Time scales in the context of General Relativity

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First operational Caesium standard

July 1955: Essen and Parry (National Physical Laboratory (U.K.))

Inaccuracy (relative) 10^{-9} to 10^{-10}

(Irregularities of Earth rotation, and UT1, 10^{-7} to 10^{-8})

OPENED A NEW ERA FOR THE MEASURE OF TIME

Almost immediate application: construction of integrated atomic times for the study of the short term irregularities of the rotation of the Earth.

One of these scales became the International Atomic Time TAI (1972)

The first meeting of CCDS in June 1957 was mostly devoted to atomic frequency standards, but the definition of the ephemeris second (based on the orbital motion of the Earth) was under way and became official in 1960.

It was abrogated in 1967…

Towards 1967 : Introduction in Time measurement of corrections predicted by the Theory of General Relativity

- Corrections for the **gravitational frequency shift** : the frequencies are referred at fixed points on the rotating geoid (sea level)
- Adoption of a **convention of "coordinate synchronization"**
- Some **tests** to check if atomic clocks behave as predicted by the theory of GR:

  Caesium clocks in circumnavigation by air, in both directions

  Flight of an hydrogen maser in a rocket.

  The predicted effects are confirmed **great psychological importance**…
- Corrections were applied when needed, but **without** insertion in a **global treatment** of space time reference system, **until 1991**.
- Many misunderstandings – some still persist

The complexity of relativistic models

Relativistic modeling is now essential for celestial mechanics, space navigation, geodesy, with important applications in geophysics, oceanography, etc. and also for everyday needs of positioning on the Earth

To match the measurement accuracy, essentially due to time and frequency techniques, very complex definitions and mathematical developments are required.

These developments are based on definitions of reference systems provided by the International Astronomical Union in Resolutions in 1991, then in 2000.

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The metric

In GR, gravitation appears as a property of space-time described by a metric tensor. The contact with metrology is made through **the proper time** of the observer.

Proper time is a theoretical concept, but it is assumed to be measurable locally in the usual sense of metrology by any observer carrying a « good clock. »

By postulate, we consider that atomic clocks as « good clocks » providing the proper time

(up to now this postulate has not led to inconsistencies)

The choice of coordinates for expressing the components of the metric tensor is largely arbitrary and is guided by the criterion of lesser complexity.

Coordinates are not directly measurable. Some authors consider them as dimensionless (« telephone numbers » of events (Misner et al., 1970))

IAU Resolution A4(1991) Recommendation I

Coordinates t, x^1, x^2, x^3

in each coordinate system centered at the **barycentre of any ensemble** of masses

should be selected so that

$d\tau^2$ is expressed with a **minimum** degree of approximation in the form

$$d\tau^2 = (1 - 2U/c^2)\, dt^2 - c^{-2}(1+2U/c^2)\, [(dx^1)^2 + (dx^2)^2 + (dx^3)^2]$$

τ proper time

t coordinate time

c velocity of light

U sum of the gravitational potentials of the ensemble of masses

IAU Resolution A4(1991) Recommendation II

Considering

…

c) **That the same physical units should be used in all coordinate system**,

recommends that

…

2. **The time coordinate be derived from a time scale realized by atomic clocks operating on the Earth,**

3. **The basic physical units of space-time in all coordinate systems be the second of the International System of Units (SI) for proper time, and the SI meter for proper length, connected to the SI second by the value of the velocity of light *c* = 299 792 458 ms^{-1}.**

The metric adopted in 1991

The coordinates adopted by the IAU are close, but not equal, to Cartesian coordinates for space and to the Newtonian time of the classical dynamics (Post Newtonian approximation). They have clearly the dimension of time and length and therefore **should be expressed in SI units**.

The IAU metric of 1991 was insufficient for celestial mechanics and some space projects. It was completed by higher order terms in 1/c in 2000.

The metric can cover, at the requested accuracy, only a limited region of space.

Two basic coordinates systems are needed in the context of this discussion:

(1) The **Barycentric Celestial Reference System**, centered at the barycenter of the solar system

(2) The **Geocentric Celestial Reference System**, centered at the center of mass of the Earth (including its fluid envelopes)

Hence two coordinate times **TCB Barycentric Coordinate Time**

TCG Geocentric Coordinate Time

In addition

a **Geocentric Terrestrial Reference System (GTRS),**

co-rotating with the Earth,

Is required in geodesy and geophysics.

It is related to the GCRS by a spatial rotation which takes into account the Earth rotation parameters

This rotation is still treated in the classical geometry for space coordinates

It keeps the coordinate time TCG without change

NOTE

The terminology used in this presentation is that officially adopted by the International Astronomical Union

Relativistic definition of TAI

TAI, as a time reference for the Earth and its surrounding, must be a realized coordinate time.

Why we have not chosen to realize Geocentric Coordinate Time TCG ?

For historical reasons, of course, (initially TAI was conceived in the classical context, using clocks on the ground),

The TCG frequency is higher to that of caesium standards at sea level by about 7×10^{-10} in relative value (22 ms/year).

To avoid this inconvenience, a new theoretical time called

Terrestrial Time TT is defined as a conventional linear function of TCG, implying a rate correction

The frequency of TT is that of proper time on an equipotential (gravitational + rotational) surface close to the geoid

Then **TAI is defined as equal to TT + 32.184 s (exactly)**

The time offset is a fossil of the history of time

Coordinated Barycentric Time TCB versus TAI

The difference *TCB – TT* includes a linear term and small periodic terms (smaller than 2 ms)

The mean rate of TCB is faster than the rate of TT by about 0.5 s/year.

This was judged inconvenient, so that a

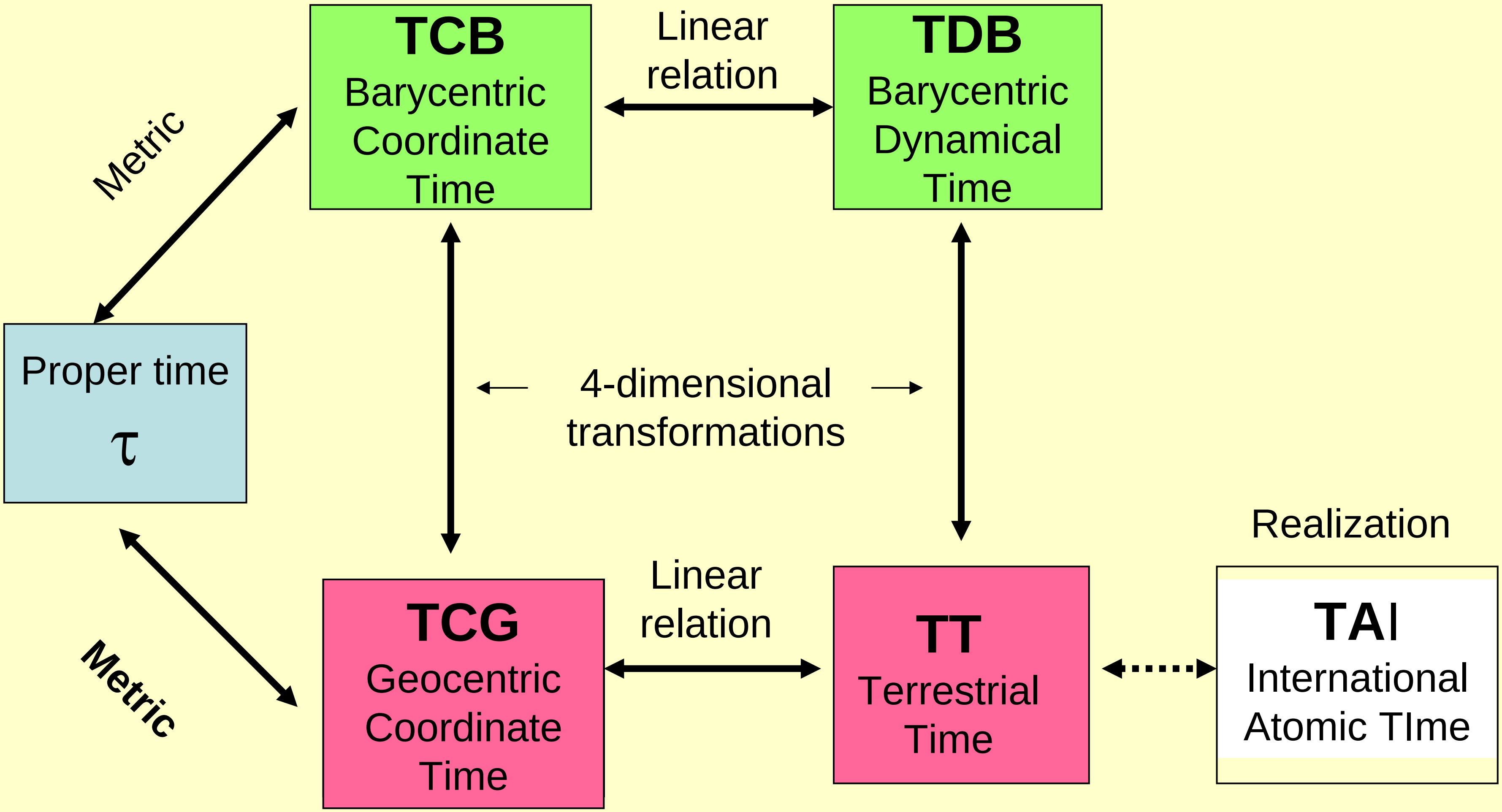
Barycentric Dynamical Time TDB

was defined by a conventional linear relation with TCB in order to follow approximately TT (and TAI) in the long term

The use of TT and especially TDB are sources of controversies because

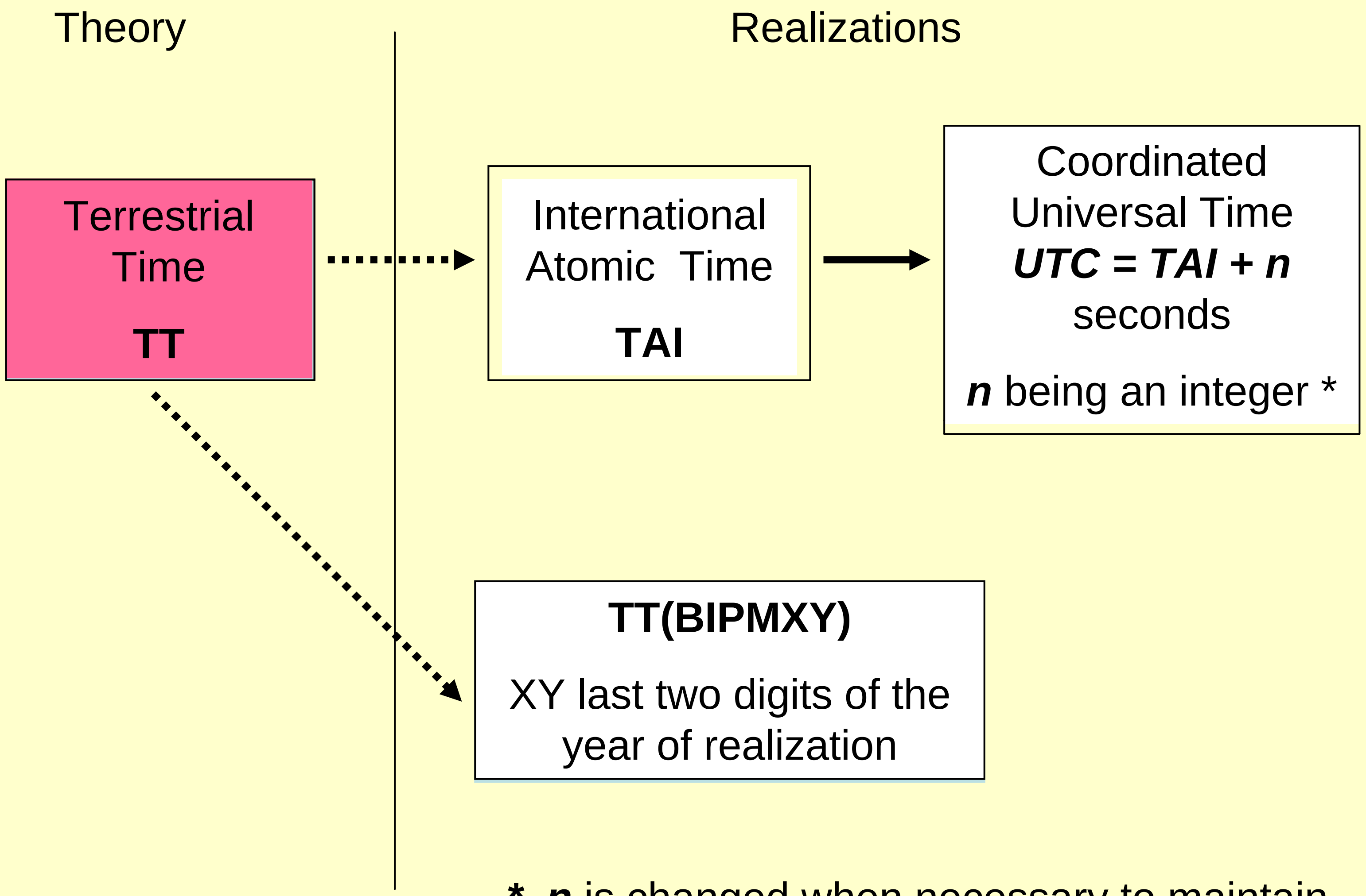
they change the definition of quantities appearing in the metrics

(sometimes this is considered as a change of units, which is worse!)



Relativistic times, their relations and realizations

Realized Time scales



* ***n*** is changed when necessary to maintain ***UT1 - UTC* < 0.9 s**

The unit of time and the scale unit of TAI

In order to satisfy the needs of physicists in local experiments

the second must be defined and realized as the unit of proper time

But what is obtained by dissemination of TAI or UTC is the scale unit of a coordinate time

When working in a fixed laboratory at the surface of the Earth, the access to the second of proper time through TAI requires the local value of the gravitational potential

The potential can be obtained as a function of the altitude, but that may be a limiting factor

The magnitude of the effect is in relative value 10^{-13} per km of altitude

The convention of synchronization

By convention, a coordinate simultaneity of distant events occurs when their dates are equal in a stated coordinate time

This is not an absolute concept; it depends on the adopted reference system.

Hence the definition of difference of clock readings

and of clock synchronization

Questions under consideration

1. Units of coordinates

Clearly, the unit of proper time is concretely the second as defined in the SI.

The metre is also defined as a proper unit : its definition must be applied to a sufficiently small sub-multiple, so that the relativistic effect over the length remains negligible.

Difficulties arise with coordinates whose units are often designated by a special names such as TCB-second, TCG-second.

After long discussions several astronomers recommended to use exclusively the second as sole base unit for all quantities having the dimension of time.

This is in agreement with the metrological rules:

- the unit does not define a quantity,

- the quantity calculus ensures that equations between quantities are valid with their numerical values.

However, it is not yet unanimously accepted.

2 . Problems arising from the scaling factors for TDB and TT

No indication from the IAU on the metric to which these coordinate time pertain

Hence two interpretations

a. **One may introduce new quantities: GM* instead of GM, x* instead of x; which are expressed in second and metre. The inconvenient is that GM becomes dependent on the coordinate system where it is used.**

b. **This can be seen as a change of units. For example, the term TDB-second is sometimes used. Then the same form of the metric is kept, but this introduces new units (non SI) for proper time and length**

Interpretation **a** is favored, but not unanimously and not officially.

3. The astronomical unit of length

In the dynamics of the solar system, the masses *M* of celestial bodies appear only through the products *GM* where *G* is the constant of gravitation.

GM has the dimension $L^3 T^{-2}$

In the past, angular measurements could provide GM_{Sun} with a relative uncertainty of only 10^{-4}

This led to adopt a conventional value of GM_{Sun} and to define an **« astronomical unit of length » whose relation to the metre was experimental**

A consequence of range measurement and of the 1967 definition of the metre is that now the SI units may be used in the solar system and that the **astronomical unit might be defined by a conventional relation to the metre and as a unit of proper length**

This new definition (now proposed by some astronomers) would have the consequences

1. that the GM_{Sun} would become a measured quantity (+/- 10^{-11}). Then possible variations of the mass on the Sun and/or of the constant of gravitation could be directly detected
2. to bring dynamical astronomy in the SI as recommended by UAI