祖冲之
与大明历法

ZU CHONGZHI
AND THE
CHINESE CALENDAR
REFORM OF 462 AD

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Cover Picture: A modern artist's impression of Zu Chongzhi

SECTION 1:INTRODUCTION

A BRIEF BIOGRAPHY OF ZU CHONGZHI

Zu Chongzhi (429-500 AD) is one of the most famous ancient Chinese mathematicians in both China and the Western world, but his contributions in the field of astronomy are largely ignored outside Asia, partly due to ignorance of the Chinese system of astronomy. This project seeks to address that gap in understanding, and also to explore the interesting question of whether Zu Chongzhi's achievements, while under-rated in the West, might also be over-rated in his own homeland.

First of all, a description of Zu's life and work will be necessary. Zu Chongzhi was born in a family with a tradition of scholarship and scientific research. The Zu clan had their ancestral home in Fanyang, to the southwest of what is now Beijing. During the barbarian rebellion of the early 300s (more of which will be spoken in the next section), however, they were forced to flee south across the Yangzi River and settle in the new southern capital of Jiankang (present-day Nanjing). Chongzhi's grandfather Zu Chang was himself a skilled engineer and inventor who held the post of Minister of Construction (Da Jiang Qing) in the imperial court. His father Zu Shuozhi held the post of Minister by Appointment (Feng Chao Qing), somewhat equivalent to a technical consultant.

From his youth, Zu Chongzhi had a keen interest in mathematics and astronomy. His inquiring nature led him to read widely into scientific works both old and new, but always with a critical and analytical eye. In his 20s, he was appointed by the reigning emperor Liu Jun to serve in the Imperial Institute (Hualin Xueshen), the largest research body within the state. During this time, based on the work of previous experts from the Han and Wei Dynasties, he derived the most refined and accurate value for π in the world up to that time – between 3.1415926 and 3.1415927. This value, correct to 7 decimal places, was far more exact than the previous value of 3.14 derived by Liu Hui in the Wei Dynasty, and in fact was not surpassed until the 15th century, when a value to 17 decimal places was obtained.

Zu Chongzhi was later posted out of the capital, as a staff officer to the governor of the province of South Xuzhou, based in Jingkou (present-day Zhenjiang). While holding this post, he researched into the workings of the Yuanjia calendar which had been in use since 443, discovered its flaws, and privately formulated a new calendar to replace it. Upon its completion in 462, he submitted it to the imperial court. However, due to political reasons (which will be explained in Sections 5 and 6), it did not come into use until the year 510. By then, Zu Chongzhi had been dead for10 years.

Zu Chongzhi was a man of many talents. Besides his work in astronomy and mathematics, he was said to be a master of court music and the mathematical games 'Bo' and 'Sai'. He also engaged in several feats of engineering and invention in his later life. From his post in South Xuzhou, he was promoted first to magistrate of the county of Louxian, and then to Executive Secretary of Protocol (Yezhe Pushe) at the imperial court. Then, in 478, the powerful Grand Marshal Xiao Daocheng (who usurped the throne the next year) commissioned Zu to reconstruct the famous "south-pointing" chariot (Zhinan Che) which had last been built by the genius inventor Ma Jun in the Wei Dynasty, 250 years ago. This chariot incorporated the figure of a man, arm outstretched, who would rotate to point south no matter which direction the chariot faced. However, the relevant technology for building it (said by modern researchers to involve advanced differential gears) had been lost in the tumultuous years since the Wei. Zu Chongzhi succeeded in this task, but a traveling "expert" from the North, named Suo Yulin, then claimed to also know how to construct such a chariot. The emperor therefore held a demonstration for both chariots in the Imperial Gardens, whereupon Suo's chariot proved much inferior and was confiscated and burnt.

In the 490s, Zu was promoted to Captain of the Zhang River (Zhangshui Xiaowei), one of 5 Captains in the elite Palace Guards. He wrote a Treatise on Preserving Peace on the Borders (An Bian Lun), advising the imperial court to promote farming in the frontier regions, including by the army garrisons, so as to stabilize the economy and population. The reigning emperor Xiao Luan then sent him on a tour of the provinces, to find appropriate sites for irrigation works which could benefit agriculture. This was interrupted, however, by the outbreak of war with the Northern Wei Empire, which gave the court more immediate concerns.

Zu Chongzhi's other projects included a reconstruction of the Wooden Oxen and Rolling Horses invented by another genius from 250 years ago, Zhuge Liang of the Shu-Han Kingdom. These have been variously interpreted by modern commentators as either simple wheelbarrows (unknown in the rest of the world before the 11th century) or the earliest robots in history[1]. He also built a "self-powered boat" (probably a paddle-wheel), which traveled more than 50km in a day when demonstrated on a small river just south of Jiankang, and a hydraulic millstone which was demonstrated in the Imperial Gardens.

Zu Chongzhi was a prolific writer who wrote annotations to various classic texts: the mathematical classic "Jiuzhang Suanshu", the Daoist writings of Laozi, Zhuangzi and the Yi Jing (Book of Changes), as well as the Confucian classics Lun Yu (the Analects) and Xiao Jing (Book of Fillial Piety). He wrote a book of ghost stories (a genre which was becoming very popular at the time) entitled "Accounts of the Strange and Bizarre" (Su Yi Ji). But his most famous written work is undoubtedly the "Zhui Shu", a mathematical manual co-written with his son Zu Geng. It was made a standard textbook for the Imperial Academy in the Tang Dynasty, and was even used in Japan and Korea. Unfortunately, no copies of this book have survived to the present day.

[1] Refer, for example, to the website http://www.taiwaninfo.org/info/sinorama/8511/511032e1.html

SECTION 2

HISTORICAL BACKGROUND ON THE NORTHERN AND SOUTHERN DYNASTIES

In order to understand the cultural and political issues behind the calendar reform of 462 AD, we will need to take into account the history of the period in Chinese history known as the Wei, Jin and Northern and Southern Dynasties (Wei Jin Nanbei Chao). It would be impossible to relate in detail the many events of this grim but fascinating 400-year period within the scope of this project, but we will try to outline the key developments up to the year 462.

The story begins with the decline of the once-mighty Han Dynasty in the late 2nd century. While the imperial court was paralyzed by factional struggles, governors and generals in the provinces took the chance to create their own semi-independent spheres of influence. In time, they began to war with one another for territory. One particularly shrewd warlord, Cao Cao, took control of the capital and got himself appointed as Prime Minister. Two others, Sun Quan and Liu Bei, gained control of the Yangzi delta and the Sichuan basin respectively, resulting in the military stalemate of Three Kingdoms which was made famous by the later novel written in the Ming Dynasty. In 220, upon the death of Cao Cao, his son deposed the puppet emperor and founded the Wei Dynasty. Liu Bei and Sun Quan followed suit, declaring themselves emperors of the Han (Shu-Han) and Wu kingdoms. The deadlock lasted until 263, when Shu-Han was conquered by the Wei in a surprise attack through supposedly impassable terrain. 2 years later, the powerful Wei Prime Minister Sima Zhao died, and his son Sima Yan, following in the footsteps of the Cao family, forced his emperor to abdicate and founded his own Jin Dynasty.

In 280, a Jin invasion fleet finally conquered the Wu kingdom, and the Chinese Empire was reunified. However, this was to prove short-lived and a time of even greater division was to come. Upon the death of Sima Yan in 290 and the succession of his simple-minded son Sima Zhong, a vicious power struggle ensued between various imperial relatives. For 10 years, Sima Zhong's empress Jia Nanfeng had the upper hand, but in 301 she was betrayed and executed by Sima Lun, one of Sima Yan's younger uncles. When Sima Lun tried to take the throne for himself, however, other ambitious princes of the Sima clan rose against him. Having eliminated the usurper, they then fought among themselves for the right to take his place. In the general civil war that followed, most of the Sima princes died at each other's hands. Meanwhile, a nomadic 'barbarian' race, the Xiongnu, who had once been powerful rivals of the Chinese Empire, but were defeated and subdued by the Han Dynasty, took this chance to revive their fortunes. They rebelled against the Jin and proclaimed their own independent kingdom.

The demoralized and disunited Jin armies could not stand against the swift cavalry of the Xiongnu, who over-ran the Jin capital of Luoyang in 311. A nephew of Sima Zhong escaped and established a new capital at Changan (present day Xian), but had to surrender in 316. As the Xiongnu slowly occupied all of north China, only one region remained safe from their reach: the lands south of the great Yangzi River, which could not be crossed by cavalry. Innumerable Chinese families fled south to escape war, famine and barbarian rule. The prince garrisoning Jiankang (present day Nanjing), Sima Rui, thus proclaimed himself the new emperor of the Jin. (This regime is referred to as the Eastern Jin by historians, to distinguish it from the previous "Western Jin" which had its capitals further west in Luoyang and Changan.)

In the north, another 'barbarian' race called the Jie (probably descended from the Kushans, old enemies of the Xiongnu) challenged and eventually defeated the Xiongnu in 329, founding the Zhao kingdom. In 349, Zhao collapsed into civil war, and was carved up by two other races: the Xianbei from present-day Mongolia and Manchuria and the Di from the Sichuan region. At the same time, in the northwestern Gansu corridor leading to Central Asia, the Zhang family - descendants of the last Jin governor of the region - doggedly maintained an independent kingdom while nominally pledging loyalty to the Eastern Jin. But by 376 the Di kingdom of Qin, ruled by the brilliant Fu Jian, had conquered both the other kingdoms and now turned its attention to the south.

The Eastern Jin, after enduring several rounds of civil war in its first decade, had remained politically stable for nearly 50 years. Several military campaigns had been launched to retake the north, but all had failed from poor leadership and logistics. As most of the Jin emperors succeeded to the throne as children and died in early adulthood, the court came to be dominated by a few aristocratic families, most notably the Wang and Xie clans. These "scholar-aristocrats" (shi zu) were descended from a long line of illustrious ancestors who had held high positions in government dating back to the Han. Since the beginning of the Western Jin, the scholar-aristocracy had gradually become an exclusive elite in itself and held all the key posts in the imperial court, while also monopolizing the military and the arts. When Fu Jian led a huge army to invade the south in 383, the Jin army which faced him was led by generals of the Xie family.

At the Fei River, the Jin army met a Qin vanguard force 5 times its size. However, by a combination of psychological warfare and plain luck, the Jin generals managed to convince the Qin army that they had been defeated. As the Qin retreated in panic, they were decimated by pursuing Jin soldiers. Fu Jian returned to the north to find that all the other races in his empire had seized the chance to revolt against him, and he was captured and killed in 385 by the Tibetan Qiang race, who founded their own kingdom retaining the name of Qin (known to history as the Later Qin). From then until 394, the western region around Changan came to be controlled by the Qiang, and the eastern region of the north China plain by Xianbei of the Murong family, while the Gansu corridor was ruled by Fu Jian's Di subordinate Lü Guang. In 396, however, another Xianbei tribe, the Tuoba, swept south from the Mongolian region and soundly defeated the Murong, who had to flee back to Manchuria. Around the same time, two of Lü Guang's generals rebelled and formed states of their own. This process of annexation and division continued, until by 409 north China was split into a total of 9 kingdoms.

Meanwhile, the situation was also changing in the south. Despite surviving Fu Jian's invasion and taking advantage of his downfall to regain a large stretch of territory up to the south bank of the Yellow River, the Eastern Jin nonetheless began to decline due to corruption and factionalism. Army mutinies in 397 and 398 and a peasant rebellion from 398 to 402 so weakened the central government that in 402 a provincial governor, Huan Xuan, sailed his fleet down the Yangzi, easily defeated the central army and took control of Jiankang. One year later, he deposed the emperor and proclaimed his own dynasty. However, before long a general named Liu Yu began to plot an insurrection to restore the Jin emperor to the throne. Liu was a commoner who had risen through the ranks during the campaigns against the peasant rebels, and in 404 he and a band of co-conspirators led 1,700 men in an attack on the capital, defeating Huan Xuan's army against great odds. Huan fled back up the Yangzi, but was finally killed after a series of naval battles. Liu Yu entered Jiankang as a loyalist hero, and soon rose to the position of commander-in-chief of the Jin army. In 410, he led a military expedition which conquered the kingdom of Southern Yan in the Shandong peninsula, ruled by remnants of the Murong who had not fled north. Building on the prestige gained from this victory, he then moved to systematically eliminate his former comrades in arms, who had now become potential rivals for power.

In 416, Liu Yu led another campaign against the now-ailing Qiang kingdom of Qin. By late 417, he had taken the Qin capital of Changan, but the sudden death in Jiankang of his right-hand man Liu Muzhi compelled him to return south to safeguard his political future. As a result, in 418 all the territory he had gained was lost to another 'barbarian' kingdom. However, Liu Yu's immediate ambitions were limited to the political situation in Jiankang. He got himself promoted to prime minister and ennobled as the Duke of Song, which opened the path for him to secure absolute power. In 420, this former loyalist forced the abdication of the last Jin emperor and founded the Song Dynasty, the first of the Southern Dynasties. (This dynasty is often referred to by historians as the Liu-Song, to avoid confusion with the later Northern and Southern Song Dynasties in the 10^{th} to 13^{th} centuries.)

Liu Yu died at the age of 60 after only 2 years as emperor, and was succeeded by his eldest son Liu Yifu. The 17 year-old Yifu chose to ignore matters of state, and the Tuoba state of Wei took this chance to capture several Song outposts south of the Yellow River. 3 senior Song ministers then orchestrated a coup to replace Yifu with his younger brother Yilong. In 424 Liu Yilong ascended the throne and took the reign title of Yuanjia ("beginning of goodness"). In 426 he executed the 3 powerful ministers who had put him on the throne and began to personally administer his empire. Over the next 20 years of relative peace and stability, the economy of the south grew by leaps and bounds.

In the north, the Wei empire (also known as the Northern Wei or Tuoba-Wei to differentiate it from the Wei of the Three Kingdoms), under its ambitious ruler Tuoba Tao, began to slowly conquer the other northern kingdoms, until by 439 only one remained: the Liang kingdom in the Gansu corridor, ruled by Juqu Mujian. In that year, Tuoba Tao led an invasion force to besiege the Liang capital, forcing Juqu to surrender. Northern China was thus reunified under the first of the Northern Dynasties.

The two empires of Wei and Song now faced off across a long border between the Yellow and Huai Rivers. Liu Yilong had long harboured ambitions to retake the north, and several bitter but inconclusive border wars were fought from 430 to 450. In the winter of 450, a Wei invasion force led personally by Tuoba Tao drove as far as the north bank of the Yangzi, just opposite Jiankang. The obstacle of the river proved still too great, however, and Tuoba Tao turned back. Both sides lost thousands of soldiers, and entire villages on the border were slaughtered by the retreating Wei army. The carnage did not end with Tuoba Tao's assassination by a eunuch in 452, as Liu Yilong took the chance to launch another attack on the north. This failed disastrously, and Liu Yilong himself was assassinated the next year by his own eldest son, Liu Shao.

The Crown Prince of the Song had been implicated in a conspiracy to use witchcraft against his father and, fearing for his life and position, led his bodyguard in an attack on the imperial palace. Liu Yilong was murdered in his own study, and Liu Shao then orchestrated his own succession to the throne. Liu Shao's brother Liu Jun, who was then leading an army against aboriginal tribes on the middle Yangzi, decided to rise against the unfilial usurper. With the support of his uncle Liu Yixuan and younger brother Liu Dan, he sailed his fleet to attack Jiankang. He defeated Liu Shao's army outside the capital, and then proclaimed himself emperor. Liu Shao, his co-conspirators and his wife, concubines and children were all executed.

Liu Jun took the apt reign title of Xiaojian ("establishment of filial piety"), but his reign did not enjoy a good start. By 454 he had fallen out with his uncle and former ally Liu Yixuan, who rose in rebellion on the upper Yangzi. Unlike Liu Jun, Yixuan's attack on Jiankang failed. He fled back up the river, but was captured and killed along with his 16 sons, while his daughters became Liu Jun's concubines. In 457, Liu Jun changed his reign title to Daming ("Great Brightness"), but there was little brightness to be seen on the political scene. The emperor grew increasingly suspicious of his other brothers, and by 461 had killed 4 of them, including Liu Dan. The Northern and Southern Dynasties period was a time of incredible cruelty, which thankfully has not been distorted in history by romanticism of the sort seen in the Three Kingdoms novel. Deceit and dishonour seemed to hold sway in almost every dynasty, but the Song and the Southern Qi that came after it have become notorious for the intense fratricide which saw emperors ruthlessly murdering their closest relatives, often out of simple jealousy and paranoia. Out of Liu Yu's 7 sons, only one died peacefully, though as a deeply depressed alcoholic. Of Liu Yilong's 18 sons, 6 died violent deaths, 5 were poisoned, and 1 had to defect to the Tuoba–Wei to stay alive. Liu Yilong may have started this trend by executing his brother Yikang in 451, but Liu Jun was the first to make it a regular feature of his reign.

However, Liu Jun's personality was more complex than that of a mere bloodthirsty tyrant. He made several useful reforms to the laws and tax policies of the Song, maintained relative peace with the north throughout his reign, and was a devout Buddhist and accomplished poet. He was also a shrewd politician. Being descended from a low-class family, he feared that the scholar-aristocracy would not give him their full loyalty, but knew that stripping them of their posts would provoke a revolt. He thus took measures to limit the political power of the scholar-aristocracy by gradually converting their high official posts to ceremonial ones and transferring the actual power to their subordinates, who were usually educated commoners with old, proven ties of loyalty to the emperor.

A classic example is Dai Faxing, who served as a staff officer to Liu Jun while he was still a prince, and was in 453 appointed as a Palace Secretarial Attendant (Zhongshu Tongshi Sheren). These were originally clerks serving the Palace

Secretaries (Zhongshu Shilang), who took charge of drafting all imperial edicts and therefore had regular policy discussions with the emperor. But under Liu Jun, only Dai Faxing and his colleague Chao Shangzhi, also a commoner, had the privilege of attending such discussions. The aristocratic Palace Secretaries were thus left out of the policy-making process. This exclusive access to (and influence on) the emperor's thoughts on personnel postings and promotions, rewards and punishments, resulted in a steady stream of bribes for Dai Faxing and his colleagues. As he was not constrained by the scholar-aristocracy's elaborate code of ethics, this made Dai Faxing a very rich and powerful man indeed.

Another group of commoners raised to political power were the Minute-keepers (Dianqian), originally humble filing clerks in charge of keeping records and minutes in the army and provinces. Whereas generals and governors had previously been responsible for employing their own Minute-keepers, Liu Jun began to personally appoint them from among his trusted followers. There were now 2 to 4 Minute-keepers to every general or governor's staff, with the added task of observing their superiors' actions and returning to the capital several times a year to report these observations directly to the emperor. Liu Jun used this method to keep track of the loyalty of his governors and generals, especially those who were also his brothers and relatives. The Minute-keepers thus became a feared and hated secret police whom none dared to offend, and they often took advantage of this to interfere with administrative matters. In 461, Liu Jun's 17 year-old brother Liu Xiumao, who was serving as governor of Yongzhou, was frustrated at being repeatedly prevented by his 2 Minute-keepers from having any say in provincial administration. Believing rumours that they were about to report him to the emperor for treason, he killed them and rebelled, but was later betrayed and killed by one of his own staff officers.

This, then, was the uneasy political situation when in 462 a provincial staff officer named Zu Chongzhi presented to Liu Jun, on his own initiative, a new calendar which would bring him into conflict with the powerful Dai Faxing.

SECTION 3

AN OVERVIEW OF CHINESE CALENDARS AND ASTRONOMY UP TO ZU CHONGZHI'S TIME

Having provided the background for the 462 Reform in terms of political history, we will now turn to astronomical history and briefly describe the origins of the Chinese calendar used in 462, as well as the key astronomical concepts behind it.

Chinese astronomy, like that of every other ancient civilization, had its roots in astrology. Starting from the Warring States period (480-222 BC), astrologers began to group the stars into constellations, each with a symbolic significance, in relation to which the motions of the sun, moon and planets were used as portents of earthly events. Eventually up to 283 constellations were identified, and the 28 most important were classified as lunar mansions (xiu)[2]. They were then further divided into 4 'palaces' of 7, corresponding with the 4 seasons and 4 compass directions. There was also a 'central palace' consisting of all circumpolar stars within 40 degrees from the north celestial pole.

The 28 lunar mansions later formed the basis of the Chinese astronomical coordinate system, which sliced the celestial sphere into 28 sectors like an orange, with all the lines radiating from the 'orange stem' of the north celestial pole. Each sector contained one of the lunar mansions, and its width therefore depended on the size of the constellation. As the lunar mansions were spaced out along both sides of the celestial equator, this coordinate system was usually regarded as equatorial[3]. Unlike Greek astronomy, which was based on an ecliptic coordinate system, Chinese astronomy generally ignored the horizon and the ecliptic. It has, in fact, been argued that our modern equatorial coordinate system was inspired by the Chinese, as Tycho Brahe switched to using the equatorial coordinate in the 17th century when he found the Chinese system to be more convenient than the Greek one.

The key difference from the Western system was that, rather than measuring right ascension from the vernal equinox, the Chinese used ruxiu du: degrees east of the determinative star (ju xing) in the relevant lunar mansion. The equivalent of declination was quji du: distance in degrees from the north celestial pole. As the ancient Chinese divided a circle into 365 ¼ degrees, instead of 360 as the Babylonians did, one Chinese degree was in fact equivalent to 360/365.25 of our degrees.

While the Chinese astronomical system would seem to depend upon the concept of the celestial sphere, this idea was still highly contentious in Zu Chongzhi's era. From the Han Dynasty onwards, there had been a lively debate regarding the shape of the heavens, generally split into 3 schools of thought. The oldest school was that of the Celestial Dome (gaitian shuo), which held that the sky was a slowly spinning dome covering a flat earth. At first it was said that the earth was a flat square, but when this proved mathematically unsound, it was modified to a flat circle

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[2] Refer to Appendix I for a list of the 28 lunar mansions, and Appendix III for star maps showing their positions and determinative stars.

[3] Sun and Kistemaker, however, claim that the lunar mansions followed the ecliptic, and only a few of them follow the equator.

and then a convex circle. The school of the Celestial Sphere (huntian shuo) believed that the heavens were a rotating sphere enveloping the earth as an egg envelopes its yolk. However, most scholars did not extend this analogy to consider the idea that the earth, too, might be round like an egg yolk. Lastly, the smallest school, Light and Darkness (xuanye shuo), claimed that the universe had no tangible shape and was of infinite size, and that the heavenly bodies floated in the sky, moving in their own different patterns, like the waves and tides. The earth lay beneath the universe and was neither flat nor round but had infinite depth. This idea of the universe as an ocean was the closest yet to a non-geocentric model of the heavens, though it lacked any scientific basis.

While the "shape of the heavens" debate never completely died down until the Tang Dynasty, the Celestial Sphere school gradually began to gain ascendancy, largely due to the practicality of the armillary sphere and celestial globe invented by its proponents. These two instruments, as well as the water clock and the gnomon (a long pole used to measure the shadow cast by the sun at noon), became the foundation for calendar calculations in ancient China.

The Chinese calendar was lunisolar, using the solar year (measured as the mean time between 2 winter solstices, rather than vernal equinoxes as in Western astronomy) and the lunar or synodic month (the mean time between 2 new moons). However, a solar year is about 11 days longer than 12 lunar months, and this necessitated the addition of a leap (intercalary) month slightly more than every 3 years. By the beginning of the Warring States period, the "¼ Calendar" (sifen li) had been invented, with 365 ¼ as the length of the year. It was found around the same time that 235 lunar months is nearly equal to 19 solar years, with a difference of only about 2 hours. But 19 years of 12 lunar months is still 7 months less than 235. Thus a system was formulated of inserting 7 leap months in every 19 years, bringing the mean length of the month to 29.53085 days. This system, called the zhang method in China, is better known as the Metonic cycle after the Greek astronomer Meton who first used it in the West.

The Chinese method for finding the length of a year was to first find the number of days in a given number of months - for example, 81 months contain 2392 days - and then multiply that by the Metonic cycle (i.e. 19 years contain 235 months). The number of days in a year would in this case be $235/19 \times 2392/81 = 365 + 385/1539$. This accounts for the widely varying fractions (now more commonly represented as decimals) in different calendars which started their calculations with different day-month ratios. The length of a mean lunar month was then found by dividing the length of the year by 12.

Another fundamental concept in Chinese calendars is the 24 solar terms (jieqi), which originally served as seasonal markers for farming. The solar terms cut the ecliptic into 24 sections of 15 days each, and mark the beginning of different stages in the 4 seasons. The even-numbered terms are also called zhongqi, and they include the solstices and equinoxes, among others. The present Chinese New Year usually falls on the new moon closest to the solar term of Lichun (Beginning of spring). However, during the Warring States period, different States began their year in different months, leading to 6 calendars (Huangdi, Zhuanxu, Xia, Yin, Zhou and Lu) being used at the same time, all based on the Sifen calendar. After unifying the

Empire, the Qin Dynasty used the Zhuanxu calendar, in which the year began in winter. This continued for more than a 100 years until in 104 BC, during its Taichu reign period, the Han produced a new, standardized calendar. This began the year in spring (midway between the winter solstice and the vernal equinox) and fixed the month of the winter solstice as the 11^{th} month, a practice that has been followed ever since.

The Taichu calendar took 365.2502 and 29.53086 as the year and month values, but in 85 AD the Han reverted to the original Sifen values. During the Three Kingdoms period, Shu-Han continued using the Sifen calendar, but Wu used the Qianxiang calendar and Wei the Jingchu calendar, both of which derived slightly more exact values based on closer observation of the timing of the winter solstice and better knowledge of the motion of the moon, including the asymmetrical shape of its orbit. The Jin and Liu-Song Dynasties retained the Jingchu calendar, though under different names, until the Song scholar He Chengtian submitted his new calendar to the court in 443, during Liu Yilong's Yuanjia reign period. In the North, astronomers in at least 2 'barbarian' states produced new, advanced calendars for their rulers: Jiang Ji of the Later Qin, and Zhao Fei of the Juqu-Liang (Northern Liang) kingdom. The Tuoba-Wei (Northern Wei) used the Jingchu calendar until 451, when it switched to using Zhao Fei's calendar.

We will now give some details about the Yuanjia calendar, since that was the calendar which Zu Chongzhi sought to reform. Its author, He Chengtian, was probably the pre-eminent scholar of the early Liu-Song period. His interests ranged from history to mathematics to music, and his Yuanjia calendar was the fruit of more than 40 years of research and astronomical observation. Through systematic use of the gnomon, he discovered that the Jingchu calendar had an error of 3 degrees in the position of the sun at the winter solstice, as well as an error of 3 days in the date of the solstice itself. He also proved that day and night are of equal length on the equinoxes. This debunked a theory which the Jingchu calendar had inherited from the Sifen calendar, that day is slightly longer than night on the vernal equinox as it closer to the summer solstice, while night is longer than day on the autumnal equinox because it is nearer to the winter solstice.

Another contribution made by He Chengtian was in the aspect of li yuan (first calendar year). Previous calendars had mostly started their first year from the 11^{th} month, either at the new moon or the winter solstice, such that the first year consisted of only one or two months. This followed the Chinese concept of sui (solar year, which starts and ends at the winter solstice), as opposed to nian (lunisolar year, which starts at Chinese New Year in spring). He Chengtian felt that it made more sense to start the first year in its first month, and so he chose to have the first year of his calendar start at the zhongqi of Yushui (Rainwater) in spring. After 2 years of rigorous assessment by court astronomers, the Yuanjia calendar came into official use in the first month of 445.

Starting from the Taichu calendar of the early Han, the 7 leap months of the Metonic cycle were allocated according to the "no zhongqi" rule, where the first month of a leap year in which no zhongqi fell was repeated as a leap month. Normal months themselves alternated between 29 and 30 days, with occasionally 2 long months in a row. These were all based on the mean moon (ping shuo) – an assumption of the regularity of the moon’s motion. In fact, the moon’s true motion is highly irregular, such that it is possible to have up to 4 long months or 3 short months in a row. Thus, calendars using the mean moon often encountered inaccuracy in their prediction of eclipses or phases of the moon. Full lunar eclipses did not always take place at a full moon, or full solar eclipses at a new moon, as should be the case astronomically. He Chengtian was the first Chinese astronomer to try to follow the true moon (ding shuo) with his Yuanjia calendar. However, this was opposed during the assessment process by the Imperial Astronomer (Taishi Ling) Qian Lezhi, who felt the appearance of several long months or short months in a row was too great a departure from tradition. He Chengtian backed down, and the calendar was eventually revised and adopted without the true moon. He may have hoped to get this decision overturned once the calendar came into general acceptance, but he never got the chance, as he died 2 years later in 447.

SECTION 4

ASPECTS OF THE DAMING CALENDAR REFORM

In the 6th year of the Daming reign (462 AD), the 33 year-old Zu Chongzhi submitted a petition to the Song emperor Liu Jun, beginning with these words[4]:

"Your subject[5] has visited the graves of the ancients and referred to the classics of the remote past, the movements of the celestial bodies in the time of the Five Emperors[6], the timing of the equinoxes during the time of the Three Kings[7], the months and seasons of the "Spring and Autumn Annals"[8], the eclipses recorded in the "Bamboo Annals"[9], the accounts of Sima Tan and Sima Qian[10], the records of Ban Biao and Ban Gu[11], the annotated calendars of the Wei Dynasty, and the daily imperial records of the Jin Dynasty, seeking out unusual events of the past and present and reading the important writings of both our Empire and the barbarian states. He has researched for evidence of the movements of the sun and moon, and proof of the changing positions of the stars, in all the history written since writing was invented, the history of more than two thousand harvests. He devoted himself to work and immersed himself in thought until he could give a full account of all this knowledge. In addition, he personally measured the shadow of the gnomon and observed the readings of the water clock, scrutinizing them down to the smallest unit of measurement and making exhaustive stick calculations[12], checking and verifying the results over time and carefully recording the details.

However, the ancient calendars were full of errors and inaccuracies, and different schools of astronomy disputed among themselves without grasping the key issues at hand. When He Chengtian submitted his calendar, he had the intention of making a reform, but his method was so sketchy that it has now run counter to his original purpose. Your subject has checked the calendar and found 3 major errors: an error of 3º in the positions of the sun and moon; an error of nearly one day in the measurement of the solstices; and an error of up to 40 days in the periods of visibility for the 5 planets, as well as an error of 2 lunar mansions in tracing their respective motions. If the equinoxes and solstices are not fixed at the correct time, then the solar terms and leap months will also be fixed wrongly; if the lunar mansions are not given their correct place in the sky, then astrologers will be unable to make accurate predictions. Since your subject has the good fortune of living in a reign of virtue and prosperity, he has dared take the foolish initiative of creating a new calendar. He

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[4] There are 2 slightly different versions of this text, one in Nanqi Shu and the other in Song Shu. Yan Dunjie collated the best of both versions in his book, and we have relied mostly on his new version.
[5] The ancient Chinese always referred to themselves in the third person, as Chen (your subject), when addressing the emperor. The slightly grandiose yet self-deprecating style seen here was also typical.
[6] The 5 semi-legendary founding fathers of the Chinese race: Huangdi, Zhuanxu, Ku, Yao and Shun.
[7] Probably referring to the first 3 Chinese dynasties: Xia, Shang and Zhou.
[8] Chun Qiu, a history book written by Confucius in the Spring and Autumn period (770-476 BC).
[9] Zhushu Jinian, a history book written in the Warring State of Wei, found in the tomb of a Wei king by a thief in 279 AD and later confiscated by the Jin imperial court.
[10] The father and son who wrote "Records of the Historian" (Shi Ji) in the early Han Dynasty.
[11] The father and son who wrote "History of the Han Dynasty" (Han Shu) in the Later Han.
[12] Ancient Chinese mathematicians used sticks of bamboo, wood or ivory (and sometimes chopsticks) to make calculations.

respectfully proposes the intention of making 2 changes and introducing 3 new methods to our present calendar.

The first of the changes is this: the old zhang cycle of 7 leap months in 19 years contained slightly too many leap months, leading to an error of 1 day after 200 years. This extra day caused a shift in the solar terms and leap months, necessitating corrections to the calendar, and that is why past calendars had to be revised periodically. Let us now change the cycle to 144 leap months in 391 years. The seasons thus remain the same as in the Zhou and Han Dynasties, and from now on the calendar can be used forever without any further deviation.

As for the second change: the "Classic of Yao"[13] states that "When the day is shortest and the mansion of Mao is on the meridian at sunset, that is the middle of winter." From this we may infer that in the Tang era the sun at the winter solstice was more than 50º to the left of the lunar mansion which it is in today. At the beginning of the Han Dynasty[14], the imperial court continued using the Qin calendar[15], and the winter solstice sun was 6º within Qianniu. When Emperor Wu of the Han[16] switched to the Taichu calendar, the solstice sun was recorded as being just within Qianniu[17]. By the time the Later Han started using the Sifen calendar[18], the solstice sun had shifted westwards to 22º within Nandou. During the Jin Dynasty[19], Jiang Ji used a lunar eclipse to find the position of the sun, and from this deduced that the solstice was 17º within Nandou. Now, by referring to the meridian star and verifying this with the eclipse method, we find that the sun at winter solstice is 11º within Nandou. To sum up these figures, there is a shift of 2º in slightly less than 100 years[20]. The old calendars gave a fixed yearly position for the solstice sun, but since there is indeed a shift in the heavens, the coordinates of the 7 Luminaries[21] gradually ceased to correspond to the calendar. Since the resultant error has become so prominent, it is clear that we should make a change. If we changed the figure only to suit the present situation, it would have no long-term effect, and that is precisely the reason for this constant need of reform. Now let us stipulate that the position of the winter solstice shifts slightly from year to year, verifying this by reference to the annotated writings of the Han Dynasty and our own careful observations, so that we may use the calendar for the long term without the bother of repeated revisions.

As for the 3 new methods: Firstly, we take Zi to be the first of the earthly branches, and its position to be due north. As for the lunar mansions, Xu is in the middle of the Northern Palace and therefore represents due north as well. Therefore

[13] Yao Dian, a document in the Book of History (Shu Jing) relating to the legendary Tang kingdom founded by Yao.
[14] The Han Dynasty was founded in 206 BC.
[15] Generally believed to be the Zhuanxu calendar.
[16] Liu Che, better known in the West as Han Wudi, who reigned 140-87 BC. His Taichu reign period was from 104 BC to 101 BC.
[17] That is, about 1º east of the determinative star.
[18] In 85 AD, during the Yuanhe reign period (84-88).
[19] This was actually in the Later Qin kingdom (384-417) of the north, which was later conquered by the Eastern Jin and therefore not recognized by the Jin court as a legitimate government.
[20] Zu Chongzhi seems to have reached this figure mainly from comparing the Sifen calendar with Jiang Ji's reading: a shift of 5º from 85 to around 384. Some scholars have taken this to imply that Zu's own reading was recorded wrongly in the historical texts, and should instead be 15º. However, as we will explain in Section 6, Zu Chongzhi's 11º reading was in fact the most accurate of the 3.
[21] Qi Yao: the sun, moon and 5 planets known to the Chinese.

the beginning of all things should have been from this point. A past scholar, Yu Xi, raised a similar argument in his time. For the new calendar, the sun's position at Great Year 1 is thus set at 1° within Xu. Secondly, Jiazi is the first combination of the stems and branches, and so a calendar should begin in this year. Yet, of the 11 calendars used since ancient times, none has put its Great Year 1 in a Jiazi year[22]. For the new calendar, its Great Year 1 is a Jiazi year. Thirdly, a calendar should begin the motion of the heavens with the start of Great Year 1, but the Jingchu calendar only lists the time remaining from the start of Great Year 1 to the next node or perigee. He Chengtian's calendar even sets a different Year 1 for each of the 5 planets, while retaining the compromise of "time remaining to node" and "time remaining to perigee". Thus the only conjunction is in the sun and moon. This is haphazard and far from the level of perfection that the ancients aspired to. The new calendar introduces a method by which the paths of the sun, moon and 5 planets, as well as the node and perigee, all coincide at the beginning of Great Year 1. Thus we can obtain clear evidence of the motion and conjunction of the planets, simply by calculating from a common point, and that is what your subject considers a true improvement to the methods of the ancients.

If all measurements could be done accurately and proved practically, if the signs of the heavens were easy to understand and the gnomon were easily verified, then what your subject proposes would convince anyone. But that is not the case with real calculations, which require much interpretation and adjustment. There are 2 kinds of reforms in mathematics: the simple and the complex. The simple can be accepted by all, but that does not mean that the complex is useless. Why does your subject say this? Because his theories on the cycle and precession are presented in concrete and structured terms, and are not overly complicated at all. He has measured to the minutest degree for the sake of accuracy, and spared no effort for the sake of making this calendar useful for all time. This is far better than contemplating a problem without realizing its cause, or knowing the problem but not correcting it. The only thing to fear is those men who worship the past so much that they invariably follow its ways in the present, for they would rather trust what they hear with their ears than the evidence before their eyes. So your subject has expended all his ability on understanding the sun and moon, that his results may be above the doubt of mortal men. If what he submits is worthy of Your Majesty's acceptance, Then let it be presented before the ministers of the court for their detailed assessment, that they may add their distinguished comments to this humble piece of work."

The petition went on to describe in detail the various workings of the new calendar, but we will focus on the 4 areas in which the calendar made major changes to the Yuanjia calendar specifically, and to all previous calendars in general. All 4 were mentioned in the passage quoted above, but the last (the nodal month) was not listed as a significant change by Zu Chongzhi at the time, although his calendar was the first to differentiate it from the other types of months.

[22] The 11 are the Huangdi, Zhuanxu, Xia, Yin, Zhou, Lu, Taichu, Sifen, Qianxiang, Jingchu and Yuanjia. Jiang Ji's Sanji calendar did indeed have a Jiazi year as its Great Year 1, but the Song imperial court did not recognize the legitimacy of Jiang Ji's native kingdom, the Later Qin.

1. Replacing the Metonic cycle

The difference between 235 mean lunar months and 19 solar years is only about 2 hours, making the Metonic cycle (7 leap months in 19 years) a relatively effective formula for a lunisolar calendar. However, this two-hour difference accumulated to an error of one extra day in about 220 years, which meant that the calendar had to be periodically revised to make it accurate again. Nonetheless, Chinese astronomers had stuck to the Metonic cycle, for want of a better method, for nearly a thousand years. The first to provide an alternative was Zhao Fei of the Northern Liang kingdom. He proposed a cycle of 221 leap months in 600 years, and used this in the Yuanshi calendar which he produced for the Liang in 412. In 437, 2 years before it fell to the Tuoba-Wei, the Liang sent a diplomatic mission to the Song, bearing a tribute of gifts which included Zhao Fei's calendar. In 452, the Tuoba-Wei began using this calendar, and so the new cycle again came into use in the north. In the South, however, He Chengtian did not use Zhao Fei's idea in his Yuanjia calendar, for fear that it would overly complicate his calculations, and stuck to the Metonic cycle.

Zu Chongzhi not only recognized the need for a new cycle, but further improved Zhao Fei's cycle to 144 leap months in 391 years, hoping that this would eliminate the need to revise the calendar every 200 years or so. He derived his new cycle from dividing his new value for the solar year by that for the lunar month: $(365 + 9858/39491) \div (116321/3939) = 12 + 144/391$. We can also see that the new calendar did not have the extra day after about 200 years that the Yuanjia calendar would have. Taking the lunar month value for the Yuanjia calendar and using its metonic cycle, 200 years would have: $[(12 + 7/19) \times 200] \times 22207/752 = 73049.34$ days. Taking Zu Chongzhi's month value and his new cycle, 200 years would have: $[(12 + 144/391) \times 200] \times 116321/3939 = 73048.56$ days.

2. Precession

The effect of precession was first discovered in the 2^{nd} century BC by the Greek astronomer Hipparchus. He determined the position of the star Spica to be 6º west of the autumn equinox by finding the angle between it and the moon at the time of an eclipse. However, according to his star records, Timocharis had found Spica to be 8º west of the equinox 150 years earlier. He then observed that the longitudes of other stars (following the Greek ecliptic coordinate) also changed at the slow rate of "not less than 1º in 100 years". Hipparchus concluded that the celestial sphere, besides rotating around the earth, also shifts gradually about the poles of the ecliptic.

The real reason for precession was first explained much later, by Newton. The effect is caused by the gravitational pulls of the sun and moon on the earth's equatorial bulge, in an effort to reduce the tilt of the earth's axis with respect to its own orbital plane and that of the moon. The earth responds to these gravitational tugs by slowing wobbling its rotational axis in one full round over about 25,800 years. This causes the solstices and equinoxes to shift clockwise (westward in the northern hemisphere) with respect to the stars by about 1º in 72 years (1º in 70 years according to the Chinese degree). Consequently, under the equatorial coordinate system, a star's right ascension will increase by 1 minute eastward from the vernal equinox every 20 years, and its declination by about 1.4 degrees above the celestial equator every century. This is significant as it is almost the only change in a star's equatorial coordinate over time.

Another important result of precession is that the north celestial pole shifts in relation to the stars along with the wobbling of the earth's axis. Different historical periods therefore had different polestars. The closest star to the north celestial pole at present is Polaris (Ursa Minor α or Gouchen 1 in the Chinese constellation system). In 2000 BC, however, it was Thuban in the constellation Draco (or the right pivot star, Youshu, of the Chinese constellation Ziwei). In Zu Chongzhi's time, Western and Chinese astronomers recognized different polestars: the Chinese took the "sky pivot", Tianshu (32H Camelopardalis), 5th star in the constellation of Beiji. The West took Kochab (Ursa Minor β, known to the Chinese as the 2nd star of Beiji, the emperor star), which was brighter but further from the north celestial pole[23]. In 10,000 AD, the polestar will be Deneb in the constellation Cygnus (Tianjin 4 to the Chinese), and 4,000 years later, it will be Vega in Lyra (the famous Chinese constellation Zhinu, or Weaver Girl).

The Chinese discovered precession much later than Hipparchus, mainly because under their equatorial coordinate system, they did not record the position of stars with respect to the equinoxes, but used the 28 lunar mansions which remained fixed along the equator. Thus their only clue as to the existence of precession was through observation of the positions of the equinoxes and solstices themselves. Since the Chinese solar year began with the December solstice, the position of the solstice was the most keenly-observed by the Chinese. They therefore described the effect as a precession of the solstices, rather than "precession of the equinoxes" as it is usually known in the West. The first Chinese to write about precession was the Eastern Jin astronomer Yu Xi (281-356 AD). By comparing the meridian at sunset on the winter solstice in his day with that of 2700 years ago, he discovered that it had shifted from the lunar mansion of Mao, through the mansions of Wei, Lou, and Kui, to end up in Dongbi – about 53º away (we will be using Chinese degrees from here onwards, unless otherwise stated). Yu Xi thus concluded that the winter solstice point shifts by 1º every 50 years, and termed this the "yearly difference" (sui cha). He went on to suggest that the solar year was therefore different from the sidereal year.

In fact, as early as the Han Dynasty, Chinese astronomers had begun to suspect that the solstice point was not fixed, as originally thought. Astronomers in the Warring States had observed the position of the sun on the winter solstice to be just inside the lunar mansion of Qianniu, and this was followed by subsequent calendars. But in 85 AD, the compilers of the Han version of the Sifen calendar observed it to be 21.25º in the lunar mansion Nandou[24]. However, they did not subject this phenomenon to further analysis, believing that past observational errors were to blame. By the time of the Eastern Jin, observation and calculation of the solstice point had become accurate enough for astronomers like Yu Xi to be sure of their readings. While working on his Yuanjia calendar, He Chengtian also compared previous records of the solstice point with his own reading (14.1º within Nandou) and noticed a shift of 27-28º every 2700 years or so, which is equal to 1º every 100 years. However,

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[23] Refer to Appendix II for a comparison of Western and Chinese polestars through the ages.
[24] Refer to Appendix III for a star map showing the positions of Qianniu and Nandou.

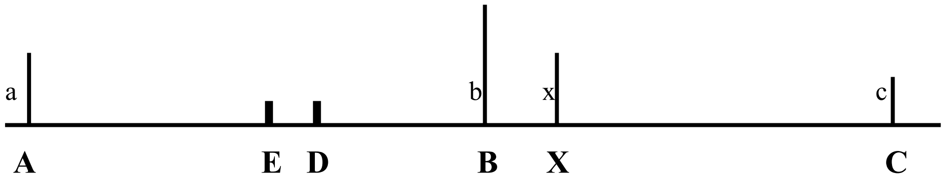
he merely made a note of this without factoring it into his calculations for the length of the year. Zu Chongzhi was the first to incorporate the concept of precession into his calendar on a practical basis.

Zu Chongzhi could be sure about precession because of two recent breakthroughs in the observation of the solstice point and the exact time of the solstice. The first was made by Jiang Ji in the northern kingdom of Later Qin. Previously, the winter solstice point had to be derived through a complicated mathematical process. First, the stars observed on the meridian at sunset and sunrise of a certain day were used, along with the time of sunset and sunrise as measured by the water-clock, to calculate the position of the sun at sunrise and sunset. From this, one would derive the position of the sun at midnight on that day as a median point, and then add the number of days between that day and the solstice (assuming the apparent motion of the sun to be 1º a day) to find the solstice point.

Jiang Ji devised a much more efficient method, in which the position of the moon during a lunar eclipse was reversed to find the position of the sun at that time (since the sun is always directly opposite the moon during a full lunar eclipse), and the number of days from the eclipse to the winter solstice was then added to find the corresponding solstice point. Using this method, Jiang Ji calculated the solstice point to be 17 º within Nandou. When Liu Yu conquered the Later Qin in 417, Jiang Ji's writings, including his Sanji calendar, were brought back to the South. From then onwards, most Chinese astronomers began to use the "eclipse method", and Zu Chongzhi also compared his astronomical readings with those of Jiang Ji to ascertain their accuracy.

The second breakthrough was made by Zu Chongzhi himself. As the time of solstice can be at any time of the day, not necessarily at noon, the measurement of the shadow of the gnomon had to be done consistently on the day of the solstice, so as to find the exact point when the shadow was longest. This was a tedious and not always accurate procedure. Zu Chongzhi tackled the problem by devising a method of measuring the noon shadow 23 to 24 days before and after the solstice, computing the change in shadow length in one day, and assuming that the theoretical noon shadow length always changed at this rate. This was based on 2 assumptions that were not completely true, namely that the rate of change in shadow length is the same before and after the solstice, and that the rate of change in shadow length is the same everyday. However, we now know that in Zu Chongzhi's time, the solstice point and perihelion were about 13.5º apart, and therefore the speed of the earth's orbit changed at different rates before and after the solstice point. Furthermore, the apparent motion of the sun among the stars is not uniform, due to its northward or southward motion in some parts of the ecliptic, as well as the elliptical orbit of the earth. This difference between the mean sun and the true sun was not known to Chinese astronomers until at least 50 years after Zu Chongzhi's death.

As an illustration of Zu Chongzhi's new method, in the diagram below, A and B represent noon 23-24 days before and after the solstice, and a and b are the noon shadow readings on those days. C is noon the day after B, with c as its shadow reading. E is midnight on the day of the solstice, and therefore the midpoint between A and B in terms of time. However, b is longer than a, which means that the solstice is not at noon (E) and there is a point X between B and C where the shadow would be the same length as a. The midpoint D between A and X would therefore be the time of the solstice. If we found the distance ED, we could calculate the time of D.



$$
\begin{aligned}
ED &= AD - AE \\
&= DX - EB \\
&= (DB + BX) - (ED + DB) \\
&= BX - ED
\end{aligned}
$$

Therefore $2ED = BX$

$ED = \frac{1}{2} BX$

Since $a = x$,

$BC : BX = b\text{-}c : b\text{-}a$

$BX = BC(b-a) / (b-c)$

If we take BC to be 100,

$$
\begin{aligned}
ED &= \tfrac{1}{2} BX \\
&= \tfrac{1}{2}\ [BC(b-a) / (b-c)] \\
&= \tfrac{1}{2}\ \{100\ [(b-a) / (b-c)]\} \\
&= 50\ [(b-a) / (b-c)]
\end{aligned}
$$

Using the above two methods, and having referred to Jiang Ji's readings, Zu Chongzhi found the current solstice point to be 11º within Nandou. He therefore set the rate of precession as 1º every 45 years and 11 months, and included this in his calendar as the difference between the sidereal year and the solar year. Zu Chongzhi's value for the solar year was about 365.2428, while his sidereal year was about 365.2646. This worked out to a difference of about 0.0218º a year. This was the first time in a Chinese calendar that the sidereal and solar years appeared as different values.

3. A Standard Great Year 1

Chinese astronomers, when writing new calendars, usually chose a year in the past as a theoretical Great Year 1 (shang yuan), from which they could base all calculations of the motion of the sun, moon and planets up to Year 1 of their own calendar (li yuan) – in other words, a common point from which they could start counting. The ideal Great Year 1 was one in which all the heavenly bodies were in alignment at the beginning, and all the time from that point to the first year of the current calendar was expressed as a number of years called "Years accumulated since Great Year 1" (Shang yuan ji nian). Since the Taichu calendar of the Han, calendars had taken the winter solstice as the beginning of the solar or astronomical year, the new moon as the beginning of the lunar month, midnight as the beginning of a day. Besides this, the winter solstice was also taken as the starting point for the motion of all the heavenly bodies, and the Jiazi combination was taken as the first in the system of Heavenly Branches (Tiangan) and Earthly Branches (Dizhi). Therefore, the Great Year 1 had to begin at midnight on the new moon of the 11th month, on a Jiazi day, in which the heavenly bodies would all be in alignment at the winter solstice point. Obviously, such a year would be incredibly hard to find, and it would take a lot of astronomical guesswork, not to mention the likelihood of inaccuracy in older astronomical records. This might explain why, although a description of a Great Year 1 was given at the beginning of almost every Chinese calendar, the exact method of deriving it was never shown.

Besides He Chengtian's reform of li yuan, as mentioned in the previous section, he also made a significant change in the use of shang yuan. When it came to calculating the motion of the planets, he recognized that they moved at different speeds, and thus rejected as impractical the idea that they had to be in conjunction at the beginning of a Great Year 1. He instead set a different Year 1 for each of the 5 known planets in his Yuanjia calendar, choosing whichever year was most convenient for calculating the motion of each planet: 326 for Uranus, 435 for Mars, 434 for Saturn, 348 for Mercury, and 425 for Venus. This method came to be known as hou yuan ("later Year 1"), and it greatly improved previous calculations for the cycles of conjunction in the planets.

Strangely, Zu Chongzhi did not accept this practical method, and instead argued for a return to a standard Great Year 1. For his calendar, he used a standard Great Year 1 of 51939 years before 463 AD (i.e. 51476 BC). He also proposed some new criteria for the selection of shang yuan, hoping to make the process easier and more accurate. The winter solstice point at the beginning of this year would have to be just 1° within the lunar mansion of Xu[25], as this was in the middle of the Northern Palace, and north was believed to be the first among the 4 compass directions. The year should be a Jiazi year, and besides the conjunction of the sun, moon and planets, the perigee of the moon and one of the nodes (intersections of the ecliptic and the moon's orbit) should also occur at this time. This was a step further than the Jingchu calendar, which had only indicated the time to the next node and perigee at the start of Great Year 1. In one sense, Zu Chongzhi was narrowing down the choices for shang yuan, but in another sense, he was making the search for the ideal Great Year 1 even more difficult. Yet amazingly, without using the hou yuan method, the calculations for planet conjunction in Zu's calendar were still more exact than those in the Yuanjia calendar.

4. The Nodal Month

The ancient Chinese recognised 4 kinds of months: the lunar (or synodic) month, the sidereal month, the anomalistic month and the nodal month. The solar month, which the Gregorian calendar generally uses, was never used in China. The lunar month had been used since the 6 ancient calendars, and the sidereal month since the early Han. The anomalistic month refers to the mean time taken by the moon's passage from one perigee (the point in the orbit of the moon when it is nearest to the earth) to the next, which is approximately 27.55 days. The Qianxiang calendar was the first to use this month, as its author Liu Hong was the first Chinese astronomer to realize that the moon had an elliptical orbit. The nodal month refers to the mean time

[25] Xu is shown on the star map in Appendix III.

taken for the moon to cross the same node twice. The nodes are the points of intersection between the lunar orbit and the ecliptic (the earth's or sun's orbit, depending on one's point of view), which are tilted 5º (Western degrees) apart. There are two nodes, each of which the moon crosses once a month, and the straight line between them is called the line of nodes (refer to Appendix IV for a diagram of this). The node where the moon is travelling southwards (i.e. below the ecliptic) is called the descending node, while the node where it is travelling northwards (i.e. above the ecliptic) is called the ascending node.

The nodes are important because eclipses only occur when the new or full moon is on or around these points. In the same way, the nodal month is important because it can be used to predict when an eclipse will occur. The prediction of eclipses has always been used by Chinese as the truest test of whether a calendar is accurate, and the accuracy of this prediction can literally make or break a calendar. Knowledge of the nodal month, therefore, is a big step towards making calendars more accurate. However, before Zu Chongzhi's calendar, there had never been a distinct value for the nodal month in Chinese calendars, and many people even assumed that the anomalistic month and nodal month were the same.

Zu Chongzhi deduced correctly that the nodal month had a value of its own, which he calculated as 27.21223035 days. He also tried to refine the measurement of the angle between the planes of the ecliptic and the lunar orbit. Previous calendars like the Qianxiang and the Yuanjia had taken this angle to be (6 + 1/12)º (Chinese degrees), but Zu Chongzhi found a smaller value of 6º (equivalent to 5.91 Western degrees). Another important value to Chinese astronomers was the "range of no partial eclipse" (bu pian shi xian), which is the angle above or below the line of nodes outside which even a partial lunar eclipse would not occur – in other words, the greatest possible range for eclipses around the nodes. The Qianxiang calendar had given an angle of 14.55º (14.76 Chinese degrees), based on half the length of its lunar month, and later calendars all followed this. But Zu Chongzhi decided to calculate a new value based on his value for the nodal month. Taking the difference between the lengths of the lunar and nodal months and dividing this by half, then multiplying the result by the mean daily distance traveled by the moon (13.36875º), he derived an angle of 15.19º (15.27 Chinese degrees).

We will evaluate the validity of Zu Chongzhi's reforms in Section 6, but first we must make a lengthy detour into the political controversy surrounding the new calendar.

SECTION 5

THE ZU CHONGZHI – DAI FAXING DEBATE

As Zu Chongzhi had requested, Liu Jun called upon all astronomers and calendar experts in the Song Empire to debate the merits of the new calendar, in order to reach a decision as to whether it should be adopted. However, since the deaths of He Chengtian and Qian Lezhi, there were no qualified astronomers or mathematicians left in the imperial court, and no one dared to raise any argument either for or against Zu's calendar. The only exception was Dai Faxing, who then held the posts of Palace Secretarial Attendant and Guard Commander of the Crown Prince's Bodyguard. Believing himself to have sufficient learning of his own, and confident of being in favour with the emperor, he wrote a lengthy critique of Zu's suggestions. Zu Chongzhi was given a copy of this, and after reading it was convinced of one thing: that his opponent might have had some mathematical training, but knew next to nothing about astronomy. He was neither impressed nor intimidated by Dai Faxing's liberal quoting of classic texts to support his conservative views and soon submitted a point-by-point rebuttal to the imperial court.

We have translated the arguments made by both men, which are found in the Calendrical Section of the Song Shu (History of the Liu-Song Dynasty)[26], but changed the format for greater ease of understanding. Originally, Dai Faxing's critique came first, followed by Zu Chongzhi's rebuttal containing extensive quotations of Dai's words. We have now juxtaposed their arguments directly, according to the order in which Zu Chongzhi made his rebuttals. While we do not understand many of the astronomical concepts used in the arguments, we have tried to translate them as accurately as we can, omitting only the most obscure phrases. We have also preserved the style of ancient Chinese court language as much as possible.

Dai Faxing: Citing merely 3 figures as proof and taking the conjunction of 5 orbits as a basis is hardly sufficient conjecture, and one can presume to correct the past, reform the present and rectify the use of the gnomon and lunar mansions only by extensive study of the changing lengths of the noon shadow. With reference to Zu Chongzhi's proposal, wherever there is a factual error, your humble subject shall apply his limited understanding to identify and question it in the order by which it appears.

Zu Chongzhi: Your subject has, since youth, devoted his limited intelligence to an interest in mathematics, collecting both ancient and recent works and examining them thoroughly. He has analysed all the material relating to astronomy in the Book of History and studied the calendars of the Zhou and Han Dynasties. He has done all the calculations that can be done, and found out which are more accurate and which are less so. For example, the method for finding the surface area of a sphere in the "9 Chapters on the Mathematical Art"[27] was wrong, and Zhang Heng[28] knew this in the Later Han but did not correct it. Wang Mang[29] had a bronze measuring flask made,

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[26] With much reference to the annotations, corrections and translation into simplified Chinese by Yan Dunjie in his "Annotated Collation".

[27] Jiuzhang Suanshu, an ancient mathematical classic.

[28] An important mathematician and astronomer of the Later Han.

[29] A Han Prime Minister who usurped the throne from 9-20 AD but was overthrown by loyalists who eventually established the Later Han.

inscribed with mathematical calculations which contained an inaccurate value of π, but Liu Xin[30] tried to cover this up by distorting the figures. These were all serious flaws in the mathematics of the time. The Qianxiang and Jingchu calendars contained methods for calculating the exact dates of the new moon and full moon, but were either inaccurate in their readings or overly complicated in their calculations. These were mistakes made by the authors of earlier calendars. Zheng Xuan, Kan Ze, Wang Fan and Liu Hui[31] all made improvements to mathematics, but they also made many errors. Your subject used his free time in the past to rectify their miscalculations, and he has detailed evidence to prove it. Your subject therefore has just enough confidence and knowledge not to blindly worship the work of his predecessors.

With reference to He Chengtian's calendar, his dates for the solstices are earlier than they should be, and his leap months thus shift forward by a month. The periods of the 5 planets are off by more than 40 days, and there is a redundant category relating to the varying speed of the moon: namely, the difference between the daily movement of the moon and that of the day before. Furthermore, the movement of the moon on the 7th and 21st days of the anomalistic month is still calculated using the mean moon. Your subject has corrected all these past errors in his new calendar. Since we now understand the history of past calendars, we should make efforts to discard whatever obstructs progress, so that what remains is made even more accurate, be it the observation of the celestial bodies in the sky or that of the shadow of the gnomon on the ground. Is it not a shame, then, that such efforts are instead subjected to calumny and ridicule? Looking at the 6 arguments made by Faxing, none of them address the crux of the matter or even make sense at all. These arguments are as listed below:

First, relating to the precession of the solstice point. This was overlooked by all previous calendars, but your subject gathered evidence from historical texts to prove the existence of this phenomenon. Dai Faxing has quoted 3 passages from the Books of Poetry and History to criticise your subject's position, but he is mistaken in all 3. **Second**, your subject used the length of the noon shadow to determine the length of a solar year, and changed the old zhang cycle based on this. Faxing can find no argument against this, and so can only claim that your subject's arguments are "superficial". **Third**, relating to the accusation that the new calendar shifts the positions of the 12 zones of Jupiter, there is no such element in the calendar at all, and Faxing is deliberately distorting the truth. **Fourth**, the Great Year 1 of the new calendar is a Jiazi year, and the method for deriving this is concrete and efficient, but Faxing calls it an arbitrary decision, a charge which is dubious indeed. **Fifth**, your subject has chosen the Great Year 1 as a starting point for all the 7 Luminaries, an arrangement with which no fault can be found, but Faxing claims that this "cannot be measured by any ordinary man". **Sixth**, the anomalistic and nodal months are of different lengths, but Faxing does not understand this and ignorantly insists that they are the same. These arguments involve either deliberate misinterpretations of historical data for the purpose of mocking your subject, or total fabrications used to drown out the truth. Not one of them is correct or satisfactory. Now your subject respectfully makes his replies to each argument, to the very limit of his ability, and submits them for Your Majesty's perusal.

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[30] A historian and astronomer of the Later Han.

[31] All scholars and mathematicians of the Later Han and Three Kingdoms periods.

Dai Faxing: According to Chongzhi's new calendar, "the position of the winter solstice shifts slightly from year to year". Your subject Faxing is of the opinion that the two solstices are the northernmost and southernmost positions of the sun, the sun travels a fixed distance along the ecliptic of 1º a day, and the positions of the 28 lunar mansions do not change. The solstice point for all ancient calendars was in the constellation of Jianxing[32].

Zu Chongzhi: From the last years of the Zhou Dynasty to the beginning of the Han, there were no professional astronomers, and so a jumble of theories emerged, along with many schools of prognostication. Some used the names of rulers to increase their prestige, others used the names of sages to sanctify themselves. As a result, much of this prognostication was a mere sham, and Huan Tan[33] was well aware of its tricks. These falsehoods mixed their way into the ancient calendars, and Du Yu[34] of the Western Jin had already suspected their inaccuracy.

According to Liu Xiang's "Treatise of the 5 Eras"[35], the Huangdi calendar had 4 methods of calculation, while the Zhuanxu, Xia and Zhou calendars had 2 methods. Each method is different, with no way of knowing which is right, and that is the **first** reason why the ancient calendars are unreliable. The Xia calendar, unlike all the rest, took the 7 Luminaries as all moving westwards, and so Liu Xiang believed that it was a hoax from much later than the Xia Dynasty. That is the **second** reason. The length of a lunar month in the Yin calendar was 29 + 499/940 days, but the "Prognostication of Changes"[36] says that it was 29 + 43/81. If the latter is correct, then the Yin calendar's figures must be fake. That is the **third** reason. The Zhuanxu calendar began in a Yimao year, but the "Introduction to the Setting of Calendars"[37] says that it began 1 year earlier, in Jiayin. This is the **fourth** reason. The "Spring and Autumn Annals" records 26 solar eclipses which occurred at the new moon, and this should be based on either the Zhou or Lu calendars. But if we calculate using the Zhou calendar, 25 of these eclipses would not fall on the new moon, and 13 if we use the Lu calendar. Both calendars do not match the historical record completely, and obviously at least one of them was not in fact used at that time. This is the **fifth** reason. The 6 ancient calendars all used the Sifen method, but when this method has been used for long, the winter solstice starts to fall later than in reality. If we verify this using the eclipses, which must fall on a new moon, we find that there is an error of 1 day after about 300 years.[38] If we use the ancient calendars to calculate our present year, we find that the date given for the new moon is late by about 2 days. This implies that the ancient calendars were written no longer than 700 years ago, between the Zhou and the Han, and not in the time of Huangdi, Zhuanxu or the Xia and Shang Dynasties as was claimed. If we further test the calendars against the Spring and Autumn period, the

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[32] A constellation slightly to the northeast of the lunar mansion Nandou, both of which are part of the Western constellation Sagittarius. Refer to the star map in Appendix III.

[33] A scholar of the Later Han, who displeased the founding emperor Liu Xiu by trying to dissuade him from using divination as a guide in state affairs.

[34] A scholar and general of the Western Jin.

[35] Wu Ji Lun. Liu Xiang was the father of Liu Xin and a noted scholar himself.

[36] Yi Wei, a book of astrology and divination.

[37] Ming Li Xu.

[38] A lunar month was 29.53085 days in the Sifen calendar and 29.53059 days in Zu Chongzhi's calendar. Taking the difference multiplied by the length of the Sifen solar year, and then by 300, we get: $(29.53085 - 29.53059) \times (12 + 7/19) \times 300 \approx 1$

new moon falls earlier than it should. This again proves that they did not originate before the Xia, Shang or Zhou. This is the **sixth** and last reason why the ancient calendars cannot be quoted reliably.

The Calendrical Section of the "History of the Han Dynasty" states that in the Early Han, the solstice point was between the lunar mansions of Nandou and Qianniu, and that is indeed near Jianxing. But this was not a permanent rule laid down by the gods, and the armillary sphere and water clocks were imperfect and prone to error. Thus we should not take the statement about Jianxing as the unquestionable truth.

Dai Faxing: In the time of the Warring States, there was disunity in the land, and court historians were unable to keep complete records. The beginning of the Han did not see much improvement in observations, and astronomers could only vaguely determine that the sun was 21º within Nandou at the winter solstice. In the Yuanhe reign of the Later Han, the Sifen calendar continued using this figure and thus remained faithful to the ancient calendars. Even up to the Jingchu calendar, no change was made to this rule.

Zu Chongzhi: There are many questionable points about the ancient calendars, and it is now impossible to know the details of their workings. Only the accuracy of the Zhuanxu calendar can be verified, since the Qin Dynasty used it effectively. The Taichu calendar reform came after thorough discussions and the calibration of the astromical instruments and water clocks, and this is recorded in the "History of the Han Dynasty". Their methods for determining the positions of the stars should also be relatively accurate. Now Dai Faxing emphasises the strengths of the early calendars while ignoring their flaws, and blindly uses them to criticise your subject. He accuses your subject of using the methods of the present to violate the traditions of the past, but this is a one-sided argument far inferior to the views of your subject. The method of the Jingchu calendar is clearly wrong in determining the paths of the 5 planets, for they are now exactly 180º away from where the calendar calculates they would be. The same deviation can be seen in the times of the solstices. This is because when Yang Wei made this calendar, he simply made slight adjustments to the values of the year and month, rather than making detailed observations. In terms of the water clock readings for the lengths of day and night, and the stars culminating at dawn and dusk, he followed the Sifen calendar completely. Nor did he make any correction to the disparity in shadow lengths on the equinoxes. It is therefore not surprising that he should also be mistaken regarding the solstice point.

Dai Faxing: The "Book of History"[39] says that "When the day is shortest and the mansion of Mao is on the meridian at sunset, that is the middle of winter." If we link the sequence of the 12 months with the stars on the meridian at sunset on the equinoxes and solstices, we find that the meridian star was always in the division directly opposite of the State of Wei[40]. This was how the ancients told the time, and it

[39] Shu Jing, also known as Shang Shu, one of the Confucian classics. It includes a collection of historical documents from the time of Huangdi to the Zhou Dynasty.

[40] Ancient Chinese astrologers divided the path of Jupiter, which completes one round in about 12 years, into 12 zones (ci), somewhat similar to the Western zodiac. Each zone formed a division (fen ye) corresponding to one of the Warring States and one of the earthly branches (dizhi). The zone of Zouzi corresponded to the State of Wei and its opposite was Chunwei, corresponding to the State of Chu. Refer to Appendix I for a list of the 12 zones and their corresponding mansions, states and branches.

should not be changed even after ten thousand generations. Now Chongzhi says that the solstice point in the Tang era was more than 50º left of the lunar mansion where it is today, and so he groundlessly adds degrees to the original figure, trying vainly to remove the law of the heavens.

Zu Chongzhi: The "Book of History" used the star culminating at due south at dusk for determining the equinoxes and solstices because it believed that the ruler of a country should sit facing south. Furthermore, it is easier to determine the directions of north and south, and to observe the movement of the stars around the meridian. That is how previous scholars interpreted the "Book of History", but Dai Faxing believes instead that the Book describes the meridian stars on the seasonal markers as being always in the southeastern division of Si[41]. If these stars were to move forward, they would no longer be in the south but in the east. Clearly he has distorted the words of the Book to support his own arguments. If in this case he does not use the division of Wu, which is due south, as the meridian, but uses Si instead, does that not mean that none of the stars will culminate on Wu? If he uses only the middle lunar mansion in a Palace to tell the time, does that mean that all the other 6 lunar mansions are of no use? Even if he claims that one lunar mansion is enough to confirm the positions of all 7, he is just being evasive and saying as good as nothing. If we look carefully at the words of the Book, it does say that the meridian star was in the lunar mansion of Mao, but it does not say that it was opposite the division of Wei. So where did the latter theory come from? Your subject can only presume that it is pure nonsense, and sighs in exasperation at Faxing's inability to offer any constructive opinion about determining the solstices and equinoxes.

Dai Faxing: Zu Chongzhi's calendar is off by more than 15º in its solstice point, which is roughly an error of 1º for every 45 years and 9 months.

Zu Chongzhi: Dai Faxing is right about the solstice point being 21º in Nandou in the time of the Sifen calendar, but he is wrong to equate this with the ancient calendars' solstice point in Jianxing. Your subject has used his calendar to recalculate this and obtained the same result as the Sifen calendar. Yet Dai Faxing, finding no grounds to criticise your subject's calendar, makes up an amusingly astonishing story about it being off by 15º. Furthermore, the rate of precession in your subject's calendar is 1º every 45 years and 11 months, not 9 months as Faxing says. This mistake is typical of his standard of mathematics.

During a full lunar eclipse, the sun must be directly opposite the moon, and we can use this to determine the coordinates of the sun. Allow your subject to use this method to prove who is accurate and who is not. According to the records of the Imperial Astrologer, there was a full lunar eclipse on the night of the 16th of the 12th month of the 13th year of Yuanjia[42]. During the eclipse, the moon was 4º within Yugui (168º) and the sun would then have been 6º within Qianniu (350º). According to Dai Faxing's method, the sun would be 8º within Xunu (360º). There was another full lunar eclipse on the 15th of the 5th month of the 14th year of Yuanjia[43]. The moon was 26º within Nandou (344º), and the sun would then have been 30º within Dongjing (161º). According to Dai Faxing's method, the sun would be 3º within Liu (171º).

[41] Chunwei, the zone opposite the division of Wei, corresponded to the earthly branch of Si.
[42] 7.53 pm on 8 January 437, according to the Western calendar.
[43] 2.14 am on 4 July 437.

There was an eclipse on the 15th of the 8th month of the 28th year of Yuanjia[44]. During the eclipse, the moon was 11º within Kui (62º), and the sun would then have been 2º within Jiao (245º). According to Dai Faxing's method, the sun would be 12º within Jiao (255º). There was an eclipse on the 15th of the 9th month of the 3rd year of Daming[45]. During the eclipse, the moon was at the eastern boundary of Wei (93º), and the sun would then have been 12º within Di (276º). According to Dai Faxing's method, the sun would be 2º within Xin (286º). All these 4 eclipses correspond exactly to your subject's calendar, but Dai Faxing's method is off by 10º in each. Clearly he is the one distorting the movement of the heavens. It is a fact that the heavens are shifting gradually, and we should recognize this. The truth is plain for all to see, so what point is there in believing blindly in tradition while doubting the evidence of the present?

Dai Faxing: In the "Book of Poetry"[46], the poem "Movement of the Fire Star in the 7th month" is set at the beginning of autumn, while "Choosing a site for the palace" takes place in the solar term of Xiaoxue[47]. If this theory of the shifting winter solstice were true, then the Fire Star[48] would have started shifting west on the summer solstice instead, and the state of Chu would actually have started building its palace in the solar term of Hanlu[49], and that is totally preposterous.

Zu Chongzhi: Dai Faxing is wrong on both counts. The "Book of Poetry" probably uses the term "movement" to mean that the Fire Star had already begun shifting west and was then on the meridian, as a sign that the cold season was beginning. It does not mean that the Star only began to shift in the 7th month. Even if it were so, taking the solstice point to be 21º within Nandou, the Fire Star would have culminated before the solar term of Dashu[50], and not in the 7th month. This is criticism for its own sake, without any intention of correcting a real mistake. The Xia calendar says that the Fire Star culminates at dusk in the 5th month. Is that not contradictory to Dai Faxing's claim that the meridian star on the summer solstice is in the zone of Zouzi, opposite the division of Wei[51]? He also says that according to your subject's calendar, the Chu palace would have been built in Hanlu, at the beginning of the 9th month. Annotations of this poem have always stated that the meridian star at dusk in this time comprised the stars between Yingshi and Dongbi, forming a square shape. From this we know that the meridian was 8º within Yingshi. Calculating this with the new calendar, we find that this was the case 4 days after the solar term of Lidong[52] in that year, which would have been the beginning of the 10th month, and not Hanlu as charged by Dai Faxing. He has probably confused the Zhou Dynasty with the time of Yao, causing a major error of 50º. As for his assertion that the second poem takes place in Xiaoxue, that is his own wishful thinking, as there is no evidence for it in the poem.

[44] 2.45 am on 27 September 451
[45] 10.30 pm on 27 October 459
[46] Shi Jing, another Confucian classic which includes many ancient poems.
[47] "Slight Snow", in early winter.
[48] Antares, the 2nd star in the lunar mansion of Xin.
[49] "Cold Dew", in late autumn.
[50] "Great Heat", in late summer.
[51] The lunar mansion of Xin, which contained Antares, was in the zone of Dahuo.
[52] "Beginning of Winter"

Dai Faxing: In the "Annals of Zuo"[53], there is an account of a plague of locusts in the 10th month. When asked for advice, Confucius said, "I have heard that when the Fire Star sets in the west, the plague will end. Now the Fire Star is still moving west, and those in charge of the calendar should be reprimanded for not realizing this fact before I did." If we were to follow Chongzhi's mistake, then there would no longer be any way of tracking the stars using the 12 zones of Jupiter[54], or of determining the directions using the Eight Trigrams[55]. All the astronomical terms of the past would no longer apply to the present, and the rites and institutions of the state would no longer make sense. The 11th and 12th months in the time of Yao would become the 1st and 2nd months in the present, while in terms of the zones of Jupiter, the present zone of Shouxing would be equivalent to the zone of Chunwei during the Zhou Dynasty. The lunar mansion of Dongbi would now have moved from the Northern Palace to the Western Palace, while Zhen would have moved from the Southern Palace to the Eastern. That is the true extent of this slander against the heavens and violation of the classics.

Zu Chongzhi: The polestar lies in the centre of the sky, and all the other stars turn around it. Everything has its direct opposite and is governed by the elements[56], which is why we can find the directions for fire and water based on the Southern and Northern Palaces, or east and west based on the Green Dragon and White Tiger[57]. The validity of these terms does not depend on the position of the sun. How do we know this? The first trigram of the 64 in the "Book of Changes" states that the solar terms begin with the winter solstice, which is in the 11th month, in the Northern Palace, with Xu at due north exactly. If we use the armillary sphere to find the directions based on the position of the sun, then by the time the winter solstice is in the north, our present south would have shifted to the east. What point would there be in that? If we used the seasons to determine the directions, that would be totally against common sense! From this we know that the division of the sky into the lunar mansions has nothing to do with the 4 seasons, while the varying shadow length at the winter solstice from year to year tells us that the solstice point is not fixed.

As for the Classics' use of the rising and setting of the meridian star to tell the time and seasons, this was because of the difficulty of expressing the time in numbers. It was clearer and easier to use the motion of the heavenly bodies, and so each dynasty recorded the patterns of motion as seen in its time. Therefore, the rules of one dynasty are not equivalent to those of another, and the laws of both heaven and mankind are constantly changing. Likewise astronomy as an institution of the state cannot remain fixed while the times change.

Dai Faxing seems to be using the solar terms to determine the names of the months. However, names must accord with reality. Because of the precession effect, we cannot ascertain which month it is based only on which lunar mansion the handle of the Big Dipper points at. In verifying the records of the Han, we find that we have since shifted by 15º, so what is the use of going by the Big Dipper? Perhaps he is not

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53 Zuo Zhuan, a historical text from the Spring and Autumn Period.
54 Called Mu Xing, the Wood Planet, by the Chinese.
55 The Ba Gua, found in the Book of Changes (Yi Jing) and used for divination.
56 The Chinese believed that the balance between Metal, Wood, Water, Fire and Earth governed all human affairs. They also represented directions: West, East, North, South and Centre respectively.
57 The animal names for the Eastern and Western Palaces respectively.

even referring to any of the astronomical classics, and is only relying upon the deceitful books of prognostication. The 12 zones of the path of Jupiter are based on the 4 compass directions, and do not follow the same rules as the lunar mansions. The times of the equinoxes and solstices are changing year to year, so how can it make sense that the solstice point is not moving as well? Does Faxing think that your subject cannot even tell north from south or east from west? He should consider carefully before mocking your subject's knowledge of astronomical terms.

As for his postulation of the lunar mansion of Dongbi moving from the Northern Palace to the Western Palace, and Zhen moving from the Southern Palace to the Eastern, this is indeed correct according to your subject's observations. In the Yuanjia calendar, the boundary of the zone of Shouxing had in fact moved west to the lunar mansion of Yi, in the Southern Palace[58], implying that Zhen has shifted into the Eastern Palace. It is the 28 lunar mansions that are moving, not the 12 zones. If we go further back, many more such examples can be found in the writings of the Jin Dynasty. The heavens have changed every hundred years or so, and only if Dai Faxing has the power to make it such that the sun does not reach its southernmost point on the winter solstice, or is not opposite the moon when the moon is full, only then will his arguments stand. Otherwise, he will have to admit that the changing position of the sun has nothing to do with the 12 zones, and he is only conjuring a baseless argument, thus proving that your subject's calendar is in fact correct and above such criticism.

Your subject's position, on the other hand, is based on historical fact. He has referred to everything from the astronomical writings in the "Book of History" to the calendars written in the Han. However, he has chosen to disregard those books of prognostication which contain mere myth and hearsay. This, he believes, is what makes his words trustworthy. For example, using the eclipse method to find the position of the sun is the best available way, and this method can only be found out by those authorized to read the historical records in the imperial library. Without the proper knowledge and research, what we get is such nonsense as the 4 meridian stars all being in the Si division, and the sun's position still being the same as in the Sifen calendar. It should now be clear who is really slandering the heavens and violating the classics.

Dai Faxing: In addition, Chongzhi proposes to change the zhang cycle to 144 leap months in 391 years. Your subject Faxing believes that the sun is in Nandou at the winter solstice, but not always at the same degree. That is because the sun does not always move at the same speed[59], and so some years are longer than others. When the ancients created the zhang (Metonic) cycle, they chose the most appropriate formula, which is 7 leap months in 19 years. There may be some differences in the length of the shadow at noon on the winter solstice, but the cycle cannot be changed. Chongzhi wants to reduce the number of leap months and thus destroy the cycle. If we subtract the fractional value of Chongzhi's year from that of the Sifen calendar, we find that

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[58] This zone originally had its western boundary at Zhen while extending eastwards up to Di in the Eastern Palace.

[59] Dai Faxing seems to have, perhaps unwittingly, stumbled upon a fact of which even Zu Chongzhi was unaware: the non-uniform motion of the true sun. This was only understood by Chinese astronomers about 100 years later.

there will be one day missing after 139 days and 2 months.[60] If we subtract his cycle from that of the Sifen calendar, we also find one leap month missing after 7429 years[61]. A missing day affects the accuracy of the time, while a missing month affects the activities of the entire year. The livelihood and productivity of the people depend upon work schedules based on our present calendar. I foolishly suggest that Chongzhi's superficial arguments provide no reasonable basis for changing it.

Zu Chongzhi: From the "History of the Later Han"[62] and the "Qianxiang Treatise", we know that although the Sifen calendar established the zhang cycle and bu[63] cycle in 85 AD, the noon shadow lengths for the various solar terms were only established in 174 AD. The Sifen calendar says clearly that the noon shadow was 1 zhang long at the solar term of Lidong in 174, and 9 chi 6 cun[64] at Lichun[65] the following year. Going by the fact that the noon shadow is longest on the winter solstice, and that the length of time from Lidong to the winter solstice is the same as that from the solstice to Lichun, the noon shadow should be the same length on Lidong and Lichun. Yet we now find that the shadow on Lidong is longer than that on Lichun by 4 cun. This implies that the solstice fell earlier than the date given by the Sifen calendar. On average, there was a change of 9.5 fen a day from Lidong to Lichun. Based on this rate, if both solar terms shifted back by 2 days and 2 ke[66], this would shorten the Lidong shadow and lengthen the Lichun shadow, such that both are 9 chi 8 cun. We would then have the true dates of Lidong and Lichun. From that we can tell that the winter solstice in the Sifen calendar is late by 2 days and 2 ke. The calendar puts it on the 13th day of the 11th month, at the 50th ke, but it should really be at the 38th ke of the11th day[67].

Your subject has been measuring noon shadows for about 10 years, always down to the very fen and cun. The length of a bronze gnomon is not affected by weather, nor does the smoothness of its dial allow for any mistake in reading. The noon shadow on the 10th day of the 10th month in 461AD (last year) was 1 zhang, 7 cun and 7.5 fen long. Finding the average, we know that the winter solstice was on the 3rd day of the 11th month, at the 31st ke[68]. The Yuanjia calendar fixes the date as the 2nd day of the 11th month, which is 1 day early.

Your subject has used this method to calculate the noon shadow for many years, and always obtained consistent results no matter which year it was used on. Therefore he formulated a new value for the solar year based on these readings, discarding the old zhang cycle. If one should now use the new calendar to calculate the solstice timing for last year, one will get exactly the same result that your subject did. This proves that the new calendar is practical and should be used henceforth.

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60 $1 \div (1/4 - 9589/39491) = 157964/1135 = 139 + 199/1135 \approx 139 + 2/12$

61 $7/19 - 144/391 = 1/7429$

62 Hou Han Shu

63 A bu contained 4 zhang cycles of 19 years.

64 The zhang, chi, cun and fen were all Chinese units of measurement, each a tenth of the former. We are unable to provide useful Western equivalents because different standards were used in different parts of China at this time, and the value also varied from one dynasty to the next.

65 "Beginning of Spring"

66 The ke was a unit based on the water clock. There were 100 ke in a day (one midnight to the next).

67 $13.50 - 2.12 = 11.38$

68 Originally, Zu Chongzhi made a lengthy description at this point of his new method for finding the solstice time. Since we have already explained this in the previous section, we now omit it.

The 6 ancient calendars all followed the Sifen method, taking the solar year to be 365 ¼ days. If we used this value for a long time, we would begin to deviate from the actual movement of the sun and moon. After about 300 years, the new moon would be late by a day[69]. That is why in the 400 years of the Han, solar eclipses were always recorded on the last day of a month rather than the first day of the next month, at the new moon. The Wei Dynasty realized this error and thus did not continue using the old method, but that never incurred any criticsm at the time because it clearly accorded with astronomical reality. As for the zhang cycle, its flaws are even more evident. It appeared at about the same time as the Sifen method, but is not mentioned in any of the astronomical classics. Now Dai Faxing says that because the ancients used this cycle, we cannot make any change to it. It is ridiculous to say that we should continue using an ancient method forever despite its inaccuracy, and if such a view can be accepted, then does Dai Faxing also want us to revert to using the Sifen method? Your subject can make no sense of this. If he says that your subject's reforms contain errors and inadequacies, then he has not provided any evidence for it.

When He Chengtian made the Yuanjia calendar, he originally intended to change the cycle, but he feared that the calculations would become too complicated, and eventually all he did was improve the accuracy slightly by subtracting a value from the noon shadow on Yushui. That is why his calendar remains problematic. As for his leaving some leap months out of his calendar to compensate for their being too many, that is quite a reasonable thing to do as well. If, as Dai Faxing says, the cycle cannot be changed and not a single fen can be added or subtracted, then He Chengtian's method is wrong as well. If He Chengtian's calendar is wrong, then perhaps we should follow the timings for solar terms in the Jingchu calendar? But the solstice dates in the Jingchu calendar are off by 3 days! A solar term is wrong by 3 days, and he says nothing about it but instead claims that your subject's calendar has problems. He may harp about a missing day affecting the time, but does he realize that an error in the solar terms affects the allocation of a whole leap month? Having no astronomical experience whatsoever, he is far from qualified to talk about the finer points of calendars. He goes on about livelihood and productivity, but your subject thinks he is just waffling to hide his ignorance.

At the beginning of his argument, Faxing also mentioned that we can correct the mistakes of the past and reform the present only by extensive study of the changing lengths of the noon shadow. But now he says that despite differences in the length of the noon shadow on the winter solstice, we cannot take this as a reason to change the cycle. Which of these contradictory statements should we follow? If our observations are inaccurate and changed to suit our own whims, then how would we ever have a believable calendar? Ever since the "Spring and Autumn Annals", we have been using solar eclipses to verify the dates of the new moon, and this has always proved reliable. This shows that the motion of the sun is constant and predictable. Your subject has measured the noon shadow for many years, with the greatest care and precision, and the results entirely match those of past records. Mencius spoke truly when he said that one can know the solstice points of a thousand years without leaving one's seat. As for the statement that the sun moves at varying

[69] If we compare Zu Chongzhi's lunar month with the Sifen month, there will be a difference after 300 years of $(29.53085 - 29.53059) \times (12 + 7/19) \times 300 = 0.96471 \approx 1$. The result is the same if we use Zu Chongzhi's new cycle instead of the Metonic cycle.

speeds, we have never seen any proof of that. Such hollow and superficial arguments are hardly enough to intimidate your subject.

Dai Faxing: Chongzhi also wants to set a new rule that the Great Year 1 should begin on a day when the sun is 1º within Xu, saying that Xu is the middle lunar mansion in the Northern Palace. Your subject Faxing thinks: since Chongzhi claims that the winter solstice precesses, he is ignoring an object only to niggle about its shadow, and we should not be taken in by him. Why is this so? There can be no light when there is no sun in the sky, and so we can only tell the sun's position at night in relation to Nandou. Suppose the winter solstice were once in Xu, then it would have been further from the ecliptic than it is today, and so the northeast would have become due north, and the northeastern mansions of Yingshi and Dongbi would have shifted with it. How then could Xu still be in the middle? If one makes the equinoxes and solstices shift positions, but expects the zones to remain in place, how are we to use our astronomical instruments to plot the movements of the 7 Luminaries, how would we use the constellation of Sheti to determine the seasons[70], and how would we find out the directions corresponding to the 5 elements?

Zu Chongzhi: Your subject has already addressed the above points in detail earlier. All this talk of the 12 zones shifting and the directions changing positions and Xu no longer being in the middle of the Northern Palace is simply a mess of contradictions. While it is true that Xu is no longer in the middle of the Northern Palace because of precession, this has no effect on the zones and directions, because they do not precess. The mistake lies in Dai Faxing's own misconceptions, not your subject's calendar. As for tracking the 7 luminaries using astronomical instruments, Wang Fan and Zheng Xuan have already written clearly enough about this in the past. Although there were some alternative theories later on, these were much less convincing.

Dai Faxing: Chongzhi says that the Great Year 1 should be a Jiazi year as well. Your subject Faxing observes that various calendars began at different times of the year, some to follow the prophecies of astrology and divination, some to suit the specific needs of the time. Chongzhi claims that "the ancient calendars were full of errors and inaccuracies, and different schools of astronomy disputed among themselves without grasping the key issues at hand". However, Huangdi began his calendar in a Xinmao year, and there was no error in days and months; Zhuanxu began his calendar in a Yimao year, but the seasons were always on time; the Jingchu calendar began in a Renchen year, and the last night of the month was always as dark as it should be; the Yuanjia calendar we now use began in a Gengchen year, and the shadow at noon is always the right length. Is this not following the will of heaven? Chongzhi, in insisting on beginning with a Jiazi year, can be said to be trying to bend heaven to his own whims.

Zu Chongzhi: A calendar should be based on astronomy rather than schools of mysticism, and so it does not matter whether it corresponds to divination and superstition. These prophecies may seem accurate for a time, but in the long term they cease to have any relevance, therefore your subject puts no trust in them. Beginning the calendar in a Jiazi year is a sensible and logical thing to do. We do not know why

[70] The 6 stars of this constellation are in line with the handle of Beidou, the Big Dipper, and the direction in which they point was used to indicate the beginning of the seasons. 3 of the stars lie on the left of the star Arcturus (Dajiao to the Chinese), and 3 on the right. Refer to star map in Appendix III.

the Huangdi calendar began in a Xinmao year, but as your subject explained earlier, the ancient calendars contained many errors and inaccuracies. Rattling off a few calendar names without considering the truth about these calendars is hardly the right spirit for a scholar in search of knowledge. The calendars that Dai Faxing mentioned began in a year that fitted the circumstances of the time, but that is exactly why they could not be used in perpetuity. The first year of a calendar serves as the starting point for calendrical calculations, and not as a determinant of the 4 seasons.

Let your subject explain this further. Up to the Xia and Shang Dynasties, we have no records to go upon, but the "Spring and Autumn Annals" and the historians of the Han began to record the dates of solar eclipses. Using the solar eclipse to find the date of the new moon is the best available method, and calculations using your subject's calendar have all corresponded to the records. This shows that the new calendar is so exact that we can follow the same calculations even for dates a thousand years apart, and thus be sure of the events of the distant past. The calendars of the past never attained this level of accuracy – some were 3 days off for the new moon, while others had an error of 7 days in the solar terms, and none remain useful in the present. If the calendar began in an Yichou year, Dai Faxing would ask: "Why the second combination and not the first?"[71] But now that the calendar begins in Jiazi, he believes it is an arbitrary choice. A year must have a name, whether we like the name or not, otherwise we might as well stop writing our history books and calendars. If Faxing thinks your subject is being subjective, then he must be the most objective one of all, and your subject looks forward to seeing some proof of his objectivity.

Dai Faxing: Chongzhi wants the orbits of the sun, moon and 5 planets, as well as the node and perigee of the moon, to all coincide with the start of Great Year 1. Your subject Faxing feels that when the sun and moon intersect on the line of nodes, a full solar eclipse will indeed occur; however, the varying speed of the moon cannot be measured by any ordinary man. In the Later Han, Jia Kui caught a hint of this phenomenon, and Liu Hong incorporated an approximation of it into his Qianxiang calendar. As to how the speed changes exactly, no one knows for sure. The orbit of the 5 planets also increases and decreases in length. For example, when Jupiter is in the lunar mansion of Zhen, it is said to have exceeded 7 zones by one. Since calendar experts can use mathematics to retrace the movement of the planets from the present, then it is clearly possible to find out their positions at any time in the past and future, without the need for a standard Great Year 1. The Jingchu calendar indicated the time remaining to the next node or perigee at the beginning of each Ji[72], while the Yuanjia calendar took different, more recent starting points from which to calculate the movement of each planet. These are practical solutions for saving time and simplifying the calculations, much better than arbitrary decisions which only complicate matters. Chongzhi has not only gone against heaven's will by making his changes, but he is also introducing new methods based on his own preferences alone. Your subject foolishly suggests that there is no greater mistake to be made in creating a calendar.

[71] Jiazi is the first combination of stems and branches; Yichou is the second.
[72] The ancient Chinese grouped a certain number of years as a Ji, but this number often changed from calendar to calendar. Zu Chongzhi's calendar took 39491 years as 1 Ji, 39491 being the denominator (Ji fa) for the fractional value of his solar year.

Zu Chongzhi: The varying speed of the moon is not something strange and supernatural, and it can be both verified with observations and predicted with calculations. Since Jia Kui and Liu Hong have already written about it, then it is up to us to further improve on this foundation. Dai Faxing says that "the orbit of the 5 planets also increases and decreases in length" and "when Jupiter is in the lunar mansion of Zhen, it is said to have exceeded 7 zones by one." He is referring to the fact that Jupiter moves through an average of 85/84 zones in a year, which is slightly more than one zone. After 7 years, the extra 1/84 would have accumulated to 1/12, which is one additional zone, bringing Jupiter forward into Zhen. Although the calendar may have been changed 10 times to suit contemporary circumstances, it always describes the movement of Jupiter as following this pattern, and the historical and astronomical records support this. This shows that though the paths of the planets may vary, they do so according to certain patterns and not at random. If Faxing thinks that the orbits increase and decrease in length, then why is it that we only see an increase and no decrease? Whoever observes the paths of the planets should record it down to the degrees and minutes, and then use it to verify the readings of the past and predict the readings of the future. He should prove his findings with proper records and true historical sources. Any arguments without such substantiation are either nonsense or rhetoric. Even the astronomical writings of the ancient Gan and Shi schools[73] contradicted each other, and so we need not expect experts to always agree. But arguments based on a single sentence or word, arising from bias and aiming to distorting the truth, are certainly unsatisfactory and can be done without.

In recent times, the different calendars have generally used similar methods of calculation. However, the Jingchu calendar invented the 2 expressions of "time to next perigee" and "time to next node", while the Yuanjia calendar took a recent Year 1 for each planet. This was mainly because their various readings could not be reconciled with one another or with those of the past, forcing them to adopt a workable compromise. It is not always necessary to deviate from the past in order to formulate a sound theory. As long as the truth is borne out by the records and figures, that should be sufficient to establish its value. The ancient method of the Great Year 1, from which all calculations begin, is a simple, clear and logical concept which deserves no ridicule. It is those who disparage it who are the ridiculous ones. Even though the Yuanjia calendar took a different Year 1 for each of the Luminaries, it still began numbering its Ji with Jiazi[74] and using this to arrange its solar terms, did it not? Can that be only a minor inconsistency? Fixing a Great Year 1 for nothing and pretending that it is the beginning of the calendar, while the names of the years and days do not correspond to the name of the Great Year 1, and the conjunctions of the 5 planets do not correspond to the date of the winter solstice – is that what we should hold up as a standard calendar to be followed? To say that your subject is introducing new methods according to his personal views and preferences is correct, and surely there is nothing wrong with this. But to say that he is going against heaven by doing so is a totally groundless accusation.

[73] Gan De and Shi Shen, 2 astronomers of the Warring States, compiled the 2 main Chinese constellation systems. These were later combined by Chen Zhuo in the Three Kingdoms period.
[74] The Ji in a calendar were numbered in the same style as years, by combinations of stems and branches (of which Jiazi was the first). Zu Chongzhi is being rather facetious here, because the numbering of Ji and years has nothing to do with the Great Year 1.

Dai Faxing: Your subject Faxing believes that the sun has 8 paths which combine into a single path, while the moon has one path which is split into 9 different paths. On the left is a path between the ascending and descending nodes, while on the right is a path between the perigee and apogee, so that the moon travels half on one path and half on the other. From this we know that the anomalistic month and the nodal month have the same length, since both are halves of the same circle. Chongzhi's nodal month is shorter than his anomalistic month by only 9040/26377 days, and even after slightly more than 79 revolutions, the difference would not accumulate up to a single anomalistic month. This leads to the moon speeding up when it should be slowing down, and to the month being longer when it should be shorter.

Zu Chongzhi: Dai Faxing's words may be vague and without proof, but one can guess the source of his ideas. When he says that the sun has 8 paths, he is probably speaking in relation to the 9 paths of the moon. This refers to the 4 parts of the moon's orbit: when the speed changes from fast to average, from average to slow, from slow to average, and from average to fast. These parts are further divided into half each, and if we include the nodes, when the path of the moon can be said to be the same as the sun (the ecliptic), then the moon does indeed have 9 paths altogether. However, there must be a specific range around the nodes where eclipses can occur, and it does not help to say that it is sometimes in Nandou and sometimes in Qianniu, like the sun at the solstice. If, at a node, the moon and the sun are on the same path, then they should have the same position in the lunar mansions. In that case, the moon should have a fixed distance from the North Celestial Pole (Quji di) during the nodes. How can it sometimes be north of the ecliptic and sometimes south, as it is in reality? Therefore it is pointless to liken the sun to the moon and claim that it has the same paths, but excluding the nodes.

As for the theory that the left of the lunar orbit is between the ascending and descending nodes, while the right is between the perigee and apogee, that sounds rather absurd. There are two possible meanings for what Faxing is trying to say: either that the node is opposite the perigee, or that the node is right at the beginning of the path from average speed to fast (which ends at the perigee). If the latter, then the nodes are 7 days and 21 days from the perigee respectively. In that case, the eclipses should always occur on those days, and there could not be any problem of the month being too long or too short. If the former meaning is correct, then the perigee starts exactly opposite to the node, and there could not be any problem of changing numbers of days before perigee. Nor does your subject understand where he got the notion that the moon travels half on one path and half on the other. Your subject has read many books on calendars, both ancient and recent, and never seen such a theory. It both distorts the teachings of the past and contradicts the astronomical realities of the present. Your subject is humbly concerned that it may become a cause of great confusion among the ignorant.

It is because the anomalistic and nodal months are different that eclipses do not always occur at the same time of day. Past writings have long mentioned this fact, and past scholars have spoken at length about it. Dai Faxing, however, says that the two have exactly the same value. Your subject deduces that he does not know what he is talking about, and his mistakes are so obvious that they need no further debate. He says that the moon speeds up when it should slow down, and vice versa, yet provides no numerical data to prove this. Perhaps he realizes his own lack of understanding and deliberately makes himself ambiguous. Furthermore, when comparing your subject's values for the nodal and anomalistic months, he mistakenly compares the numerical difference between the two with the anomalistic month directly. He even gets his mathematics wrong again: the difference should be 9033/26377, not 9040/26377, and the figure of 79 revolutions should in fact be 80. And he makes these errors in trying to prove that your subject's calculations are in error!

As a whole, Dai Faxing's arguments are not just implying that your subject's calendar is inaccurate, but also that He Chengtian's calendar is even more so. If your subject's calendar should be rejected, then He Chengtian's calendar, which we are now using, is even more unacceptable. If Faxing thinks that he is the most knowledgeable about calendars, then let him create a new one of his own. As for his arguments that the noon shadows on the solstices are not the longest or the shortest, and that the sun is not directly opposite the moon at a solar eclipse[75], your subject is interested in seeing what other new and wonderful theories he has to offer.

[75] Zu Chongzhi is probably indulging in a bit of exaggerated rhetoric himself, as Dai Faxing does not, in fact, seem to have made these arguments.

SECTION 6

HOW EFFECTIVE WAS THE DAMING REFORM?

In the debate between Zu Chongzhi and Dai Faxing, Zu clearly had the stronger argument, but all the ministers of the court chose to agree with Dai Faxing out of fear of his political power. The only one who supported Zu Chongzhi was, strangely enough, Dai Faxing's colleague Chao Shangzhi. Perhaps, being himself a Palace Secretarial Attendant in the emperor's favour, Chao was keen on showing that he was Dai Faxing's equal. Whatever the case, Chao Shangzhi insisted that the new calendar was worth adopting, and the emperor was gradually won over to this point of view. Apparently, Liu Jun himself had a taste for unusual things, especially those with a sense of antiquity about them. Zu Chongzhi's advocacy of the ancient idea of the Great Year 1 may have appealed to him. He decreed that the new calendar would come into use in the following year, since he had planned to change his reign title anyway. By this time, the debate had dragged on for 2 years, and it was now 464 AD (the 8th year of Daming). Therefore the new calendar would be used from 465 onwards.

However, things did not go as planned. Later in the year, Liu Jun died at the age of only 35 and was succeeded by his 16 year-old son, Liu Ziye. Any hopes that the new emperor would follow his father's wishes were dashed when Ziye turned out to be a mentally-unstable teenager, bent on using his absolute power in the most destructive ways possible. He killed his last surviving great-uncle Liu Yigong, 2 of his brothers and 2 of his best generals, and had his last few surviving uncles imprisoned and tortured sadistically. When Dai Faxing tried to restrain him from his excesses, Liu Ziye decided to get rid of the powerful minister as well. Dai's enemies had already whispered rumours to the emperor of how Dai Faxing was seen by the people as the real power behind the throne, and in 465 Liu Ziye sent Dai a cup of poison and an order to commit suicide. Dai Faxing had no choice but to obey; Chao Shangzhi, too, was relieved of his post. Meanwhile, Liu Ziye's servants, tired of being constantly threatened with execution by the emperor, conspired to assassinate him. That winter, they hacked Liu Ziye to death and placed his eldest surviving uncle on the throne. A civil war then broke out between Liu Ziye's uncles on one side and his brothers on the other, which the uncles won by the end of 466. They executed all 15 of Liu Jun's surviving sons, but this frightened those border generals who had supported the wrong side into defecting to the north. By 469, the Northern Wei had taken all Song territory between the Yellow and Huai rivers.

In the midst of this turmoil, Zu Chongzhi's calendar was completely forgotten, and Zu was not so foolhardy as to continue pressing the matter. Over the next decade, the Liu-Song Dynasty steadily declined until the throne was usurped in 479 by one of its generals, Xiao Daocheng. Xiao founded the Southern Qi Dynasty (so termed to differentiate it from the Northern Qi which rose later in the north) and ruled until his death in 482, upon which he was succeeded by his son Xiao Ze. The next decade was one of relative stability, and the Crown Prince Xiao Zhangmao managed to get his hands on Zu Chongzhi's calendar in 492. He was highly impressed and wrote a recommendation to the emperor, but unfortunately died of illness early in the next year, followed a few months later by Xiao Ze himself. With these untimely deaths, Zu Chongzhi again lost his chance. The throne passed to Xiao Zhangmao's son Xiao

Zhaoye, another decadent and irresponsible young man. One of Xiao Daocheng's nephews, Xiao Luan, launched a palace coup, killed the emperor and eventually usurped the throne in 494. Xiao Luan went on to murder almost all of Xiao Daocheng's and Xiao Ze's sons. With most of its imperial house dead and its borders under incessant attack by the Northern Wei, the Southern Qi soon declined after only 20 years in power.

In 500 AD, Zu Chongzhi passed away at the old age of 71. By this time, mutinies were breaking out all over the empire. That winter, Xiao Yan, a provincial governor and distant relative of the imperial house, rose in rebellion and succeeded within a year. In 502, he founded the Liang Dynasty. Another 2 years later, Chongzhi's son Zu Geng (known as Zu Gengzhi in some sources) was appointed as a Junior Consultant (Sanqi Shilang) to the Liang court. Zu Geng had inherited all of his father's knowledge of mathematics, astronomy and calendars. Just as Chongzhi had done in 462, he wrote a petition to point out the mistakes in the Yuanjia calendar, adding that "in the Daming period of the Song, your subject's late father examined the ancient methods and produced a better calendar, and all its astronomical predictions have indeed proven true since then." He got no response from the court, and spent the next few years on closer study of the gnomon and water clock. In 509, he again submitted his findings in a petition, together with a copy of his father's calendar. This time, Xiao Yan was sufficiently persuaded to grant his approval, and in 510 the Daming Calendar at last came into use.

Zu Geng himself became the most prominent mathematician and engineer of his time, and led a relatively eventful life. His greatest contribution to Chinese astronomy was measuring the polestar to be 1° south of the north celestial pole, when previous astronomers had assumed the star to be at the pole itself. In 514, while serving as General of Engineers (Caiguan Jiangjun), he was put in charge of building a dam on the Huai river to be used to flood a Northern Wei stronghold on its banks. After conducting a geological survey, Zu Geng concluded that the soil on the riverbed was unsuitable for damming, but his advice was ignored. The Wei city continued to hold out until, in 516, the dam did collapse with heavy losses to the Liang army. Zu Geng was imprisoned for his 'responsibility' in this disaster. After his release, he was reassigned as Minister of River Transport (Dazhou Qing). In 525, he was involved in a military campaign against the north, led by one of Xiao Yan's sons. When this son unexpectedly defected to the Northern Wei, Zu Geng was captured as well. He served for some time as a scientific expert in the Wei court, and then returned to the south in a prisoner exchange.

Zu Geng's son Zu Hao was a skilled mathematician and calendar expert as well. In 547, a general named Hou Jing, who has earlier defected from the north, rebelled and laid siege to Jiankang. The capital fell in 549, and Xiao Yan was imprisoned and starved to death in his own palace. Early in 550, Zu Hao tried to start a resistance movement in the nearby city of Guangling (present-day Yangzhou), but was soon captured and executed in an extremely cruel fashion: tied to a post and shot with arrows, and then torn apart by 5 horse-carts. The entire population of Guangling was buried from the waist down and used for target practice by Hou Jing's cavalry. Hou Jing himself was finally defeated and killed in 552, but his rebellion was a blow from which the south never fully recovered, and although the Liang was replaced by the Chen Dynasty in 557, its territorial, economic and military strength were much diminished. The Sui Dynasty rose to power in the north in 581, and launched a full-scale attack which conquered the Chen in 589. China was reunified after 300 years of division; the Southern Dynasties came to an end, and with it the use of the Daming calendar.

How effective was the Daming reform? We have seen that for political reasons, it was not effective at all within Zu Chongzhi's lifetime. It was then used in the south for 79 years, from 510 to 589, after which it was replaced by the much inferior Kaihuang calendar of the Sui. From this perspective, its record seems rather unimpressive. However, closer analysis will show that the Daming calendar did in fact have a great impact on the accuracy of later calendars from the Sui onwards. This impact was positive in some aspects and negative in others. We will now evaluate the various reforms in the Daming calendar, pointing out along the way how it influenced later calendars.

1. **The solar year and lunar month:** Zu Chongzhi gave a value of 365.2428 days for the solar year, which was far more accurate than previous values (the actual length of the solar year is about 365.24219878 days). The Daming calendar and Zhao Fei's Yuanshi calendar (which gave a value of 365.2443 and was used for 70 years in the north) can be said to have started a downward trend in solar year lengths. While the solar year had hovered in the range of 365.2462-69 for the last 200 years, almost all calendars after the Daming calendar moved to a range of 365.2430-49 (refer to the table in Appendix V). Unfortunately, none surpassed the accuracy of Zu Chongzhi's value all the way until 1199 AD, when the Song (note that this was a different dynasty from the Liu-Song) astronomer Yang Zhongfu gave a value of 365.2425 in his Tongtian calendar.

   Similarly, Zu Chongzhi's lunar month value (in exact terms) of 29.5305915 days was only 0.0000056 days longer than the actual value, which equates to an extra 0.485 seconds a day. This level of accuracy was again not exceeded until the Song, when Zhou Zong's Mingtian calendar gave a value of 29.530589743 days. Thus, we can see that while the Daming calendar was extremely accurate for its time, the effectiveness of this accuracy was sadly limited to a period of only 79 years, after which it was replaced by less accurate calendars for the next 500 years.

2. **The leap cycle:** Zu Chongzhi's accurate values for the solar year and lunar month were very much due to his cycle of 144 leap months every 391 years. If we divide the actual value of the solar year by that of the lunar month, we find that there should be about 12.36826678 months a year. Zu Chongzhi's cycle, which gave a result of 12.36828644 months, was more accurate than that of Zhao Fei, which gave 12.36833333. In fact, it was the most accurate of all leap cycles used in Chinese calendars, as the collective effect of Zu Chongzhi's calendar in the south and Zhao Fei's calendar in the north was that most calendars from the 6th century onwards abandoned the Metonic cycle and either formulated cycles of their own or borrowed those of earlier calendars. A table of the various kinds of cycles is given in Appendix VI, but only the first calendar in which each cycle appeared is listed. Actually, Zu Chongzhi's cycle was later used by the Tianhe calendar of the Northern Zhou. The Datong cycle

was borrowed by Liu Xiaosun and Zhang Mengbin in the Northern Qi (although all 3 of these calendars were never used in the end), and the Tianbao cycle was borrowed by Liu Chuo for his Huangji calendar (which was never used) and Fu Renjun for his Wuyin Yuan calendar.

The Wuyin Yuan calendar deserves more attention at this point, because it was the first calendar used by the Tang Dynasty, and also the last to have a leap cycle. The reason for this is that from the Tang onwards, Chinese calendars switched to using the true moon (ding shuo), which made the "no zhongqi" rule much more reliable and thus eliminated the need for a cycle. As mentioned in Section 4, He Chengtian had first suggested this move in 443 but was unsuccessful. A century later, in 544, the Imperial Astronomer Yu Kuang created a new Datong calendar for the Liang court, in which he proposed to use the true moon based on He Chengtian's original method. However, the Hou Jing rebellion broke out a few years later and the calendar was never used. The northern astronomer Xindu Fang (who, incidentally, had worked together with Zu Geng during his short stay in the north) also privately wrote a Lingxian calendar using the true moon, but died before he could finish it. It was only in 619, when the Tang adopted Fu Renjun's calendar, that the true moon was used for the first time. Even then, there were hiccups: when 4 long (30-day) months occurred in a row 20 years later, the court was so alarmed that they reverted to the mean moon. In 665, when Li Chungfeng's Linde Calendar replaced the Wuyin Yuan, the true moon came into use again, and has been used ever since.

In a sense, Zu Chongzhi was on the wrong track when he tried (successful though he was) to improve the accuracy of the leap cycle. If he had carried on He Chengtian's efforts to switch to the true moon, then there would not have been a need for an accurate cycle at all. Instead, he concentrated on Zhao Fei's method of "breaking the zhang cycle" while still using the mean moon, and this may have led Chinese calendars into a detour for 50 years.

3. **Measurement of the solstice time:** Since the Zhou Dynasty, measurements of the solstice time had usually taken the day with the shortest noon shadow as the day of the solstice. Because of optical error and the fact that the solstice mostly does not occur exactly at noon, such measurements were routinely in error by 2 or 3 days. The Taichu calendar was a significant exception, with an error of only 24 ke when it fixed the solstice at midnight on the Jiazi day of the 11^{th} month of 104 BC. Modern scholars have theorized that this exceptional result was a deduction made when the noon shadows on that day and the day before happened to be the same length, and were also the longest in the year.

   He Chengtian improved on this method by making round-the-clock observations of the gnomon around the time of the solstice, but the true breakthrough came with Zu Chongzhi's new, simplified method for measuring the solstice time. While He Chengtian managed to reduce his error to 50 ke, Zu Chongzhi's readings in 461 and 462 both had an error of only 20 ke (refer to the table in Appendix VI). Zu's method continued to be used by astronomers until the Song Dynasty, and during this time, readings had an average error of 20 ke although particularly skilled astronomers like Li Chunfeng and Yixing managed an error of only 1 to 2 ke. The Song astronomer Zhou Zong further improved upon Zu's method by specifying Lidong and Lichun (about 45 days before and after the solstice respectively) as the A and B points for noon shadows to be taken, thus nearly doubling Zu Chongzhi's range of measurement. This brought the average error down to just 10 ke. Later improvements by Yang Zhongfu and Guo Shoujing brought the error to less than 1 ke. These astronomers all used Zu's method as a starting point.

However, it must again be pointed out that Zu Chongzhi's method assumed an uniformity in the apparent motion of the sun which was not borne out by reality. Zu Chongzhi had no knowledge of the difference between the mean and true sun, and the first Chinese astronomer to discover this difference did so about 30 to 60 years after Zu's death. In the late 520s, as rebellions erupted in the declining Northern Wei, a scholar named Zhang Zixin took refuge on an offshore island. For the next 30 years, he spent his time on the island doing astronomical observations, from which he found that the sun slows down after the spring equinox and speeds up after the autumn equinox. He later passed on this knowledge to other astronomers in the new empire of Northern Qi. This was how the true sun came to be known to the Chinese, and it played a growing part in calendars from the Sui Dynasty onwards. Although the Chinese did not actually switch to following the true sun until the Qing Dynasty, they took it into account in measuring the solstice time - by increasing the measuring range, for example.

4. **Measurement of the solstice point:** We have stated that Jiang Ji's eclipse method simplified the measurement of the solstice point. Some credit must also be given to Zu Chongzhi for helping to popularize this method in the south, such that even up to the time of Guo Shoujing in the Yuan Dynasty, astronomers were still relying mainly on it. However, as can be seen from the table in Appendix VI, the eclipse method did not substantially increase the accuracy of measurement, because numerous other factors remained: inaccuracies in the armillary sphere, in the water clock, in the determination of true north and true south, in reading the positions of the moon and planets, in the non-uniform motion of the sun, and most importantly in measuring the solstice time and the distance between the lunar mansions. These last two errors have been added to the solstice point error in our table, to derive the "total error" in the solstice point for each calendar. From this table, we can see that Zu Chongzhi's solstice point of 11° within Nandou was more accurate than most earlier calendars, but was still less accurate than the Yuanjia calendar. Furthermore, the unfortunate fact that Zu's errors were all negative meant that they accumulated to a larger total error than all the earlier calendars except the Taichu calendar. This inaccuracy in measuring the solstice point is one of the reasons why the rate of precession derived by Zu Chongzhi was so much faster than the actual rate.

5. **Precession:** Yu Xi's precession rate of 1° every 50 years was faster than the actual rate of 1° every 70 years (or 72 in terms of Western degrees), while He Chengtian's rate seems to have been half that rate (1° every 100 years), which

was too slow. Zu Chongzhi, rather than taking the median of the two (75 years), which would have been almost correct, instead felt that Yu Xi's rate was still slightly too slow. As we have explained above, he was partly misled by inaccuracy in his reading of the solstice point. Another important factor was that all Chinese astronomers up to then had observed the solstice point based on the equatorial coordinate system of the lunar mansions. As the ecliptic is not parallel to the celestial equator, the rate of precession relative to the celestial equator (chidao sui cha) is actually about 1º every 77 years (78 years and 1 month in Western degrees) Furthermore, the ecliptic is 23.5º from the celestial equator at the winter solstice. This invariably had an adverse effect on accuracy, compared to the Greek ecliptic system which measured the equinoxes and solstices by their position along the ecliptic, and the stars in turn by their distances from the equinoxes. The modern Western equatorial system is in effect taking the best of both worlds by measuring from the vernal equinox, a point shared by both the ecliptic and celestial equator.

The Liang astronomer Yu Kuang later found Zu Chongzhi's rate of 1º every 45 years and 11 months to be too fast, but again went to the other extreme by suggesting 1º every 186 years in his Datong calendar. The Kaihuang calendar of the Sui Dynasty, which also displaced the Daming calendar in the south after reunification, was even worse in this respect. Its author, Zhang Bin, was a Daoist priest with little astronomical training, and he simply took the Yuanjia calendar and made some changes to it to produce a 'new' calendar. Not only did he continue using the Metonic cycle, he also totally ignored the discovery of precession and assumed that the solstice point was fixed. Not surprisingly, better Sui astronomers protested in outrage. They pointed out that Zu Chongzhi's calendar had already recognized the effect of precession for 80 years in the south, and this was proof of Zhang Bin's ignorance. After 10 years of debate and political struggle, Zhang Zhouxuan's Daye calendar replaced the Kaihuang in 596. Zhang Zhouxuan, who was a great admirer of Zu Chongzhi, sought a more balanced precession rate at 1º every 83 years, which is relatively reasonable if measured along the celestial equator. But the real breakthrough came with the Huangji calendar written by Zhang Zhouxuan's arch-rival Liu Chuo in 600.

Liu Chuo proposed, for the first time, the concept of measuring precession along the ecliptic, in degrees starting from 1º within the mansion of Xu (probably for similar reasons as why Zu Chongzhi chose it as the starting point for his Great Year 1). One historical source (Xin Tang Shu, the "New History of the Tang Dynasty") states that he merely took the median (75 years) of Yu Xi's and He Chengtian's rates as his precession rate, but this is clearly a misconception. For one thing, the two earlier rates were measured along the celestial equator, and thus could hardly form a basis for Liu Chuo's new method of measurement. For another, according to the Huangji calendar itself, the rate was 609.5/46644º a year, which is about 1º every 76.53 years. This is relatively close to the actual rate (for Chinese degrees) of 70 years. Relative to the celestial equator, this rate can be converted to 1º every 83.5 years, which is slightly less accurate than Zhang Zhouxuan's calendar. Thus we can see that if Zhang had adopted an ecliptic measurement, he would have gotten an even better value himself

.

The Huangji calendar's values for the anomalistic, sidereal and nodal months were also far more accurate than all previous values. However, Liu Chuo had even less luck than Zu Chongzhi, as his calendar was never used. Opposed at every turn by Zhang Zhouxuan, he died in 610 after 5 unsuccessful attempts to get his calendar accepted by the Sui court. In 618, the Tang Dynasty replaced the Sui and began using the Wuyin Yuan calendar. In 665, it switched to using Li Chunfeng's new Linde calendar. The otherwise brilliant Li Chunfeng for some reason did not believe in precession at all, and left it out of his calendar. Only in 728, with the adoption of the Dayan calendar written by the monk-astronomer Yixing, did "precession along the ecliptic" (huangdao sui cha) gain a permanent place in the Chinese calendar.

6. **The Great Year 1:** The most curious aspect of the Daming reform was the issue of shang yuan, the Great Year 1. Here, Zu Chongzhi and Dai Faxing seemed to switch roles: Zu became the champion of antiquity and tradition, while Dai argued for practicality and common sense. The most likely reason for this is that Zu Chongzhi was by nature a perfectionist. He saw He Chengtian's use of hou yuan as a compromise for the sake of expediency, "far from the level of perfection that the ancients aspired to". He also had such faith in mathematics that he believed a reliable method of finding a true, standard Great Year 1 was possible. However, we now know that such a method is not only unnecessary, but also inherently unworkable. Errors in observation were prevalent in early astronomy, while knowledge about irregularities in the motion of the sun, moon and planets was imperfect or incomplete. Even Zu Chongzhi's calendar was not above these weaknesses, despite his assertion that it could be used perpetually. Therefore it is impossible to be certain that one's projection of a Great Year 1 will be of any value in calculating the motions of the planets. It is testimony to Zu Chongzhi's skill in astronomy that his measurements of the conjunction of the planets were more accurate than those of He Chengtian. However, the fact is that from a scientific point of view, He Chengtian was right in simplifying the process of calculation with the use of hou yuan, while Zu Chongzhi was wrong in trying to make it even more complicated by adding yet more criteria to it. In any case, Chinese astronomers stuck to the anachronistic concept of the Great Year 1 until 1280, when Guo Shoujing's Shoushi calendar discarded it once and for all.

7. **The nodal month:** Zu Chongzhi gave a value of 27.21223035 days for the nodal month, which is only 0.00001514 days less than the actual length of 27.21221521 – an error of just 1.308 seconds a day. The Daming calendar was the first to give a value to the nodal month, but neither the Kaihuang or Daye calendars followed this example after reunification by the Sui Dynasty. Liu Chuo gave an even more accurate value of 27.212222 days in his Huangji calendar, but as this never came into use, nodal months did not become a common feature of Chinese calendars until the Tang Dynasty, starting with Li Chunfeng's Linde calendar in 665. It used a value of about 27.21223, similar to Zu Chongzhi's value. The next was Yixing's Dayan calendar, which had an even worse value of 27.21200. 6 of the 7 calendars used in the Tang Dynasty

had nodal months, but Liu Chuo's nodal month was not exceeded in accuracy until the last Tang calendar, the Chongxuan calendar of 893.

As for the "range of no partial eclipse" found by Zu Chongzhi (15.19º in Western degrees), it was in fact less accurate than that given by the Qianxiang calendar (14.55º in Western degrees), as we now know the actual range to be 12.37º. After reunification, the Kaihuang calendar of the Sui and the Dayan and Xuanming calendars of the Tang all used Zu Chongzhi's figure, while others used even less accurate ones, until the Chongxuan calendar found a new value of 14.59º, which approached the accuracy of the Qianxiang calendar. The closest that any calendar ever came was 12.38º, in the Jiyuan calendar, written in 1106 in the Song Dynasty. However, no other calendar followed this example.

Another astronomical phenomenon of which Zu Chongzhi was unaware was the precession of the lunar orbit, and the consequent shifting of the line of nodes (refer to Appendix IV for a diagram of this). In his debate with Dai Faxing, Zu Chongzhi could only vaguely state that the difference between the nodal and anomalistic months, and the fact that the line of nodes is not perpendicular to the line between the perigee and apogee, is the reason why eclipses occur at different times and dates. In fact, the nodes, perigee and apogee all precess together, but to a different extent, since they are on different parts of an elliptical orbit. The nodes precess by about 19.3° in a year, and the perigee by about 40.7º a year. This difference in the rates of precession is the actual reason why eclipses occur on different dates in successive years. By the time of the Dayan calendar, Yixing knew that the moon does not return to the same point after one round in its orbit. That Zu Chongzhi's eclipse predictions were highly accurate despite not knowing about lunar precession is, again, indicative of his skill in mathematics.

A final area that warrants some attention is the theory suggested by some scholars that Zu Chongzhi's recognition of precession was a victory for the Xuanye (Light and Darkness) school in the Shape of the Heavens debate. They argue that only an astronomer who believed the Xuanye theory that the heavenly bodies are not "tied to the same roots" could have accepted that the solstice point shifts relative to the stars. They also point to the fact that Yu Xi, the original Chinese discoverer of precession, was a major proponent of the Xuanye school, and wrote a "Treatise on Stabilising the Heavens" (Antian Lun) which became the key Xuanye text. Ge Hong, a prominent Daoist alchemist of the Eastern Jin, criticized Yu Xi's theories with words which sound remarkably like Dai Faxing's: "If the stars and the lunar mansions are not attached to the heavens, then the heavens serve no useful purpose, and hence we can assert that the heavens do not exist at all. What then is the purpose of postulating a heaven that remains stationary?"

However, there are some flaws in this theory that we should note. Firstly, Zu Chongzhi never expressed any opinion about the shape of the heavens in his writings, nor did he make any reforms that questioned the validity of the Celestial Sphere theory. Secondly, Zu Chongzhi's own son Zu Geng wrote a "Treatise on the Celestial Sphere" (Huntian Lun) which asserts that among all the theories about the shape of the heavens, "the principle of the celestial sphere has been proven to be the most believable". Since Zu Geng received all his astronomical training from his father, we can assume that Zu Chongzhi accepted the celestial sphere as well. Thirdly, the most advanced versions of the Huntian school had, by Zu Chongzhi's time, recognized that not all the heavenly bodies were fixed on a rotating sphere, but some of them moved in their own paths as well. The Gaitian school even experienced a kind of revival in the Liang Dynasty, as the emperor Xiao Yan was a devout Buddhist and was thus influenced by Indian astronomical traditions (contained in the Buddhist texts) to promote the Celestial Dome as the true shape of the heavens. Meanwhile, the Xuanye school faded into obscurity, until the only surviving historical record of its existence was a passage in the astronomical section of the Jin Shu ("History of the Jin Dynasty"), written by Li Chunfeng. If Zu Chongzhi's concept of precession had validated the Xuanye school, then the theory would not have died out among later astronomers who believed in precession.

Based on our evaluation of Zu Chongzhi's reforms, it would seem that the effectiveness of the Daming calendar has been over-stated by modern Chinese. The accurate solar year and lunar month values were only used for 79 years, and otherwise caused only a minor improvement in later calendars. The leap cycle was a step in the wrong direction, as was the Great Year 1. The precession rate, as well as the methods for measuring the solstice point and solstice time, were all hampered by inaccuracy and gaps in astronomical knowledge. The nodal month was ahead of its time, and did not come into regular use until 200 years later. The "range of no partial eclipse" was a step backwards that had adverse effects on calendars for more than 400 years. Given these facts, it might appear strange that Zu Chongzhi is famous at all, other than for refining the value of π. His confidence in declaring his calendar applicable for all eternity looks laughable with hindsight.

However, that would be missing the point in a way. The value of Zu Chongzhi's achievements has always been more symbolic than practical. To the Chinese, he deserves credit for including precession in his calendar, even if his figure was inaccurate, just as he deserves praise for his value for π even though it has since been surpassed. What is important is that he managed to achieve it in his time, under those limitations. Moreover, his courageous stand against Dai Faxing has been lauded through the centuries as an exemplar of the conviction for truth embodied in the scientific spirit. That conservative, Confucian China even had a scientific spirit to speak of is a great source of pride and comfort to today's Chinese scientists, and this understandably makes Zu Chongzhi a hero for many of them. Western scientists recognize this fact, which is one reason why the larger craters on the moon, which are named after "famous deceased scientists, scholars and artists", include one named after Zu Chongzhi. He is the only Chinese to be thus commemorated in the heavens.

APPENDIX I

The 28 Lunar Mansions

Eastern Palace: Qinglong 青龙 (Green Dragon)	Northern Palace: Xuanwu 玄武 (Black Warrior)	Western Palace: Baihu 白虎 (White Tiger)	Southern Palace: Zhuque 朱雀 (Red Bird)
1. Jiao 角	8. Nandou 南斗	15. Kui 奎	22. Dongjing 东井
2. Kang 亢	9. Qianniu 牵牛	16. Lou 娄	23. Yugui 舆鬼
3. Di 氐	10. Xunu 须女	17. Wei 胃	24. Liu 柳
4. Fang 房	11. Xu 虚	18. Mao 昴	25. Qixing 七星
5. Xin 心	12. Wei 危	19. Bi 毕	26. Zhang 张
6. Wei 尾	13. Yingshi 营室	20. Zi 觜	27. Yi 翼
7. Ji 箕	14. Dongbi 东壁	21. Shen 参	28. Zhen 轸

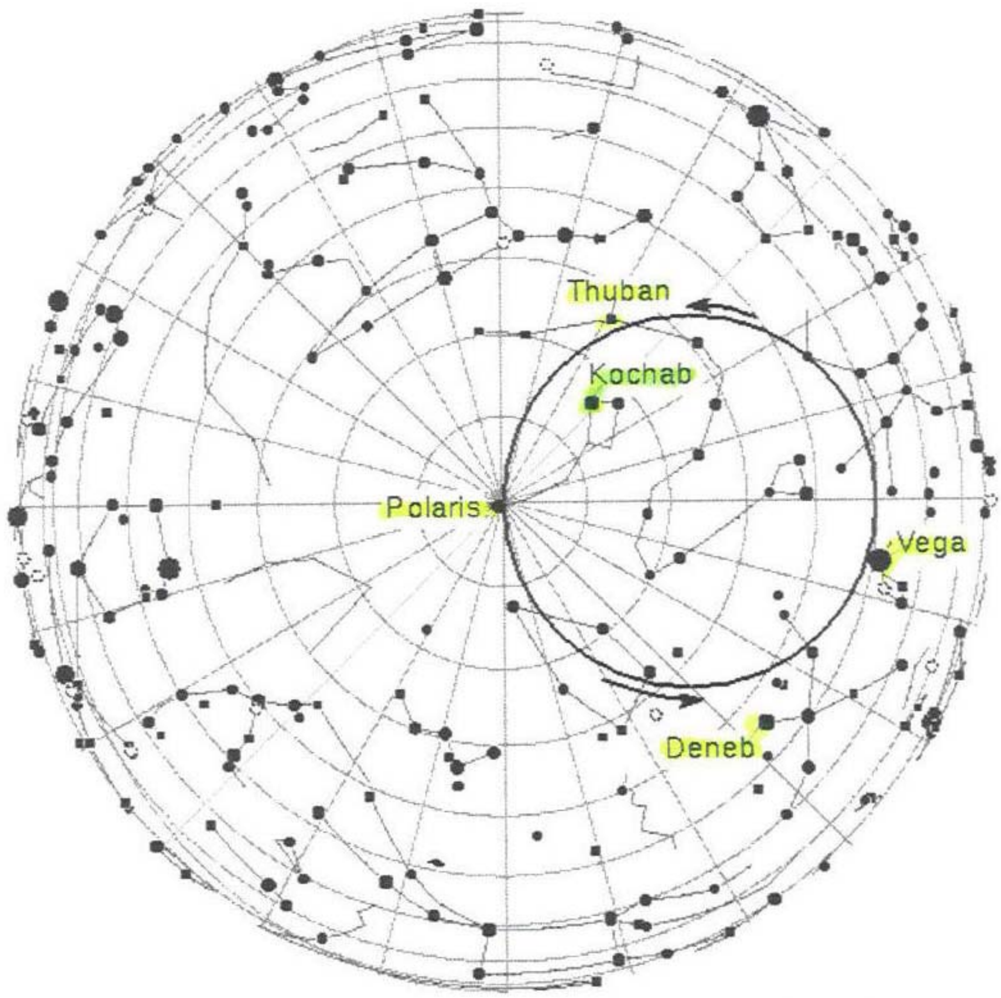
The 12 Zones of Jupiter

Zone	Lunar mansions	Earthly Branch	Corresponding State
Shouxing 寿星	Zhen 轸 to Di 氐	Chen 辰	Zheng 郑
Dahuo 大火	Di 氐 to Wei 尾	Mao 卯	Song 宋
Ximu 析木	Wei 尾 to Nandou 南斗	Yin 寅	Yan 燕
Xingji 星纪	Nandou 南斗 to Xunu 须女	Chou 丑	Wu 吴 and Yue 越
Xuanxiao 玄枵	Xunu 须女 to Wei 危	Zi 子	Qi 齐
Zouzi 诹訾	Wei 危 to Kui 奎	Hai 亥	Wei 卫
Jianglou 降娄	Kui 奎 to Wei 胃	Xu 戌	Lu 鲁
Daliang 大梁	Wei 胃 to Bi 毕	You 酉	Zhao 赵
Shishen 实沈	Bi 毕 to Dongjing 东井	Shen 申	Wei 魏
Chunshou 鹑首	Dongjing 东井 to Liu 柳	Wei 未	Qin 秦
Chunhuo 鹑火	Liu 柳 to Zhang 张	Wu 午	Zhou 周
Chunwei 鹑尾	Zhang 张 to Zhen 轸	Si 巳	Chu 楚

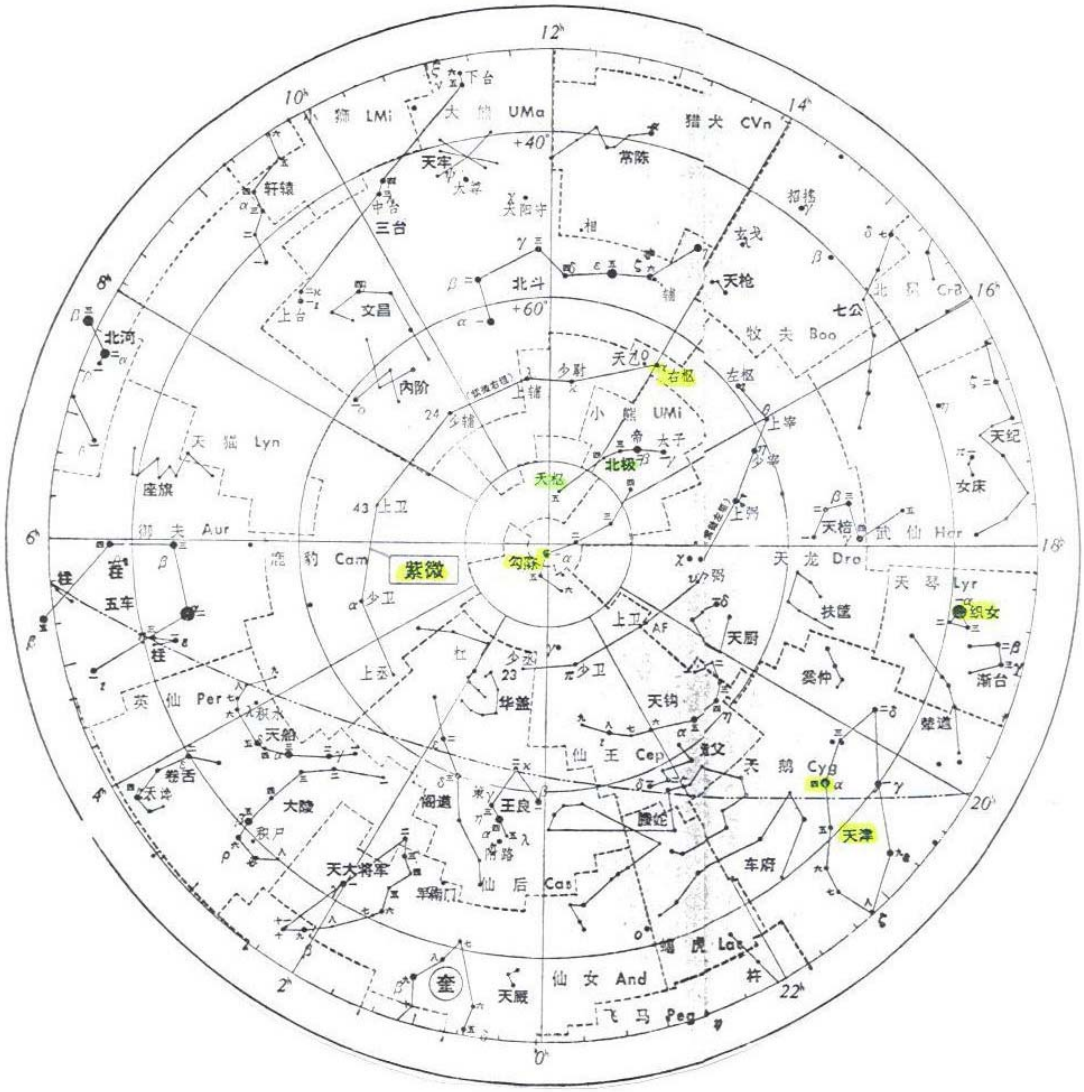
APPENDIX II

The effect of the path of precession on the polestar

The Western polestars and constellations near the north celestial pole (as seen from the North Pole) are shown on this page, and the Chinese ones on the next. Polestars and their constellation names are highlighted with yellow, except for Kochab (Western), which was the Western polestar in the 1st Century AD, and Tianshu (Chinese), which was the Chinese polestar at that time. They are in green.



The path of the precession of the Earth's rotation axis.
It takes 26,000 years to complete a full 360° wobble.

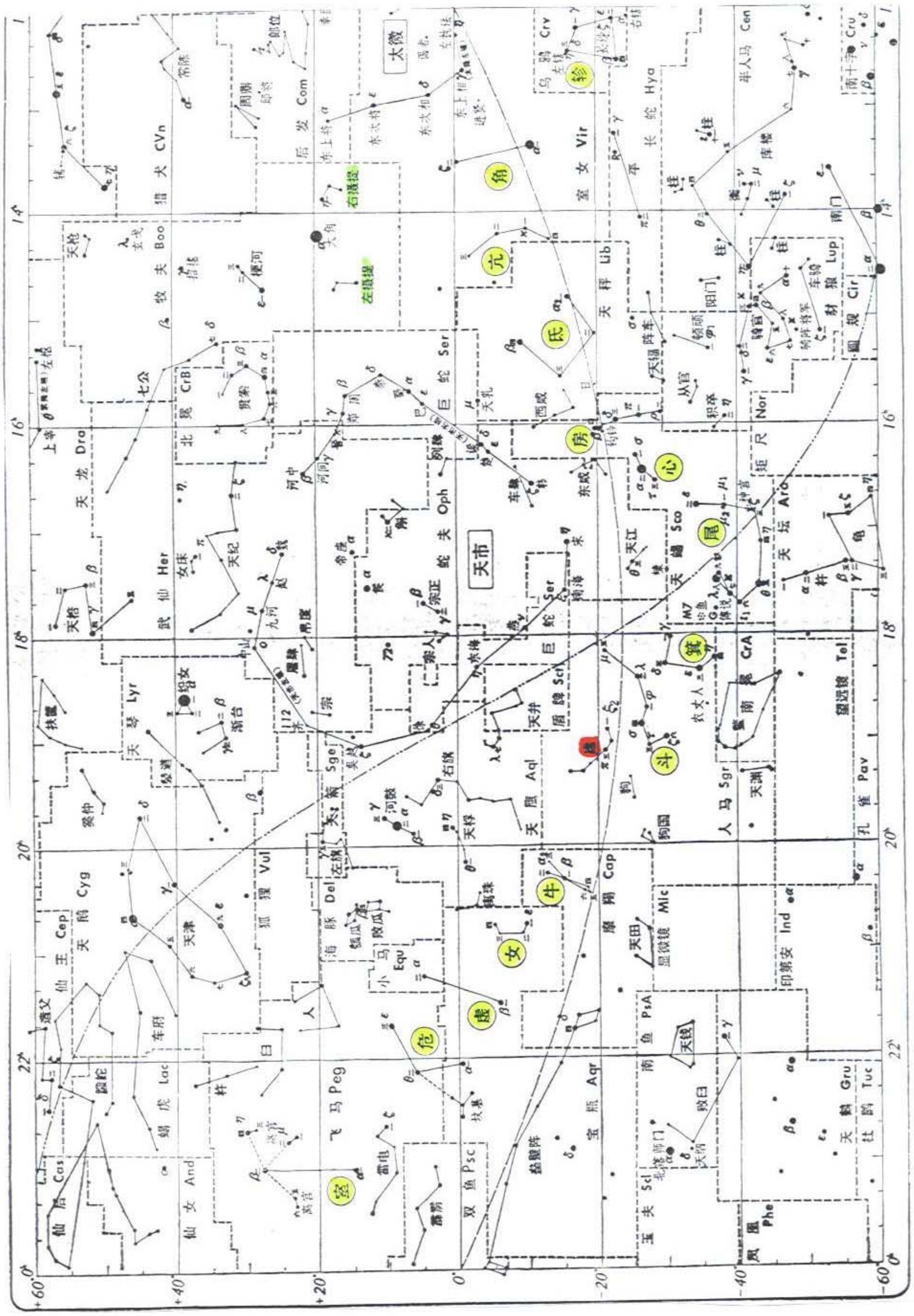


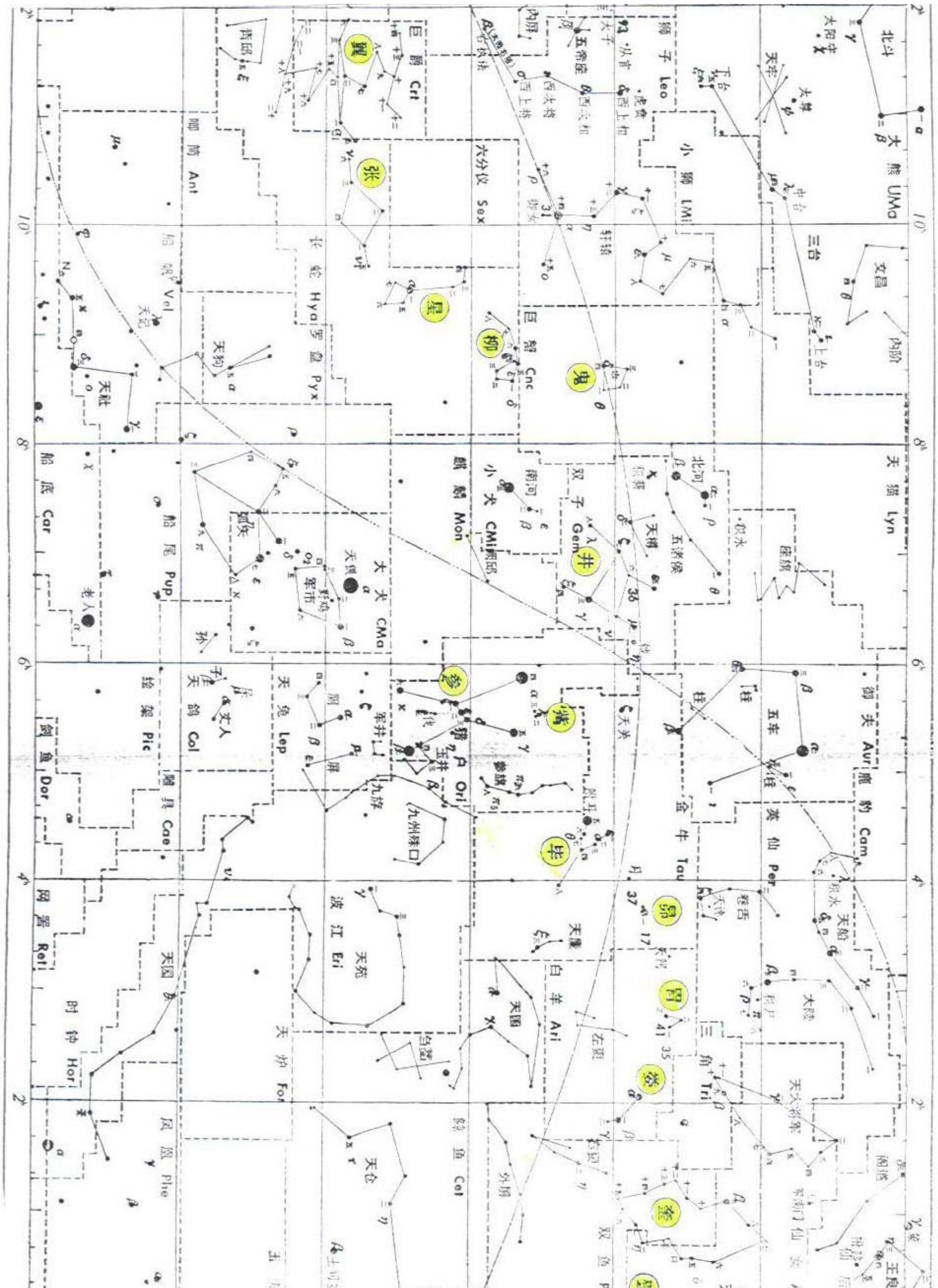
12h
10h
14h
16h
18h
20h
22h
0h
2h
6h
8h
+40°
+60°
小狮 LMi
大熊 UMa
猎犬 CVn
下台
天牢
中台
三台
常陈
轩辕
相
摄提
玄戈
北斗
天枪
文昌
上台
北冕 CrB
七公
牧夫 Boo
内阶
少尉
天乙
右枢
左枢
上辅
少辅
小熊 UMi
帝
太子
上宰
少宰
天纪
天猫 Lyn
天枢
北极
女床
座旗
上卫
天棓
武仙 Her
御夫 Aur
鹿豹 Cam
紫微
勾陈
天龙 Dra
天琴 Lyr
北河
柱
五车
少卫
扶筐
织女
天厨
奚仲
渐台
英仙 Per
积水
上丞
杠
华盖
天钩
辇道
天船
仙王 Cep
造父
天鹅 Cyg
卷舌
大陵
阁道
王良
螣蛇
天津
积尸
车府
天大将军
仙后 Cas
蝎虎 Lac
杵
奎
仙女 And
天厩
飞马 Peg

APPENDIX III

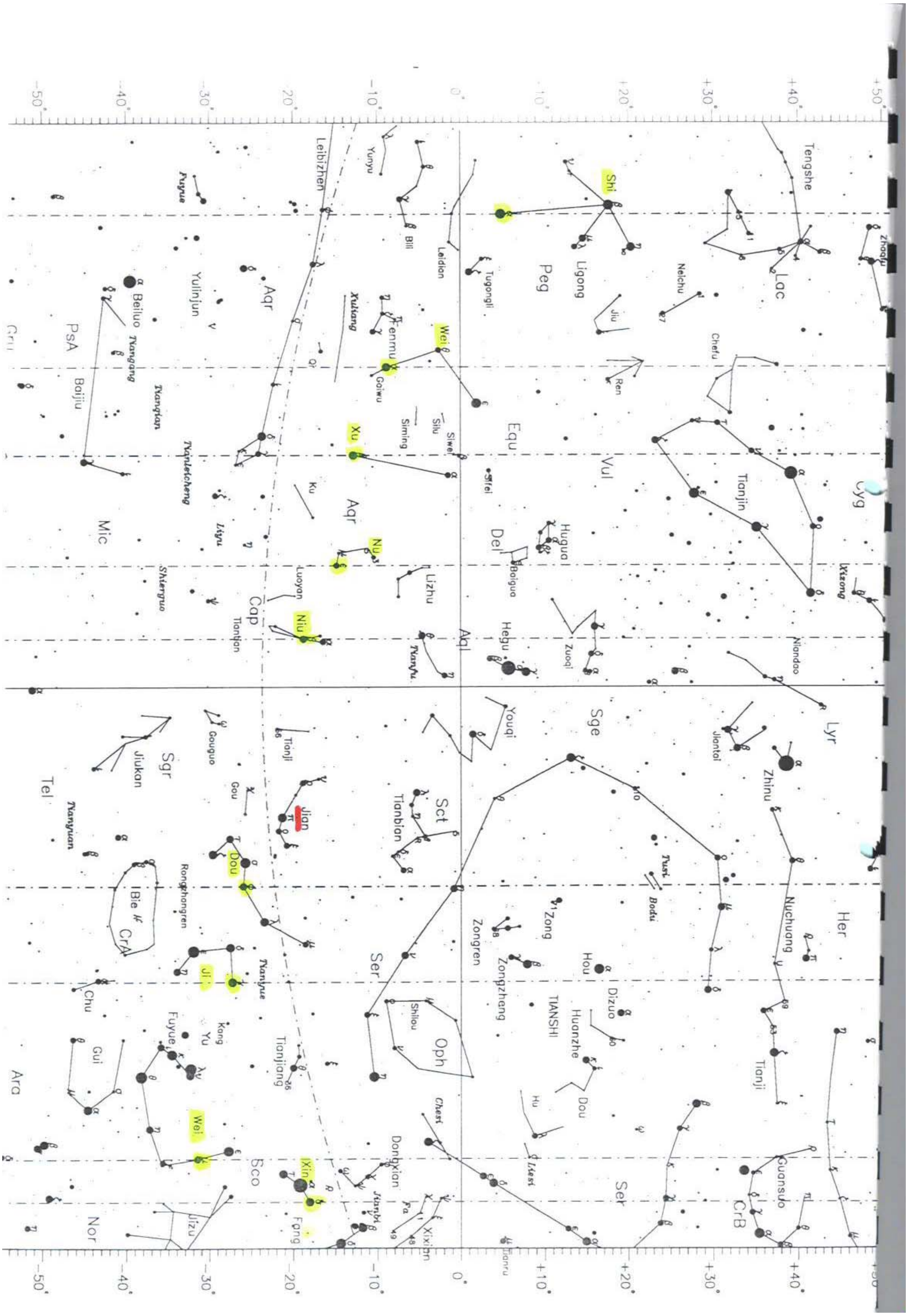
Star maps of the lunar mansions

Mansions from Jiao to Yingshi in the Eastern and Northern Palaces, as well as Zhen in the Southern Palace, are shown from right to left (west to east) on this page. The next page continues from Dongbi to Yi. The constellation of Sheti (split into left and right) is highlighted in green, and Jianxing (Jian) in red. The third map shows, in English, the mansions from Xin to Yingshi with their determinative stars highlighted. Note that some mansions are in their short forms here: Nandou as Dou, Qianniu as Niu, Xunu as Nu, Yingshi as Shi, Dongbi as Bi, Dongjing as Jing, Yugui as Gui, and Qixing as Xing.

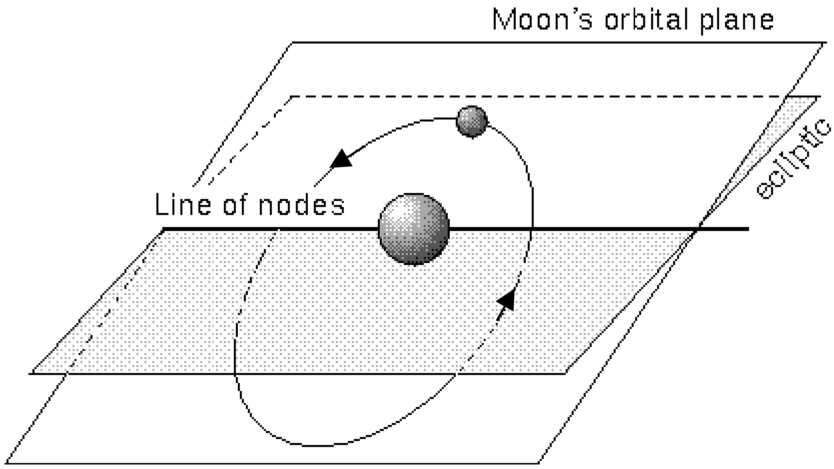




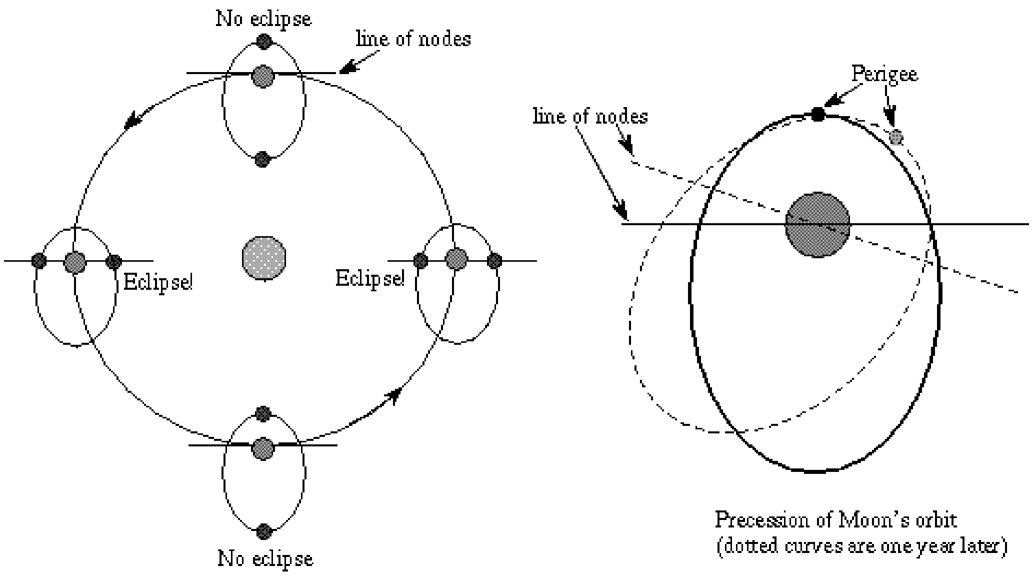
北斗
大熊 UMa
三台
文昌
天猫 Lyn
狮子 Leo
小狮 LMi
六分仪 Sex
巨爵 Crt
唧筒 Ant
长蛇 Hya
罗盘 Pyx
船帆 Vel
巨蟹 Cnc
双子 Gem
小犬 CMi
麒麟 Mon
大犬 CMa
船尾 Pup
船底 Car
御夫 Aur
鹿豹 Cam
英仙 Per
金牛 Tau
猎户 Ori
天兔 Lep
天鸽 Col
绘架 Pic
剑鱼 Dor
雕具 Cae
网罟 Ret
波江 Eri
白羊 Ari
三角 Tri
时钟 Hor
天炉 For
鲸鱼 Cet
凤凰 Phe
双鱼 Psc
翼
张
星
柳
鬼
井
参
觜
毕
昴
胃
娄
奎
壁


Tengshe
Lac
Shi
Ligong
Peg
Neichu
Chefu
Jiu
Ren
Tugongli
Leidian
Bili
Yunyu
Leibizhen
Fuyue
Yulinjun
Aqr
Beiluo
PsA
Tiangang
Baijiu
Xuliang
Fenmu
Wei
Goiwu
Equ
Vul
Tianjin
Cyg
Xu
Siming
Silu
Siwei
Sifei
Ku
Tianqian
Tianlecheng
Mic
Ligu
Huagua
Del
Baigua
Nu
Lizhu
Luoyan
Cap
Tiantian
Shiergua
Niu
Tianfu
Aql
Hegu
Zuoqi
Niandao
Xizong
Lyr
Jiantai
Zhinu
Sge
Youqi
Sgr
Jiukan
Tel
Gouguo
Tianji
Gou
Jian
Tianbian
Sct
Tianyuan
Dou
Ronghongren
Bie
CrA
Tianyue
Ji
Chu
Kang
Yu
Fuyue
Gui
Tianjiang
Ara
Wei
Sco
Xin
Fang
Nor
Jizu
Ser
Zongren
Zong
Zongzheng
Hou
Dizuo
TIANSHI
Huanzhe
Dou
Hu
Shilou
Oph
Donghxian
Xixian
Her
Nuchuang
Tianji
Guansuo
CrB
Tunü
Bodu
Chesi
Liesi
-50°
-40°
-30°
-20°
-10°
0°
+10°
+20°
+30°
+40°
+50°

APPENDIX IV
The Line of Nodes



The Moon's orbit is tilted by 5° with respect to the ecliptic (the Earth's orbital plane). The **line of nodes** is the intersection of the two planes.



Top view of Moon's orbit and Earth's orbit

The Earth-Moon system at four points in their orbit around the Sun. The **line of nodes** is the intersection of the Moon's orbit with the ecliptic (Earth's orbit). The Moon crosses the ecliptic twice a month. An eclipse happens when the Moon is in either new or full phase **AND** at the line of nodes. The Moon's orbit is slightly elliptical (exaggerated a lot in this picture) and the Moon may not be exactly on the line of nodes so a solar eclipse may or may not be total.

The Moon's orbit precesses so the line of nodes shifts by about 19.3° in one year and the perigee (closest point to Earth) of the Moon's orbit shifts by about 40.7° in one year. The line of nodes completes one full precession in 18.61 years. This precession explains why eclipses occur at different dates in successive years.

APPENDIX V

Ancient Chinese Calendars (Up to the Dayan calendar)

Name of calendar	Author	Year written	Years used	Dynasties using	Solar year	Lunar month	Anomalistic month
Huangdi 黄帝, Xia 夏, Yin 殷, Zhou 周, Lu 鲁	Unknown	Unknown	Unknown	Unknown	365.25	29.53085	-
Zhuanxu 颛顼	Unknown.	Unknown	221(?) – 105BC	Qin, Han	365.25	29.53085	-
Taichu 太初	Deng Ping, et al	104BC	104BC – 84AD	Han	365.2502	29.53086	-
Sifen 四分	Li Fan, et al	85AD	85 – 263AD	Han, Shu-Han	365.25	29.53085	-
Qian xiang 乾象	Liu Hong	206AD	223 – 280AD	Wu	365.2462	29.53054	27.55336
Jingchu 景初	Yang Wei	237AD	237 – 451AD	Wei, Jin, Song, Northern Wei	365.2469	29.53060	27.55450
Sanji 三纪	Jiang Ji	384AD	384 – 417AD	Later Qin	365.2468	29.53060	-
Yuanshi 元始	Zhao Fei	412AD	412 – 439AD, 452 – 522AD	Northern Liang, Northern Wei	365.2443	29.53060	-
Yuanjia 元嘉	He Chengtian	443AD	445 – 509AD	Song, Southern Qi, Liang	365.2467	29.53059	27.55452
Daming 大明	Zu Chongzhi	462AD	510 – 589AD	Liang, Chen	365.2428	29.53059	27.55468
Zheng guang 正光	Zhang Longxiang	521AD	523 – 565AD	Northern Wei, Northern Zhou	365.2437	29.53059	-
Xinghe 兴和	Li Yexing	540AD	540 – 550AD	Eastern Wei	365.2442	29.53060	-
Tianbao 天保	Song Jingye	550AD	551 – 577AD	Northern Qi	365.2446	29.53060	-
Tianhe 天和	Jia Luan	566AD	566 – 578AD	Northern Zhou	365.2443	29.53071	-

Daxiang 大象	Ma Xian	579AD	579 – 583AD	Northern Zhou, Sui	365.2438	29.53063	-
Kaihuang 开皇	Zhang Bin	584AD	584 – 596AD	Sui	365.2434	29.53061	27.55451
Daye 大业	Zhang Zhouxuan	597AD	597 – 618AD	Sui	365.2430	29.53059	27.55455
Wuyin Yuan 戊寅元	Fu Renjun	619AD	619 – 664AD	Tang	365.2446	29.53060	27.55454
Linde 鳞德	Li Chunfeng	665AD	665 – 728AD	Tang	365.2448	29.53060	27.55456
Dayan 大衍	Yixing	728AD	729 – 761AD	Tang	365.2444	29.53059	27.55453

APPENDIX VI

Leap cycles in Chinese calendars

Dynasty	Calendar	Years	Leap Months	Note
Warring States	Sifen	19	7	-
Northern Liang	Yuanshi	600	221	-
Liu - Song	Daming	391	144	-
Northern Wei	Zhengguang	505	186	-
Eastern Wei	Xinghe	562	207	-
Liang	Datong	619	228	Never used
Northern Qi	Tianbao	676	249	-
Northern Qi	Jiayin	657	242	Never used
Northern Zhou	Daxiang	448	165	-
Sui	Kaihuang	429	158	-
Sui	Daye	410	151	-

Errors in calendar readings

Name Of Calendar	Year	Solstice time recorded in ke	Actual time	Error	Error in degrees (A)	Solstice point recorded in degrees	Actual point	Error (B)	Lunar mansions error (C)	Total error (A+B+C)
Taichu	104BC	Midnight	76	-24	-0.2	Qianniu 6	Nandou 21.6	4.6	-0.2	4.2
Sifen	85AD	50	11	-239	-2.4	Nandou 21.25	Nandou 18.6	2.6	-0.5	-0.3
Qianxiang	206AD	6	88	-218	-2.2	Nandou 21	Nandou 16.8	4.1	-0.7	1.2
Jingchu	237AD	87	66	-221	-2.2	Nandou 21.25	Nandou 16.3	4.9	-0.8	1.9
Sanji	384AD	90	13	-277	-2.8	Nandou 17	Nandou 14	3.0	-1.1	-0.9
Yuanjia	443AD	99	49	-50	-0.5	Nandou 14.1	Nandou 13.1	1.0	-1.2	-0.7
Daming	462AD, 463AD	31, 55	11, 35	-20	-0.2	Nandou 11	Nandou 12.7	-1.7	-1.2	-3.1

GLOSSARY OF IMPORTANT CHINESE TERMS AND NAMES

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Chao Shangzhi 巢尚之: An influential minister in the reign of Liu Jun
Chen 陈: Dynasty which ruled southern China from 549 to 589 AD
Ci 次: 12 zones, each covering the distance traveled by Jupiter in one year of its orbit
Dai Faxing 戴法兴: An influential minister in the reign of Liu Jun, executed in 465
Eastern Jin 东晋: Dynasty which ruled in southern China from 317 to 420 AD
Gaitian shuo 盖天说: The ancient Chinese theory of the Celestial Dome
He Chengtian 何承天: A scholar and astronomer of the early Liu-Song period
Huntian shuo 浑天说: The ancient Chinese theory of the Celestial Sphere
Jiang Ji 姜岌: An astronomer of the Later Qin kingdom
Jiaodian Yue 交点月: Nodal month (time between 2 ascending or descending nodes)
Jindian Yue 近点月: Anomalistic month (time between 2 perigees)
Later Qin 后秦: A Qiang kingdom which ruled in western China from 384 to 417 AD
Li Chunfeng 李淳风: An astronomer of the Tang Dynasty
Liang 梁: Dynasty which ruled in southern China from 502 to 549 AD.
Liu Chuo 刘绰: An astronomer of the Sui Dynasty
Liu Jun 刘骏: Emperor of the Liu-Song Dynasty from 453 to 464 AD
Liu-Song 刘宋: Dynasty which ruled in southern China from 420 to 479 AD
Liu Ziye 刘子业: Son of Liu Jun and emperor of the Liu-Song from 464 to 465 AD
Northern Liang 北凉: A kingdom in northwestern China from 397 to 439 AD
Northern Wei 北魏: Dynasty which ruled in northern China from 386 to 556 AD
Qian Lezhi 钱乐之: Imperial Astronomer in the early Liu-Song period.
Shangyuan 上元: Great Year 1, the theoretical Chinese "beginning of time"
Southern Qi 南齐: Dynasty which ruled in southern China from 479 to 502 AD
Sui 隋: Dynasty which unified China in 589 AD and ruled until 618 AD.
Sui Cha 岁差: Chinese term for the precession of the solstices (or equinoxes)
Wei 魏: Dynasty of the 3 Kingdoms which ruled northern China from 220 to 265 AD
Western Jin 西晋: Dynasty which unified China in 281 AD and ruled until 317 AD
Xiao Daocheng 萧道成: Founding emperor of the Southern Qi, reigned 479-482 AD
Xiao Luan 萧鸾: Nephew of Xiao Daocheng and emperor from 494 to 498 AD
Xiao Yan 萧衍: Founding emperor of the Liang Dynasty, reigned 502-547 AD
Xiao Ze 萧赜: Son of Xiao Daocheng and Southern Qi emperor from 482 to 493 AD
Xiao Zhangmao 萧长懋: Eldest son and Crown Prince of Xiao Ze, died in 493 AD
Xiu 宿: Lunar Mansions, a group of 28 major Chinese constellations
Xuanye shuo 宣夜说: The ancient Chinese theory of Light and Darkness
Yixing 一行: Monk-astronomer of the Tang Dynasty
Yu Kuang 虞邝: Astronomer of the Liang Dynasty
Yu Xi 虞喜: Astronomer of the Eastern Jin Dynasty
Zhangfa 章法: Chinese term for the Metonic cycle of 7 leap months in 19 years
Zhao Fei 赵 : Astronomer of the Northern Liang kingdom
Zu Chongzhi 祖冲之: Astronomer-mathematician of the Liu-Song and Southern Qi
Zu Geng 祖 : Son of Zu Chongzhi, himself a mathematician in the Liang Dynasty

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