd is presented and qualitatively analyzed. A notable feature of the model is that it allows for the importation of asymptotically-infected cattle (into the herd) due to re-stocking from outside sources. Rigorous analysis of the model shows that the model has a globally-asymptotically stable disease-free equilibrium whenever a certain epidemiological threshold, known as the reproduction number, is less than unity. In the absence of importation of asymptotically-infected cattle, the model has a unique endemic equilibrium whenever the reproduction number exceeds unity (this equilibrium is globally-asymptotically stable for a special case). It is further shown that, for the case where asymptotically-infected cattle are imported into the herd, the model has a unique endemic equilibrium. This equilibrium is also shown to be globally-asymptotically stable for a special case.

### A nonlocal equation arising from a travelling wave problem

**José M. Arrieta\***, Universidad Complutense de Madrid, Spain [arrieta@mat.ucm.es]

**Abstract:** Under an appropriate change of variables we transform a 1-d parabolic problem into an evolution problem containing a non local term. This change of variables transforms a travelling wave (with unknown speed of propagation) into a standing wave (zero speed travelling wave). We analyze the relation between the two problems in the whole real line. We also study the behavior of this non local problem in a bounded interval and show that it has a unique equilibria which is asymptotically stable. (Join work with Maria Lopez-Fernandez and Enrique Zuazua)

### Mathematical methods for modeling of lightning and thunderstorm electrification

**Beyza Aslan\***, University of North Florida, USA [beyza.aslan@unf.edu]

**Abstract:** In climate change research and other areas concerning weather, lightning and the gases and energy it produces is a big interest. To be able to have better estimates related to anything lightning produces, one needs to understand lightning better. In our work, we try to accomplish two goals: Modeling the electric potential and computing the charge density deposited by a flash. These two parameters are the two most important parameters in computing the lightning flash energy, which in turn provides other helpful information about lightning as well. In this talk, I will briefly discuss the mathematical methods we use to achieve these goals, and present some applications.

## Rational decay rates for fluid-structure interaction PDE models

George Avalos\*, University of Nebraska-Lincoln, USA [gavalos@math.unl.edu]

**Abstract:** In this talk we shall derive certain delicate decay rates for a partial differential equation (PDE) system which comprises (parabolic) Stokes fluid flow and a (hyperbolic) elastic structural equation. The appearance of such coupled PDE models in the literature is well-established, inasmuch as they mathematically govern many physical phenomena; e.g., the immersion of an elastic structure within a fluid. The coupling between the distinct hyperbolic and parabolic dynamics occurs at the boundary interface between the media. In previous work, we have established semi- group wellposedness for such dynamics, in part through a nonstandard elimination of the associated pressure variable. For this PDE model, we provide a uniform rational decay estimate for solutions corresponding to smooth initial data; viz., for initial data in the domain of the semigroup generator. The attainment of this result depends upon the appropriate use of a recently derived operator semigroup result of A. Borichev and Y. Tomilov.

## Lipschitz control of geodesics in the Heisenberg group

Robert Berry\*, Sandia National Laboratories, USA [rdberry@sandia.gov]

**Abstract:** Monge first posed his ( $L^1$ ) optimal mass transfer problem: to find a mapping of one distribution into another, minimizing total distance of transporting mass, in 1781. It remained unsolved in  $R^n$  until the late 1990's. This result has since been extended to Riemannian manifolds. In both cases, optimal mass transfer relies upon a key lemma providing a Lipschitz control on the directions of geodesics. We will discuss the Lipschitz control of geodesics in the (subRiemannian) Heisenberg group. This provides an important step towards a potential theoretic proof of Monge's problem in the Heisenberg group.

## A mathematical model for understanding local ant community responses to climatic change

Sharon Bewick\*, NIMBioS, University of Tennessee, USA [sharon\_bewick@hotmail.com]

**Abstract:** When predicting the sensitivity of ant assemblages to climatic change, it may be important to consider trade-offs that both currently allow coexistence between ant species in a community and also are likely to change as a result of global warming. In particular, differences in thermal tolerance will likely play a key role in determining ant community composition under climatic warming, and a dominance thermal-tolerance relationship has been proposed in several systems. In order to mathematically interpret and predict shifts in ant species abundance that occur as a result of climatic warming, we take the basic assumption of linear transitive dominance hierarchies from a mathematical dominance-discovery model proposed by Adler et. al. (2007), and then extend the model by including terms to describe species specific seasonal foraging patterns, which we use as a proxy for species specific thermal tolerances. We apply our 'dominance-thermal tolerance model' to a system of three sympatric ant species (*Paratrechina tereticauda*, *Aphaenogaster rudis* and *Prenolepis imparis*) in an eastern hardwood forest. Our model predicts coexistence assuming parameter estimates made from data collected under current climatic conditions. We then consider potential changes in these parameters that might occur as a result of climatic warming. In particular, we focus on altered ant behavior, food availability and competitor abundance, and use our model to predict the effects that these changes will have on ant community composition.

# Existence of square mean almost periodic mild solutions to non-autonomous stochastic differential equations

Paul Bezandry\*, Howard University, USA [pbezandry@gmail.com]

**Abstract:**In this talk, we discuss the existence of square-mean almost periodic solutions to some class of abstract non-autonomous stochastic differential equations in a real separable Hilbert space by the means of the well-known Schauder fixed point principle.

## Exact solutions for a generalized Benjamin-Bona-Mahony equation

Maria Santos Bruzón\*, Universidad de Cádiz, Spain [matematicas.casem@uca.es]

M. L. Gandarias, Universidad de Cádiz, Spain [marialuz.gandarias@uca.es]

**Abstract:** The methods of point transformations are a powerful tool in order to find exact solutions for nonlinear partial differential equations. The classical theory of Lie point symmetries for differential equations describes the groups of infinitesimal transformations in a space of dependent and independent variables that leave the manifold associated with the equation unchanged. In the last few years a great progress has been being made in the development of methods and their applications to nonlinear ordinary differential equations for finding exact solutions [1, 3, 5].

In this paper we consider the Generalized Benjamin-Bona-Mahony equation,

$$u_t + bu_x + a(u^m)_x + (u^n)_{xxt} + c(u^k)_{xx} = 0,$$

where  $a, b, c$  are arbitrary constants. When  $c = 0$ , the equation reduces to Benjamin-Bona-Mahony equation, and in [1] the authors analyzed its classical and nonclassical symmetries and in [2], obtained the corresponding exact traveling wave solutions by using the  $(\frac{G'}{G})$ -expansion method. We apply the Lie-group formalism [4] to deduce symmetries of the above equation depending on the values of the parameters  $a, b, c, m, n, k$ . Some exact solutions can be derived by applying a direct method. These solutions yield new solutions to the Benjamin-Bona-Mahony equation.

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## Derivation of stochastic correlated random walk models

Ummugul Bulut\*, Texas Tech University, USA [gul.bulut@ttu.edu]

**Abstract:** Derivation of stochastic correlated random walk models. Two well-known random walk models are studied. Specifically, the telegraph equation in one-dimension and the linear transport equation in two dimensions. These equations are useful, for instance, in animal movement. In present investigation, stochastic telegraph and linear transport are derived from basic principles. In particular, dynamical systems, with time discrete, are studied to determine the different independent random changes. As the time interval decreases, the discrete stochastic models lead to certain stochastic differential equation systems. Then Brownian sheets are appropriately substituted for Wiener processes in SDE systems. When intervals in the secondary variables go to zero, the final SPDE models are derived.

## Applications of general inequalities to some engineering problems

Jonathan Burns\*, University of South Florida, USA [jtburns@mail.usf.edu]

**Abstract:** The Cauchy-Schwarz and Hölder inequalities arise naturally in both a theoretic and numerical context. This talk will explore how these and more general inequalities [1] can be applied through differential and integral equations [2] to both classic engineering problems such as mass-spring systems and to more complex systems which must be evaluated numerically. Several inequalities will be compared for their efficiency in cases where an exact solution is known.

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## A look at the bv space as an extension of the (1, 1)-Newtonian space in metric measure spaces

Chris Camfield\*, Hendrix College, Conway, USA [camfield@hendrix.edu]

**Abstract:** We will look at the space of functions of bounded variation in a metric measure space as defined by Miranda Jr., and show its relationship to the (1,1)-Newtonian space defined by Shanmugalingam. In Euclidean spaces, the BV space is an extension of the (1, 1)-Sobolev space with  $W^{1,1}$  being the subspace of functions whose variation measure is absolutely continuous with respect to the Lebesgue measure. We will show that the analogous relationship holds in metric measure spaces with doubling measures supporting a (1, 1)-Poincaré Inequality.

## Equation of state mixing rules for gas and liquid mixtures

Scott Campbell\*, University of South Florida, USA [campbell@usf.edu]

**Abstract:** An equation of state relates the pressure of a fluid (gas or liquid) to its volume, temperature and chemical composition. Through various theoretical integral and derivative relations, many



other thermodynamic properties can be calculated for fluid mixtures once the equation of state is specified. The composition dependence of the equation of state enters the equation through "mixing rules" in which parameters appearing in the equation are expressed as functions of the mole fractions of the species present in the mixture. Classical equations of state use mixing rules that have the theoretically proper composition dependence for gases at low pressure but they do not adequately represent thermodynamic properties (particularly the Helmholtz free energy) of liquid mixtures at higher pressure. Approaches have been developed that base mixing rules on semi-theoretical expressions for the Helmholtz free energy at infinite pressure but these methods either violate the conditions for the low pressure limit or use an ad-hoc procedure to satisfy one of the conditions at the expense of the others. This presentation explores the possibility that perhaps both the composition dependence and density dependence of the equation of state must be considered when trying to match the equation to theoretical or semi-theoretical expressions for composition dependent properties at low and high pressure extremes.

## Runge-kutta methods for nonlinear fractional differential equation

Xuenian Cao\*, Xiangtan University, P.R. China [cxn@xtu.edu.cn]

**Abstract:** Consider the nonlinear fractional differential equation of the form

$$\begin{cases} {}_0^C D_t^\alpha y(t) = f(t, y(t)), & 0 \leq l-1 < \alpha < l, \quad 0 \leq t \leq T, \\ y^{(i)}(0) = y_0^i, & i = 0, 1, \dots, l-1, \end{cases}$$

where  ${}_0^C D_t^\alpha y(t)$  denotes the Caputo fractional derivative of the function  $y(t)$ ,  $f : [0, T] \times R \rightarrow R$  is a given mapping satisfying the Lipschitz condition

$$|f(t, y(t)) - f(t, z(t))| \leq L|y(t) - z(t)|, \quad \forall t \in [0, T].$$

Using the relationship Caputo fractional derivative and Riemann-Liouville fractional derivative, and applying a high order approximation of Runge-Kutta methods for Riemann-Liouville fractional derivative advanced by Lubich and Ostermann, several classes high order Runge-Kutta methods for solving nonlinear fractional differential equation are constructed in this paper. Consistency, convergence and stability of these methods are obtained. Especially, for several classes commonly used Runge-Kutta methods and the existed fractional BDF methods, the compares of the computational accuracy and computational speed are given. Furthermore, in order to decrease the computational cost, implementation of Runge-Kutta methods combining the short memory principle is chosen. The Theoretical analysis and numerical experiments show that the proposed methods are high accuracy and efficient for solving nonlinear fractional difference equation.

## An integrable system from complex geometry

Alex Castro\*, University of Toronto, Canada [alex.lucio.castro@gmail.com]

**Abstract:** In this joint work with Richard Montgomery, we have explored a classical problem from complex geometry and several complex variables from the point of view of geometric mechanics. The smooth boundary of a complex domain in  $\mathbb{C}^2$  has a very interesting intrinsic geometry nicknamed "Cauchy-Riemann" (CR) geometry due to its interaction with the ambient complex structure. Surprisingly enough there's Hamiltonian system naturally associated to these structures, and the trajectories of this system when interpreted correctly give rise to the CR-analog of geodesics, containing also information about the curvature of these spaces. Our studies approached on the left-invariant deformations of the

standard CR structure on the three-sphere and the output was a neat new example of integrable system. We shall sketch the architecture of our proof, and show some pictures and diagrams. Our paper can be found at The Pacific Journal of Mathematics.

### **Line-solitons of KP II and chord diagrams**

**Sarbarish Chakravarty\***, University of Colorado, Colorado Springs, USA [chuck@math.uccs.edu]

**Abstract:** The line-soliton solutions of the KP II equation can be enumerated by a particular type of permutations called derangements. One of many ways to represent the derangements is via chord diagrams. The set of derangements is a disjoint union of subsets invariant under the action of the cyclic group. This action induces a transformation which takes a given line-soliton solution of KP II to another solution described by the same number of free parameters but with a different resonant interaction pattern.

### **Multi-scale mathematical modeling for combinatorial control of DNA transcription**

**Chichia Chiu\***, Michigan State University, USA [chiu@math.msu.edu]

**Abstract:** Unlike the earlier description of regulation of DNA transcription as a biological switch which simply turns on and off, scientists now understand that DNA transcription is a much more complicated process. It may depend on many transcriptional factors (TF). These TFs may consist of several signaling proteins which interact with each other to regulate one gene or several genes. Often, the transcription result depends on the concentration levels of these proteins. Continuous variation of the signal levels derives different transcription results. For life organisms, the continuous variation of signaling protein levels is a dynamical event which can be described as a regulatory network. Other than the interaction of signaling proteins, there is another important factor for DNA transcription that is the DNA regulatory element (RE) constructs. These constructs are differed by: the stoichiometry of the regulatory binding sites, the arrangement (position) and spacing of the sites, the covering range of these sites and the affinity of the sites. These factors together serve as the blue print or code for DNA transcription. By considering both the DNA regulatory elements and the related transcriptional signals, we would be able to understand better about the DNA transcription process. For this purpose, we propose a 2-scale mathematical model for computer simulations of the DNA transcription process: 1. Mathematical potential functions for integrating all regulatory elements (REs). 2. A partial differential equation system for protein-DNA interactions of TFs and REs to transcribe the gene(s).

### **Antibiotic treatment of bacterial biofilms**

**Nick Cogan\***, Florida State University at Tallahassee, USA [cogan@math.fsu.edu]

**Abstract:** Bacterial biofilms have become widely recognized as the cause of a variety of recurrent infections. Because of the multi-layered protective mechanisms, it is very difficult to treat biofilm infections with standard courses of antibiotics. This talk will outline some recent advances in the development of treatments designed through the derivation and analysis of mathematical models. In particular, we will focus on analysis of ‘persister’ cell dynamics in a one-dimensional PDE setting where sharp bound estimates can be found. We will also consider host response effects on the treatment.

## Almost automorphic solutions for equation with piecewise constant argument

William Dimbour\*, Université des Antilles et de la Guyane, Guadeloupe (FWI) [eddimb@msn.com]

**Abstract:** By using spectral theory we obtain sufficient conditions for the almost automorphy of bounded solutions to differential equations with piecewise constant argument of the form  $x'(t) = A(t)x([t]) + f(t); x \in \mathbb{R}$ .

## Combinatorics of eigenvectors of rational matrix functions and applications

Anton Dzhamay\*, University of Northern Colorado, USA [adzham@unco.edu]

**Abstract:** In this talk I will describe some interesting relationships between the eigenvectors of a rational matrix functions. Understanding such relationships is important for describing possible factorizations of rational matrix functions and the refactorization transformations. This work is motivated by the fact that for many discrete integrable systems, the Lax-pair representation can be thought of as a refactorization transformation of its Lax matrix  $L(z)$ . I will also explain how to use this relations to find good coordinate systems on the space of rational Lax matrices and consider some applications to discrete integrable systems and their Lagrangian description.

## Global and blow up solution in a mutualistic model

Peng Feng\*, Florida Gulf Coast University, USA [pfeng@fgcu.edu]

**Abstract:** TBA

## Symmetry reductions and travelling wave solutions for a higher order wave equation of the KdV type

M. L. Gandarias\*, Universidad de Cádiz, Spain [marialuz.gandarias@uca.es]

M. S. Bruzón, Universidad de Cádiz, Spain [matematicas.casem@uca.es]

**Abstract:** The classical KdV equation models weakly nonlinear unidirectional long waves, and arises in various physical contexts. The higher order wave equations of the KdV type model strongly nonlinear long wavelength and the short amplitude waves. It is a just reason for the strongly nonlinear character and integrability of these equations attracting many researchers to study them. In this work we study a higher order wave equations of the KdV type from the point of view of the theory of symmetry reductions in partial differential equations. We obtain the classical symmetries admitted by the equation, we use the transformations groups to reduce the equations to ordinary differential equations. Some exact solutions are also provided. Among them we obtain travelling wave solutions.

In [3] a general theorem on conservation laws for arbitrary differential equation which does not require the existence of Lagrangians has been proved by N.H. Ibragimov. This new theorem is based on the concept of adjoint equations for non-linear equations. We prove that the higher order wave equation is not self-adjoint however it is quasi-self-adjoint.

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## A simple time-dependent Hamiltonian system with piecewise constant acceleration

**Mohammadreza Ghasemi\***, **Vadim Zharnitsky**, **Tamas Kalmar-Nagy**,  
Texas A&M University, USA [reza\_742@tamu.edu]

**Abstract:** We investigate the behavior of the simple time-dependent Hamiltonian system  $\ddot{x} = (-1)^{([t]+[x])}$ . It is motivated by the numerous applications of KAM theory to establish boundedness of energy in time-periodic Hamiltonian systems. This system is not sufficiently smooth so KAM theory does not directly apply and the question arises as to how regular are the large energy orbits. Of particular interest is the question whether there exist trajectories reaching arbitrary large velocities. We reformulate the problem as a discrete 2-D map and analyze the critical curves, invariant sets and grazing boundaries to obtain insights into the behavior of the system.

## Boundary layers for the linearized Navier-Stokes equations in a curved domain

**Gung-Min Gie\***, University of California, Riverside, USA [gungmin@ucr.edu]

**Abstract:** We study, in a curved bounded domain in  $\mathbb{R}^3$  with a characteristic boundary, the asymptotic behavior of the linearized Navier-Stokes equations (LNSE) when the viscosity is small. Using the curvilinear system, we show that the solutions of the LNSE behave like the corresponding Euler solutions except in a thin region, near the boundary, where a certain heat solution is added as a corrector. This is a joint work with Makram Hamouda and Roger Temam.

## Convolutions and integro-differential inequalities

**Arcadii Z. Grinshpan\***, University of South Florida, USA [agrinshp@usf.edu]

**Abstract:** The author's recent work on general inequalities with the negative binomial weights [1] leads to the following weighted inequality for the convolution of complex-valued functions:

$$\begin{aligned} & \left[ \int_a^b (x-a)^{(\alpha+\beta-1)(1-\tau)} (b-x)^{\lambda-1} \left| \int_a^x f(t)g(x-t)dt \right|^\tau dx \right]^{1/\tau} \\ & \leq K \left[ \int_a^b (x-a)^{(\alpha-1)(1-p)} (b-x)^{\beta+\lambda-1} |f(x)|^p dx \right]^{1/p} \\ & \quad \times \left[ \int_0^{(b-a)} x^{(\beta-1)(1-q)} (b-a-x)^{\alpha+\lambda-1} |g(x)|^q dx \right]^{1/q}, \end{aligned}$$

where  $\alpha, \beta, \lambda > 0$ ;  $p > 1$  ( $1/p + 1/q = 1$ ),  $\tau = \min(p, q)$ ; and  $K = K(\alpha, \beta, \lambda, p, b-a)$  is known explicitly.

Among other things, this inequality implies sharp integral estimates for solutions of various convolution integral equations which, in turn, result in new integro-differential inequalities.

[1] A. Z. Grinshpan, Weighted inequalities and negative binomials, *Adv. in Appl. Math.* **45** (2010) pp. 564-606 (doi:10.1016/j.aam.2010.04.004).

## An application of global attractors in spatial ecology: how to predict the success of intervention against invasive species

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**Abstract:** The problem of invasive species is thought to be second only to habitat destruction as a threat to biodiversity. Eradication strategies applied over spatial domains can take in many cases up to decades before achieving local extinction of the targeted invasive species. These practical efforts demand correct estimation of the outcome of the strategy before committing substantial economic and political resources over long periods of time. This talk discusses how the existence of global attractors in an infinite dimensional dynamical system, representing genetic control of an invasive species, can be used in spatial ecology to determine a state of local extinction. It is shown that in some cases it is possible to determine for a finite time the existence of a state of local extinction, and the conditions under which this happens. It is also shown that some eradication strategies do not produce local extinction. In general, a method to determine extinction in population dynamics is presented.

## Boundary layers for the primitive equations

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**Abstract:** We present in this lecture some convergence results related to the Linearized Primitive Equations (LPEs) as the viscosities go to zero. The (full nonlinear) Primitive Equations read:

$$\begin{cases} \frac{\partial \tilde{v}}{\partial t} + (\tilde{v} \cdot \nabla) \tilde{v} + \tilde{w} \frac{\partial \tilde{v}}{\partial z} + f k \times \tilde{v} + \frac{1}{\rho_0} \nabla \tilde{p} - \mu_{\tilde{v}} \Delta \tilde{v} - \nu_{\tilde{v}} \frac{\partial^2 \tilde{v}}{\partial z^2} = F_{\tilde{v}}, \\ \frac{\partial \tilde{p}}{\partial z} = -\tilde{\rho} g, \\ \nabla \tilde{v} + \frac{\partial \tilde{w}}{\partial z} = 0, \\ \frac{\partial \tilde{T}}{\partial t} + (\tilde{V} \cdot \nabla) \tilde{T} + \tilde{w} \frac{\partial \tilde{T}}{\partial z} - \mu_{\tilde{T}} \Delta \tilde{T} - \nu_{\tilde{T}} \frac{\partial^2 \tilde{T}}{\partial z^2} = Q_{\tilde{T}}, \\ \tilde{\rho} = \rho_0 (1 - \alpha (\tilde{T} - T_0)). \end{cases}$$

One of our aims, among others, is to give the limit solution associated with the Linearized system of (PEs) that we obtain by dropping the nonlinear terms. However, a difficulty for the limit LPEs system (that is we set the viscosities to be equal to zero) lies in the fact that no set of local boundary conditions ensures its well-posedness. Several choices of nonlocal boundary conditions are possible. Hence, in view of the uniqueness, our aim is to give an asymptotic expansion of the solution of the LPEs at small viscosities confirming thus our choice for the boundary conditions of the limit solution.

## Rogue wave solution for the variable coefficient Nonlinear Schrödinger type equation

Jingsong He\*, Ningbo University, Zhejiang, P.R. China [hejingsong@nbu.edu.cn]

**Abstract:** In this talk, the variable coefficient nonlinear Schrödinger equation (VCNLSE) and derivative nonlinear Schrödinger equation (VCDNLSE) are discussed. By using a transformation which maps a VCNLSE (or VCDNLSE) to the well known usual NLSE (or DNLSE) equation, the rogue wave and soliton solutions of the former are given from known solutions of the latter. Several figures for these solutions are plotted to understand intuitively its dynamical evolution. This is a joint work with Prof. Yishen Li and my students Shuwei Xu and Youying Wan.

## A periodic system dependent on parameters

Min He\*, Kent State University at Trumbull, OH, USA [mhe@kent.edu]

**Abstract:** This work is concerned with a periodic system that is dependent on parameters. We study differentiability with respect to parameters of the periodic solution of the system. Applying a fixed point theorem and the results regarding parameters for  $C_0$ -semigroup, we obtained some convenient conditions for determining differentiability with respect to parameters of the periodic solution. An application of the obtained results to a periodic boundary value problem is also discussed.

## Problem on minimum wave speed of traveling waves for a Lotka-Volterra competition model

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**Abstract:** Consider a reaction-diffusion system that serves as a 2-species Lotka-Volterra competition model with each species having logistic growth in the absence of the other. Suppose that the corresponding reaction system has one unstable boundary equilibrium  $E_1$  and one stable boundary equilibrium  $E_2$ . Then it is well known that there exists a positive number  $C_*$ , called the minimum wave speed, such that, for each  $c$  larger than or equal to  $C_*$ , the reaction-diffusion system has a positive traveling wave solution of wave speed  $c$  connecting  $E_1$  and  $E_2$ , and the system has no nonnegative traveling wave with wave speed less than  $C_*$ . It has been shown that the minimum wave speed for this system is identical to another important quantity - the speed of the population spread towards the stable equilibrium. Hence to find the minimum wave speed  $C_*$  not only is of the interest in mathematics but is of the importance in application. Although much research work has been done to give an estimate of  $C_*$  and some partial results have been obtained, the problem on finding an algebraic or analytic expression for the minimum wave speed remains unsolved in general. In this talk we will give algebraic condition of linear determinacy that is weaker than those given previously. We also show that the minimum wave speed in general cannot be determined by the linearization at the unstable equilibrium point.

## Almost linear Volterra integral equations and the existence of bounded solutions

Muhammad Islam\*, University of Dayton, Dayton, OH, USA [muhammad.islam@notes.udayton.edu]

**Abstract:** A set of conditions are applied on the functions involved in certain Volterra type equations. The equations will be called almost linear when these conditions hold. Then the existence of bounded continuous solutions of these equations is studied. Krasnosel'skii's fixed point theorem is used in the analysis as the primary mathematical tool.

## Analytical solution of the nonlinear fractional differential equations using variational iteration method

Hossein Jafari\*, , University of Mazandaran, Iran [jafari@umz.ac.ir]

**Abstract:** Fractional calculus has been used to model physical and engineering processes that are found to be best described by fractional differential equations. For that reason we need a reliable and efficient technique for the solution of fractional differential equations. In this article, we implement the variational iteration method for solving fractional differential equations such as the Bagley Torvik equation which arises in the modelling of the motion of a rigid plate immersed in a Newtonian fluid. The fractional derivatives are described in the Caputo sense.

## Higher order Miles theory of shallow water waves

Yuhan Jia\*, The Ohio State University, USA [yuhanjia@math.osu.edu]

**Abstract:** We consider the Miles theory of solitary wave interactions in shallow water in terms of the normal form theory. We first confirm that the Miles theory is equivalent to the KP theory, and then derive the normal form including the next order corrections to the KP equation. We then show that the normal form provides an excellent model to describe the interaction properties. In particular, we discuss the Mach reflection problem observed in real experiments and the numerical simulations.

## A determining form for the 2-D Navier-Stokes equations

Michael Jolly\*, Indiana University, USA [msjolly@indiana.edu]

**Abstract:** The determining modes for the incompressible Navier-Stokes equations (NSE) are shown to satisfy a well-posed differential equation which preserves the solutions of the NSE. This new infinite-dimensional system is shown to be dissipative; an estimate for the radius of an absorbing ball is derived in terms of the number of modes and the Grashof number. Applications to the Kuramoto-Sivashinsky and Lorenz equations are also discussed.

## Blow-up in a subdiffusive medium with advection

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Colleen M. Kirk\*, California Polytechnic State University, CA, USA [ckirk@calpoly.edu]

C. A. Roberts, College of the Holy Cross, Worcester, MA, USA

**Abstract:** Blow-up due to a localized high energy source within a subdiffusive medium with advection is considered. This problem is studied within the framework of a fractional diffusion equation. Spatial domains of infinite extent in one, two, and three dimensions are considered. It is shown that a blow-up always occurs, independent of spatial dimension, thermal properties of the material, or advection speed. Results also suggest that increasing the advection speed will delay the time to blow-up, even though it does not prevent a blow-up. These results are in distinct contrast with the analogous classical diffusion problem, in which blow-up can be avoided by sufficiently increasing the advection speed.

## Robust control of stochastic systems with noise dependent states and inputs under markovian switching

Michael Knap\*, Tennessee State University, USA [michael.j.knap@gmail.com]

**Abstract:** A problem of state feedback stabilization of discrete-time stochastic processes under Markovian switching and random diffusion (noise) is considered. The jump Markovian switching is modeled by a discrete-time Markov chain. The control input is simultaneously applied to both the rate vector and the diffusion term. Sufficient conditions based on linear matrix inequalities (LMI's) for stochastic stability is obtained. The robustness results of such stability concept against all admissible uncertainties are also investigated. An example is given to demonstrate the obtained results.

## Soliton webs and triangulations of polygons

Yuji Kodama\*, Ohio State University, USA [kodama@math.ohio-state.edu]

Lauren Williams, University of California at Berkeley, USA

**Abstract:** A soliton web is a pattern in the  $xy$ -plane generated by a soliton solution of the KP equation. Triangulations of a (convex)  $n$ -gon are related to cluster algebras of type  $A_{n-3}$ . In this talk, we will show an interesting connection between these two objects, based on consideration of the totally positive part of the Grassmannian  $Gr(2, n)$ .

## Reliable analysis for the peristaltic transport of an incompressible viscous fluid in an asymmetric channel under the effect of transverse magnetic field with slip boundary conditions

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Ahmet Yildirim, Ege University, Turkey

**Abstract:** In this study, we studied the effects of both magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel using the homotopy perturbation method (HPM). The channel asymmetry is generated by propagation of waves on the channel walls travelling with different amplitudes, phases but with the same speed. We considered the long wavelength and low Reynolds number assumptions in obtaining solution for the flow. Also we investigated the flow in a wave frame of reference moving with velocity of the wave. Closed form expressions have been obtained for the stream function and the axial velocity component in fixed frame. Numerical example is presented to illustrate the efficiency, simplicity and reliability of the method.

## The cone property for parabolic partial differential equations

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**Abstract:** We show that a class of parabolic partial differential equations satisfy a cone property. This result implies that the projection from the global attractor to a finite dimensional Fourier space is injective and thus the global attractor becomes a finite dimensional graph.

## Integrability of homogeneous differential system

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**Abstract:** TBA



# Solutions of bigraded Toda hierarchy

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**Abstract:** In this paper, we prove BTH (bigraded Toda hierarchy) has a symmetry between  $(N, M)$ -BTH (whose infinite Lax matrix has  $N$  upper and  $M$  lower nonzero diagonals) and  $(M, N)$ -BTH which explain the symmetry between their solutions later. Because  $(N, M)$ -BTH is equivalent to  $(N, M)$  band bi-infinite matrix-formed Toda hierarchy, so we consider its reduction, i.e., the semi-finite and finite matrix form of BTH. Then we give all the primary Hirota equations of BTH, meanwhile we give solutions of BTH using orthogonal polynomials in matrix form. We also give some rational solutions of BTH and corresponding Young diagrams.

## Recent advances in $B$ -theory of Runge-Kutta methods for nonlinear stiff Volterra functional differential equations

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**Abstract:** For convenience of the reader, we first list the related existing results as follows.

### 1. Properties of the true solution $y(t)$ of the problem in VFDEs:

$$y'(t) = f(t, y(t), y(\cdot)), \quad t \geq 0; \quad y(t) = \varphi(t), \quad t \leq 0$$

satisfying the condition

$$\begin{aligned} & \langle f(t, u_1, \psi(\cdot)) - f(t, u_2, \psi(\cdot)), u_1 - u_2 \rangle \leq \alpha \|u_1 - u_2\|^2, \quad \forall t \geq 0, \quad u_1, u_2 \in \mathbf{R}^m, \quad \psi \in \mathbf{C}_m(\mathbf{R}); \\ & \|f(t, u, \psi_1(\cdot)) - f(t, u, \psi_2(\cdot))\| \leq \beta \max_{t-\mu_2(t) \leq \xi \leq t-\mu_1(t)} \|\psi_1(\xi) - \psi_2(\xi)\|, \quad \forall t \geq 0, \quad u \in \mathbf{R}^m, \quad \psi_1, \psi_2 \in \mathbf{C}_m(\mathbf{R}). \end{aligned}$$

**Theorem 1**  $y(t)$  is stable on finite interval  $[0, T]$  provided  $[\alpha + \beta]_+$  is of moderate size.

**Theorem 2**  $y(t)$  is contractive on the interval  $[0, +\infty)$  provided  $\alpha + \beta \leq 0$ .

**Theorem 3**  $y(t)$  is strictly contractive and asymptotically stable on the interval  $[0, +\infty)$  provided that  $\alpha + \beta < 0$  and  $\lim_{t \rightarrow +\infty} (t - \mu_2(t)) = +\infty$ .

### 2. Properties of numerical solution $\{y_n\}$ of Runge-Kutta method for VFDEs:

$$\begin{aligned} & y^h(t) = \Pi^h(t; \psi, y_1, y_2, \dots, y_{n+1}), \quad t \leq t_{n+1}; \\ & Y^{(n+1)} = ey_n + h_n AF(Y^{(n+1)}, y^h(\cdot)), \quad y_{n+1} = y_n + h_n b^T F(Y^{(n+1)}, y^h(\cdot)); \end{aligned} \quad (*)$$

which is always assumed to satisfy a canonical condition.

**Theorem 4** If the method (\*) is algebraically stable and diagonally stable, then this method is  $B$ -stable on finite interval  $[0, T]$  provided that  $\alpha_+$ ,  $\beta$  and  $T$  are of moderate size.

**Theorem 5** Assume that the method (\*) is algebraically stable, and satisfies the  $\mathcal{W}(C_1, C_2)$  condition. Then

(1) this method is contractive on the interval  $[0, +\infty)$  provided  $\alpha + q\beta \leq 0$ , where  $q := \frac{c_\pi}{\sqrt{c_1 - c_2}}$ , the constant  $c_\pi$  depends only on the interpolation operator  $\Pi^h$ ;

(2) this method is strictly contractive and asymptotically stable on the interval  $[0, +\infty)$  provided that  $\alpha + q\beta < 0$  and  $\lim_{t \rightarrow +\infty} (t - \mu_2(t)) = +\infty$ .

### 3. Efficient Runge-Kutta methods for VFDEs:

**Theorem 6** All the  $s$  ( $s \geq 1$ ) stage Gauss, Radau IA, Radau IIA and the two stage Lobatto IIIC Runge-Kutta methods of the form (\*) for VFDEs satisfy the assumption of Theorem 4. All the  $s$  stage Radau IA, Radau IIA and Lobatto IIIC Runge-Kutta methods of the form (\*) for VFDEs satisfy the assumption of Theorem 5.

In this report, the following new results on  $B$ -theory of Runge-Kutta methods for VFDEs will be presented.

**Theorem 7** If the method (\*) is algebraically stable, diagonally stable and satisfies the  $\mathcal{W}(C_1, C_2)$  condition, then this method is  $B$ -stable on finite interval  $[0, T]$  provided that  $[\alpha + q\beta]_+$  and  $T$  are of moderate size.

**Remark 1** The original Theorem 4 does not admit  $\beta$  taking large value, i.e., there does not admit stiffness of the functional part of the problem, whereas Theorem 7 only requires  $[\alpha + q\beta]_+$  being of moderate size, which is more perfect numerical analogue of the result of Theorem 1.

**Theorem 8** If the method (\*) is algebraically stable,  $B$ -consistent of order  $p$ , and satisfies the  $\mathcal{W}(C_1, C_2)$  condition, then this method is optimally  $B$ -convergent of order  $p$  on the infinite integration interval  $[0, +\infty)$  provided that  $\alpha + q\beta < 0$  and that  $h_{\max}\beta$ ,  $1/|\alpha + q\beta|$  and  $h_{\max}/h_{\min}$  are of moderate size.

## On stepanov-like (pseudo) almost automorphic functions

Jin Liang\*, Shanghai Jiao Tong University, P.R. China [jinliang@sjtu.edu.cn]

**Abstract:** This talk will present new results, especially new composition theorems on Stepanov-like almost automorphic and Stepanov-like pseudo almost automorphic functions. Applications to evolution equations with Stepanov-like pseudo almost automorphic perturbations are also presented.

## Bosonizations, Fermionizations and dark parameterizations

Sen-Yue Lou\*, Ningbo University, P.R. China [lousenyue@nbu.edu.cn]

**Abstract:** Bosonization approach to the fermion systems such as the super integrable systems and supersymmetric systems is presented. By introducing the multi-fermionic parameters in the expansions of both the boson and fermion fields, we can solve fermion systems via boson systems. The method is illustrated by taking the  $\mathcal{N} = 1$  supersymmetric KdV (sKdV) equations as a simple example. By solving the coupled bosonic equations, various novel types of exact solutions can be explicitly obtained. Actually, any kind of exact solutions of the usual KdV equations can be extended to those of the sKdV in fruitful forms. The inverse version of the bosonization procedure (we call it fermionization) provides a new method to extended the usual integrable systems to supersymmetric integrable ones. The appearance of the “dark Grassmann” parameters hints us to find much more integrable systems by introducing a “dark parameterization approach”.

## On chaos synchronization of two nonlinear gyroscope systems with and without a periodical disturbance

Fuhong Min, Albert C. J. Luo\*, Southern Illinois University, Edwardsville, USA [aluo@siue.edu]

**Abstract:** In this paper, the synchronization of two gyroscope systems with and without a periodical disturbance is studied using a simple control law. Analytical conditions for two gyro systems synchronization are achieved through the theory of discontinuous dynamical systems, and the onset and vanishing conditions of synchronization are used to develop the parameter space for the partial, full synchronizations and none-synchronization. The switching scenarios between desynchronized and synchronized states are presented numerically. The control parameter map for synchronization is produced. Finally, partial and full synchronizations of the two chaotic gyros are illustrated. Through this paper, an alternative way is presented to investigate dynamical system synchronization.

## Stochastic models in epidemiology

S. Sathananthan, I. Lyatuu\*, Tennessee State University, USA [ilyatuu@gmail.com]

**Abstract:** Both deterministic and stochastic models of SIS and SIR type are considered where birth and death occurs at equal rates with all newborns being susceptible. In an SIS model the effect of births and deaths is to raise the threshold which determines the number of infective as time approaches infinity. The asymptotic analysis of the equilibrium state or points of both deterministic and stochastic models are investigated for SIS and SIR epidemic models.

## Non-integrability and partial non-integrability of the three body problem

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Maria Przybylska, University of Zielona Góra, Poland

**Abstract:** We consider the planar problem of three bodies which attract mutually with the force proportional to a certain negative integer power of the distance between the bodies. We show that such generalization of the gravitational three body problem is not integrable in the Liouville sense. We show also that the problem is not partially integrable. That is it does not admit a single additional first integral. Our considerations are based on our criterion for partial integrability deduced from an analysis of the differential Galois group of variational equations around certain particular solutions of the problem.

## On semigroups with scaling invariance

Yasunori Maekawa\*, Kobe University, Japan [yasunori@math.kobe-u.ac.jp]

**Abstract:** In this talk we discuss the abstract theory on semigroups which possess an invariant property with respect to the action of the multiplicative group of positive real numbers (scaling). Some results on self-similar solutions and large time behavior of solutions will be also presented.

## Separable hamiltonian systems as a source of (old and new) solutions of soliton equations

Krzysztof Marciniak\*, Linköping University, Sweden [krzma@itn.liu.se]

**Abstract:** Starting from paper [1] many relations between finite- and infinite- dimensional integrable systems have been discovered. Among them a particular interest has been paid to restricted and stationary flows of soliton hierarchies which lead to Liouville integrable Hamiltonian systems of Stäckel type. In my talk I will show that also an opposite way is possible. More precisely, I will demonstrate how to produce various soliton hierarchies as well as some classes of their solutions from appropriately chosen Stäckel systems of so called Benenti type. In my talk I will focus on three types of soliton hierarchies: multicomponent Korteweg-de Vries hierarchy, multicomponent local Harry Dym hierarchy and multicomponent nonlocal Harry Dym hierarchy. The method however is general and can be applied for generating large varieties of integrable hierarchies of PDE's of evolutionary type. This talk will be an extension and overview of our results obtained in papers [2-4] below.

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## Tropical determinants and soliton solutions of (2+1)-dimensional ultradiscrete soliton equations

Kenichi Maruno\*, The University of Texas-Pan American, USA [kmaruno@utpa.edu]

**Abstract:** We discuss determinant-type (tropical determinant) solutions of  $(2 + 1)$ -dimensional ultradiscrete soliton equations. As examples, we study  $N$ -soliton solutions of ultradiscrete KP and ultradiscrete 2-dimensional Toda systems. The tropical determinant form of  $N$ -soliton solutions can be obtained systematically from totally non-negative determinant solutions of discrete KP and 2-dimensional Toda equations by taking the ultradiscrete limit.

Applying reductions, we obtain solutions of 1-dimensional ultradiscrete soliton systems. We discuss ultradiscrete (tropical) Casorati-type and Gram-type solutions of ultradiscrete soliton equations. The detail of line soliton interactions of ultradiscrete KP and ultradiscrete 2-dimensional Toda equations will be discussed.

## Existence and uniqueness of solutions for fractional stochastic differential equations driven by martingales

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**Abstract:** In this work, we discuss the existence and uniqueness of solutions for a class of fractional stochastic differential equations with non-Lipschitz coefficients. Moreover, we obtain estimates for the

solution processes. Examples are presented to illustrate our results.

## Cauchy biorthogonal polynomials and discrete integrable systems

**Hiroshi Miki\***, Kyoto University, Japan [miki@amp.i.kyoto-u.ac.jp]

**Abstract:** Cauchy biorthogonal polynomials (CBOPs) appear in the Hermite-Padé type approximation problem which is originally introduced in the study of Degasperis-Procesi equation. In this conference, we show that some discrete integrable system is obtained via the Christoffel-Darboux transformations of CBOPs. We also show that the discrete Toda molecule equation is the special case of this system. This can be naturally explained from the fact that the same equation is obtained from the Christoffel-Darboux transformations of quasi-orthogonal polynomials.

## Decay estimates for quasi-linear evolution equations

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**Abstract:** TBA

## Superintegrability in classical and quantum Hamiltonian systems

**Willard Miller Jr.\***, University of Minnesota, USA [miller@ima.umn.edu]

**Ernie G. Kalnins**, University of Waikato, New Zealand

**Jonathan M. Kress**, University of New South Wales, Australia

**George S. Pogosyan**, Yerevan State University, Armenia

**Abstract:** Superintegrable systems in classical and quantum mechanics are Hamiltonian systems that admit the maximal possible number of globally defined constants of the motion. They have remarkable properties: They can be solved explicitly, essentially without quadratures, and the representation theory of their symmetry algebras, usually not Lie algebras, gives important spectral information about the associated quantum systems. Anharmonic classical and quantum oscillators, the classical Kepler system and the quantum hydrogen atom are among the simplest superintegrable systems. The Hohmann transfer, widely used for orbital maneuvering of spacecraft, is based on superintegrability. Recently there has been explosive growth in the discovery of superintegrable systems with generating symmetries of arbitrarily high order, e.g., differential operators with orders in the trillions. We describe methods for verifying superintegrability for such systems and uncovering their structure.

## Optimal control of fractional diffusion equation

**Gisèle Mophou\***, Université des Antilles et de la Guyane, Guadeloupe (FWI) [gmophou@univ-ag.fr]

**Abstract:** In this paper we apply the classical control theory to a fractional diffusion equation in a bounded domain. The fractional time derivative is considered in Riemann-Liouville sense. We first study the existence and the uniqueness of the solution of the fractional diffusion equation in a Hilbert space. Then we show that the considered optimal control problem has a unique solution. Interpreting the Euler-Lagrange first order optimality condition with an adjoint problem defined by means of left fractional Caputo derivative, we obtain an optimality system for the optimal control.

## A tropical approach to KP line soliton combinatorics

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**Folkert Müller-Hoissen\***, Max-Planck-Institute for Dynamics and Self-Organization, Germany  
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**Abstract:** The simplest class of KP-II line soliton solutions can be qualitatively described, via a tropical approximation, as a chain of rooted binary trees. It turns out that they correspond to maximal chains in *Tamari lattices* (which are poset structures on associahedra). An essential step is the computation of the “critical events” at which a transition to a different rooted binary tree takes place, furthermore the computation of coincidences of such events, and of corresponding higher order critical events. They can be nicely expressed in terms of higher evolution variables of the KP hierarchy (which reparametrize the solutions). A general line soliton solution can be written as a certain exterior product of solutions from the simplest class, generalized to allow for certain singular solutions (since only the total solution has to be regular). Generically such a solution turns out to be well approximated by a superimposition of solutions from the simplest class, however. This yields a characterization of possible evolutions of line soliton patterns on a shallow fluid surface (provided that the KP-II approximation applies). All this is based on our paper *arXiv:1009.1886*.

## Bidifferential calculus and soliton equations

**Aristophanes Dimakis**, University of the Aegean, Greece [dimakis@aegean.gr]  
**Folkert Müller-Hoissen\***, Max-Planck-Institute for Dynamics and Self-Organization, Germany  
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**Abstract:** In this talk we briefly summarize recent results and present some new results on an approach to integrable partial differential and difference equations within the framework of bidifferential graded algebras. This may be regarded as a generalization of the structure underlying the self-dual Yang-Mills equation. It allows to study relations between various integrability aspects and solution generating methods on a universal level.

## $\lambda$ -Symmetries and solvable structures

**Concepcion Muriel\***, **J. L. Romero**, University of Cádiz, Spain [concepcion.muriel@uca.es]

**Abstract:**  $\lambda$ -Symmetries of ordinary differential equations are applied to prove the existence of solvable structures for equations that admit three-dimensional symmetry algebras. This fact implies the integrability by quadratures of equations which admit the non-solvable symmetry algebras  $sl(2, \mathbb{R})$  and  $so(3, \mathbb{R})$ . We also derive an algorithm to construct these solvable structures.

## Ultradiscretizing for some determinants

**Hidetomo Nagai\***, Waseda University, Japan [hdnagai@aoni.waseda.jp]

**Abstract:** Some determinants which can be ultradiscretized are proposed. They can be expressed by ultradiscrete permanent (UP), which is defined by ultradiscretizing permanent. Moreover, they have ultradiscrete analogue of Plücker relation and solve some soliton equations.

## Measure of noncompactness and nondensely defined semilinear fractional functional differential equations

Gaston N'Guérékata\*, Morgan State University, USA [Gaston.N'Guerekata@morgan.edu]

**Abstract:** In this talk we will investigate the existence of integral solutions for a nondensely defined semilinear functional differential equation involving the Riemann-Liouville derivative in a Banach space. The arguments are based upon Mönch's fixed point theorem and the technique of measures of noncompactness.

## Dispersive quantization

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**Abstract:** The evolution, through linear dispersion, of piecewise constant periodic initial data leads to surprising quantized structures at rational times, and fractal, non-differentiable profiles at irrational times. Similar phenomena have been observed in optics and quantum mechanics, and lead to intriguing connections with exponential sums arising in number theory. Ramifications of these observations for numerics and nonlinear dispersion are proposed as open problems.

## A reduced model of dopaminergic neuronal dynamics: nicotinic effects on burst firing

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**Abstract:** The dopaminergic (DA) neurons within the ventral tegmental area (VTA) are thought to signal reward prediction error through their tonic and phasic. However, recent studies suggest that burst firing of DA neurons may also convey reward-related information. Moreover, dopamine release during burst firing events is substantially greater than during regular spiking. In addition, DAergic neuronal bursting is influenced by the nicotinic acetylcholine receptors (nAChRs). All of this suggests that burst firing is a key component of the VTA neuronal dynamics. Here we introduce a single compartmental model of a VTA DA neuron. We demonstrate that this model captures the essential qualitative behavior of DAergic neuronal dynamics, yet is simple compared to existing, detailed, multi-compartmental models. Moreover, we extend the model to include the detailed dynamics of two prevalent nAChR subtypes ( $\alpha 4\beta 2$  and  $\alpha 7$ ) so that we can examine how nicotine alters DAergic neuronal dynamics. We hope the model will aid in the identification of the mechanisms for tonic to burst activity induced by nicotine application. The model results are compared with experimental findings on the physiological properties of VTA DAergic responses to nicotine stimulation in both control and nAChR specific knock-out/knock-in mice.

## On the fourth order of accuracy difference scheme for the Bitsadze-Samarskii type nonlocal boundary value problem

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**Abstract:** The Bitsadze Samarskii type nonlocal boundary value problem

$$\left\{ \begin{array}{l} -\frac{d^2 u(t)}{dt^2} + Au(t) = f(t), \quad 0 < t < 1, \\ u(0) = \varphi, \quad u(1) = \sum_{j=1}^J \alpha_j u(\lambda_j) + \psi, \\ \sum_{j=1}^J |\alpha_j| \leq 1, \quad 0 < \lambda_1 < \lambda_2 < \dots < \lambda_J < 1, \end{array} \right.$$

for the differential equation in a Hilbert space  $H$  with the self -adjoint positive definite operator  $A$  is considered. The well-posedness of the problem in Hölder spaces with a weight is established. The coercivity inequalities for the solution of the nonlocal boundary value problem for elliptic equations are obtained. The fourth order of accuracy difference scheme for approximate solution of the problem is presented. The well-posedness of this difference scheme in difference analogue of Hölder spaces is established. In applications, the stability, the almost coercivity and the coercivity estimates for the solutions of difference schemes for elliptic equations are obtained.

## Use of numerical differential equations in Biomorphic Robotics

Mayur Palankar\*, University Of South Florida, USA [mpalanka@mail.usf.edu]

**Abstract:** Biomorphic Robotics is an emerging sub-discipline of Robotics focused on building robots which emulate biological systems. The field is inspired by principles of biological systems and tries to imitate the mechanical system, the sensor system as well as the behavior of an animal. The idea is to use simple hardware which performs rhythmic functions controlled by input from the environment and sensors. One of the current research areas in this field is emulating the walking of an insect or an animal. Research scientists are using data generated by biologists studying animal or insect locomotion to generate cyclic functions for each joint in the system. One of the methods used is to process the data collected during walking experiments where the animals or insects run on a treadmill to collect data like force, torque etc for each joint. Numerical differential equations, which study the numerical solution of ordinary differential equations, have been used to solve the equations. This paper talks about the possible use of numerical analysis to find solutions in the field of Biomorphic Robotics by providing a few examples.

## Multiple constant sign and nodal solutions for nonlinear neumann eigenvalue problems

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**Abstract:** We consider a nonlinear Neumann eigenvalue problem driven by a possibly nonhomogeneous differential operator which incorporates as a special case the  $p$ -Laplacian. We assume that the right-hand side nonlinearity is  $(p - 1)$ -superlinear, but need not satisfy the Ambrosetti-Rabinowitz condition or to be monotone. We show that, for all values of the parameter  $\lambda$  in an upper half line, the problem has two positive and two negative solutions. Subsequently, for the case of the  $p$ -Laplacian, we also produce a nodal solution. Finally, for the semilinear case we show that the problem has two nodal solutions.



## On the long time dynamics of the Trojan Y Chromosome model

Rana Parshad\*, Clarkson University, USA [rparshad@clarkson.edu]

**Abstract:** An invasive species is one that often times invades the habitat of a native species. These species once established are difficult to control and/or eradicate. They have been known to cause immense economic and ecological damage, both globally, and in various parts of the U.S, such as Florida. Most Government and industrial efforts at controlling invasive species have not had much success. We recently proposed a novel genetic strategy, aimed at the eradication of aquatic invasive species. This is the so called Trojan Y Chromosome strategy. This strategy avoids the use of harmful piscicides and is thus eco friendly. In this work we prove the well posedness of the model. We further study the model's long time dynamics and prove the existence of finite dimensional global attractor, which is compact and invariant in the  $H^2$  topology, and attracts uniformly in the  $H^2$  metric. We also present some numerical results. This is joint work with Dr Juan B. Gutierrez at the Mathematical Biosciences Institute at Ohio State University.

## Integrable Hamiltonian hydrodynamic chains associated with Dorfman Poisson brackets

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**Abstract:** This talk is devoted to a description of integrable Hamiltonian hydrodynamic chains associated with Dorfman Poisson brackets. Three main classes of these hydrodynamic chains are selected. Generating functions of conservation laws and commuting flows are found. Hierarchies of these Hamiltonian hydrodynamic chains are extended on negative moments and negative time variables. Corresponding three dimensional quasilinear equations of the second order are presented. Multi-parametric solutions of these equations are constructed. Relationships between distinct hydrodynamic chains are discovered.

## Wave breaking in Ostrovsky-Hunter equation

Dmitry Pelinovsky\*, McMaster University, Canada [dmpeli@math.mcmaster.ca]

**Abstract:** TBA

## Application of Exp-function method to the $(2 + 1)$ -dimensional Zakharov-Kuznetsov equation

Zehra Pinar\*, Ahmet Yildirim, Ege University, Turkey [zehra.pinar@hotmail.com]

**Abstract:** In this paper, the exp-function method is used to obtain generalized solitary solutions of the  $(2 + 1)$ -dimensional Zakharov-Kuznetsov (ZK) equation. It is shown that the exp-function method, with the help of symbolic computation, provides a powerful mathematical tool for solving nonlinear evolution equations in mathematical physics.

## Recent advances in evolution equations and applications

Doriano-Boris Pougaza\*, University of Paris-Sud 11, France [pougaza@gmail.com]

**Abstract:** TBA

## Searching for integrable perturbations of integrable homogeneous potentials

**Maria Przybylska\***, University of Zielona Góra, Poland [Maria.Przybylska@astri.uni.torun.pl]

**Abstract:** Let  $V_k$ , where  $k \in \mathbb{Z}^*$ , be a homogeneous integrable potential of degree  $k$ , with  $n$  degrees of freedom. We consider its perturbations of the form

$$V = V_k + V_{k+m},$$

where  $m \in \mathbb{N}^*$  and  $V_{k+m}$  is a homogeneous potential of degree  $k + m$ . Let us assume that there exists a solution  $\mathbf{d} \neq \mathbf{0}$  of the algebraic systems

$$V'_k(\mathbf{d}) = \alpha \mathbf{d}, \quad \text{and} \quad V'_{k+m}(\mathbf{d}) = \mathbf{d},$$

for some  $\alpha \in \mathbb{C}^*$ . Then one can find necessary conditions for the integrability in the Liouville sense of natural Hamiltonian system governed by

$$H = \frac{1}{2} \sum_{i=1}^n p_i^2 + V,$$

analyzing the differential Galois group of variational equations obtained from linearization of Hamilton equations along a nonequilibrium particular solution. These conditions are expressed by means of conditions on admissible eigenvalues of Hessians  $V''_k(\mathbf{d})$  and  $V''_{k+m}(\mathbf{d})$ . The case of perturbations of radial potentials  $V_{2s} = |\mathbf{q}|^{2s}$  for  $s = -1/2$  and  $s = 1$  with  $n = 2, 3$  degrees of freedom is analyzed in details. Also some integrability results for homogeneous perturbations of general quadratic forms  $V_2 = \mathbf{q}^T S \mathbf{q}$  with symmetric two- and three-dimensional matrices  $S$  are announced.

## An ODE model for the accumulation of collagen during the wound healing process

**Angela Reynolds\***, Virginia Commonwealth University, USA [areynolds2@vcu.edu]

**Abstract:** The wound healing process involves many complex interactions, which complicate the development of more effective treatments. Therefore, we have turned to mathematics to explore the interactions between both the key components at the wound and the immune cells recruited to the wound. Understanding the interplay between these components is key to developing more effective treatment options. To gain understanding of this system, we developed an ordinary differential equation model for the accumulation of collagen in a wound during the healing process. Collagen is measured experimentally and its levels are used to validate the model. With this model we look at the ability for a wound to heal with and without infection. Wound healing in diabetic patients is prolonged and often results in chronic wounds. We also use this model to explore the effects of reduced oxygen levels at the wound site, as seen in diabetic patients, on the healing process.

## Stabilization of nonautonomous parabolic linear and nonlinear equations

Anibal Rodriguez-Bernal\*, U. Complutense de Madrid, Spain [arober@mat.ucm.es]

**Abstract:** TBA

## On the determinant solution to the Cauchy problem for the KdV equation with a general step-like initial data

Alexei Rybkin\*, University of Alaska Fairbanks, USA [arybkin@alaska.edu]

**Abstract:** We are concerned with the initial value problem for KdV equation on the whole line with initial profiles which are decaying sufficiently fast on the right half plane and arbitrary enough on the other one. By modifying the inverse scattering transform to this setting, we prove well-posedness in a very strong sense for a broad class of initial profiles with no particular pattern of behavior at minus infinity. We also show that the solution can always be presented in the determinant form which readily yields some interesting properties of the solution (e.g., real analyticity) and extends some relevant results due to Cohen, Kappeler, Khruslov, Kotlyarov, Venakides, Zhang and others.

## Stochastic switching systems: stability and control

Sivapragasam Sathananthan\*, Tennessee State University, USA [satha@coe.tsuniv.edu]

**Abstract:** A problem of robust guaranteed cost control of stochastic discrete-time systems with parametric uncertainties under Markovian switching is considered. The jump Markovian switching is modeled by a discrete-time Markov chain and the noise or stochastic environmental disturbance is modeled by a sequence of identically independently normally distributed random variables. Using linear matrix inequalities (LMI's) approach, the robust quadratic stochastic stability is obtained. The proposed control law for this quadratic stochastic stabilization result depended on the mode of the system as well as the environmental disturbances. This control law is developed such that the closed-loop system with a cost function has an upper bound under all admissible parameter uncertainties. The upper bound for the cost function is obtained as a minimization problem. A numerical example is given to demonstrate the potential of the proposed techniques and obtained results.

## Linear regression models and discrete integrable systems

Hiroto Sekido\*, Kyoto University, Japan [sekido@amp.i.kyoto-u.ac.jp]

**Abstract:** In statistical design of experiments, optimal designs are sets of observation points which give the highest accuracy estimators based on a particular optimality criterion. D-optimal designs for polynomial regression and trigonometric regression were calculated by Studden in 1980 and by Lau-Studden in 1985, respectively. In these calculation, the objective functions of optimization problems corresponding D-optimal designs were expressed in terms of canonical moments. In this talk, relationships between canonical moments and discrete integrable systems are found. Then the method for calculating D-optimal designs for polynomial regression and trigonometric regression with prior information are shown.

## Elliptic modular double of $sl_n$ and corresponding solvable

## Euclidean field theory

Sergey Sergeev\*, Australian National University, Australia [sms105@rsphysse.anu.edu.au]

**Abstract:** We present the solvable lattice Euclidean field theory (i.e. integrable lattice model of statistical mechanics with continuous configuration space) with positive Boltzmann weights parameterized in terms of elliptic gamma-functions and effective  $sl_n$  symmetry.

## Multi-scale structural mechanics for advanced aircraft design

D. Simkins\*, University of South Florida, USA [dsimkins@eng.usf.edu]

**Abstract:** Laminated composites are playing an increasingly important role in the design of high-performance vehicles ranging from aircraft to sports cars. The benefits include high-strength, low weight and resistance to corrosion. From the designers standpoint, however, laminated composites present significant challenges, including reliable computational analysis tools. The mechanical response of laminated composites is significantly more complicated than engineering metals and traditional parametrized constitutive models are inadequate for the desired coverage of the design space, in particular the problems of the onset of material failure and damage propagation. To achieve the goal of truly predictive, emergent failure, one must use physics-based models. In this context, physics-based models means only models based on fundamental physical laws and data, that one cannot use phenomenological models for the material response. Since classical mechanics governs flight, but molecular processes govern the material response, a multi-scale approach must be used. In this case, a bottom up approach is insufficient: the loading on the material comes from the macro-state, but the material response, including failure, is at the micro-scale. The failure of the material at the micro-scale opens up cracks in the material and affects the macro-state. Thus, two-way information sharing is required to solve the design-level problem of structural response of aircraft. This talk will discuss recent methodology and algorithms based on the Reproducing Kernel Particle Method and a crack morphology and propagation algorithm coupled with a micro-mechanically enhanced material model developed by Buchanan, et. al. that relates the macro-state and micro-state of a representative volume element (RVE) for laminated composites using glassy polymers. This results in multi-scale computations with two-way information sharing for structural response and fracture.

## Reduction of symmetries of hamiltonian systems

Jedrzej Sniatycki\*, University of Calgary, Canada [sniat@math.ucalgary.ca]

**Abstract:** Presence of symmetries in Hamiltonian systems simplifies the task of solving equations of motion. If the action of  $G$  on  $P$  is proper, the orbit space  $P/G$  is stratified by symplectic manifolds. The Hamiltonian equations of motion induce the reduced Hamiltonian equations on symplectic strata. A solution of the original Hamiltonian equations is reconstructed from the corresponding solution of the reduced equations. If the action of  $G$  on  $P$  is not proper, there is no general theory of reconstruction of solutions of Hamiltonian systems. I shall discuss examples of improper actions.

## An expert system for partial differential equations

Arthur Snider\*, University of South Florida, USA [snider@usf.edu]

Sami Kadamani, Hillsborough Community College, USA

**Abstract:** The execution of the solution, by the separation of variables process, of the Poisson, diffusion, and wave equations (homogeneous or nonhomogeneous) in rectangular, cylindrical, or spherical coordinate systems, with Dirichlet, Neumann, Robin, singular, periodic, or Sommerfeld boundary conditions, can be carried out in the time, Laplace, or frequency domains by a decision-tree process, using a library of eigenfunctions. We describe an expert system, USFKAD, that has been constructed for this purpose.

## On symplectic forms in the soliton theory

Fedor Soloviev\*, University of Toronto, Canada [soloviev@math.toronto.edu]

**Abstract:** It is known that the KdV equation possesses 2 equivalent Hamiltonian formulations. One Poisson bracket is known as the Gardner-Zakharov-Faddeev bracket, and the other one is called the Magri bracket. Several Hamiltonian formulations are possible for most integrable equations with a Lax representation. All Hamiltonian structures arise from Krichever-Phong's universal formula. In this talk we will discuss some new results in the finite-dimensional case, i.e., when the Lax function is a meromorphic matrix function either on the Riemann sphere, or on an elliptic curve. In particular, we show that a multiplicative representation linearizes the so-called quadratic Poisson bracket (the Magri bracket for the KdV).

## Non-integrability of the Painlevé V equation as a hamiltonian system and stokes phenomenon

Tsvetana L. Stoyanova\*, Sofia University, Bulgaria [cveti@fmi.uni-sofia.bg]

**Abstract:** The Painlevé V equation (as well as all the six Painlevé equations) can be expressed as a Hamiltonian system dependent on parameters. It is well known that this equation possesses rational solutions for some special values of the parameters. For these particular families of the fifth Painlevé equation I will present non-integrable results from the point of view of the Hamiltonian dynamics. I explicitly compute "by hand" formal and analytic invariants of the higher variational equations which generate topologically the differential Galois group. In this way our calculations and Ziglin - Morales - Ramis - Simó method yield to the non-integrable results.

## Robust stability and stabilization of a class of non-linear discrete-time stochastic systems with state and controller dependent noise

Andre Strong\*, Tennessee State University, USA [astrong@mytsu.tnstate.edu]

**Abstract:** problem of robust state feedback stability and stabilization of nonlinear discrete-time stochastic processes is considered. The linear rate vector of a discrete-time system is perturbed by a nonlinear function that satisfies a quadratic constraint. Our objective is to show how linear constant feed- back laws can be formulated to stabilize this type of nonlinear discrete-time systems and, at the same time maximize the bounds on this nonlinear perturbing function which the system can tolerate without becoming unstable. The state dependent diffusion is modeled by a normal sequence of identically

independently distributed random variables. The new formulation provides a suitable setting for robust stabilization of nonlinear discrete-time systems where the underlying deterministic system satisfy the generalized matching conditions. Our method which is based on linear matrix inequalities (LMI's) is distinctive from the existing robust control and absolute stability techniques. Examples are given to demonstrate the results.

## Universality of fibonacci patterns

Zhiying Sun\*, University of California at Irvine, USA [zhiyings@math.uci.edu]

**Abstract:** Pattern patterns, or phyllotaxis, the arrangements of phylla (flowers, leaves, bracts, florets) in the neighbourhood of growth tips, have intrigued natural scientists for over four hundred years. Prominent amongst the observed features is the fact that phylla lie on families of alternately oriented spirals and that the numbers in these families belong to subsets of the integers defined by the Fibonacci rule  $m_{j+1} = m_j + m_{j-1}$ . The corresponding patterns, which we call Fibonacci patterns, are widespread and universal on plants. Unlike the vast majority of research focusing on discrete mechanisms, Our goal is to stem from actual physical and biochemical mechanisms experienced by the plant, which are governed by continuous PDEs, and ask if any pattern forming system which is dominated by quadratic nonlinearities and in which the pattern is laid down annulus-by-annulus, by a generative front, will lead to Fibonacci patterns. We have shown is that the Fibonacci patterns are fixed-point, free energy minimizing solutions of the order-parameter equations arising from a wide class of pattern-forming PDEs, and that these Fibonacci patterns have certain universal features which resemble what is seen in plants. What we have not shown is that the free energy minima corresponding to the Fibonacci patterns have large basins of attraction and can effectively compete with the many other local minima in the energy landscape. Direct simulations of the PDEs themselves seem to indicate that under certain circumstances Fibonacci patterns easily undergo Eckhaus-like instabilities and lead to textures which are part Fibonacci and part non-Fibonacci with many point (hepta-penta type) defects. In this case, the property that plants freeze their patterns once they are formed also seems to be very important.

## $p$ -harmonious functions with drift

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**Abstract:** Consider a connected finite graph  $E$  with set of vertices  $X$ . Choose a nonempty subset  $Y \in X$ , not equal to the whole  $X$ , and call it the boundary  $Y = \partial X$ . We are given a real valued function  $F : Y \rightarrow \mathfrak{R}$ . Our objective is to find function  $u$  on  $X$ , such that  $u = F$  on  $Y$  and  $u$  satisfies the following equation for all  $Y \in X$ .

$$u(x) = \alpha \max_{y \in S(x)} u(y) + \beta \min_{y \in S(x)} u(y) + \gamma \left( \frac{\sum_{y \in S(x)} u(y)}{\#(S(x))} \right)$$

Where  $\alpha, \beta$ , and  $\gamma$  are some predetermined non-negative constants such that  $\alpha + \beta + \gamma = 1$ , for  $x \in X$ ,  $S(x)$  is the set of vertices connected to  $x$  by an edge, and  $\#(S(x))$  denotes the cardinality of  $S(x)$ . We prove uniqueness and existence of the solution of the above Dirichlet problem using game-theoretic approach and non-linear average operator approach. We also study qualitative studies of the properties of the solutions on different types of graphs

## Max-plus analysis of digital particle systems

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**Abstract:** In this talk, 1+1D digital particle systems are discussed. In the systems, time and space variables are both integers and the state variable is also. Moreover, the sum of all state variables is conserved through time evolution and we can consider that the value of every state variable denotes the number of particles. Thus the time evolution rule of the system determines the rule of motion of particles.

One of the famous such systems is the elementary cellular automaton of rule number 184. This system expresses a simple traffic flow model and the phase transition is observed from the free-flow phase to the congestion phase at a certain density of particles. Such transition is clearly viewed by the fundamental graph, the graph of density vs average velocity.

In the talk, various digital particle systems collected by the numerical experiment are shown and their exact properties are discussed from a unified point of view; the asymptotic behavior of general solution, the fundamental diagram and the phase transition. The max-plus representation of general solution is an important key of the analysis of their properties. The representation is very effective to analyze the digital systems such as ultradiscrete soliton systems. We utilize the efficiency of the representation and propose the common mathematical structure of solutions of general digital particle systems.

## A classification of two dimensional integrable mappings and rational elliptic surfaces

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T. Takenawa\*, Tokyo University of Marine Science and Technology, Japan [takenawa@kaiyodai.ac.jp]

**Abstract:** We classify two dimensional integrable mappings by investigating the actions on the fiber space of rational elliptic surfaces. While the QRT mappings can be restricted on each fiber, there exist several classes of integrable mappings which exchange fibers. We also show an equivalent condition when a generalized Halphen surface [1] becomes a Halphen surface of index  $m$  [2]. Instead of the elliptic integral in classical theory, we use so called the period map which parameterize the generalized Halphen surfaces.

## Wronskian determinant solutions of the (3+1)- dimensional Jimbo-Miwa equation

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Wen-Xiu Ma, University of South Florida, USA

Wei Xu, Northwestern Polytechnical University, P.R. China

**Abstract:** A set of sufficient conditions consisting of systems of linear partial differential equations is obtained which guarantees that the Wronskian determinant solves the (3+1)- dimensional Jimbo-Miwa equation in the bilinear form. Upon solving the linear conditions, the resulting Wronskian formulations bring solution formulas, which can yield rational solutions, solitons, negatons, positons and interaction solutions.

## On the long-time h1-stability of the implicit Euler scheme for the 2d thermohydraulics equations

Florentina Tone\*, University of West Florida, USA [ftone@uwf.edu]

**Abstract:** In this article we consider the two-dimensional thermohydraulics equations, we discretize these equations in time using the implicit Euler scheme and with the aid of the classical and uniform discrete Gronwall lemmas, we prove that the scheme is H1-uniformly stable in time.

## Symmetry applications for debris flow models

Carmen Mineo, M. Torrissi\*, University of Catania, Italy [mineo@dmi.unict.it; torrissi@dmi.unict.it]

**Abstract:** We consider here the class of balance equations of the form:

$$\begin{aligned}u_t + h^1 + (h^2(x, u, v))_x &= 0, \\v_t + h^3 + (h^4(x, u, v))_x &= 0.\end{aligned}$$

This class has been extensively studied in some previous papers [1]-[4] in order to look for equivalence transformations and to find symmetries for special subclasses. Here bearing in mind the search for solutions we analyze, in the same framework, some quasilinear PDE  $2 \times 2$  systems concerned with mathematical models describing debris flow avalanches (see e.g. [5] and references inside).

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## Kelvin's matrix via Lie group method

Rita Tracinà\*, University of Catania, Italy [tracina@dmi.unict.it]

**Abstract:** It has been observed that the fundamental solutions of the linear partial differential equations of mathematical physics are often group-invariant solutions. This led a new approach to fundamental solutions that combines the philosophy of Lie symmetries with the theory of distributions (generalized functions). Here we extend this approach to compute the fundamental solution (Kelvin's matrix) of the system that describes the static equilibrium in classical elasticity of an isotropic and homogeneous medium under no body forces.



The explicit expressions of fundamental solutions in the theory of elasticity were obtained in different way in the literature.

For general operators with constant coefficients to compute the fundamental solutions can be quite complicated, we will see that the Lie group method provides a method largely algorithmic.

## Stability of solutions of neutral SPDEs

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**Abstract:** In this paper, we study neutral stochastic partial differential equations (SPDEs). Assuming the existence and uniqueness of a mild solution, our aim here is to investigate the almost sure exponential stability of mild solutions of SPDEs by using the Ousgood and linear growth conditions on the nonlinear terms.

## A systematic method for generating new integrable systems via inverse Miura maps

**Takayuki Tsuchida\***, Okayama Institute for Quantum Physics, Japan [tsuchida@poisson.ms.u-tokyo.ac.jp]

**Abstract:** We provide a new natural interpretation of the Lax representation for an integrable system; that is, the spectral problem is the linearized form of a Miura transformation between the original system and a modified version of it. On the basis of this interpretation, we formulate a systematic method to identify modified integrable systems that can be mapped to a given integrable system by Miura transformations. Thus, this method can be used to generate new integrable systems from known systems by inverse Miura maps; it can be applied to both continuous and discrete systems in  $1 + 1$  dimensions as well as in  $2 + 1$  dimensions. The effectiveness of the method is illustrated using various examples.

## UC Hierarchy and monodromy preserving deformation

**Teruhisa Tsuda\***, Kyushu University, Japan [tudateru@math.kyushu-u.ac.jp]

**Abstract:** The UC hierarchy is an extension of the KP hierarchy, which possesses not only an infinite set of positive time evolutions but also that of negative ones. Through a similarity reduction, the UC hierarchy yields a broad class of Schlesinger systems including (higher order) Painleve VI and Garnier systems, which describe monodromy preserving deformations of Fuchsian linear differential equations with certain spectral types. The above class of Schlesinger systems has interesting features as polynomial Hamiltonian structure, algebraic solutions, hypergeometric solutions, Weyl group symmetry, etc. Also we present a q-difference analogue of this construction.

## Explicit flow equations and recursion operator of the ncKP hierarchy

**Junyi Tu\***, University of South Florida, USA [junyi@mail.usf.edu]  
**Jingsong He, Xiaodong Li, Lihong Wang**, Ningbo University, P.R. China

**Abstract:** The explicit expression of flow equations of the noncommutative Kadomtsev-Petviashvili (ncKP) hierarchy is derived. By comparing with the flow equations of the KP hierarchy, our result shows that the additional terms in the flow equations of the ncKP hierarchy consist of commutators of dynamical coordinates  $u_i$ , indeed. The recursion operator for the flow equations under the n-reduction is presented. Further, under 2-reduction, we calculate a nonlocal recursion operator of the noncommutative

Korteweg-de Vries hierarchy, which generates a hierarchy of local, higher order flows. Thus we solve the open problem suggested by P. J. Olver and V. V. Sokolov (Commun. Math. Phys. 193 (1998), 245-268).

## Numerical simulation of Burgers' and good Boussinesq equations using HPM-Padé technique

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**Abstract:** In this paper, we used HPM-Padé technique for approximating the solution of the Burgers' and good Boussinesq equations which include various models. Generally, the truncated series solution of homotopy perturbation method (HPM) is adequate only in a small region when the exact solution is not reached. We overcame this limitation by using the Padé techniques, which have the advantage in turning the polynomials approximation into a rational function, are applied to the series solution to improve the accuracy and enlarge the convergence domain. Using this combined technique, solutions of Burgers' and good Boussinesq equations are constructed with better accuracy and better convergence than by using the HPM alone. Results derived from our method are shown graphically.

## Approximate symmetries for nonlinear viscoelastic media

Antonino Valenti\*, University of Catania, Italy [valenti@dmi.unict.it]

**Abstract:** We consider the third order partial differential equation

$$w_{tt} = [\sigma(w_x) + \lambda(w_x)w_{tx}]_x, \quad (2.1)$$

where  $\sigma$  and  $\lambda$  are smooth functions,  $w(t, x)$  is the dependent variable and subscripts denote partial derivative with respect to the independent variables  $t$  and  $x$ .

Equation (2.1) can describe one-dimensional motion in nonlinear viscoelastic media.

Some mathematical questions related to equation (2.1), as the global existence, uniqueness and stability of solutions can be found in Refs. [1, 2]. Moreover, shear wave solutions are found in Ref. [3], where some explicit examples of blow up for boundary-values problems with smooth initial data are shown. While a symmetry analysis of equation (2.1) and some exact solutions are shown in Refs. [4, 5].

As it is well known, a small dissipation is able to prevent the breaking of the wave profile allowing to study the so called "far field" and a technique widely used is the perturbation analysis performed by expanding the dependent variables in power series of a small parameter (may be a physical parameter or often artificially introduced). Having in mind to perform an "approximate symmetry analysis" Ref. [6, 7], we introduce in (2.1) a small parameter  $\varepsilon$ , namely

$$w_{tt} = f(w_x)w_{xx} + \varepsilon[\lambda(w_x)w_{tx}]_x. \quad (2.2)$$

In the frame work of the approximate method proposed in Refs. [8, 9], we obtain the symmetry classification of the functions  $f(w_x)$  and  $\lambda(w_x)$  through which equation (2.2) is approximately invariant. Approximate solutions are calculated by using the approximate generators of the first-order approximate symmetries.

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### Non classical solution to a linear second order ODE

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**Abstract:** Using techniques of generalized functions we consider solutions of the linear second order differential equation  $-au'' + bu' + cu = f$  with different boundary conditions that cannot be handled in the classical theory. Numerical examples are given justifying our method.

### On the linearization problem for neutral equations with state-dependent delays

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**Abstract:** Neutral functional differential equations of the form  $x'(t) = g(\partial x_t, x_t)$  define continuous semiflows  $G$  on closed subsets in manifolds of  $C^2$ -functions, under hypotheses designed for the application to equations with state-dependent delay. Differentiability of the solution operators  $G(t, \cdot)$  in the usual sense is not available, but for a certain variational equation along flowlines the initial value is well-posed. Using this variational equation we prove a principle of linearized stability which covers the prototype

$$x'(t) = A(x'(t + d(x(t)))) + f(x(t + r(x(t))))$$

with nonlinear real functions  $A, d < 0, f, r \leq 0$ . Special cases of the latter describe the interaction of two kinds of behaviour, namely, following a trend versus negative feedback with respect to a stationary state.

### Two-scale network dynamic human epidemic model

**Divine Wanduku\***, University of South Florida, USA [wandukudivine@yahoo.com]

**Abstract:** The uneven global spread of emergent infectious diseases of humans is closely associated with the large-scale structure of the human population, and the human mobility process in the population structure. The mobile population becomes the vector for the disease. We present an SIRS deterministic dynamic epidemic process in a two scale structured population. We investigate the role of population scale structure and the mobility process on the emergence, propagation and resurgence of the disease. The presented results are demonstrated by numerical simulation results.

## Optimal control of generalized bolza problem for semilinear evolution inclusion

Dong Wang\*, Fayetteville State University, USA [dwang@uncfsu.edu]

**Abstract:** This paper studies a general optimal control problem of Bolza for semilinear evolution inclusions with initial conditions and endpoint constraints in reflexive and separable Banach space. We employ the method of discrete approximations and advanced tools of generalized differentiation in infinite-dimensional spaces to derive necessary optimality condition in the extended Euler-Lagrange form.

## Spreading speeds and traveling waves of a model arising from epidermal wound healing

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**Abstract:** Systems of reaction-diffusion equations have been proposed to study epidermal wound healing. The process of wound healing involves cell migration across the surface of an epidermal wound. Approximation of traveling waves which describe the cell migration into the wound has been studied by a number of researchers. We will investigate asymptotic behavior and speed of the travel wave solutions of the system of reaction-diffusion equations. Analysis of the model system reveals biological interactions in the wound healing process and the role of the model parameters in determining spreading speeds.

## A new trajectory generation and tracking control design for nonholonomic mobile robots using complex potentials

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**Abstract:** Autonomous control of mobile robots has attracted a lot of research interests due to its potential in real-world applications as seen in the missions such as search and rescue, information acquisition in hazardous environments, industry for factory floor automation, and so on. Fundamentally, the control problem of mobile robots exploring or moving within a dynamic environment boils down to make the robot move from the starting point to the final point while avoiding collisions with obstacles in the environment. Therefore, it is of paramount importance to design real-time trajectory generation and trajectory tracking control algorithms in order to make the robot move autonomously in a dynamically changing environment.

The issue of trajectory planning and control for nonholonomic mobile robots has been known as a difficult one, since the inherent kinematic constraints associated with mobile robot system dynamics make time derivatives of some configuration variables non-integrable, and thus a collision-free path in the configuration space may not be achievable for robots. An intuitive example can be found in the parallel parking experience of a car. The traditional methods for the trajectory planning of holonomic mobile robots in the structured and/or unstructured environment, such as vector field histogram, may not be applicable to the trajectory generation and control of mobile robots with nonholonomic constraints.

In this paper, we propose a new trajectory generation and tracking control design for mobile robots moving in a dynamic environment by using complex potentials. The robot under consideration is a wheeled mobile robot with nonholonomic constraints and its dynamics are governed by the following equations:

$$\dot{x} = v \cos \theta, \quad \dot{y} = v \sin \theta, \quad \dot{\theta} = \omega$$

where  $(x, y) \in \mathbb{R}^2$  is the robot's position in the 2-dimensional plane,  $\theta$  is the orientation angle,  $v \in \mathbb{R}$  and

$\omega \in \mathfrak{R}$  are the linear velocity of the wheel and its angular velocity around the vertical axis, respectively, which are taken as the control inputs to be designed. Let the trajectory of the moving target be represented by  $(x_t(t), y_t(t))$ , the control objective is to design control inputs  $v$  and  $\omega$  such that mobile robot can asymptotically track the moving target while avoiding any obstacles in the working environment. The desired trajectory will be designed using complex potentials in a 2-dimensional plane while avoiding obstacles. The complex potential of the motion is defined by

$$w(z) = \phi(x, y) + i\psi(x, y)$$

where  $w$  is a holomorphic function of the complex variable  $z = x + iy$  in any region where  $\phi$  and  $\psi$  are one-valued, and  $\phi$  and  $\psi$  are called the velocity potential and stream function, respectively. We will explicitly define the complex potential to goal, the complex potential to a wall shape obstacle, the complex potential to circular obstacle, and the complex potential potentials to elliptic obstacles. For instance, given the goal position  $(z_t = x_t + iy_t)$ , it can be shown that function  $\phi(x, y) = 1/2 \ln((x - x_t)^2 + (y - y_t)^2)$  is harmonic in  $\mathfrak{R}^2$  excepting at point  $(x_t, y_t)$ . Thus, it can be derived that the complex potential for the goal  $(x_t, y_t)$  as

$$\begin{aligned} f(z) &= \frac{\lambda}{2} \ln((x - x_t)^2 + (y - y_t)^2) + i\lambda \tan^{-1} \frac{y - y_t}{x - x_t} \\ &= \lambda \ln(z - z_t) \end{aligned}$$

Simulation and experimental results will be provided in the paper to validate the proposed design.

## Global mathematical analysis of an HIV infection model with full logistic growth and saturation incidence

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**Abstract:** In this research, we study an HIV infection mathematical model with a full logistic growth and a saturation incidence. Both local and global mathematical analysis is carried out. By identifying two numbers  $R_1$  and  $R_2$  such that  $R_1 < R_2$ ; the existence and stability of the uninfected steady state  $P_0$  and the unique infected steady state  $P^*$  are established in terms of  $R_1$  and  $R_2$ . We show that if  $R_1 < R_2 \leq 1$ ;  $P_0$  is the only equilibrium in the feasible region, it is globally asymptotically stable. Therefore, no HIV-1 infection persists and infected T cells and HIV-1 virus are cleared over time. However, if  $R_1 \leq 1 < R_2$ ;  $P_0$  is still the only equilibrium but it becomes unstable. If  $1 < R_1 < R_2$ ; then the unique infected steady state  $P^*$  emerges. We show that the system is uniformly persistent and HIV infection persists. The unique infected steady state  $P^*$  is globally asymptotically stable under some conditions.

## Monotonicity and asymptotic behavior of solutions for nonlinear differential equations of second order

Lianwen Wang\*, University of Central Missouri, USA [lwang@ucmo.edu]

**Abstract:** In this talk we deal with the monotonicity and asymptotic behavior of solutions for a class of second order nonlinear differential equations in the form  $(p(t)\Psi(x)f(x'))' = g(t, x, x')$ . All solutions in their maximal intervals of existence are classified into three disjoint classes. Conditions for the existence of each solution class are also discussed. Furthermore, necessary and sufficient conditions for boundedness of all solutions are established.

## Inequalities of functional differential equations

## by Lyapunov's second method

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**Abstract:** Lyapunov's second method with conditions like  $W_1(|\varphi|) \leq V(t, \varphi)$ , and  $V'(t, \varphi) \leq \eta(t)W_2(|\varphi|)$  has been used to study stability and boundedness of solutions of differential equations. With similar conditions, we also obtain inequalities of solutions of functional differential equations. Some examples are given to show the applications of our results.

## Stability analysis of neutral functional differential equations and their numerical discretization

Wansheng Wang\*, Changsha University of Science and Technology, P.R. China [w.s.wang@163.com]

**Abstract:** This report is concerned with the stability of the theoretical solutions to a class of nonlinear neutral functional differential equations (NFDEs) which includes as an important special case the Volterra functional differential equations (VFDEs) and their numerical discretization.

1. We introduce the test problem classes  $\mathcal{L}_{\lambda^*}(\alpha, \beta, \gamma, L, \tau_1, \tau_2)$  and  $\mathcal{D}_{\lambda^*}(\alpha, \beta, \gamma, \varrho, \tau_1, \tau_2)$  with respect to the initial value problems of nonlinear NFDEs in Banach spaces. A series of stability, contractivity, asymptotic stability and exponential asymptotic stability results of the theoretical solutions to nonlinear NFDEs in Banach spaces are obtained.

1.1. By means of the perturbed method, we obtained some stability, contractivity and asymptotic stability results of the theoretical solutions to the most general NFDEs in Banach spaces. These results can be regarded as an extension of the stability theory of nonlinear stiff VFDEs in Banach spaces established by S. F. Li in 2005.

1.2. By generalizing the Halanay inequality, we obtained the exponential asymptotic stability results of the theoretical solutions to nonlinear NFDEs with bounded delays in Banach spaces.

1.3. Applying our findings to the special cases, neutral delay differential equations and neutral delay integro-differential equations, we obtained some more deeper and more general results than the existed results.

From a numerical point of view, it is important to study the potential of numerical methods in preserving the qualitative behaviour of the underlying system. Consequently, these results presented in this report provide the theoretical foundation for analyzing the stability of the numerical methods when they are applied to these systems.

2. The contractivity and asymptotic stability properties of the implicit Euler method for nonlinear functional differential equations (FDEs) are discussed. These properties are first analyzed for VFDEs and then the analysis is extended to the case of NFDEs. Such an extension is particularly important since NFDEs are more general and have received little attention in the literature. The main result we establish is that the implicit Euler method with linear interpolation can completely preserve these stability properties of the theoretical solution to such FDEs.

## Superintegrability in classical and quantum mechanics

Pavel Winternitz\*, Universite de Montreal, Canada [wintern@CRM.Umontreal.CA]

**Abstract:** We review the present status of superintegrable systems, i.e., finite dimensional Hamil-

tonian systems with more integrals of motion than degrees of freedom. The emphasis is on conceptual questions and on new developments such as the discovery of infinite families of superintegrable systems with integrals of motion of arbitrary order in the momenta.

## Discrete integrable systems and their Lagrangian formulation

Pavlos Xenitidis\*, University of Leeds, UK [p.xenitidis@leeds.ac.uk]

**Abstract:** Recently a Lagrangian formulation in terms of two-forms has been suggested for multi-dimensionally consistent discrete equations. In this talk, we review these results and show that similar structures exist for the discrete integrable systems of Boussinesq type and their continuous counterparts.

## New integrable couplings of evolution equations hierarchies with self-consistent sources

Tiecheng Xia\*, Shanghai University, P.R. China [xiatc@shu.edu.cn]

**Abstract:** A kind of integrable couplings of soliton equations hierarchy with self-consistent sources associated with  $\widetilde{sl}(4)$  is presented. Based on this method, we construct two new integrable couplings of the AKNS and JM hierarchies with self-consistent sources by using of loop algebra  $\widetilde{sl}(4)$ . In this paper, we also point out that there exist some errors in the literature and have corrected these errors and set up new formula. The method can be generalized to other soliton hierarchies with self-consistent sources.

## Stability of solutions to integro-differential equations in Hilbert spaces

Ti-Jun Xiao\*, Fudan University, P.R. China [tjxiao@fudan.edu.cn]

**Abstract:** We investigate the uniform exponential stability of solutions to abstract integro-differential equations in Hilbert spaces by the theory of operator semigroups and Laplace transforms of vector-valued functions. New criteria are given based on the growth property of associated vector-valued functions on the right half plane. Examples are presented to illustrate our results.

## Post-transition dynamics of reaction-diffusion models in connection with heart disease problems

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**Abstract:** Reaction-diffusion equations have been instrumental in modeling heart disease problems; in this connection, most of the numerical and analytical studies heavily involve bifurcation analysis and study of qualitative behavior of solutions in the vicinity of critical bifurcation values. In many instances, numerical simulations seem to have taken precedence over our analytical understanding of such problems. In this talk we aim to present an analytical framework which not only verifies certain numerical achievements but also provides further insights about posttransition dynamics of reaction diffusion models. It will be shown that by a ‘dynamic’ reduction approach we will be able to drive more information about the system, in particular, about phase transition types, which consequently enables us to study the dynamics beyond the so called Turing instabilities. The focus will be particularly on dynamics arising from steady-state mode interactions and the explicit role of nonlinearities.

## An efficient numerical algorithm for solving

## the system of nonlinear integro-differential equations

Ahmet Yildirim\*, Ege University, Turkey [ahmetyildirim80@gmail.com]

**Abstract:** In this study, we present Homotopy perturbation method (HPM) for solving the system of nonlinear integro-differential equations. We compared our solution with the exact solution. The results show that HPM is an appropriate method in solving nonlinear integro-differential equations. The mathematical technique employed in this paper is significant in studying some other problems of science and engineering.

## The boubaker polynomials expansion scheme for solving nonlinear science problems

Ahmet Yildirim\*, Ege University, Turkey [ahmetyildirim80@gmail.com]

**Abstract:** The Boubaker polynomials are investigated in this study. We propose analytical and numerical solutions to nonlinear science problems using the Boubaker Polynomials Expansion Scheme (BPES). The advantage of the used protocol: the BPES, is to yield continuous and integrable expressions which can be easily incorporated in analytical models. The results have been compared to several studies. The accuracy and the asymptotic behaviors of the solutions are discussed.

## Necessary conditions for integrability and super-integrability of Hamiltonian systems with homogeneous potential

Haruo Yoshida\*, National Astronomical Observatory of Japan, Japan [h.yoshida@nao.ac.jp]

**Abstract:** Necessary condition for integrability of Hamiltonian systems with homogeneous potential was given by Morales and Ramis, which is based on differential Galois theory. Later on, necessary condition for super-integrability of the same systems was obtained by Maciejewski et al., which has a form of subset of condition for integrability. Application of this necessary condition to known series of integrable potentials shows that they cannot be super-integrable whenever the degree of homogeneous potential  $k$  is greater than 2 or less than -2. As for the integrable centrifugal potential, it is shown that the system can be super-integrable only when  $k = -1$  (Kepler problem) and  $k = 2$  (isotropic harmonic oscillation), which reproduces the classical Bertrand's theorem. The necessary condition has a special form when the degree  $k$  of potential is +2 and -2. When  $k = -2$ , this condition naturally reproduces the so-called Tremblay-Turbiner-Winternitz (TTW) system without harmonic potential term in a simple way. This is a joint work with Andrzej Maciejewski and Maria Przybylska.

## Homotopy analysis method for the solution of of the generalized Burgers-Fisher equation

Ugur Yücel\*, Pamukkale University, Turkey [uyucel@pau.edu.tr]

**Abstract:** In this work, the homotopy analysis method (HAM) is applied to obtain the approximate analytical solution of the generalized Burgers-Fisher equation without any discretization. The HAM solutions contain an auxiliary parameter which provides a convenient way of controlling the convergence region of the series solutions. Comparison with Adomian decomposition method (ADM) reveals that HAM is very effective and convenient.



## A discrete analogue of the generalized Weierstrass representation

Dmitry Zakharov\*, Stony Brook University, New York, USA [zakharov@math.sunysb.edu]

**Abstract:** The Weierstrass representation is a way of describing arbitrary embedded surfaces in  $R^3$  using solutions of the Dirac equation. A variant of this construction, using a hyperbolic Dirac equation, describes surfaces embedded in pseudo-Euclidean spaces and parametrized along isotropic directions. I construct a discretization of this hyperbolic Dirac equation by considering a deformation of the algebro-geometric data corresponding to the generalized Dirac operator. Using this discrete Dirac operator, I construct a discrete version of the generalized Weierstrass representation of surfaces parametrized along isotropic directions in  $R^{2,1}$ ,  $R^{3,1}$  and  $R^{2,2}$ . The corresponding lattices have isotropic edges. I show that any lattice in these spaces satisfying a general monotonicity condition and having isotropic edges admits such a representation.

## Stochastic dynamical model for photosynthesis

Tadesse Zerihun\*, University of South Florida, USA [tzerihun@yahoo.com]

**Abstract:** A simple dynamical model of the effect of radiant flux density and  $CO_2$  concentration on the rate of photosynthesis in light reactions and dark reactions and as well as enzyme reactions are proposed. These reactions are coupled with the enzymatic reactions and the two coupled non linear normalized equations are solved numerically for some values of rate constant and radiant flux density. we used Matlab to solve the system numerically and plot the variables of the system. more over with the assumption that the concentration of  $CO_2$  is random we will solve a system of stochastic differential equations numerically.

## Direct fixed point mapping and periodic solutions of neutral functional differential equations

Bo Zhang\*, Fayetteville State University, USA [bzhang@uncfsu.edu]

**Abstract:** We study the existence of periodic solutions of neutral functional differential equations by the method of direct fixed point mapping. A common way of applying fixed point theory to such a problem consists of writing the differential equation as an integral equation which then defines a mapping; if the mapping has a fixed point in  $P_T$ , the Banach space of continuous T-periodic functions, then it is a periodic solution of the equation. The conversion to an integral equation produces numerous problems for nonlinear equations. In this paper, we write the solution as an integral equation and avoid those difficulties. This allows us to construct a mapping function directly from the right-hand side of the equation and show that it has a fixed point which is a periodic solution of the equation.

## Bifurcation problem of the discrete nonlinear Schrödinger equations with unbounded potentials

Guoping Zhang\*, Alexander Pankov, Morgan State University, USA  
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**Abstract:** In this talk we will present our recent results on the bifurcation phenomenon of the discrete Schrödinger equation with unbounded potentials. For the case of self-focusing and defocusing nonlinearity all eigenvalues of Schrödinger operator are bifurcation points and our estimates near the bifurcation points are optimal for the linking solutions. For the case of sign changing nonlinearity all eigenvalues of Schrödinger operator are also bifurcation points. However we can only obtain the bifurcation estimate near the first eigenvalue.

## Similarity transformation theory for nonautonomous nonlinear Schrödinger equation and applications

Jie-fang Zhang\*, Zhejiang Normal University, P.R. China [jf\_zhang@zjnu.cn]

**Abstract:** The nonautonomous (inhomogeneous) nonlinear Schrödinger equation, which has several applications in many branches of physics, such as trapped Bose-Einstein condensates and optical solitons in fibers, are more realistic model compared to the standard NLS equation. Recently, much interest has arisen on the study of inhomogeneous nonlinear Schrödinger (NLS) equations.

In this paper, the general theory for similarity transformations of the nonautonomous (inhomogeneous) nonlinear Schrödinger equation is presented based on the work of Juan Belmonte-Beitia et al. Using this similarity transformation methods, the integrability conditions of nonautonomous nonlinear Schrödinger equations are derived and explicit solutions with an interesting nontrivial behavior are constructed. Further, several differential kinds of nonautonomous nonlinear Schrödinger equations with potentials and nonlinearities depending both on time or on the spatial coordinates and discuss application in physics fields are investigated by using it. The nonautonomous nonlinear Schrödinger equations for two-spatial dimensions are also studied.

## Exact solutions for an $N$ -dimensional Hamiltonian system

Jinshun Zhang\*, Huaqiao University, P.R. China [jszhang@hqu.edu.cn]

**Abstract:** An  $N$ -dimensional Hamiltonian system is studied. The system is reduced from a Lax pair of a solitonian equation. It is proved that the system is Liouville integrable. Some exact solutions are obtained.

## Mathematical model of biofilm induced calcite precipitation

Tianyu Zhang\*, Montana State University at Bozeman, USA [zhang@math.montana.edu]

**Abstract:** TBA

## Integrable discretizations and soliton solution of KdV and mKdV equations

Yi Zhang\*, Zhejiang Normal University, P.R. China [zy2836@163.com]

**Abstract:** A method of discretizing soliton equations is presented. The method is based on the bilinear formalism. From the bilinear forms of the KdV equation and mKdV equation, we can obtain kinds of new bilinear forms through properly substituting the hyperbolic operator into Hirota operator. Meanwhile we can get the soliton solutions of these new equations by Hirota's method.

## Upper semicontinuity of global attractor for a two-dimensional non-Newtonian fluid

Caidi Zhao\*, Wenzhou University, P.R. China [zhaocaidi@yahoo.com.cn]

**Abstract:** This talk is concerned with the upper semicontinuity of global attractors of a non-Newtonian fluid when the spatial domains vary from  $\Omega_m$  to  $\Omega = R \times (-L, L)$ , where  $\Omega_m$  is an expanding sequence of simply connected, bounded and smooth subdomains of  $\Omega$  such that  $\Omega_m \rightarrow \Omega$  as  $m \rightarrow \infty$ . Let  $A$  and  $A_m$  be the global attractors of the fluid corresponding to  $\Omega$  and  $\Omega_m$ , respectively, we establish that for any neighborhood  $O(A)$  of  $A$ , the global attractor  $A_m$  enters  $O(A)$  if  $m$  is large enough.

## Semilinear schroedinger equation with a magnetic potential

Shijun Zheng\*, Georgia Southern University at Statesboro, USA [szheng@georgiasouthern.edu]

**Abstract:** TBA

## On the continuous limits and integrability of a new coupled semidiscrete mKdV system

Zuo-nong Zhu\*, Hai-qiong Zhao, Shanghai Jiao Tong University, P.R. China [znzhu2@yahoo.com.cn]

**Abstract:** In this paper, aiming to get more insight on the relation between semidiscrete coupled mKdV and the coupled mKdV equations, we propose a new coupled semidiscrete mKdV system. The Lax pairs, the Darboux transformation, soliton solutions and conservation laws for the coupled semidiscrete mKdV system are given. The coupled mKdV theory including the Lax pairs, the Darboux transformation, soliton solutions and conservation laws is recovered through the continuous limits of corresponding theory for the new semidiscrete mKdV system.

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