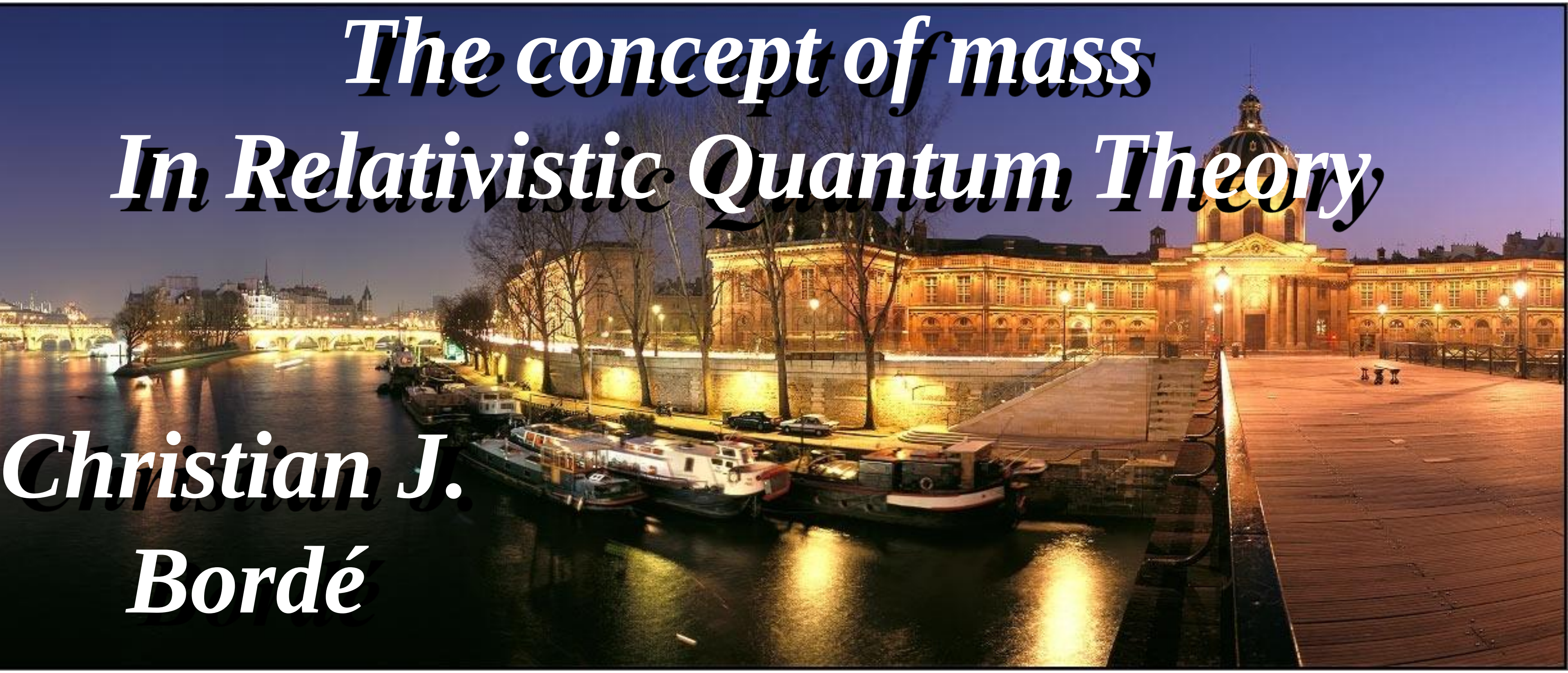
From strings to the $\mathfrak{K}$



The concept of mass In Relativistic Quantum Theory

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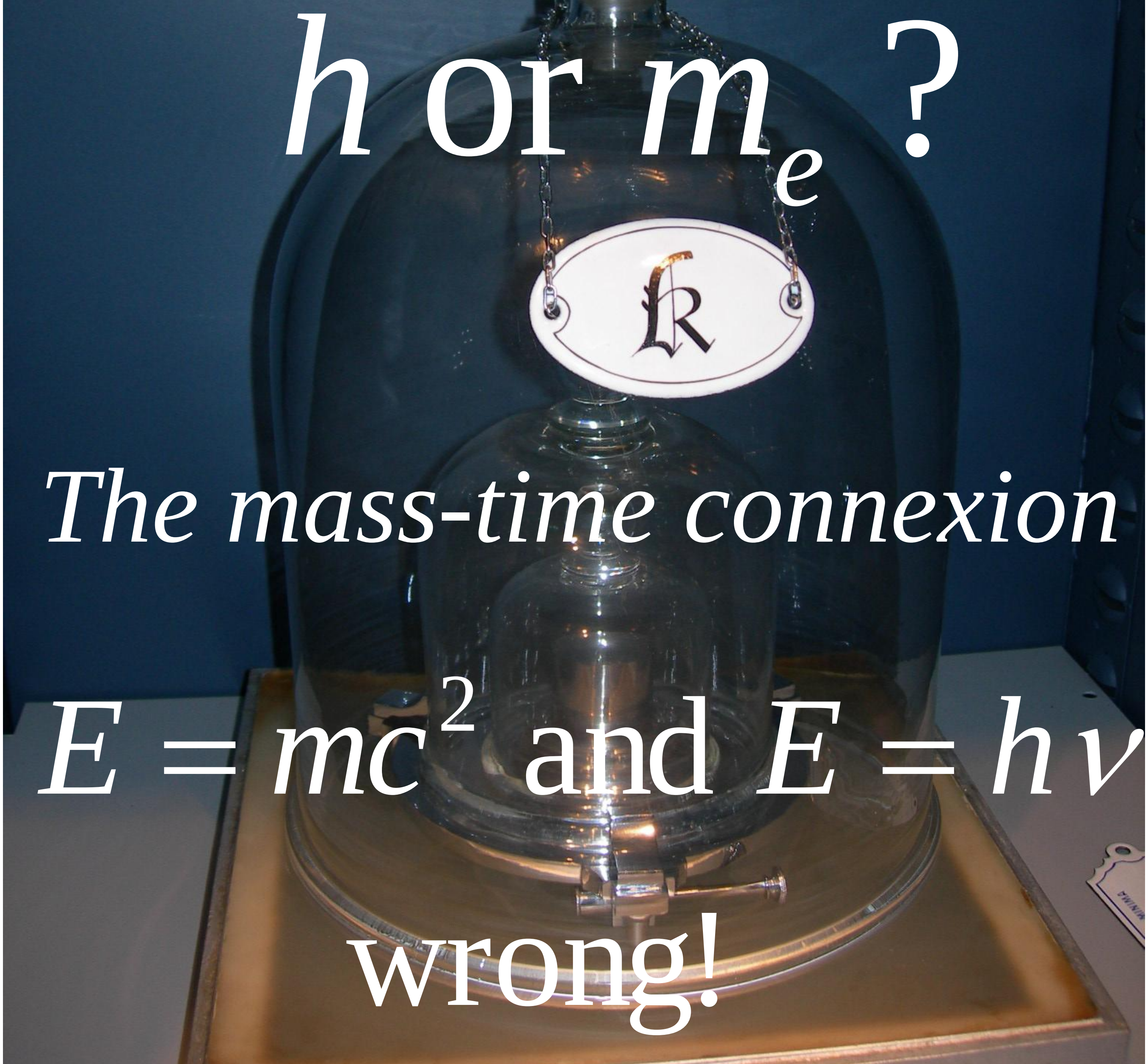
Académie des Sciences

SYRTE & LPL


h or m_e ?
The mass-time connexion
$E = mc^2$ and $E = h\nu$
wrong!

ENERGY

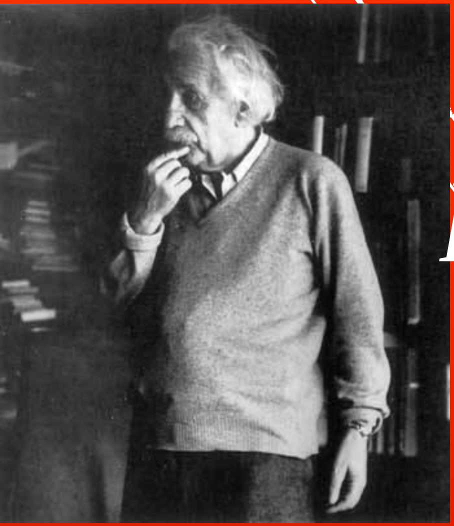
E(p)

$$p^{\mu} = (E/c, p_x, p_y, p_z)$$

$$p_{\mu} = (E/c, -p_x, -p_y, -p_z)$$

$$p^{\mu} p_{\mu} = E^2/c^2 - p^2 = m^2 c^2$$

$$E(\vec{p}) = \sqrt{m^2 c^4 + p^2 c^2} = m_R c^2$$



1905

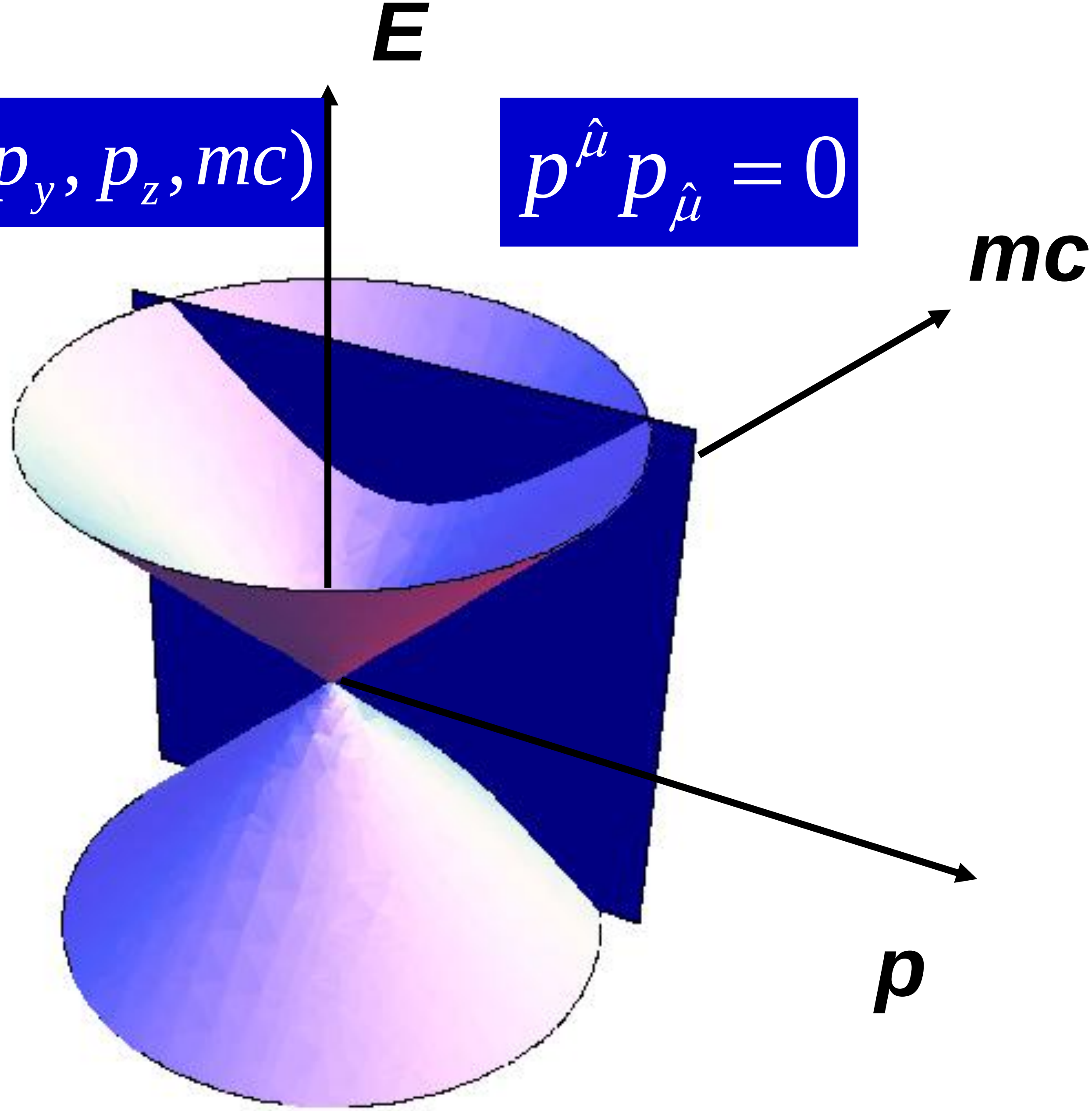
Mass

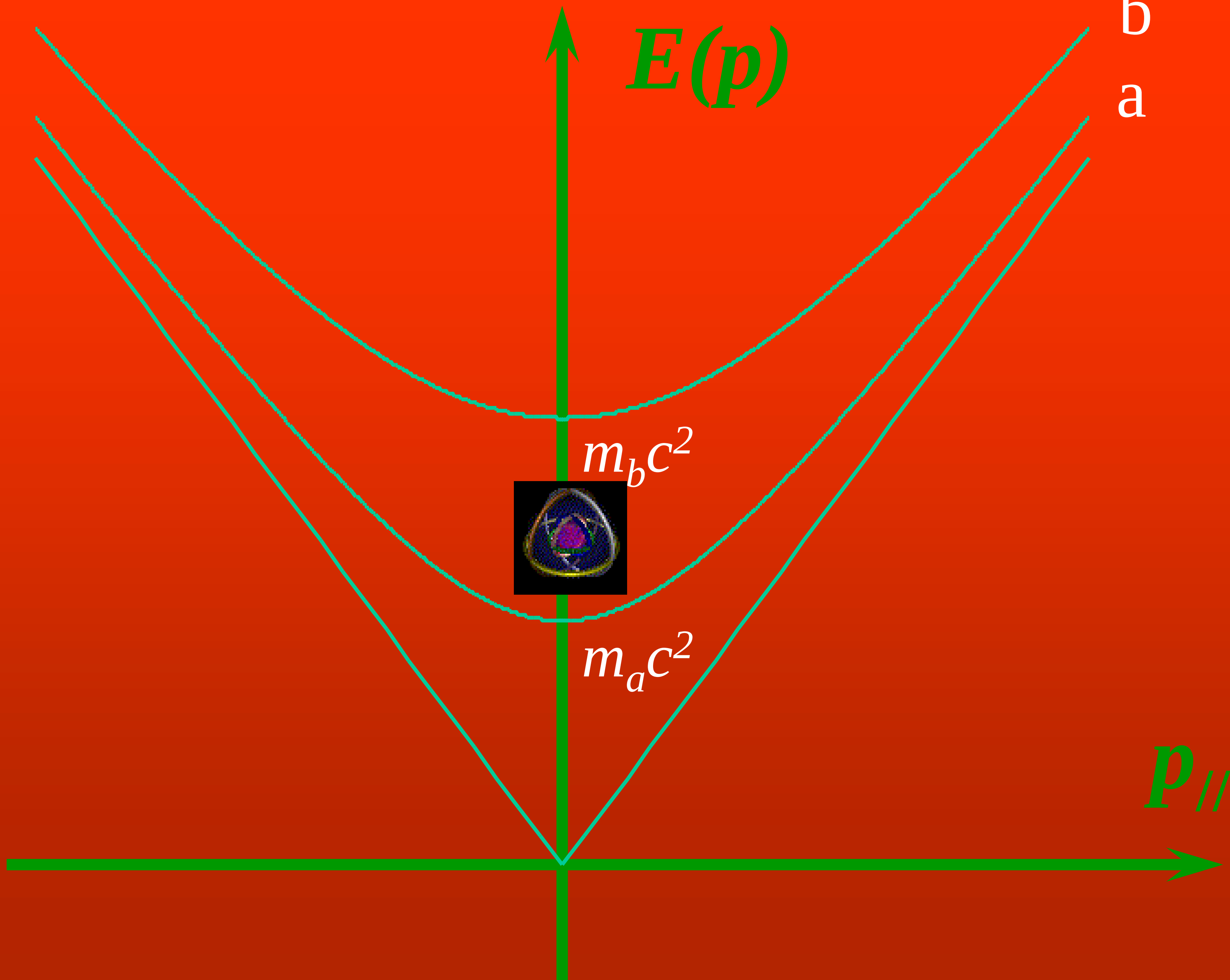
MOMENTUM

$$p^{\hat{\mu}} = (E/c, p_x, p_y, p_z, mc)$$

$$\hat{\mu} = 0,1,2,3,4$$

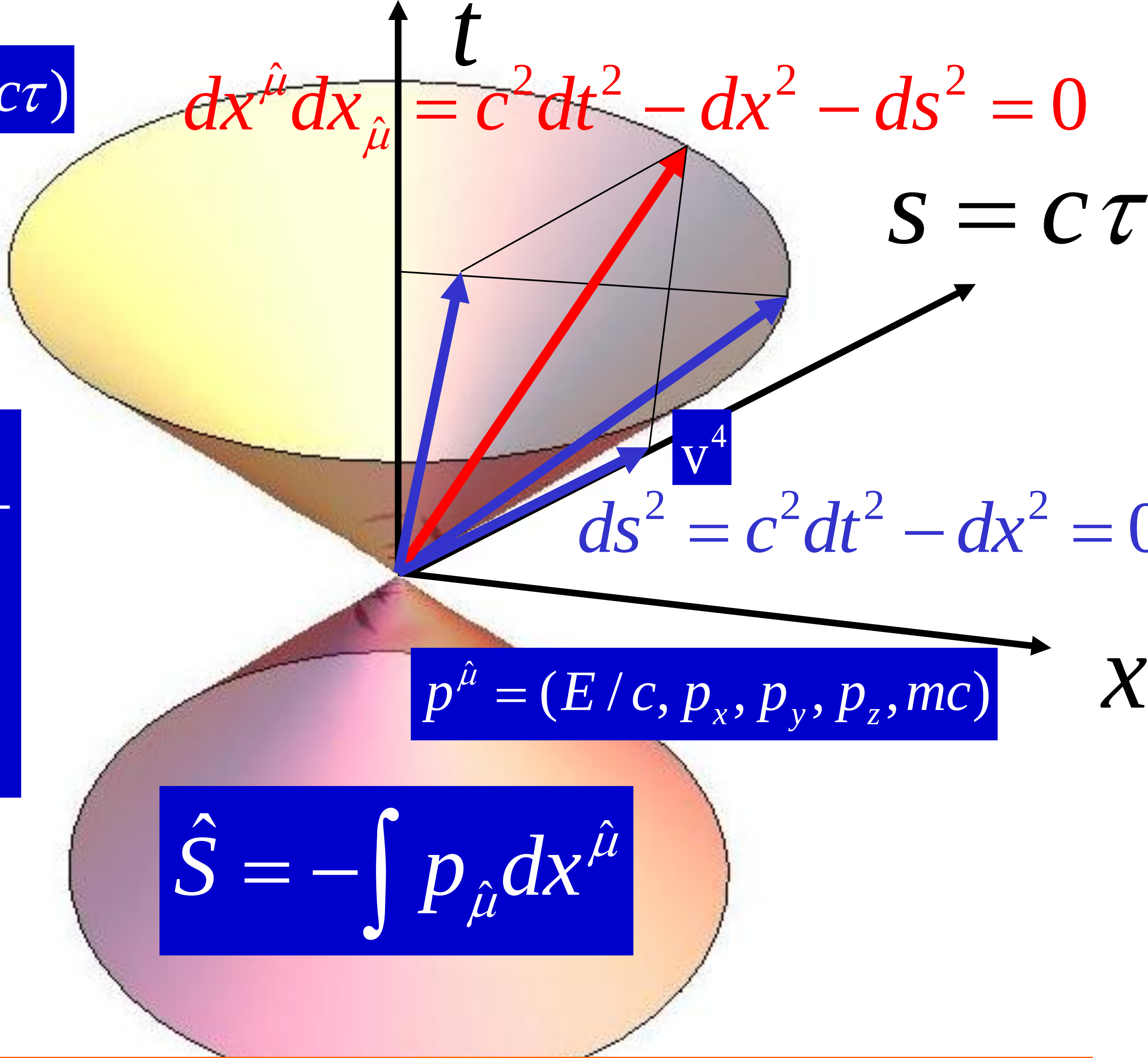
$$p^{\hat{\mu}} p_{\hat{\mu}} = 0$$





E(p)
b
a
$m_b c^2$
$m_a c^2$
$p_{//}$

$$x^{\hat{\mu}} = (ct, x, y, z, c\tau)$$



$$dx^{\hat{\mu}} dx_{\hat{\mu}} = c^2 dt^2 - dx^2 - ds^2 = 0$$

$$ds^2 = c^2 dt^2 - dx^2 = 0$$

$$\mathrm{v}^{\mu} = \frac{dx^{\mu}}{dt} = \frac{p^{\mu}}{m_R}$$

$$\mathrm{v}^{4} = \frac{dc\tau}{dt} = \frac{mc}{m_R}$$

$$p^{\hat{\mu}} = (E/c, p_x, p_y, p_z, mc)$$

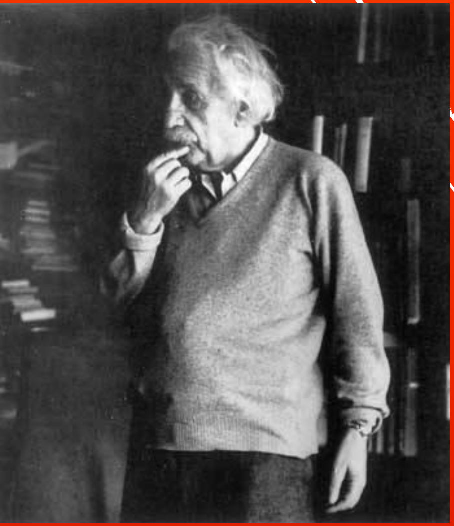
$$\hat{S} = -\int p_{\hat{\mu}} dx^{\hat{\mu}}$$

$$\hat{S} = \hbar\phi \quad \rightarrow \quad g^{\hat{\mu}\hat{\nu}} \partial_{\hat{\mu}}\phi \partial_{\hat{\nu}}\phi = 0$$

ENERGY $\mathbf{E(p)} \rightarrow \hbar\Omega$

$$p^{\mu} \rightarrow i\hbar \frac{\partial \varphi}{\partial x_{\mu}} = p^{\mu}_{op}\varphi$$

$h\nu_{dB}$



1905

$$\Box \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta$$

Mass

1923

$$p^{\mu}p_{\mu} = m^2c^2 \rightarrow \Box\varphi + \frac{m^2c^2}{\hbar^2}\varphi = 0$$

h/λ_{dB}

MOMENTUM $\mathbf{p} \rightarrow \hbar K$

— *Ondes et quanta* (1). Note de M. **Louis de Broglie**, présentée par M. Jean Perrin.

Considérons un mobile matériel de masse propre m_0 se mouvant par rapport à un observateur fixe avec une vitesse $v = \beta c$ $(\beta < 1)$. D'après le principe de l'inertie de l'énergie, il doit posséder une énergie interne égale à $m_0 c^2$. D'autre part, le principe des quanta conduit à attribuer cette énergie interne à un phénomène périodique simple de fréquence ν_0 telle que

$$h\nu_0 = m_0 c^2,$$

c étant toujours la vitesse limite de la théorie de relativité et h la constante de Planck.

Pour l'obsèrv … orrespondra une fréquence $\nu =$ … ve le phénomène périodique inte … tribuera une fré-

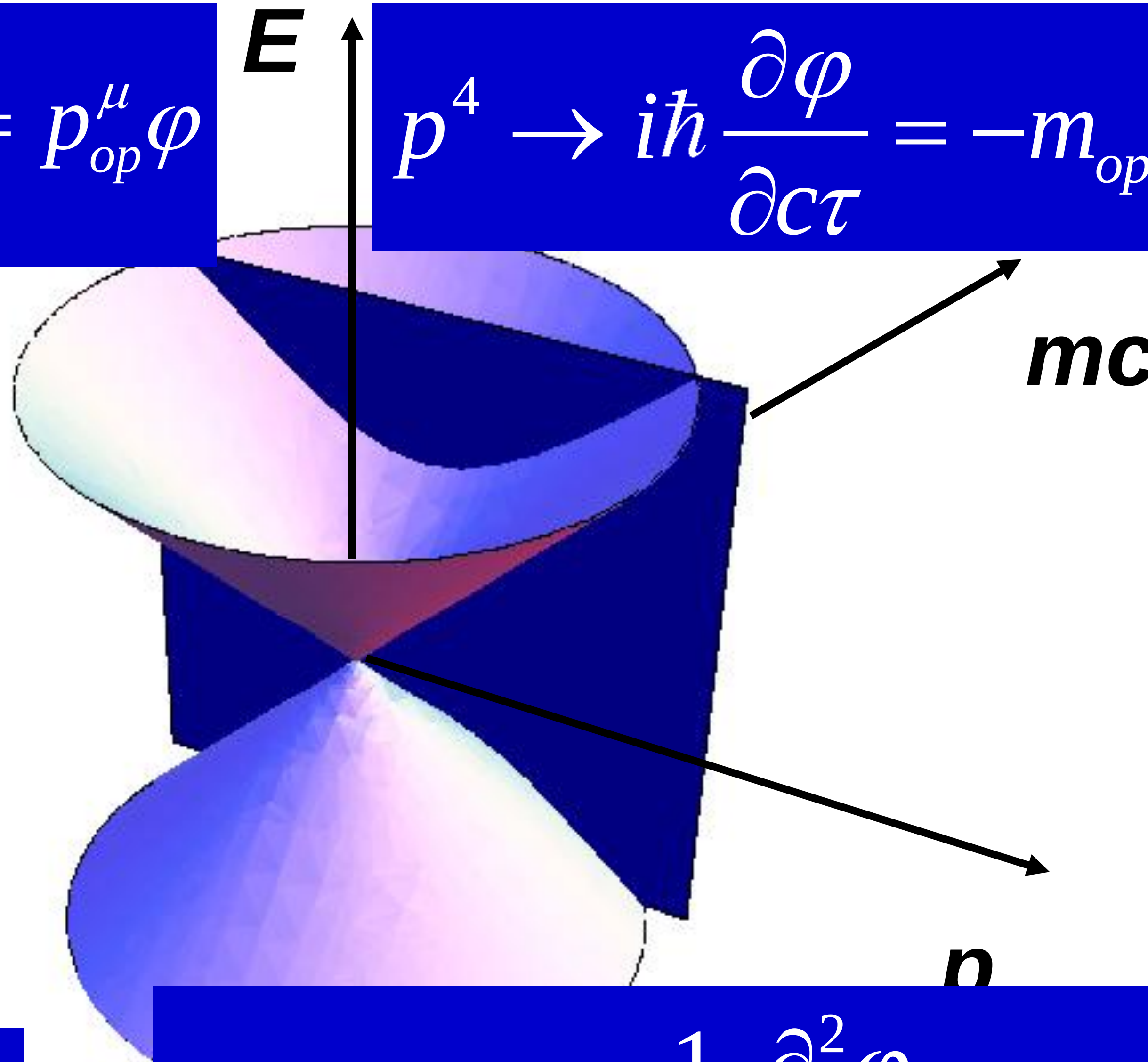
de Broglie - Compton frequency

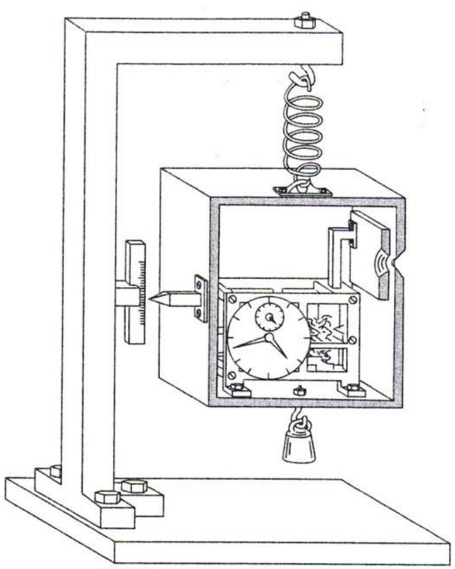
$$\nu_0 = \frac{mc^2}{h}$$

FROM 3 TO 4 SPATIAL DIMENSIONS

$$p^{\mu} \to i\hbar \frac{\partial\varphi}{\partial x_{\mu}} = p^{\mu}_{op}\varphi$$

$$p^{4} \to i\hbar \frac{\partial\varphi}{\partial c\tau} = -m_{op}c\varphi$$

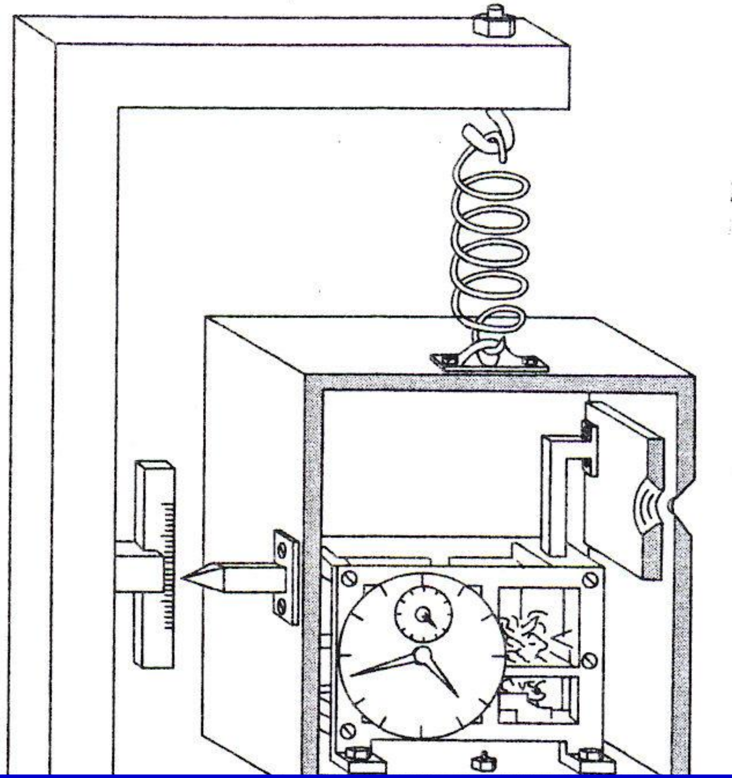




$$p^{\hat{\mu}} p_{\hat{\mu}} = 0 \to$$

$$\hat{\Box}\varphi \equiv \Box\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial\tau^2} = 0$$

Solvay-1930

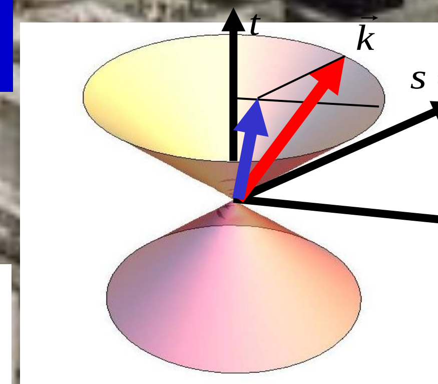


$$\left[c\tau_{op}, m_{op}c \right] = i\hbar$$

$$\Delta(mc)\Delta(c\tau) \geq \hbar / 2$$

de Broglie wavelength and de Broglie-Compton frequency

$$\hat{\Box}\varphi \equiv \Box\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial\tau^2} = 0$$



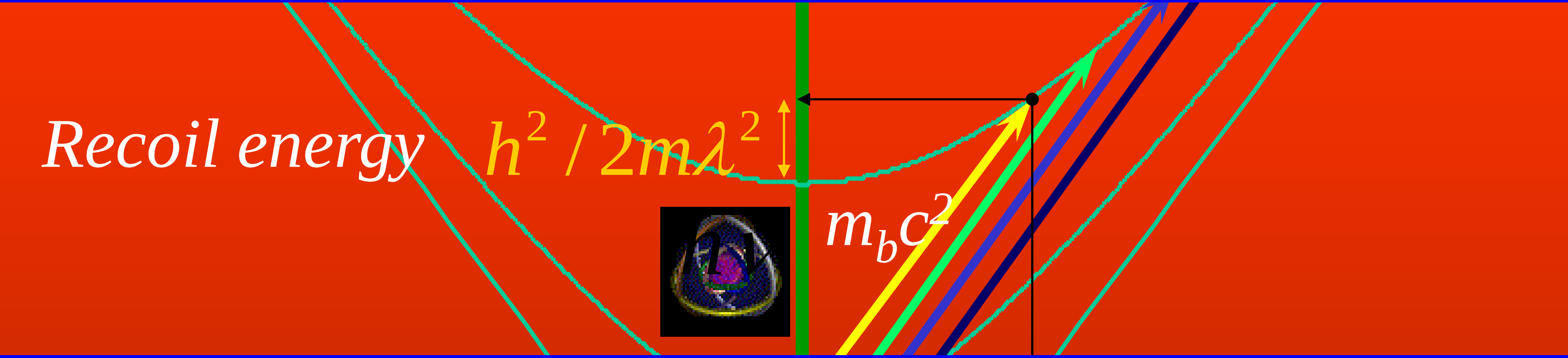
$$k^2 = \frac{1}{\lambdabar^2} = \left(\frac{E}{\hbar c}\right)^2 = \frac{1}{\lambdabar^2_{dBC}} + \frac{1}{\lambdabar^2_{dB}}$$

$$\lambda_{dBC} = \frac{h}{mc} \qquad \lambda_{dB} = \frac{h}{p}$$

No clock at the de Broglie-Compton frequency (possible in the future)

The Bohr frequency of the transition is the difference between the de Broglie - Compton frequencies of both states

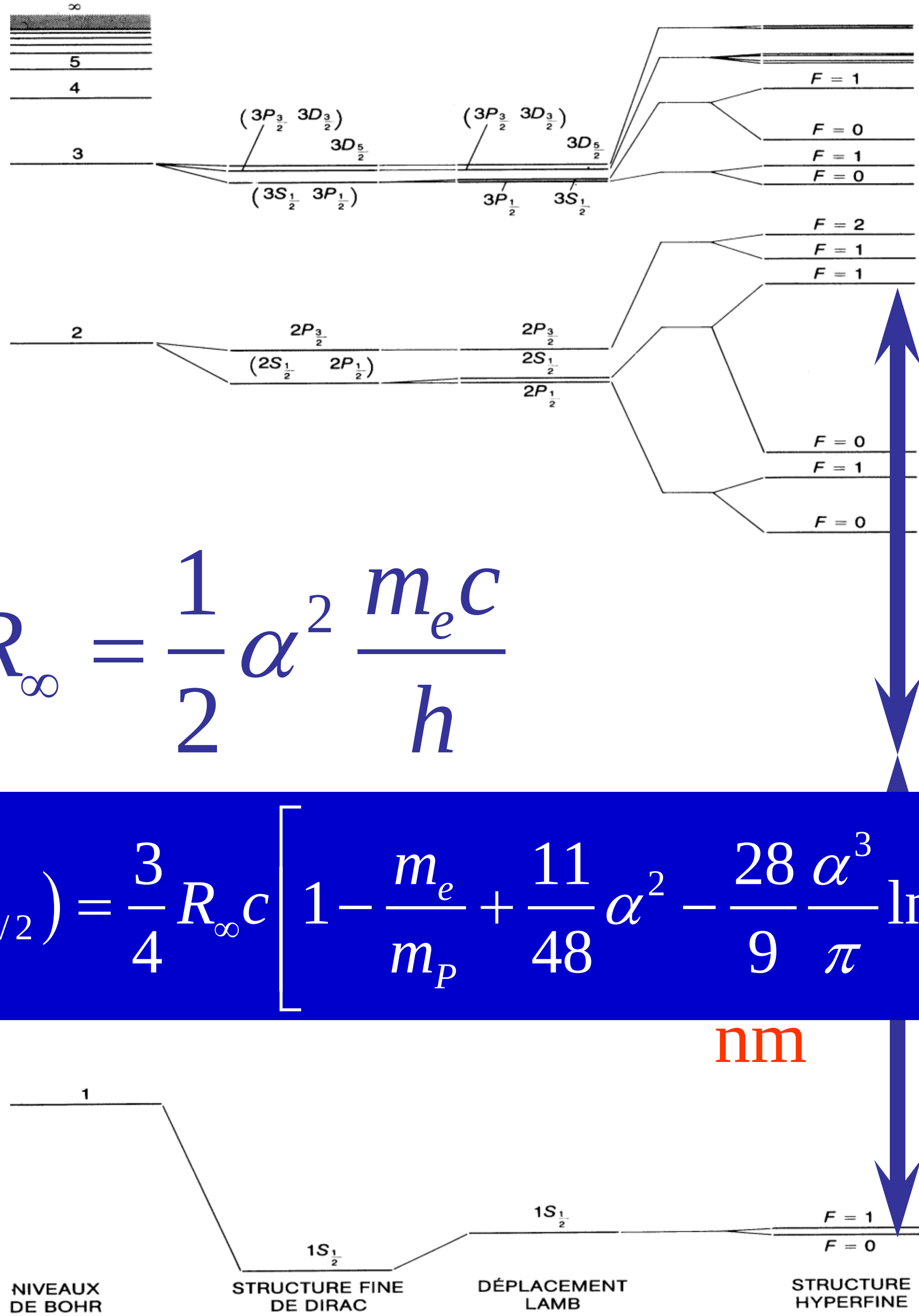
$$\nu_{ba} = \frac{m_b c^2}{h} - \frac{m_a c^2}{h}$$



"The kilogram is the unit of mass, it is the mass of $c^2 / h\,\nu_{\text{Bohr}}$ massive particles without mutual interactions with a mass equal to the mass difference between the two internal states which define the unit of time"

This definition has the effect of fixing the value of the Planck constant, h, to be 6.626 069 3 x 10^{-34} joule second exactly..

HYDROGEN ATOM



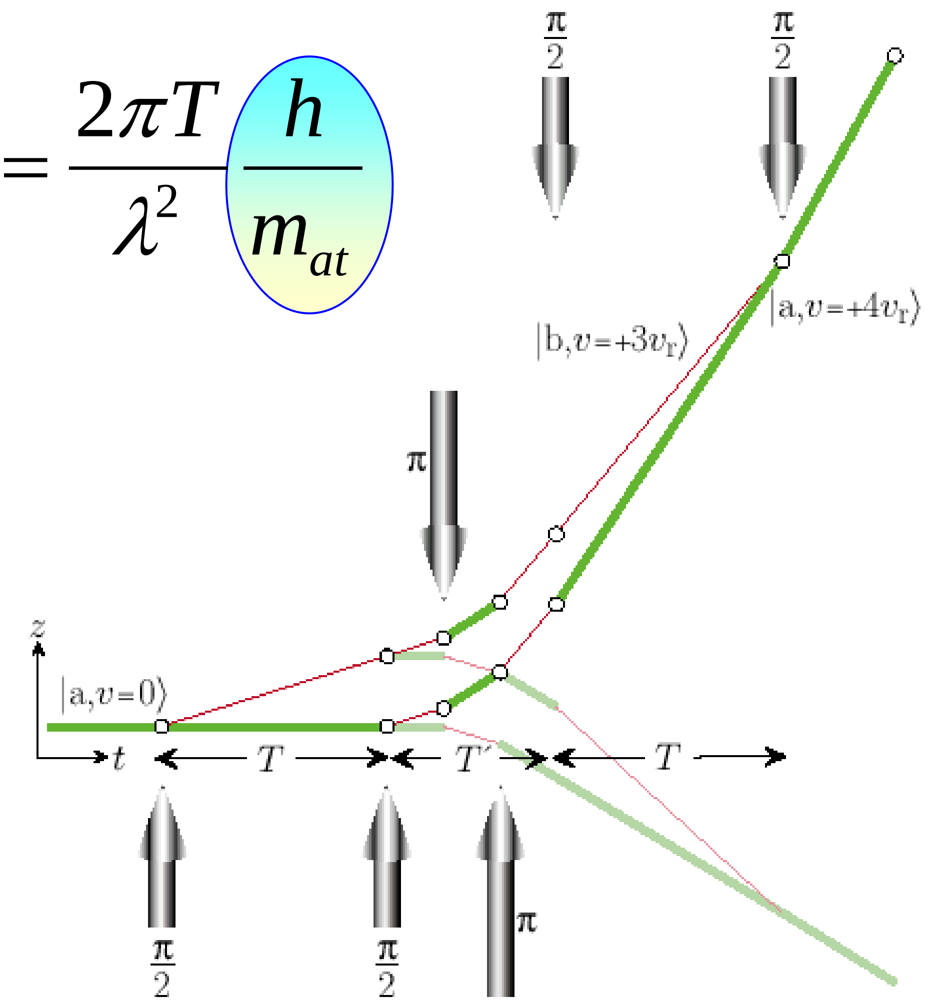
$$R_\infty = \frac{1}{2}\alpha^2 \frac{m_e c}{h}$$

$$\nu_H\left(1S_{1/2} - 2S_{1/2}\right) = \frac{3}{4}R_\infty c\left[1 - \frac{m_e}{m_P} + \frac{11}{48}\alpha^2 - \frac{28}{9}\frac{\alpha^3}{\pi}\ln\alpha^{-2} - \frac{14}{9}\left(\frac{\alpha R_P}{\bar{\lambda}_C}\right)^2 + \dots\right]$$

nm

Determination of h/m_{at} by Bordé-Ramsey atom interferometry

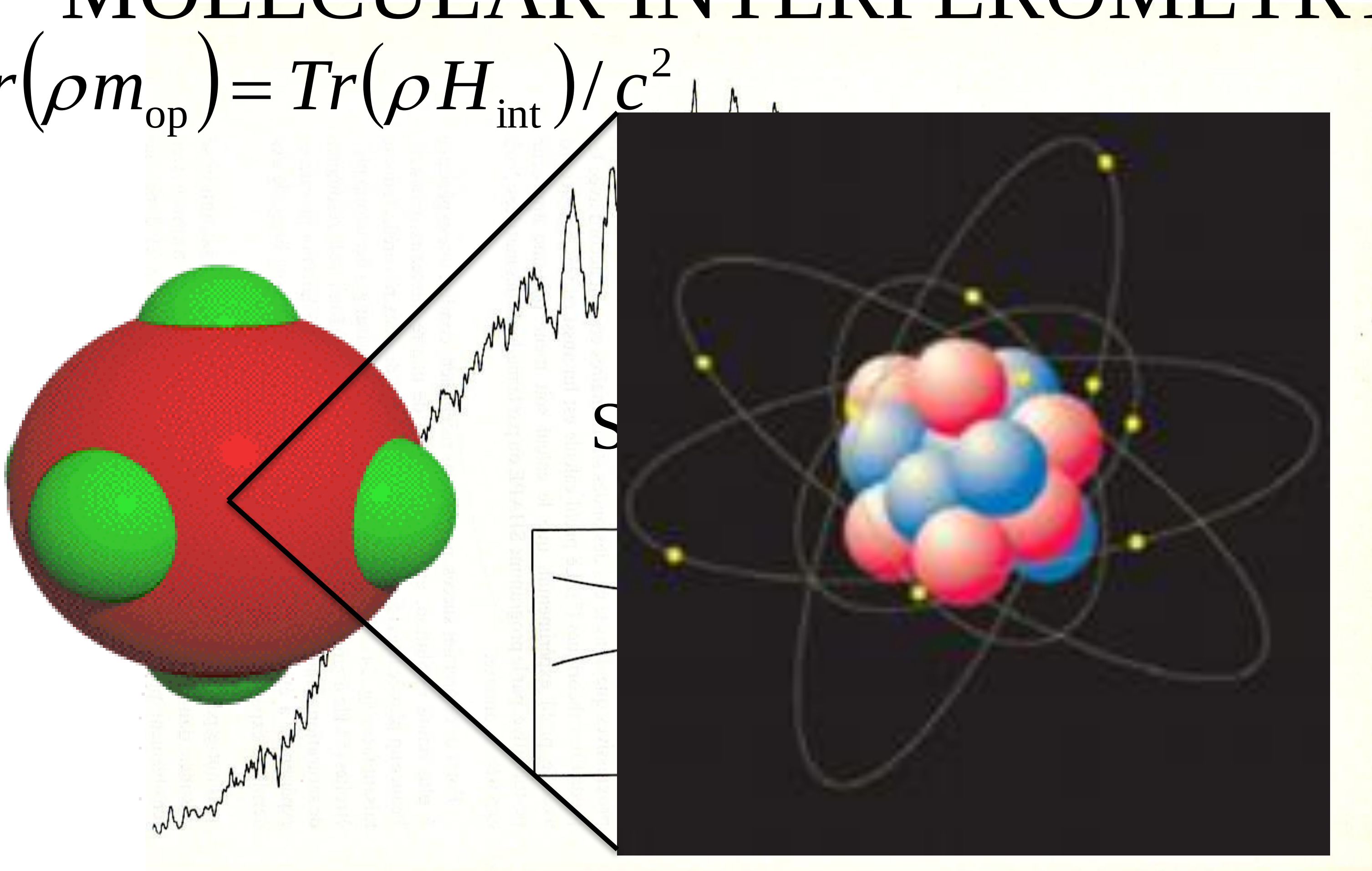
$$\Delta\varphi = \frac{2\pi T}{\lambda^2}\frac{h}{m_{at}}$$



Atomic recoil: Hall and Bordé, Chu, Biraben

MOLECULAR INTERFEROMETRY

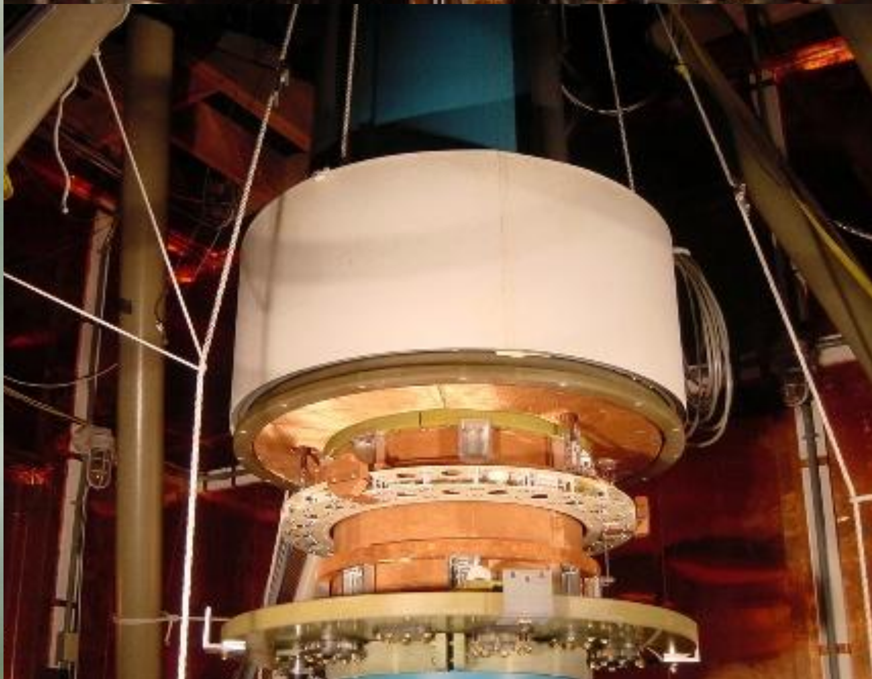
$$m = Tr\left(\rho m_{\mathrm{op}}\right) = Tr\left(\rho H_{\mathrm{int}}\right)/c^2$$



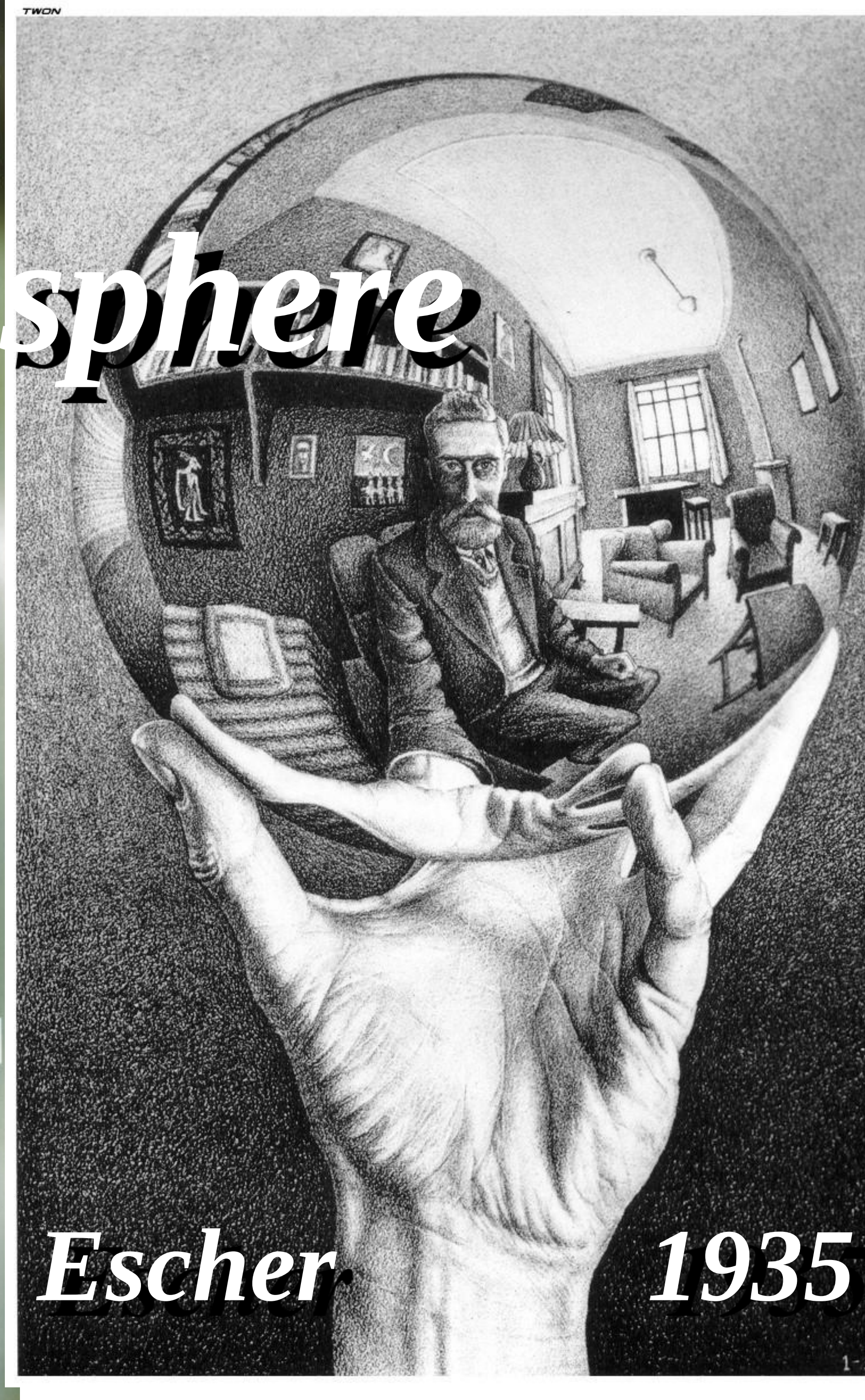
$$\hbar\varphi = \sum_j \left[-\int p_{j\mu} dx_j^{\mu} + c^2 \int \mu_j d\tau_j\right]$$

$$= -\int P_{\mu} dX^{\mu} + \int Mc^2 d\tau = -\int P_{\hat{\mu}} dX^{\hat{\mu}}$$

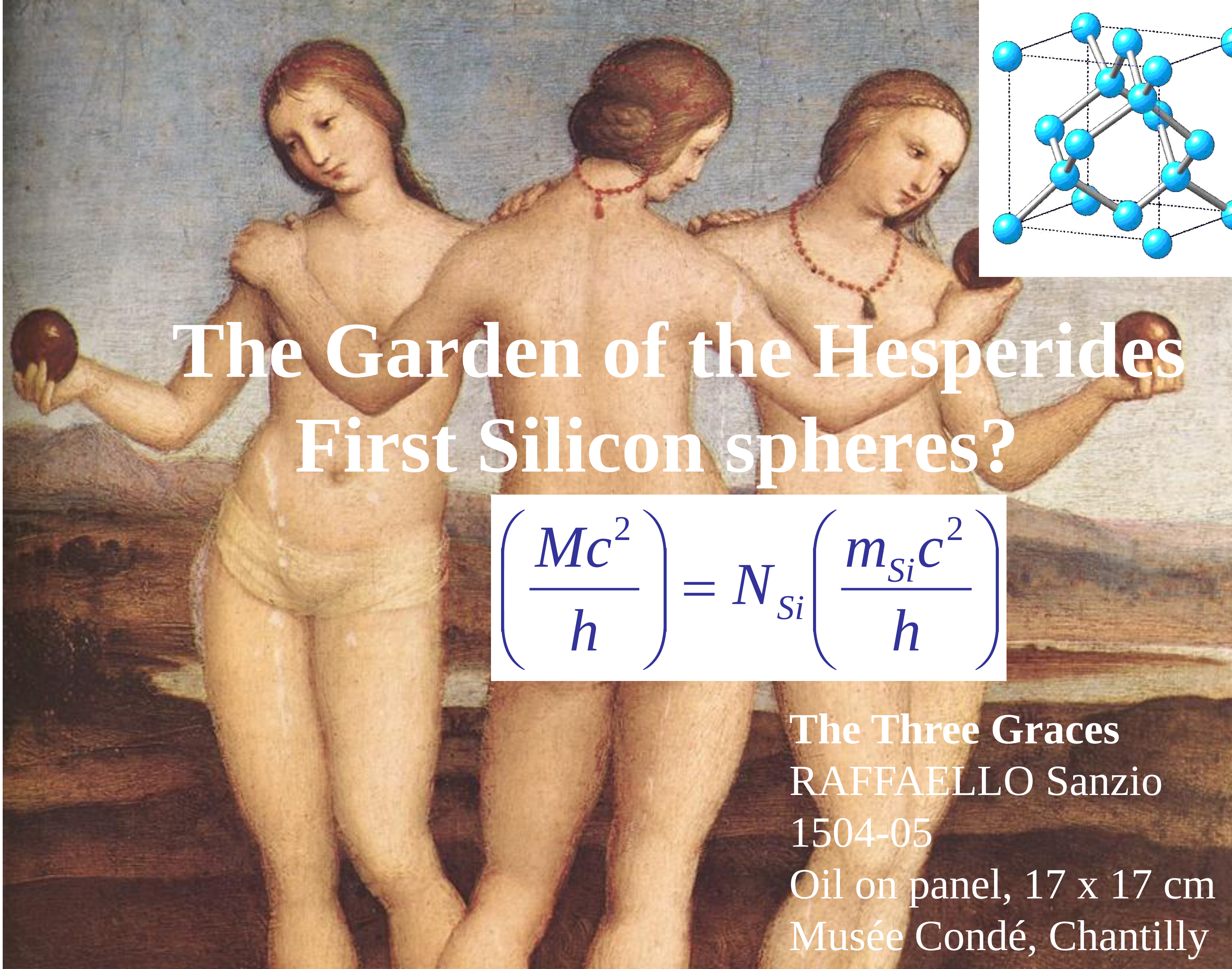
Bridging the gap to the macroscopic world



The Silicon sphere



Escher 1935

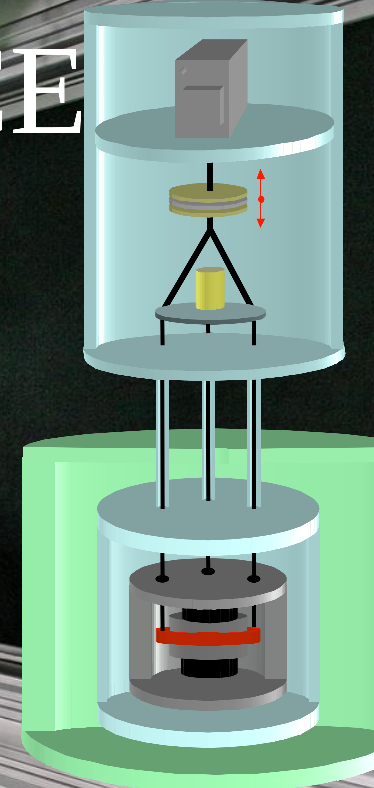


The Garden of the Hesperides
First Silicon spheres?

$$\left(\frac{Mc^2}{h}\right) = N_{Si}\left(\frac{m_{Si}c^2}{h}\right)$$

The Three Graces
RAFFAELLO Sanzio
1504-05
Oil on panel, 17 x 17 cm
Musée Condé, Chantilly

BIPM WATT BALANCE



$$M_{\mathcal{K}} g v = IU$$

$$M_{\mathcal{K}} c^2 \left(\frac{gvT}{c^2} \right) = N_{2e} h f_{\mathrm{J}}$$

CONCLUSION

Mass and proper time are entangled concepts which correspond to conjugate variables in classical mechanics and to non-commuting observables in quantum mechanics.

Their respective units thus require a joint definition in which the unit of mass is defined from the mass difference of the two levels involved in the definition of the unit of time.

A compatible mise en pratique requires to associate a quantum clock with a mass through a phase measurement either by atom interferometry and atom counting or in the watt balance.

The Avogadro number is then obtained directly from the measurement of the de Broglie-Compton frequency of the carbon atom