Offline and Online Identification of Hidden Semi-Markov Models

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***Abstract*—We present a new signal model for hidden semi-Markov models (HSMMs). Instead of constant transition probabilities used in existing models, we use *state-duration-dependant* transition probabilities. We show that our modeling approach leads to easy and efficient implementation of parameter identification algorithms. Then, we present a variant of the EM algorithm and an adaptive algorithm for parameter identification of HSMMs in the offline and online cases, respectively.**

***Index Terms*—Expectation maximization (EM) algorithm, recursive maximum likelihood (RML), recursive prediction error (RPE), semi-Markov models.**

I. Introduction

Markovian signal models have proven to be a powerful tool in signal modeling. Hidden Markov models (HMMs) are the most popular class of Markovian signal models [1], [2]. Hidden semi-Markov models (HSMMs) are a generalization of HMMs and are useful in many engineering applications, such as speech processing, signal estimation, queuing networks, and many others [3], [4]. Generally speaking, HSMMs are more powerful than HMMs in modeling physical signals; however, HSMMs lead to more complex parameter identification methods. This paper addresses the parameter identification of HSMMs in the offline and online cases.

The HSMM offline identification approaches are mainly a generalization of the Baum–Welch algorithm for parameter identification of HMMs [2]–[6], except that they require much higher computational load [7].

In [8], an HSMM with N states is reformulated as an HMM with $N \times D$ states, where D is the maximum state duration for all states, and then the Baum–Welch algorithm is used to estimate the model parameters. In other approaches, which are based on the state-duration-dependant transition probabilities [9], the state transition matrix is replaced with an $N \times N \times D$ tensor. The drawback of the methods in [8] and [9] is the addition of a large number of extra parameters to the model, which must also be estimated in addition to the usual HMM parameters.

Online identification of HMMs has been studied in [1] and [10]–[14]. These approaches are based on either the recursive maximum likelihood (RML) or the recursive prediction error (RPE) [15] techniques. However, for HSMMs, there is no online identification method reported.

In [16] and [17], a parameter estimation method is presented for a more general semi-Markovian signal model. In this model, the hidden state process is a discrete semi-Markov chain with Poisson-distributed transition times, and the signal observations are assumed to be *filtered by a linear and casual system*, mixed with white Gaussian noise. In this case, a Viterbi algorithm cannot be used for maximizing the likelihood function. The approach taken in [16] and [17] is to formulate this estimation problem as a constraint optimization problem and use a combination of a maximum *a posteriori* and the maximum likelihood estimation procedures. Because it is difficult to solve this optimization problem theoretically, a numerical procedure called integer most likely search (IMLS) is used. However, this method requires exponentially increasing memory. There are methods that aim to avoid this memory problem by using suboptimal algorithms such as pseudo-Baysian algorithm or interacting multiple-model (IMM) algorithm [18].

In this paper, we present a novel signal model for HSMMs, which leads to easier and more computationally efficient parameter identification algorithms than existing ones. We use *state-duration-dependent* transition probabilities, where the state-duration densities are modeled with parameterized probability mass functions. Our modeling scenario can be encapsulated as a time homogeneous first-order infinite state Markov model. Although our approach is similar to [8] in this regard, it differs in three ways. First, the state durations in [8] are assumed to be bounded, and hence, the encapsulated Markov model is finite state. Second, in [8], the state durations are not modeled with parameterized probability mass functions, and third, constant transition probabilities are used in [8]. This method has the disadvantage of overparameterizing the model. Our approach, however, does not overparameterize the model.

We then present a novel version of the Baum–Welch algorithm for offline identification of HSMMs in Section III. In Section IV, we present a method for online identification of HSMMs. Numerical results from implementations of our algorithms for offline and online identification of HSMMs are presented in Section V.

II. Signal Modeling

We consider a signal model where the state of the signal at time t, $\boldsymbol{s}_t$, $t \in \mathbb{N}$, is determined by a finite-state discrete-time semi-Markov chain with N distinct states. We assume the initial state $\boldsymbol{s}_1$ is given or its distribution is known. Without loss of generality, we assume $\boldsymbol{s}_t$ takes its values from the set $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots, \boldsymbol{e}_N\}$, where $\boldsymbol{e}_i$ is a $N \times 1$ vector with unity as the

Manuscript received October 3, 2003; revised July 21, 2004. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Abdelhak M. Zoubir.

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Digital Object Identifier 10.1109/TSP.2005.850344

ith element and zeros elsewhere. Suppose $\boldsymbol{s}_t = \boldsymbol{e}_j$, and let $d_t(j)$ denote the duration spent in the jth state prior to time t. Then, we define the *state-duration* vector $\boldsymbol{d}_t$ of size $N \times 1$, where all elements of $\boldsymbol{d}_t$ are equal to unity, except the jth element, which is equal to $d_t(j)$. $\boldsymbol{d}_t$ is easily constructed from $\boldsymbol{d}_{t-1}$ as $\boldsymbol{d}_t = \boldsymbol{s}_t \odot \boldsymbol{d}_{t-1} + \mathbf{1}$, where $\odot$ denotes the element-by-element product, and $\mathbf{1}$ is a $N \times 1$ vector with all unity elements.

We model the state-duration densities with a parametric *probability mass function* (pmf) $\phi_i(d)$. That is, the probability that $\boldsymbol{s}_t$ stays exactly for d time units in state i is given by $\phi_i(d)$. $\phi_i(d)$ should be selected such that it adequately captures the properties of the signal under study. Hence, the selection of $\phi_i(d)$ should be justified by some evidence from samples of the signal. Even though state durations in a semi-Markov chain are inherently discrete, it is noted in many studies that continuous parametric density functions are also suitable for modeling state durations in many applications, including speech processing [4], [5]. In this approach, state durations are modeled with the best fitting parametric *probability density function* (pdf), and then the discrete counterpart of this density function is taken as the best pmf. That is, if $\phi_i(x)$ is the continuous pdf for state duration of the ith state, then the probability that the signal stays in state i for exactly d time units is given by $\int_{d-1}^{d} \phi_i(x)dx$. Since negative state durations are not physically meaningful, it is usually more appropriate to select $\phi_i(x)$ from the family of exponential distributions [5]. Specifically, the family of Gamma distributions are considered in [4] for speech processing applications. In this paper, we assume that $\phi_i(x)$ is a *Gamma* distribution function with *shape parameter* ν_i and *scale parameter* η_i, that is,

$$\phi_i(d) = \frac{\eta_i^{\nu_i}}{\Gamma(\nu_i)} d^{\nu_i - 1} e^{-\eta_i d} \quad (0 < d < \infty). \tag{1}$$

The mean and variance of ϕ_i are ν_i/η_i and ν_i/η_i^2, respectively [19]. Note that our signal model is applicable with minor changes to HSMM signals whose state-duration densities are modeled with a pdf other than Gamma. Also, let $\Phi_i(x)$ denote the cumulative distribution function of $\phi_i(x)$, i.e., $\Phi_i(d) = \int_0^d \phi_i(x)dx$.

We construct our model for HSMMs using state-duration-dependant transition probabilities. We define the state transition matrix $\boldsymbol{A}_{\boldsymbol{d}_t}$ as $\boldsymbol{A}_{\boldsymbol{d}_t} = [a_{ij}(\boldsymbol{d}_t)]$, where $a_{ij}(\boldsymbol{d}_t) = \mathbb{P}(s_{t+1} = \boldsymbol{e}_j | s_t = \boldsymbol{e}_i, d_t(i))$. Clearly, $a_{ij}(\boldsymbol{d}_t)$'s are not constants and do change with time; however, we will denote $a_{ij}(\boldsymbol{d}_t)$ with a_{ij} for notational simplicity. It can be easily shown that for the diagonal elements of $\boldsymbol{A}_{\boldsymbol{d}_t}$, a_{ii}'s, we have

$$a_{ii} = \frac{1 - \Phi_i\left(d_t(i)\right)}{1 - \Phi_i\left(d_t(i) - 1\right)}. \tag{2}$$

The probability that the state process $\boldsymbol{s}_t$ stays in the ith state for exactly d time units is given by $(1 - a_{ii}(d)) \cdot \prod_{k=1}^{d-1} a_{ii}(k)$. By substituting a_{ii} from (2), it is easily shown that the pdf of the state-space durations is actually equal to the selected model $\phi_i(d)$. For $i \neq j$, $a_{ij} = (1 - a_{ii})a_{ij}^o$, where $a_{ij}^o = \mathbb{P}(s_{t+1} = \boldsymbol{e}_j | s_t = \boldsymbol{e}_i, i \neq j)$. We write the matrix $\boldsymbol{A}_{\boldsymbol{d}_t}$ as $\boldsymbol{A}_{\boldsymbol{d}_t} = \boldsymbol{P}(\boldsymbol{d}_t) + (\boldsymbol{I} - \boldsymbol{P}(\boldsymbol{d}_t))\boldsymbol{A}^o$, where $\boldsymbol{A}^o = [a_{ij}^o]$ is a constant matrix representing the nonrecurrent state transition probabilities, and $\boldsymbol{P}(\boldsymbol{d}_t) = [p_{ij}(\boldsymbol{d}_t)]$ is a diagonal matrix representing the recurrent state transition probabilities. p_{ij}'s are given as

$$p_{ij}(\boldsymbol{d}_t) := \begin{cases} 0, & i \neq j \\ \frac{1-\Phi_i(d_t(i))}{1-\Phi_i(d_t(i)-1)}, & i = j \end{cases}. \tag{3}$$

Note that a_{ij}^o are constrained to $\sum_{j=1}^{N} a_{ij}^o = 1$. Since $\boldsymbol{P}(\boldsymbol{d}_t)$ is a diagonal matrix and all the diagonal elements of $\boldsymbol{A}^o$ are zero, one can show that $\sum_{j=1}^{N} a_{ij}(\boldsymbol{d}_t) = 1$ for all t. One can also easily show that our model reduces to an HMM if the state transition probabilities, a_{ii}'s, do not depend on state durations $\boldsymbol{d}_t$. Hence, the hidden state process $\boldsymbol{s}_t$ evolves in time as

$$\begin{aligned} \boldsymbol{s}_{t+1} &= \boldsymbol{A}_{\boldsymbol{d}_t} \cdot \boldsymbol{s}_t + \boldsymbol{v}_{t+1} \\ \boldsymbol{A}_{\boldsymbol{d}_t} &= \boldsymbol{P}(\boldsymbol{d}_t) + (\boldsymbol{I} - \boldsymbol{P}(\boldsymbol{d}_t)) \cdot \boldsymbol{A}^o \\ \boldsymbol{d}_{t+1} &= \boldsymbol{s}_{t+1} \odot \boldsymbol{d}_t + \mathbf{1} \end{aligned} \tag{4}$$

where $\mathbf{1}$ is a $N \times 1$ vector with all unity elements, and $\boldsymbol{v}_{t+1}$ is a martingale increment.

We observe the *observation process* y_t, where the probabilistic distribution of y_t is determined by $\boldsymbol{s}_t$. In this paper, we assume that for each state i, y_t has a normal distribution. That is, $\mathbb{P}(y_t | \boldsymbol{s}_t = \boldsymbol{e}_i) = \mathcal{N}(y_t; \mu_i, \sigma_i^2)$, where μ_i and σ_i^2 are the mean and standard deviation of the observation process y_t for state i. We denote the probability of observing y_t in state i with $b_i(y_t)$, that is,

$$b_i(y_t) = \mathbb{P}(y_t | \boldsymbol{s}_t = \boldsymbol{e}_i). \tag{5}$$

Therefore, y_t may be written as $y_t = \langle \boldsymbol{\mu}, \boldsymbol{s}_t \rangle + \langle \boldsymbol{\sigma}, \boldsymbol{s}_t \rangle w_t$, where $\boldsymbol{\mu} = [\mu_1, \mu_2, \cdots, \mu_N]'$, $\boldsymbol{\sigma}^2 = [\sigma_1^2, \sigma_2^2, \cdots, \sigma_N^2]'$, $\langle .,. \rangle$ denotes inner product, and w_t is Gaussian white noise with zero mean and variance 1.

We define $\boldsymbol{\theta}$ as a vector of size $N^2 + 3N$ containing all the model parameters

$$\begin{aligned} \boldsymbol{\theta} = (&\mu_1, \mu_2, \cdots, \mu_N, \sigma_1^2, \sigma_2^2, \cdots, \sigma_N^2, a_{12}^o, a_{13}^o, \cdots, a_{N-1,N}^o \\ &\nu_1, \nu_2, \cdots, \nu_N, \eta_1, \eta_2, \cdots, \eta_N)'. \end{aligned} \tag{6}$$

III. Offline Identification of HSMMs

Given a set of observations from an HSMM signal $\mathcal{Y}_T \triangleq \{y_1, y_2, \ldots, y_T\}$, we wish to estimate $\boldsymbol{\theta}$, the parameters of the HSMM model. The algorithm we use is a variant of the expectation maximization (EM) algorithm [20] and, in essence, is very similar to the Baum–Welch algorithm for identifications of HMMs presented in [2] and [8]. We define the "forward" and "backward" variables $\alpha_t(i)$ and $\beta_t(i)$ as in [2] and [8]. Let $\hat{\boldsymbol{d}}_t = [\hat{d}_t(1)\ \hat{d}_t(2)\ \cdots\ \hat{d}_t(N)]'$, where $\hat{d}_t(i) = \mathbb{E}(d_t(i) | \boldsymbol{s}_t = i, \boldsymbol{\theta}, y_1, y_2, \ldots, y_t)$ is our estimate of the state-duration variable for state i at time t. $\hat{\boldsymbol{d}}_t$ is initialized to $[1\ 1\ \cdots\ 1]'$ for $t = 1$. We reconstruct $\hat{d}_{t+1}(i)$ iteratively as

$$\begin{aligned} \hat{d}_{t+1}(i) &= 1 + \mathbb{E}\left(\boldsymbol{s}_t(i) | y_1 y_2 \ldots y_t, \boldsymbol{\theta}\right) \cdot \hat{d}_t(i), && 1 \leq i \leq N \\ &= 1 + \frac{\alpha_t(i)}{\sum_{i=1}^{N} \alpha_t(i)} \cdot \hat{d}_t(i), && 1 \leq i \leq N. \end{aligned} \tag{7}$$

The state transition matrix $\boldsymbol{A}_{\boldsymbol{d}_t}$ is updated for each t as

$$\boldsymbol{A}_{\hat{\boldsymbol{d}}_t} = \boldsymbol{P}(\hat{\boldsymbol{d}}_t) + \left(\boldsymbol{I} - \boldsymbol{P}(\hat{\boldsymbol{d}}_t)\right) \boldsymbol{A}^o. \tag{8}$$

Our algorithm starts by initializing $\boldsymbol{\theta}$ to an initial guess. Each iteration of the algorithm consists of two steps. In the E step (see [20]), we use the readily calculated recursive formulae presented in [2], in conjunction with (7) and (8), to calculate the forward and backward variables α_t's and β_t's. Note that a_{ij}'s in recursion equations in [2] should be replaced with $a_{ij}(\hat{\boldsymbol{d}}_t)$, as calculated in (8). In the M step, the model parameters are updated to the maximum-likelihood estimate of the model parameters computed from the forward–backward variables in the E step. The update equations for parameters a^o_{ij}, μ_i, and σ_i are similar to the equations presented in [2]. Let $\mu_{i,s}$ and $\sigma^2_{i,s}$ be the mean and variance of the state duration for state i, respectively. It can be easily shown that $\mu_{i,s}$ and $\sigma^2_{i,s}$ are estimated as

$$\mu_{i,s} = \frac{\sum_{t=1}^{T-1} \alpha_t(i) \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} b_j(y_{t+1}) \beta_{t+1}(j) \right) \hat{d}_t(i)}{\sum_{t=1}^{T-1} \alpha_t(i) \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} b_j(y_{t+1}) \beta_{t+1}(j) \right)}$$

$$\sigma^2_{i,s} = \frac{\sum_{t=1}^{T-1} \alpha_t(i) \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} b_j(y_{t+1}) \beta_{t+1}(j) \right) \left(\hat{d}_t(i) - \mu_{i,s} \right)^2}{\sum_{t=1}^{T-1} \alpha_t(i) \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} a_{ij} b_j(y_{t+1}) \beta_{t+1}(j) \right)} \tag{9}$$

where ν_i and η_i, are given in terms of $\mu_{s,i}$ and $\sigma^2_{i,s}$ as $\nu_i = \mu^2_{i,s}/\sigma^2_{i,s}$ and $\eta_i = \mu_{i,s}/\sigma^2_{i,s}$.

The algorithm stops when $\boldsymbol{\theta}$ converges to a constant vector. The forward–backward algorithm has the computational complexity of $\mathcal{O}(N^2T)$ per pass and can be shown to require a memory of $3NT$.

IV. Online Identification of HSMMs

In this section, we use the state-space signal model presented in Section II and set up the problem of online identification of HSMMs such that the general recursive prediction method can be applied. Let $\boldsymbol{\theta}_t$ denote the estimate of the model parameters at t. We define the objective function $\ell_t(\boldsymbol{\theta}_t) = \log \mathbb{P}(y_1, y_2, \ldots, y_t | \boldsymbol{\theta}_t)$ as the log-likelihood of the observations up to time t given $\boldsymbol{\theta}_t$. $\ell_t(\boldsymbol{\theta}_t)$ can be rewritten as

$$\begin{aligned} \ell_t(\boldsymbol{\theta}_t) &= \sum_{\tau=1}^{t} \log \mathbb{P}(y_\tau | y_1, y_2, \ldots, y_{\tau-1}; \boldsymbol{\theta}_t) \\ &= \sum_{\tau=1}^{t} u_\tau(\boldsymbol{\theta}_t) \end{aligned} \tag{10}$$

where $u_\tau = \log \mathbb{P}(y_\tau | y_1, y_2, \ldots, y_{\tau-1}; \boldsymbol{\theta}_t)$ is the log-likelihood increment. We use the recursive prediction error (RPE) method, where the parameters are updated in the Newton–Raphson direction [15]. Starting with an initial guess for $\boldsymbol{\theta}_t$ at $t = 1$, $\boldsymbol{\theta}_t$ is updated using

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \lambda_{t+1} \cdot R^{-1}_{t+1} \cdot \psi_{t+1} \tag{11}$$

where $R_t = \partial^2 \ell_t(\boldsymbol{\theta}) / \partial \boldsymbol{\theta}^2$ is an estimate of the Hessian matrix, and ψ_t is the gradient of u_t with respect to θ_t and determines the search direction [15]. λ_t is a step size.

In summary, our online algorithm consists of four steps for each time instance t: 1) estimate the hidden layer variables α_t and $\hat{\boldsymbol{d}}_t$, 2) update the gradient vector ψ_t, 3) update our estimate of the Hessian matrix R_t, and 4) update the parameter estimate $\boldsymbol{\theta}_t$ using (11). To facilitate the development of the update equations, instead of using $\boldsymbol{\theta}$ as in (6), we use

$$\boldsymbol{\theta} = (\boldsymbol{\mu}, \boldsymbol{\sigma}^2, c_{12}, c_{13}, \cdots, c_{N-1,N}, \boldsymbol{\nu}, \boldsymbol{\eta})' \tag{12}$$

where c_{ij} are simply defined as $c_{ij} = (a^o_{ij})^{1/2}$ (see [1]). For simplicity, we use $\boldsymbol{\theta}$ instead of $\boldsymbol{\theta}_t$ in our notations. We now describe the details of each step of our online algorithm.

Let $\alpha_t(i) = \mathbb{P}(y_1 y_2 \ldots y_t, s_t = i | \boldsymbol{\theta})$; then, the forward filtering recursion equation is given by

$$\boldsymbol{\alpha}_{t+1} = \boldsymbol{B} \boldsymbol{A}'_{\boldsymbol{d}_t} \boldsymbol{\alpha}_t \tag{13}$$

where $\boldsymbol{\alpha}_t = [\alpha_t(1)\ \alpha_t(2)\ \cdots\ \alpha_t(N)]'$, $\boldsymbol{B}$ is a diagonal matrix, $b_{ii} = b_i(y_{t+1})$, and $\boldsymbol{A}'_{\boldsymbol{d}_t}$ is the transpose of the state transition matrix (4). Let $\boldsymbol{\gamma}_t = \mathbb{E}(\boldsymbol{s}_t | y_1, y_2, \ldots, y_t, \boldsymbol{\theta})$ be the conditional estimate of the state at time t. It can be easily shown that $\boldsymbol{\gamma}_t = \boldsymbol{\alpha}_t \langle \boldsymbol{\alpha}_t, \boldsymbol{1} \rangle^{-1}$. Given the observations up to time t, the next state and next observation of the signal are estimated as

$$\mathbb{E}(\boldsymbol{s}_{t+1} | y_1, y_2, \ldots, y_t, \boldsymbol{\theta}) = \boldsymbol{A}'_{\boldsymbol{d}_t} \boldsymbol{\gamma}_t \tag{14}$$

$$\hat{y}_{t+1} = \left\langle \boldsymbol{\mu}, \boldsymbol{A}'_{\boldsymbol{d}_t} \boldsymbol{\alpha}_t \cdot \langle \boldsymbol{\alpha}_t, \boldsymbol{1} \rangle^{-1} \right\rangle. \tag{15}$$

The estimate of the state duration variable is updated similarly to the offline case (7) as

$$\begin{aligned} \hat{\boldsymbol{d}}_{t+1} &= \mathbb{E}(\boldsymbol{s}_t | y_1, y_2, \ldots, y_t, \boldsymbol{\theta}) \odot \hat{\boldsymbol{d}}_t + \boldsymbol{1} \\ &= \boldsymbol{\alpha}_t \langle \boldsymbol{\alpha}_t, \boldsymbol{1} \rangle^{-1} \odot \hat{\boldsymbol{d}}_t + \boldsymbol{1}. \end{aligned} \tag{16}$$

The log-likelihood increment u_{t+1} (10) is given by

$$u_{t+1} = \log \left\langle \boldsymbol{1}, \boldsymbol{B} \boldsymbol{A}'_{\boldsymbol{d}_t} \boldsymbol{\gamma}_t \right\rangle. \tag{17}$$

We update the gradient vector $\psi_t = \partial u_{t+1} / \partial \boldsymbol{\theta} |_{\boldsymbol{\theta} = \boldsymbol{\theta}_t}$ in each iteration using our estimates of the filtration parameters (i.e., $\boldsymbol{\alpha}_t$ and $\boldsymbol{d}_t$). The detailed equations for this are presented in Appendix. After finding ψ_{t+1}, the parameter vector $\boldsymbol{\theta}$ and R_t are updated recursively using (11) and

$$R_t = R_{t-1} + \lambda_t \left[\psi(t) \psi'(t) - R(t-1) \right]. \tag{18}$$

A discussion on choices for λ_t can be found in [12] and [21]–[23].

It can be shown that our online algorithm has computational complexity of $\mathcal{O}(N^2_\theta)$, where N_θ is the number of parameters employed in signal model [according to (12), $N_\theta = N^2 + 3N$].

V. Numerical Results

Here, we present the numerical results of implementing our offline and online algorithms for identifications of HSMMs.

In the first experiment, the parameters of an HSMM signal with $N = 3$ distinct states were estimated using the offline algorithm of Section III. The number of observations was $T = 10\,000$. The actual and initial values of the parameters are given

TABLE I
ACTUAL AND INITIAL VALUES OF THE PARAMETERS OF HSMM MODELS USED IN OUR SIMULATIONS

	Actual values off-line	Initial values (off-line)	Actual values (online) $1 \le t \le 5 \times 10^3$	Actual values (online) $5 \times 10^3 \le t \le 10^4$	Initial values (online)
$\boldsymbol{A}^o$	$\begin{bmatrix} 0 & 0.30 & 0.70 \\ 0.70 & 0 & 0.30 \\ 0.50 & 0.50 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.00 & 0.50 & 0.50 \\ 0.10 & 0.00 & 0.90 \\ 0.50 & 0.50 & 0.00 \end{bmatrix}$	$\begin{bmatrix} 0 & 0.50 & 0.50 \\ 0.50 & 0 & 0.50 \\ 0.30 & 0.70 & 0 \end{bmatrix}$	$\begin{bmatrix} 0.00 & 0.50 & 0.50 \\ 0.15 & 0.00 & 0.85 \\ 0.30 & 0.70 & 0.00 \end{bmatrix}$	$\begin{bmatrix} 0.00 & 0.50 & 0.50 \\ 0.50 & 0.00 & 0.50 \\ 0.50 & 0.50 & 0.00 \end{bmatrix}$
$\boldsymbol{\mu}$	$[-10 \ \ 0 \ \ 10]'$	$[-15 \ \ 3 \ \ 15]'$	$[-10 \ \ 0 \ \ 10]'$	$[-5 \ \ 0 \ \ 10]'$	$[-13 \ \ 4 \ \ 20]'$
$\boldsymbol{\sigma}^2$	$[10 \ \ 10 \ \ 10]'$	$[8 \ \ 8 \ \ 8]'$	$[4 \ \ 4 \ \ 4]'$	$[4 \ \ 4 \ \ 4]'$	$[10 \ \ 10 \ \ 10]'$
$\boldsymbol{\mu}_s$	$[10 \ \ 20 \ \ 30]'$	$[10 \ \ 10 \ \ 10]'$	$[10 \ \ 20 \ \ 30]'$	$[10 \ \ 20 \ \ 30]'$	$[5 \ \ 10 \ \ 10]'$
$\boldsymbol{\eta}$	$[5 \ \ 10 \ \ 6]'$	$[1 \ \ 10 \ \ 20]'$	$[5 \ \ 10 \ \ 15]'$	$[5 \ \ 10 \ \ 15]'$	$[8 \ \ 10 \ \ 20]'$
$\boldsymbol{\nu}$	$[50 \ \ 200 \ \ 180]'$	$[10 \ \ 100 \ \ 200]'$	$[50 \ \ 200 \ \ 450]'$	$[50 \ \ 200 \ \ 450]'$	$[40 \ \ 100 \ \ 200]'$

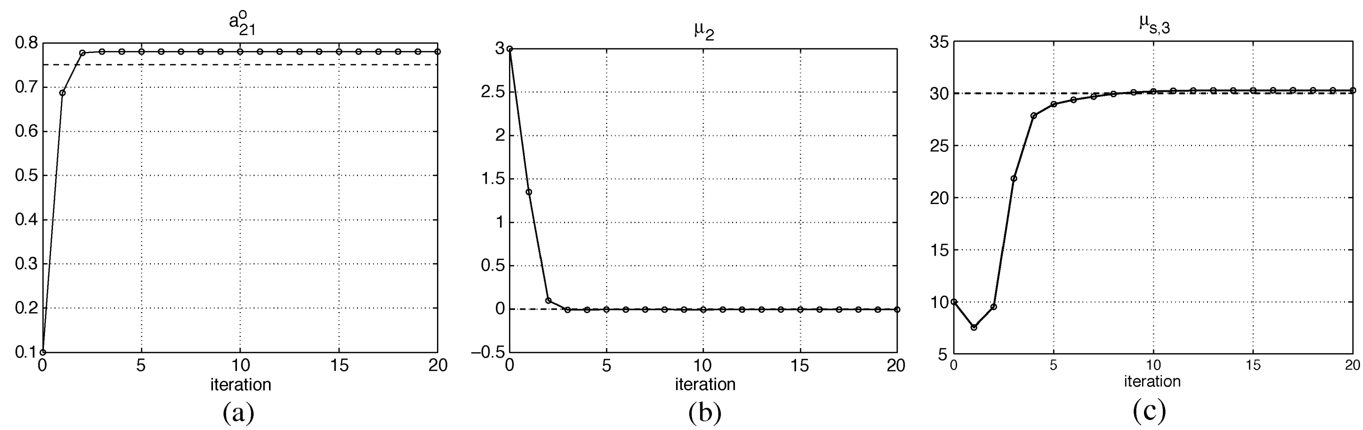

Fig. 1. Parameter estimates versus the iteration number of the offline algorithm. The dotted lines show the actual value of the parameters.

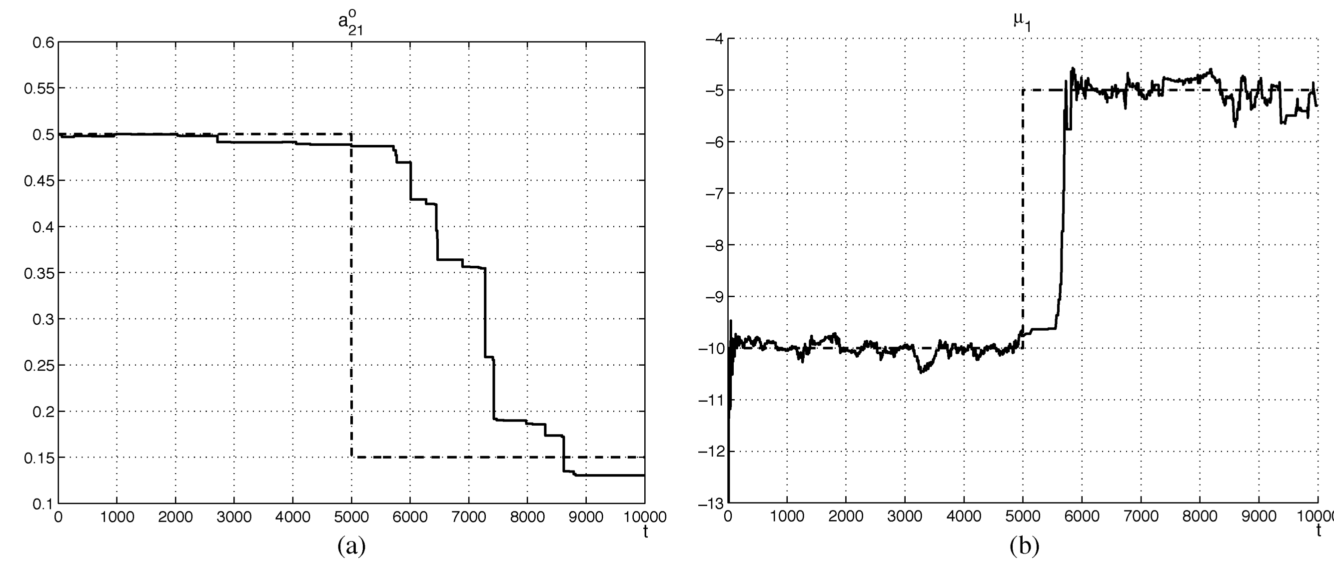

Fig. 2. Online estimation of a three-state HSMM, where the actual parameter changes at $t = 5000$. (a) State transition probability a^o_{21}. (b) Observation mean for state 1, μ_1. The dotted lines show the actual value of the parameter. The parameter estimates follow the temporal changes in the actual value of the parameter.

in Table I. Fig. 1 illustrates that the parameter estimates converge to their actual values after few iterations. We also observed that the log-likelihood of the total observation given the parameters estimate [i.e., $\log(\mathbb{P}(\mathcal{Y}_T|\boldsymbol{\theta}))$] increases in each iteration, demonstrating that the algorithm finds the maximum-likelihood estimate of the model parameters.

In the next experiment, we applied our online identification method to an HSMM signal with the parameters shown in Table I. As shown, the actual parameters of the model change at $t = 5000$. Fig. 2 illustrates that the parameter estimates converge to their actual values as t becomes large. Furthermore, the algorithm successfully tracks the temporal changes in the model parameters.

VI. CONCLUSION

We presented a novel signal model for HSMMs. This model results in easier parameter identification methods than the cur-

$$D_{\mu_i}\boldsymbol{P} = \mathrm{diag}\left(\frac{\phi(\boldsymbol{d}_t-1)-\phi(\boldsymbol{d}_t)+\phi(\boldsymbol{d}_t)\Phi(\boldsymbol{d}_t-1)-\phi(\boldsymbol{d}_t-1)\Phi(\boldsymbol{d}_t)}{(1-\Phi(\boldsymbol{d}_t-1))^2}\odot D_{\mu_i}\boldsymbol{d}_t\right) \tag{24}$$

$$D_{\mu_i}\boldsymbol{d}_t = \boldsymbol{d}_{t-1}\odot D_{\mu_i}\boldsymbol{\gamma}_{t-1} + D_{\mu_i}\boldsymbol{d}_{t-1}\odot\boldsymbol{\gamma}_{t-1}. \tag{25}$$

$$D_{\eta_i}\left(\frac{1-\Phi(d_t(i);\eta_i,\nu_i)}{1-\Phi(d_t(i)-1;\eta_i,\nu_i)}\right) = \frac{D_{\eta_i}\left[\Phi(d_t(i)-1)\right](1-\Phi(d_t(i))) - D_{\eta_i}\left[\Phi(d_t(i))\right](1-\Phi(d_t(i)-1))}{(1-\Phi(d_t(i)-1))^2} \tag{31}$$

rent signal models. We employed parameterized pdfs to model the time that a signal spends in each state.

Based on our model, we developed methods for estimating the model parameters of an HSMM signal for both the offline and online cases. Our offline method uses a version of the EM algorithm and takes advantage of our novel signal model to find the maximum-likelihood estimate of the parameters in a timely manner. Our online method adaptively updates the model parameters using a version of the RPE method, such that the likelihood of our estimate is maximized. We discussed the practical issues involved in the implementation of our methods and presented techniques to address these issues.

APPENDIX

In this Appendix, we present equations for updating the gradient vector in our online algorithm. Let $D_x(.) = \partial(.)/\partial x$ denote the derivative operator with respect to variable x. The gradient vector is written as $\psi_{t+1} = (D_{\mu_i}u_{t+1}, D_{\sigma_i^2}u_{t+1}, D_{c_{ij}}u_{t+1}, D_{\mu_{i,s}}u_{t+1}, D_{\sigma_{i,s}^2}u_{t+1})'|_{\boldsymbol{\theta}=\boldsymbol{\theta}_t}$. We assume that the probabilities of nonrecurrent transitions (i.e., c_{ij}'s), and the parameters of the state-duration pdfs (i.e., $\boldsymbol{\nu}$ and $\boldsymbol{\eta}$) do not depend on each other. Update equations for $D_{\mu_i}u_{t+1}$ are given as

$$\begin{aligned} D_{\mu_i}u_{t+1} = &\left\langle \mathbf{1}, \boldsymbol{B}\boldsymbol{A}'_{\boldsymbol{d}_t}\boldsymbol{\gamma}_t\right\rangle^{-1} \\ &\times\left(\left\langle \mathbf{1}, D_{\mu_i}(\boldsymbol{B})\boldsymbol{A}'_{\boldsymbol{d}_t}\boldsymbol{\gamma}_t\right\rangle + \left\langle \mathbf{1}, \boldsymbol{B}D_{\mu_i}\left(\boldsymbol{A}'_{\boldsymbol{d}_t}\right)\boldsymbol{\gamma}_t\right\rangle \right.\\ &\left. + \left\langle \mathbf{1}, \boldsymbol{B}\boldsymbol{A}'_{\boldsymbol{d}_t}D_{\mu_i}(\boldsymbol{\gamma}_t)\right\rangle\right) \end{aligned} \tag{19}$$

$$D_{\mu_i}\boldsymbol{\gamma}_t = D_{\mu_i}(\boldsymbol{\alpha}_t)\langle\mathbf{1},\boldsymbol{\alpha}_t\rangle^{-1} + \boldsymbol{\alpha}_t\langle\mathbf{1}, D_{\mu_i}(\boldsymbol{\alpha}_t)\rangle^{-1} \tag{20}$$

$$\begin{aligned} D_{\mu_i}\boldsymbol{\alpha}_t = &D_{\mu_i}(\boldsymbol{B})\boldsymbol{A}'_{\boldsymbol{d}_t}\boldsymbol{\alpha}_{t-1} + \boldsymbol{B}D_{\mu_i}\left(\boldsymbol{A}'_{\boldsymbol{d}_t}\right)\boldsymbol{\alpha}_{t-1} \\ &+ \boldsymbol{B}\boldsymbol{A}'_{\boldsymbol{d}_t}D_{\mu_i}(\boldsymbol{\alpha}_{t-1}) \end{aligned} \tag{21}$$

$$D_{\mu_i}\boldsymbol{B} = \left(\frac{y_t-\mu_i}{\sigma^2}\right)\cdot\boldsymbol{B}\cdot\mathrm{diag}(\boldsymbol{e}_i) \tag{22}$$

$$D_{\mu_i}\boldsymbol{A}_{\boldsymbol{d}_t} = D_{\mu_i}(\boldsymbol{P})(I-\boldsymbol{A}^o) \tag{23}$$

where we have (24) and (25) shown at the top of the page. Update equations for $D_{\sigma_i^2}u_{t+1}$ are identical to the update equations for $D_{\mu_i}u_{t+1}$ (19)–(25), except for $D_{\sigma_i^2}\boldsymbol{B}$, which is given by $D_{\sigma_i^2}\boldsymbol{B} = (((y_t-\mu_i)^2/(2\sigma_i^4)) - (1/2\sigma_i^2))\boldsymbol{B}\cdot\mathrm{diag}(\boldsymbol{e}_i)$. For $D_{c_{mn}}u_{t+1}$, we have

$$\begin{aligned} D_{c_{mn}}u_{t+1} = &\left\langle \mathbf{1}, \boldsymbol{B}\boldsymbol{A}'_{\boldsymbol{d}_t}\boldsymbol{\gamma}_t\right\rangle^{-1} \\ &\times\left(\left\langle \mathbf{1}, \boldsymbol{B}D_{c_{mn}}\left(\boldsymbol{A}'_{\boldsymbol{d}_t}\right)\boldsymbol{\gamma}_t\right\rangle \right.\\ &\left. + \left\langle \mathbf{1}, \boldsymbol{B}\boldsymbol{A}'_{\boldsymbol{d}_t}D_{c_{mn}}(\boldsymbol{\gamma}_t)\right\rangle\right) \end{aligned} \tag{26}$$

$$D_{c_{mn}}\boldsymbol{\gamma}_t = D_{c_{mn}}(\boldsymbol{\alpha}_t)\langle\mathbf{1},\boldsymbol{\alpha}_t\rangle^{-1} + \boldsymbol{\alpha}_t\langle\mathbf{1}, D_{c_{mn}}(\boldsymbol{\alpha}_t)\rangle^{-1} \tag{27}$$

$$D_{c_{mn}}\boldsymbol{\alpha}_t = \boldsymbol{B}D_{c_{mn}}\left(\boldsymbol{A}'_{\boldsymbol{d}_t}\right)\boldsymbol{\alpha}_{t-1} + \boldsymbol{B}\boldsymbol{A}'_{\boldsymbol{d}_t}D_{c_{mn}}(\boldsymbol{\alpha}_{t-1}) \tag{28}$$

$$D_{c_{mn}}\boldsymbol{A}_{\boldsymbol{d}_t} = -\boldsymbol{P}\cdot D_{c_{mn}}(\boldsymbol{A}^o) \tag{29}$$

$$D_{c_{mn}}a_{ij}^o = \begin{cases} 0, & \text{if } m\neq i \\ 2c_{ij}, & \text{if } m=i, n=j \\ -2c_{mn}, & \text{if } m=i, n\neq j. \end{cases} \tag{30}$$

Update equations for $D_{\eta_i}u_{t+1}$ are similar to (26)–(28), except for $D_{\eta_i}\boldsymbol{A}_{\boldsymbol{d}_t}$, which is given by $D_{\eta_i}\boldsymbol{A}_{\boldsymbol{d}_t} = D_{\eta_i}(\boldsymbol{P})(I-\boldsymbol{A}^o)$. $D_{\eta_i}(\boldsymbol{P})$ is a matrix with all zero elements, except the element in row i and column i, which is given by (31), shown at the top of the page. $D_{\eta_i}\Phi(d;\eta_i,\nu_i)$ is obtained by differentiating $\Phi(d;\eta_i,\nu_i)$ as defined in (1)

$$D_{\eta_i}\Phi(d;\eta_i,\nu_i) = \frac{\nu_i}{\eta_i}\left(\Phi(d;\eta_i,\nu_i) - \Phi(d;\eta_i,\nu_i+1)\right). \tag{32}$$

Update equations for $D_{\nu_i}u_{t+1}$ are identical to the update equations for $D_{\eta_i}u_{t+1}$. However, differentiating $\Phi(d;\eta,\nu)$ with respect to ν does not result in a simple form, as in (32). Fortunately, we can easily find the numerical value of $D_{\nu_i}\Phi(d;\eta_i,\nu_i)$. We have

$$\begin{aligned} D_\nu\left(\Phi(d;\eta,\nu)\right) = &(\log(\eta)-\Psi(\nu))\,\Phi(d;\eta,\nu) \\ &+\int_0^d \log(x)\phi(x;\eta,\nu)dx \end{aligned} \tag{33}$$

where Ψ is the *digamma* function [24], [25]. The numerical value of the digamma function at any point can be easily computed using the method presented in [4] and [24].

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