PBS Mathline®

Algebraic Thinking Math Project

Drip, Drop, Drip, Drop

Algebraic Thinking Focus

Modeling or simulating real world situations can help students develop the idea of function. Such experiences build an understanding of variable, the relationship between variables, and generalizing these relationships.

Lesson Objective

Students will design and carry out an experiment to model a real world situation. They express their results with multiple representations and use the results to make predictions.

Overview of the Lesson

Students design an experiment to model a leaky faucet and determine the amount of water wasted due to the leak. Using the data they gather in a table, students graph and write an equation for a line of best fit. Students then use their derived equation to make predictions about the amount of water that would be wasted from one leak over a long period of time or the amount wasted by several leaks during a specific time period.

Materials

For each group:

- Timer/stopwatches
- Pitcher or other container of water
- Paper cup and thumbtack/paper clip
- Funnel
- Graduated cylinder
- Chart paper and markers

Procedure

Prior to the Lesson: Students should have prior experience interpreting tables and graphs. They should also have worked with finding a function rule for a set of data. It would be helpful if they had designed models to simulate real world events.

1. **Introduction**: Have students close their eyes and listen to the sound of dripping water.

   See the Resources section of this lesson for an appropriate sound effects CD.

   Explain to the students that on very cold nights people often allow their faucets to drip during the night if they are concerned that the water pipes might freeze.

2. **Designing the Experiment**: Ask the students to estimate how much water such a dripping faucet would waste in one day. After they share their predictions have them design an experiment to determine a better estimate for the amount of water lost by a leaky or dripping faucet.

3. **Performing the Experiment**: Students should work in small groups to carry out the experiment. Each group should make a table and graph of their results. They should note any problems they encounter.

4. **Discussing and Interpreting the Results**: Facilitate a class discussion in which the students determine that their results do indicate a linear relationship. Then have them choose one set of data and use that data to find a line of best fit and the equation of that line.

   Examples of student work are at the end of this lesson.

5. **Displaying the Results with a Graphing Calculator**: If possible, have students use the TI-83 or another graphing calculator to enter the data in lists and plot a scattergram. In the y = menu, they should enter the equation for the line of best fit they derive. They should adjust the equation, if needed, to improve the line of best fit. Next they should use the table and/or graph to compare the results with their original predictions for the amount of water that would be wasted by one leaky faucet each day. Finally, they can predict the amount of water that would be wasted for different lengths of time by their entire city if each household had one leaky faucet.

Mathematically Speaking

"There are many payoffs for ...modeling...in the study of school mathematics. Students maintain and improve language skills. Students realize that there are serious uses for mathematics at all levels; you don't have to wait until you study calculus to answer meaningful questions... Students begin to appreciate the complexity of the world around them... Students develop connections, not just between math and contexts but within mathematics itself; modeling uses all the mathematics we know, and good questions beget new questions."

Landy Godbold in *The Nature and Role of Algebra in the K-12 Curriculum*, 1998

The leaky faucet problem deals with data in a context that has meaning for students. It relates to the issue of resource depletion worldwide. When students interpret data linked to real world situations, concepts related to variables are easier to grasp and retain. For example, in this lesson it was easy for the students in the video classroom to understand that the amount of water wasted (dependent variable) changed according to the length of time the faucet leaked (independent variable). The rate of dripping water provided a concrete meaning for slope and the students in the video classroom could easily see that a constant rate produced a linear graph. The students understood that the line of best fit provided an equation for the relationship. The equation could be used to predict other values than those they obtained in the experiment. The wider the variety of problem-solving experiences students have, the more meaning they develop for terms such as dependent and independent variable and line of best fit and the more likey they are to be successful using these concepts in a new context.

An excellent way to get students to think about the variables (time and amount of water wasted) is to ask them to estimate the amount of water wasted by a leaky faucet in one day. This step focuses students' attention on the processes they will use when they interpret the data and make predictions based on their results. Finally, comparing actual results to estimates is a good checking device for experimental results and a stimulus for conversation that leads to better estimates.

Designing the experiment is itself a worthwhile experience. Students need to consider the components of a good model. The teacher does not need to provide the details of such an experiment. Instead, students work together to design an appropriate model. This allows them to practice problem-solving skills that will be useful throughout their lives.

The students in the video used an informal way to find a line of best fit. As shown in the video, they first drew a line through the set of data points starting at (0,0) and passing through the perceived middle of the data points. Then they chose a representative point from their data (80, 30) that was on the line and used the real world meaning of that point to establish the drip rate for the leak. In other words, since it took 80 seconds to collect 30 milliliters of water, students decided to try, "Amount of water lost = .375x number of seconds faucet leaked," or "y = .375x" as the rule for the linear function. This is a meaningful way for students to find a "line of near fit" and build towards a conceptual basis for linear regression.

Examples of student work are at the end of this lesson.

The use of graphing calculators allowed students to see more precisely how the line of best fit passed through the data points from their experiment and, more importantly, made it easy for them to experiment to refine the equation. Based on their adjustments the students settled on y = .395x and used that equation for their predictions about the amount of water lost over longer periods of time and for large numbers of leaks. This activity illustrates to students the potential of mathematical equations to facilitate predictions in many real world situations. Students grades 3-8 need to realize the power of mathematics in examining real world problems and issues.

Another topic related to this lesson is discrete versus continuous functions. The video teacher made the observation that some groups connected the points on their graphs while others did not. She asked the students to address which method was more appropriate. While the students in the video classroom seemed familiar with continuous graphs and quickly agreed that this situation generated a continuous function, other students may not know these terms. Further, many students at this level have only dealt formally with continuous functions for which the coordinates are all connected. Some may not have encountered discrete functions for which there is no connected graph as there are no function values between the plotted coordinates. The question posed by the teacher provides a springboard for a discussion of discrete versus continuous functions.

Related Research Findings

Siegel (1986) has found that using real data makes mathematical concepts more accessible to students. Coordinate plotting made more sense to seventh graders when they were plotting points that represented data that they had gathered to see if there was a relationship between shoe size and family size. The use of "X" as a symbol to stand for a measurement presented no problem for sixth graders and gave them a better feeling for the meaning of the word "variable."

Writing an equation for a straight line whose graph is displayed and whose intercepts are labeled was possible for only 5% of 17-year olds in a national sample. Only 20% of students who had completed two years of algebra where able to perform this task (Carpenter, 1981). Unless students have had some extensive experiences building on y = x and other linear functions of the form y = mx, generating an equation for a line of best fit without the use of electronic technology may be very difficult. When given a choice of equations in two variables and a table of values, only 39% of eighth-grade students can select the equation that fits the values in the table (Blume and Heckman, 1997).

In the video we see the teacher use the point (80, 30) as a reference point and ask the students to determine the unit rate, or unit factor. Kaput has discussed the unit factor approach as a route into linear functions in the Related Research Findings section of the ATMP lesson guide *Bead-dazzling*.

One effective approach to help students see the relationships between tables, graphs, and equations has been developed by Robert Davis and is described in Kieran (1993, pp. 187-190). The lesson helps students find a pattern that reveals that in the equation y = mx + b, m is the slope and b is the y-intercept, so that if the slope and y-intercept are known (from the graph or from a table) the equation can be written.

An alternative to the method of finding a line of best fit in the video is described by Siegel (1986, p. 82). It involves using vertical lines to separate the points in the scattergram into three groups having approximately the same number of data points. Students then find the median value of x and the median value of y for each group of points. They plot the three points and lightly draw a line connecting the leftmost and rightmost of these three points. Then they place a ruler along that line and slide it, parallel to the line, one-third of the way toward the middlemost of the three points. The slope of the line, also the unit factor, can be determined by using the coordinates of the two points that were used to draw the initial line.

Making predictions is based on the assumption that the line of best fit approximates the data set beyond or between known values from the equation of the line or by reading the graph of the line, when x in given y can be determined.

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References

❖Carpenter, T. P., M. K. Corbitt, H. S. Kepner, Jr., M. M. Lindquist, and R. E. Reys. Results form the Second Mathematics Assessment of the National Assessment of Educational Progress. Reston: National Council of Teachers of Mathematics, 1981.

❖Kieran, Carolyn and Louise Chalouh. "Prealgebra: The Transition From Arithmetic To Algebra." Research ideas For The Classroom: Middle Grades Mathematics. Ed. Douglas T. Owens. New York: Macmillan, 1993.

❖Siegel, Murray. "Real World Help for the Middle Grades Mathematics Classroom." Teaching Mathematics: Strategies That Work. Eds. Mark Driscoll and Jere Confrey. Portsmouth: Heinemann, 1986.

Extensions

- Students can gather information about wasting and conserving water (see Technology Connections section). They can then test the validity of the information they find by designing their own experiment.

Example: The Earthworks Group, authors of *50 Simple Things Kids Can Do to Save the Earth*, claim that the average shower uses 25 gallons of water and the average bath requires 50 gallons of water.

Don't forget to have the students use their findings to determine how much water can be saved if everyone in their community takes a shower instead of a bath for one day, one week, or one year!

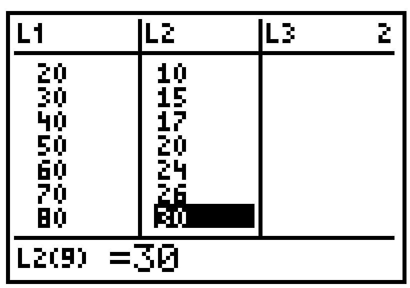
- Students can gather facts about wasting water. Based on the information they find students can make up multiple choice questions that require the reader to make estimates based on the given information. Students can then present their questions to each other and have follow-up discussions that focus on improving their number sense and estimation skills.

  Example: You can save 20,000 gallons a year by not letting the water run while you wash the dishes. That's enough to fill:
  (a) a garbage can (b) a cement truck (c) a swimming pool

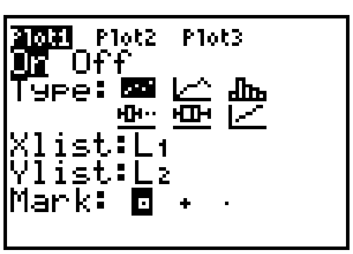
Technololgy Connections

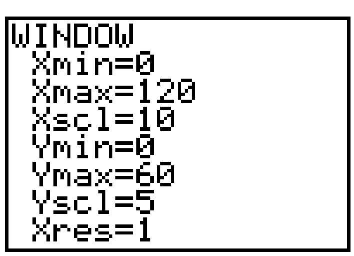
- If graphing calculators such as the TI-83 are available, it is effective to have the students plot the data from at least one of the groups, enter the equation they derive for the line of best fit, and adjust that equation, if needed.

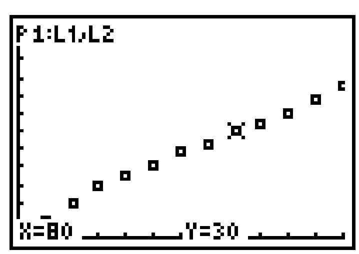
  First, enter the data for time in seconds and amount of water in milliliters into L1 and L2:



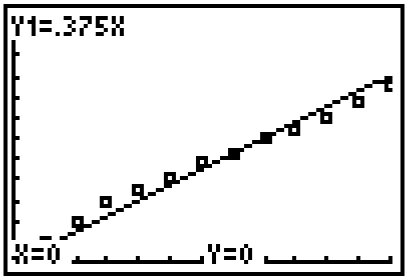
Next, set up the graph, using the STAT PLOT menu, set the viewing window, and view the scattergram:

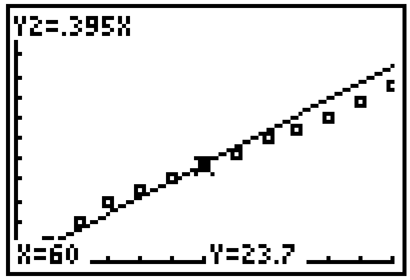






Enter the equation for the line of best fit in the y = menu, graph, and adjust the equation, if needed:





A discussion about setting the viewing window is important. Students must learn to choose appropriate values for various situations before they can use graphing calculators most effectively. Of course, for this situation, the minimum x and y values are both zeroed and the maximum number of seconds and milliliters can be used for the maximum x and y values, respectively.

Note: Detailed instructions for entering data in lists, setting up statistical plots, setting graphing windows, and graphing are available for each TI graphing calculator model. Use the Quick Search option on the TI home page (www.ti.com) to locate the appropriate guide.

- Let students search the Internet to gather information about suggested ways to conserve water. You can have them design experiments to test the validity of those suggestions. Alternatively, they can use the information to make up related questions / problems. Sites that you may find appropriate include the following:
  http://wwwga.usgs.gov/edu/sc4java.html
  http://www.n-jcenter.com/1998/Aug/27/ENV1.htm

Resources

Articles:

❖Godbold, Landy. "Why Modeling Matters." The Nature and Role of Algebra in the K-14 Curriculum. Washington, DC: National Academy Press, 1998.

❖McCoy, Leah. "Algebra: Real-Life Investigations." Mathematics Teaching in the Middle School. February 1997, 221-224.

Other:

❖3M Materials
http://www.mmm.com

❖Compact Disc with sound of water dripping (cut #14)- Sound Effects by Madacy Music Group, Inc.; PO Box 1445; St. Laurent, Quebec, Canada H4L 4Z1

❖*Principles and Standards for School Mathematics:* (Draft), Reston, Virginia: National Council of Teachers of Mathematics (NCTM), 1998.
http://www.nctm.org

❖Texas Instruments, Inc.
http://www.ti.com

Ideas for Online Discussion

Ideas are linked to the Principles and Standards for School Mathematics.

1. In this lesson, students model a leaky faucet to gather data. Compared to providing data for the students, this approach obviously requires additional time. When is this extra time worth the benefits for students?

2. When students deal with real world data, the numbers are often "messy." What effect does this factor have on student learning?

3. This lesson touched on the issue of resource depletion. Do you think it is appropriate to present students with problems that reflect real world issues? If so, what are some appropriate examples you have used?

4. What guidelines do you use in choosing problems for your students, especially when they work in groups?

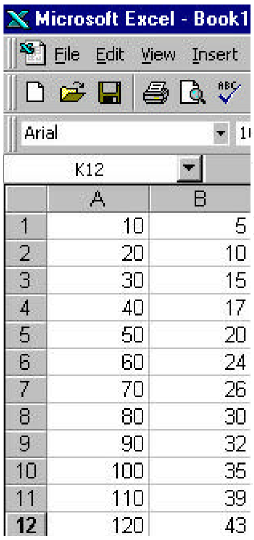
5. Graphing calculators were used in this lesson to plot the scattergram and the line of best fit. Do you think this as an appropriate use of technology? Why or why not? In general, what are the issues related to the use of graphing calculators in the grades 3-8?

6. The National Council of Teachers of Mathematics (NCTM) recommends that dealing with data is a topic that should receive increased attention. This is the case in many new mathematics program, including formal courses in algebra. Do you think this increased attention is appropriate? What challenges does this increased emphasis create for teachers in grades 3-8 and how can we address these challenges?

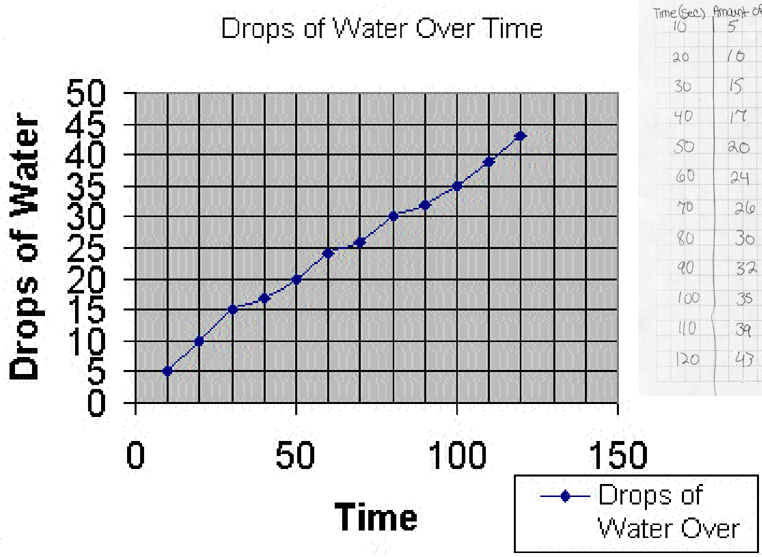
7. How can activities involving gathering, analyzing, and/or graphing data be structured to maximize the development of algebraic thinking?

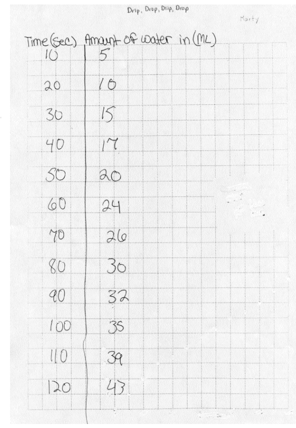
Examples of student work

Group 1



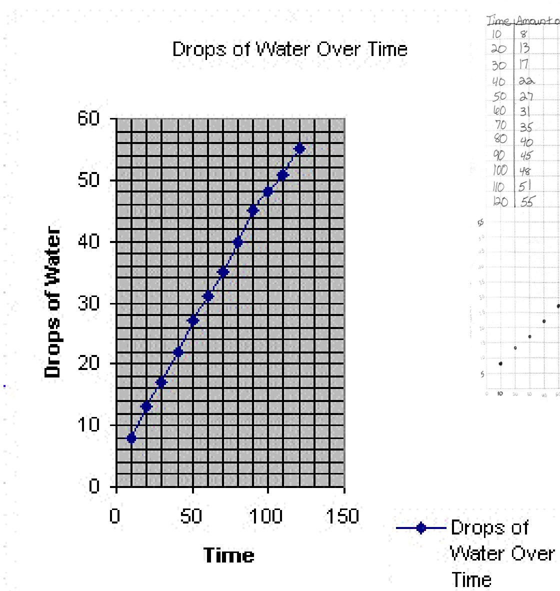
	A	B
1	10	5
2	20	10
3	30	15
4	40	17
5	50	20
6	60	24
7	70	26
8	80	30
9	90	32
10	100	35
11	110	39
12	120	43

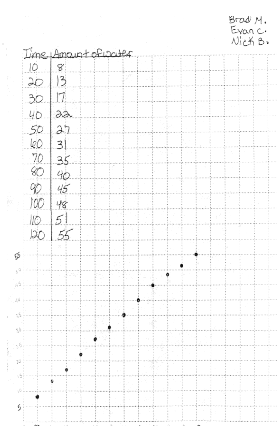




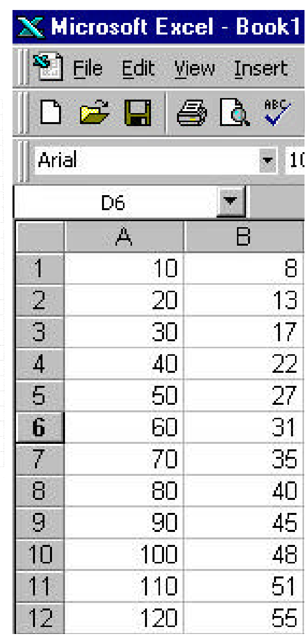
Time (Sec)	Amount of water in (mL)
10	5
20	10
30	15
40	17
50	20
60	24
70	26
80	30
90	32
100	35
110	39
120	43

Group 2





Time	Amount of water
10	8
20	13
30	17
40	22
50	27
60	31
70	35
80	40
90	45
100	48
110	51
120	55



	A	B
1	10	8
2	20	13
3	30	17
4	40	22
5	50	27
6	60	31
7	70	35
8	80	40
9	90	45
10	100	48
11	110	51
12	120	55

Students in the video did not use Excel. This has been added to show how it could be used if available.