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Investment Basics XL: Bond price volatility

1. INTRODUCTION

This is the first in a series of two articles dealing with measuring and managing the impact of interest rate changes on bond prices. This article discusses bond price volatility. The second, which will appear in a forthcoming issue, will discuss the concepts of duration and convexity.

2. BOND PRICES

All investments, including fixed-income securities and common stocks, derive their value from the cash flows they are expected to generate to the owner. Because these cash flows are received over future periods, they must be discounted in order to derive a present value or fair price for the security. In general, the fair value of any security is the present value of the stream of cash flows it is expected to generate. The cash flows from a bond consist of the periodic coupon payments (C), each of which is made every *t* periods; and the par or face value (M), which is payable at maturity of the bond after *T* periods. Hence the fair price of a straight bond (a bond with no embedded option) is determined by discounting its expected cash flows by the required yield (r) as follows:

$$\text{Bond Price} = P_0 = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{M}{(1+r)^T} \qquad (1)$$

The required yield is the rate of return offered by financial instruments of comparable risk (substitutes). In equation (1) above, the first term on the right hand-side is an annuity consisting of the periodic coupon payments that the bondholder will receive regularly every *t* periods. The second term represents the present value of the amount that will be received by the bondholder at maturity of the bond. Equation (1) above shows that a bond's price is inversely related to the required yield.

The average rate of return on a bond if bought now and held until maturity is called the yield to maturity (YTM), which must be at least as high as the required yield in order to induce investors to buy a bond. YTM is the rate of return that equates the bond's market price to the present value of the bond's cash flows if bought and held until maturity. YTM is therefore the internal rate of return on the bond investment and is found by solving for *y* in the following equation:

$$\text{Bond Price} = P_0 = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{M}{(1+y)^T} \qquad (2)$$

Equation (1) indicates that the price of a bond depends on three key variables: the coupon rate, the required yield, and the time to maturity. Whereas the coupon rate and time to maturity are set only once when the bond is initially issued, the required yield may change at any point in time depending on market conditions.

3. BOND PRICE VOLATILITY

The price of a bond is inversely related to its required yield. Therefore, as required yields in the market change, bond prices adjust accordingly. Thus for any given bond, if the required yield is equal to the coupon rate, the bond will sell at par value. If the required yield rises above the coupon rate, the bond will sell at a discount; and when the required yield falls below the coupon rate, the bond will sell at a premium.

Table 1 below portrays the volatility of bond prices arising from changes in each of the three key variables. Six hypothetical bonds, each with a par value of R100, but with different coupon rates and/or maturity are used to illustrate the volatility of bond prices. Of the six bonds, four (Bonds 1 - 4) pay semiannual coupons, whereas two (Bonds 5 and 6) are zero-coupons (C = 0%). Furthermore, the price of each bond has been determined by discounting it on a semiannual basis.

If we assume that each of the six bonds is priced initially to realize a required yield of 10%, the prices at which the bonds should be trading would be R100.00; R100.00; R78.66; R60.88; R46.65; and R2.21 respectively. In order to gauge the volatility of each bond's price to changes in the required yield, the instantaneous percentage changes in prices as the required yield changes from the initial 10% are shown in Table 2.

An interesting corollary of the inverse relationship between bond prices and the required yield is that the percentage price change as the required market yield changes is not the same for all bonds. The instantaneous percentage price changes (as the required yield changes) for each of the six hypothetical bonds are calculated in Table 2 below. The instantaneous bond price changes may be briefly summarized into five important properties of bond price volatility, which are described below.

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Property 1: The effects of small changes in the required yield

For very small changes in the required market yield (such as a change from 10% to 9,99% or from 10% to 10,01%), the absolute percentage change in the price of a given bond is approximately the same regardless of whether the required market yield increases or decreases.

Table 1: Bond price – yield relationship for six hypothetical bonds

	Bond price					
Required Yield (%)	Bond 1 C = 10% T=4 years	Bond 2 C = 10% T=20 years	Bond 3 C=6% T=4 years	Bond 4 C = 6% T=20 years	Bond 5 C = 0% T=4 years	Bond 6 C = 0% T=20 years
6,00	124,84	160,19	100,00	100,00	62,74	9,72
8,00	111,49	123,85	88,51	76,15	54,03	4,60
9,00	105,53	110,76	83,40	67,73	50,19	3,18
9,50	102,72	105,12	89,98	64,13	48,38	2,65
9,90	100,54	100,99	79,12	61,51	46,99	2,29
9,99	100,05	100,10	78,71	60,95	46,68	2,22
10,00	100,00	100,00	78,66	60,88	46,65	2,21
10,01	99,95	99,90	78,61	60,82	46,62	2,20
10,10	99,47	99,03	78,21	60,27	46,31	2,13
10,50	97,38	95,33	76,42	57,93	44,99	1,84
11,00	94,85	91,05	74,27	55,24	43,39	1,54
12,00	90,06	83,51	70,19	50,54	40,39	1,07
14,00	81,44	71,58	62,89	43,16	35,06	0,53

Table 2: Instantaneous percentage price changes for each of the six hypothetical bonds

		Percentage price change					
Required Yield (%)	Change in basis points	Bond 1 C = 10% T=4 years	Bond 2 C = 10% T=20 years	Bond 3 C = 6% T=4 years	Bond 4 C = 6% T=20 years	Bond 5 C = 0% T=4 years	Bond 6 C = 0% T=20 years
6,00	-400	24,84	60,19	27,13	64,25	34,49	340,02
8,00	-200	11,49	23,85	12,52	25,08	15,81	108,33
9,00	-100	5,53	10,76	6,02	11,24	7,58	44,09
9,50	-50	2,72	5,12	2,95	5,34	3,71	19,99
9,90	-10	0,54	0,99	0,58	1,03	0,73	3,71
9,99	-1	0,05	0,10	0,06	0,10	0,07	0,36
10,00	0	0,00	0,00	0,00	0,00	0,00	0,00
10,01	1	-0,05	-0,10	-0,06	-0,10	-0,07	-0,36
10,10	10	-0,53	-0,97	-0,58	-1,01	-0,72	-3,57
10,50	50	-2,62	-4,67	-2,84	-4,85	-3,56	-16,59
11,00	100	-5,15	-8,95	-5,58	-9,26	-6,98	-30,37
12,00	200	-9,94	-16,49	-10,76	-16,99	-13,42	-51,36
14,00	400	-18,56	-28,42	-20,05	-29,11	-24,85	-76,04

Property 2: The effects of large changes in the required yield

For large changes in the required market yield (such as a change from 10% to 9%; or from 10% to 11%), the absolute percentage change in the price of a given bond is higher for decreases in the required market yield than for increases in the yield.

Property 3: The effects of the required yield

Holding other factors constant, the higher the required yield at which a given bond is trading, the lower its price volatility. Conversely, the lower the required yield at which a given bond is trading, the higher its price volatility. This can be seen in Table 1 and Table 2 by observing the range of possible prices and percentage changes in prices for each of the six bonds: for higher yields as well as for lower yields. The relationship between the price and required yield (ranging from 2% to 40%) for Bond 1 is graphically depicted in Figure 1 below.

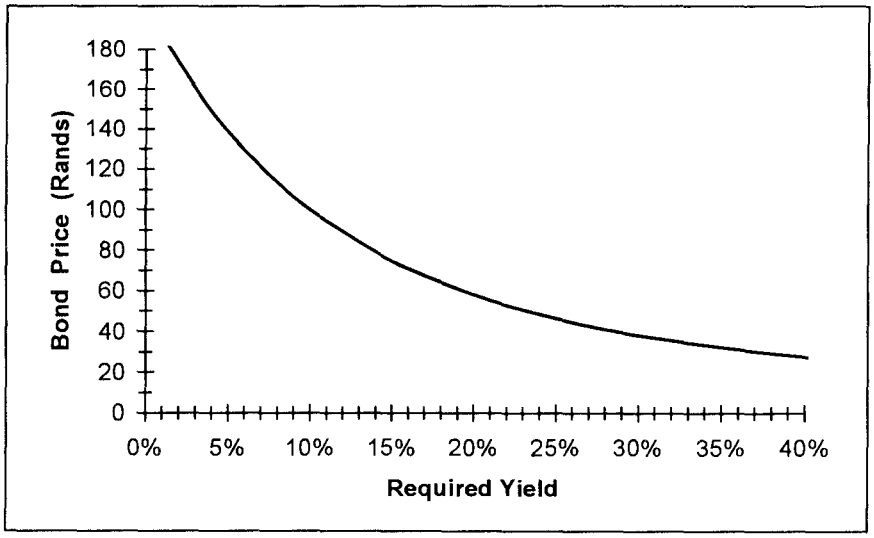

Figure 1: The relationship between price and required yield for Bond 1

Property 4: The effects of maturity

For a given coupon rate and initial yield, a bond with a longer maturity has a higher price volatility than a similar bond with a shorter maturity. Therefore, when required market yields change, the prices of longer term bonds change more than the prices of shorter term bonds, everything else being equal.

Property 5: The effects of coupon

In addition to the maturity effect, the change in the price of a bond as a result of a change in the required market yield depends on the bond's coupon rate. Given two bonds with the same par value and maturity, the price of a bond with a lower coupon rate is more sensitive to changes in the required market yield than the bond with a higher coupon rate. In sum therefore, the volatility of a bond's price is inversely related to its coupon.

4. CONCLUSION

In general, the relationship between the price of a bond and its required market yield is not linear, but convex; as depicted in Figure 1 above. As will be discussed in the next article, convexity is an important concept in the sensitivity of bond prices to changes in the required yield. Beside the required yield, the price of a bond over time may change as a result of more apparent factors such as: (i) a change in the perceived credit risk of the issuer (ii) a discount or premium bond approaching maturity (iii) a change in the market interest rate.

The required market yield depends on fundamental market conditions in the economy at any specific time, and bond investors therefore have to take it as given. Therefore, of the three key variables influencing bond price volatility, bond investors may choose (and therefore exercise control) in only two of them; namely the coupon rate and maturity. Nonetheless, the coupon rate and maturity have each a significant effect on bond price volatility.

Table 2 above summarizes the percentage changes of the six hypothetical bond prices as the required market yield changes by a given number of basis points. One measure of the change in the price of a bond if the yield changes by 1 basis point (such as from 10% to 10,01%) is called the *price value of a basis point*, which may be referred to as the rand value of a basis point. This is normally expressed in absolute rand terms (instead of the percentages shown in Table 2). Another measure of price volatility is the *change in the required market yield for a specified price change*. Obviously, it is cumbersome to calculate various possible changes based on either the price value of a basis point or the change in the yield for a specified price change.

Moreover, maturity alone is not an adequate measure of the sensitivity of a bond's price change relative to changes in the required yield because it ignores the coupon payments and the principal repayment. Investors managing bond portfolios therefore need a measure of time designed to more accurately portray a bond's average life while taking into consideration all of the bond's cash flows (coupons as well as the par value of the bond at maturity). Such a measure is called duration. As mentioned earlier, duration and convexity will be discussed in the next article.

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