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The Life and Work of Augustus De Morgan

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2006 marks the bicentenary of the birth of Augustus De Morgan

Augustus De Morgan was one of Britain's notable mathematicians of the 19th century. He made significant contributions to Logic, Algebra, Analysis, and the History of Mathematics. Known for his great sense of humour, he had a passion for collecting and posing problems, puzzles, and riddles. De Morgan was a talented teacher and made a lasting impression on many of his students.

De Morgan was born in Madura, India, on 27 June 1806. He was the fifth child of Colonel John De Morgan, who was serving with the East India Company. His mother was a descendant of John Dodson, who was known for his Table of Anti-logarithms. During De Morgan's infancy, he lost the sight in his right eye and, at seven months old, his father took the family to England. De Morgan had 'painful remembrance' of most of his early school days. He did not participate in the usual sports. Often he was the victim of pranks played by the other boys.

Figure 1 Augustus De Morgan (UCL Library Services: Special Collections).

He received his early education in private schools, where he studied Greek, Latin, and Mathematics. His talent in Mathematics became quite evident by the age of 14. According to the American mathematician George Halstead, 'he read Algebra "like a novel", and pricked out equations on the school pew instead of listening to the sermon'. De Morgan entered Trinity College, Cambridge, in 1823 and established a lifelong friendship with two of his professors, George Peacock and William Whewell. Peacock would influence De Morgan's interest in Algebra, while Whewell would foster his interest in Logic.

During his college years, De Morgan showed little interest in athletics, but was an active member of the musical club CAMUS (Cambridge Amateur Musical Union Society). He competed in The Mathematical Tripos and took fourth place which, according to many of his contemporaries, did not reflect his true mathematical abilities. In 1827 De Morgan received his BA degree. Due to his strong religious convictions, he objected to signing the theological tests and as a result was not able to obtain an MA degree or fellowship from Cambridge.

His opportunity to teach at Cambridge had been basically closed. Fortunately, University College London had just been founded and, in 1828, De Morgan was elected the first Chair of Mathematics at the age of 21. With the exception of a five-year hiatus, De Morgan served in this position for 30 years. During his career, De Morgan not only become an influential mathematician, but he established a reputation as a brilliant and inspiring teacher.

Incorporating a natural display of good humour with his mathematical prowess, De Morgan delivered clear and systematic lectures that captured his students' attention. In addition to his lectures, De Morgan developed handwritten notes, which were located in the library, for his students to supplement their studies. He was a firm believer in assigning numerous homework problems, usually at the end of each lecture, designed to foster critical thinking among his students. Several of his students who later became successful themselves include Stanely Jevons, Isaac Todhunter, E. J. Routh, and J. J. Sylvester.

De Morgan's 'unrivalled' ability as a teacher paralleled his ability in writing a tremendous number of books and articles. In 1828, he published a translation of the first three chapters of Pierre Louis Marie Bourdon's (1779–1854) *Élémens d'Algèbre*. Bourdon's treatise on algebra was extremely popular during this period in France. Likewise, De Morgan believed Bourdon's book contained material 'well adapted' for teaching the fundamental concepts of algebra.

De Morgan published *Elements of Arithmetic* in 1831. This book was primarily written because he believed most educators at the time did not see the relevance of teaching the young mind arithmetic through reason and demonstration. Modelling his approach to teaching, De Morgan's book stressed establishing 'the foundation of principles' and then developing the concepts. According to one of De Morgan's past students, Richard Hutton, 'the publication of his *Arithmetic*, a book which has not unnaturally been more useful to masters than to scholars, began a new era in the history of elementary teaching in England'.

During the summer of 1831, an administrative action by University College London on a fellow colleague affected De Morgan's relationship with the institution. The University Council dismissed the Professor of Anatomy 'without any fault of his own' (according to De Morgan). Having strong opinions of his own regarding the sanctity of professorship, De Morgan, in turn, sent a letter of resignation to the Council and left the College.

After leaving the College, De Morgan became heavily involved with the Astronomical Society, and was elected as honorary secretary in 1831. He established close relationships

with several notable members, including Sir John Herschel. De Morgan was particularly fond of Herschel, corresponding regularly with him for the next 40 years. The Astronomical Society provided De Morgan with the opportunity to establish notoriety as an historian. His lack of eyesight prevented him from experimenting with instruments, so he turned to writing articles on Astronomy and related subjects. Many of his articles were published in the *Penny Cyclopedia*, which accounted for about one-sixth of the journal's publications. During his hiatus from the College, he also wrote articles on mathematics education. These articles were published in the *Quarterly Journal of Education* beginning with the first volume in 1831. Both journals were published by the Society for the Diffusion of Useful Knowledge, of which he later became a committee member.

De Morgan returned to University College London in 1836, and shortly thereafter married Sophie Elizabeth Frend. Sophie was the daughter of William Frend who was known for his radical beliefs about religion. Frend and De Morgan had established a friendship, which lasted until Frend's death in 1841. De Morgan had his own religious convictions which Sophie respected, and the two were married in the register office rather than in a church. The first of their seven children, Elizabeth Alice, was born one year later. De Morgan's professional career was also beginning to take shape again. He was given the opportunity to present the 'introductory lecture' at the opening of the Faculty of Arts at the University.

De Morgan continued his prolific writing during this time. He published the *Connexion of Number and Magnitude* in 1836, which was devoted to using algebraic notation to express the concepts of proportion found in Euclid's fifth book. The complexity of the task was expressed by De Morgan as follows:

> The subject is one of some real difficulty arising from the limited character of the symbols of Arithmetic considered as representatives of ratios, and the consequent introduction of incommensurable ratios, that is, of ratios which have no arithmetical representation.

His next work, *The Differential and Integral Calculus*, was published in 1842 by the Useful Knowledge Society, and consisted of over 770 pages that covered a broad range of subjects. An important aspect of this book was De Morgan's 'rejection of the whole doctrine of series' in developing the foundation of both areas of Calculus. Instead, he preferred the 'theory of limits' as 'the sole foundation of the science'. De Morgan stressed the importance of number and magnitude as he discussed the use of the introduction of limits by stating 'the ideas attached to the words nothing and infinite do not permit the application of many rules in the strict and direct sense in which they are applied to numbers'. The book illustrated De Morgan's devotion to ensuring that the reader developed a 'conceptual understanding' of Calculus.

By this time, De Morgan's second son, George Campbell, had been born. Like his father, George was an extremely talented mathematician. He achieved the top prizes at University College London in 'Mathematics and Natural Philosophy'. George and Arthur Couper, in 1864, proposed the formation of the 'University College Mathematical Society', which later became 'The London Mathematical Society'. During the last two years of his short life, George was a Mathematics teacher at The University College School. He died in 1867 at the age of 26. According to Sophie, 'his father had a high opinion of the power of George's mind, which in some ways resembled his own'. Only one other child, William, achieved success somewhat comparable to his father. William became an inventor, creating ceramic tiles, and after retiring wrote a best-selling novel.

De Morgan's most significant work that brought him lasting fame was in Logic. He published *Formal Logic* in 1847, which focused on the theory of syllogism. In his work, he pointed out how the Aristotelian syllogism was limited by 'two distinct principles of exclusion'. The first exclusion suggests 'that in every syllogism the middle term must be universal in one of the premises, in order that we may be sure that the affirmation or denial in the other premise may be made of some or all of the things which affirmation or denial has been made in the first'. The other exclusion exists according to De Morgan because 'Aristotle will have no contrary terms: not-man, he says is not the name of anything'.

De Morgan ameliorated these limitations regarding 'limiting the inferences used' and the 'ambiguity of negation', by introducing terms such as 'whole' and 'universe'. Most importantly, De Morgan introduced the 'Numerically Definite Syllogism', which essentially quantified the terms in the propositions.

Although De Morgan did not truly discover the following complementation laws of set theory, he is given credit for officially introducing them as they are shown, which is why they are named after him:

$$(A \cup B)^C = A^C \cap B^C,$$
$$(A \cap B)^C = A^C \cup B^C.$$

The first law demonstrates that negating an OR makes it an AND, while the second law shows that negating an AND makes it an OR. Today, these laws are frequently used in circuit design, modern proof theory, and in software programming.

Early in the course of preparing *Formal Logic*, De Morgan wrote to Sir William Hamilton at the University of Edinburgh requesting information about 'the Aristotelian theory of syllogism'. Soon thereafter, Sir Hamilton sent a letter inferring that De Morgan's work was not original. In his letter of reply, De Morgan displays his temper and obvious wit as follows:

> I will not allude to the hasty manner in which you have expressed your suspicions of an odious charge, except to state that it does not diminish the sincere respect with which I subscribe myself.

Thus, the controversy began and continued until 1852. After the feud ceased, the two would exchange letters and books until Sir Hamilton's death. An obituary notice of him was written by De Morgan and published in the *Athenaeum*. De Morgan influenced several mathematicians and their work in the field of Logic, including George Boole and one of De Morgan's past students, William Stanley Jevons.

During the 1850's De Morgan spent a portion of his time supporting the adoption of a decimal currency in Great Britain. The idea of decimal coinage was introduced by Sir John Wrottesley to the House of Commons in 1824. De Morgan advocated the advantages and adoption of the system in articles published in *The Penny Cyclopedia* (1833) and *The Companion to the Almanac* (1841). In 1854, the Parliamentary Committee announced a favourable report regarding the decimal plan. As a result, the Decimal Association was established, of which De Morgan became a member. After a course of several meetings, the proceedings of the Decimal Association were published. De Morgan wrote an introduction stressing the major points regarding decimal coinage. The issue of a 'cents-and-

mile system' was presented again to parliament in 1855. This time, the resolution to 'address her Majesty' about finalizing the scale with the issue of a 'cents-and-mile system' was removed.

A Royal Commission was established to inquire into the subject of decimal coinage. A report was published in 1857. The debate continued as to whether the pound-and-mile system should be adopted or the present system retained. De Morgan's article, which was published in the *Literarium* in 1857, spoke to those who were in favour of the present system. He stated:

> If this school be a logical one, it ought to be prepared to maintain that a country with a decimal system already established ought to abandon its coinage, and to introduce the succession of 4, 12, 20.

This method of 'reckoning and payment' has not changed in the last 150 years. Nevertheless, De Morgan's writings on the subject were voluminous, providing information to the reader that is both instructive and historical.

In January 1865, at the age of 58, De Morgan became the first President of the 'University College Mathematical Society', which would soon thereafter become the London Mathematical Society. During his inaugural speech De Morgan spoke of the proper goals of a mathematical society. He then addressed one of his pet peeves – the Cambridge examination.

De Morgan was not fond of the typical 'hard ten-minute conundrums' as he called them, which comprised of the examination. Instead, he preferred an examination that would take the student 'two or three hours to solve', though he agreed this was not practical. Perhaps most important was De Morgan's reflection on the importance and requirement for interweaving logic and mathematics into the research function of the society. De Morgan served a two-year term as president and some of his original papers appeared in the earliest reports. In 1884, the De Morgan medal was established in his memory. The first recipient was Arthur Cayley for his contributions to mathematics. The medal has been given every third year when the year is divisible by three.

While President of the London Mathematical Society, De Morgan faced similar circumstances as he had in the past, when University College London failed to appoint Reverend James Martineau, a well-respected Unitarian Minister, for the Chair of Mental Philosophy and Logic. Upon hearing of the Council's decision not to appoint the Unitarian Minister, De Morgan resigned from University College London in November 1866. After De Morgan's resignation, the university and his colleagues did not acknowledge all of the work he had done during his 30 years of service. However, several of his past students and friends petitioned the government to give De Morgan a monetary compensation. Although he was not in favour of accepting the money, De Morgan received a pension within a year of his death.

During the last few years of his life, De Morgan divided his time between studying the *Greek Testament*, writing a history of his family and himself, and working on his *Budget of Paradoxes*. The *Budget* was a compendium of essays, which are for the most part entertaining accounts by De Morgan about 'paradoxes' written by individuals known for their 'undisciplined intellect'. According to De Morgan, 'a paradox is something which is apart from general opinion'. A portion of the book focused on material pertaining to the 'quadrature problem', 'trisecting angles', and 'squaring circles'. De Morgan was very candid, yet would mix some humour with

his remarks regarding the work of these individuals. For example, the work by Hobbes on the quadrature problem is admonished by De Morgan in the following passage:

> Hobbes, who began in 1655, was very wrong in his quadrature; but, though not a Gregory St. Vincent, he was not the ignoramus in geometry that he is sometimes supposed.

Other portions of the *Budget* were written about 'parodoxes' regarding various subjects such as astronomy, religion, and science. The book was later edited and published by De Morgan's wife after his death.

The bitter break from the college was beginning to take a toll on De Morgan. This event combined with the loss of his son, George, in 1867 and his daughter, Christiania, in 1870, marked the start of the decline of De Morgan's health. He was also suffering from 'nervous prostration' according to his wife. On 18 March 1871, he died at the age of 64.

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