SCIENCE RESEARCH REPORTS

General Science

A Chinese Root of Japanese Traditional Mathematics – Wasan

Mitsuo Morimoto

SRR# GS-0305, December 2003

International Christian University
Graduate School, Divison of Natural Sciences
3-10-2, Osawa, Mitaka-shi, Tokyo, Japan 181-8585
tel:+81-422-33-3242, fax:+81-422-33-1449
http://science.icu.ac.jp/srr
srr@icu.ac.jp

Science Research Reports
SRR# GS-0305 December 2003

A Chinese Root of Japanese Traditional Mathematics – Wasan

Mitsuo Morimoto

Abstract

The *wasan*, or Japanese traditional mathematics, was flourishing in Japan during the Edo period (1603 – 1867).
One of the great Japanese mathematicians of this epoch was Takebe Katahiro (1664 – 1739). He studied the *Introduction to Mathematics* (*Suanxue Qimeng*) written by Zhu Shijie in 1299 in China and published the *Great Commentary on the Introduction to Mathematics* (*Sangaku Keimô Genkai Taisei*) in 1690, thus establishing the algebraic foundation for the *wasan*.
This paper was read at the 11th International Conference on Finite or Infinite Dimensional Complex Analysis and its Applications, Chiang Mai, Thailand, July 27–31, 2003

Contact Address(es): mitsuo@icu.ac.jp

1 Introduction

The *wasan*, or Japanese traditional mathematics, was flourishing in Japan during the *Edo* period (1603 – 1867), on the basis of the Chinese traditional mathematics stemmed from *the Nine Chapters on the Mathematical Art* (*Jiuzhang Suanshu*) [9], which dated back at least to the first century, the *Han* Dynasty.

One of the greatest Japanese mathematicians of this epoch was Takebe Katahiro (1664 – 1739). He studied *the Introduction to Mathematics* (*Sangaku Keimô/Suanxue Qimeng*) written by Zhu Shijie in 1299 in China and published *the Great* [*accomplished colloquial*] *Commentary on the Introduction to Mathematics* (*Sangaku Keimô Genkai Taisei*) in 1690, thus establishing the algebraic foundation for the *wasan*.

In this article, we have omitted the Japanese and Chinese script, translating the name of all Japanese and Chinese books into English. (we follow [9] for the English name of a Chinese mathematics book.) We cite mostly articles and monographs in western languages. The Bibliography of Ogawa [7] contains almost all treatises on Japanese mathematics written in European languages. As general references for the history of Chinese mathematics, we refer the reader to Martzloff [5] and Li-Du [4].

A Japanese version of this article will be published from RIMS, Kyoto University.

2 Zhu Shijie and his two books

Zhu Shijie is a mathematician of the *Yuan* Dynasty (1206–1368). He published two books, *the Introduction to Mathematics* in 1299 and *the Precious Mirror of the Four Elements* (*Siyuan Yujian*) in 1303.

The last chapter of *the Introduction to Mathematics* is devoted to the "procedure of celestial element" (*tengen jutsu/tianyuanshu*), on which we shall discuss later in this note. This is a way to handle polynomials and algebraic equations of one variable with integer coefficients, while *the Precious Mirror* developed a method to handle certain kinds of algebraic equations of four variables. Therefore, the latter book is usually evaluated higher than the former in the history of Chinese Mathematics. Both books disappeared in China during the *Ming* Dynasty (1368–1644).

Because *the Introduction to Mathematics* is a systematic treatise of mathematics starting with the four rules of arithmetic, it was chosen as an important textbook for mathematics students in the Korean *Yi* Dynasty (1392 – 1910). The book was first reprinted during the reign of King Se-djong (1419–1450). This Korean reprint was imported to Japan in the late 16th century, possibly during the Japanese military expeditions to Korea, 1592-1598. (see Martzloff [5].)

3 Books on Things Small and Large

In the *Edo* period (1603 – 1867) Japan was secluded from the world. The *wasan* was investigated in the island almost independently of foreign influences. Of course, the main source of the Japanese mathematics was Chinese, but there is some speculation that there might be a Western influence through Christian missionaries.

The Portuguese drifted to Tanegashima island in 1543. Then F. Xavier, a Jesuit missionary, arrived in Kagoshima in 1549. In 1580's Jesuits were authorized to organize a "collegio" in Azuchi, capital of Japan of the time, for a few years, where mathematics was one of courses. In 1622, Môri Shigeyoshi published *the Book on Division* (*Warizan sho*). The preface of this book contained a distorted story of biblical subjects but the contents is the traditional mathematics of every day life. In 1627, Yoshida Mitsuyoshi, a disciple of Môri, published *the Book on Things Small and Large* (*Jinkôki*), more than 300 versions of which were repeatedly reproduced during the *Edo* period. The main source of this best-seller on merchant mathematics was *the Systematic Treatise on Arithmetic* (*Suanfa Tongzong*) written by Cheng Dawei (1533 – 1606) in 1592.

4 Seki Takakazu and Takebe Katahiro

Seki Takakazu (1642? – 1708) is considered to be one of founders of the *wasan.* He studied Chinese mathematics reading *the Yang Hui's Methods of Mathematics* (*Yang Hui Suanfa*) of Yang Hui, a Chinese mathematician of Southern *Song* in the late 13th century, and Zhu Shije's *Introduction to Mathematics.* In 1974, Seki's existent 27 mathematical works were compiled in [1] with explanations in Japanese as well as in English. During his life time Seki had only one publication, *the Mathematical Methods for Exploring Subtle Points* (*Hatsubi Sanpô*) in 1674.

Seki calculated the circular constant $\pi = 3.141592...$ with twelve digits accuracy. This is one of his remarkable results on the "circular principle" (*enri*), i.e., the study on the circle. (See *the Concise Collection of Mathematical Methods* (*Katsuyô Sanpô*) [1].) Seki also discovered, among others, the theory of resolvent and determinants. (See *the Methods for Solving Concealed Problems* (*Kai Fukudai no Hô*) [1].)

Takebe Katahiro, one of Seki's disciples, entered Seki's school in 1676 when he was 13 years old. His first monograph, *the Mathematical Methods for Clarifying Slight Signs* (*Kenki Sanpô*) was published in 1683. Then he published in 1685 *the Colloquial Commentary on* [*Operations in*] *the Exploring Subtle Points* (*Hatsubi Sanpô Endan Genkai*), and in 1690 *the Great Commentary on the Introduction to Mathematics.* Takebe completed all these three monographs in his twenties.

In Takebe's *Clarifying Slight Signs*, as well as in Seki's *Exploring Subtle Points*, a final solution to each problem was given in a form of algebraic equation of one variable, which was written in Chinese, without any explanation how to derive the equation.

In *the Colloquial Commentary on the Exploring Subtle Points* Takebe explained how the equations are derived by means of the "method of side writing" (*bôsho hô*), which generalizes "procedure of celestial element", and in *the Great Commentary on the Introduction to Mathematics*, Takebe explained the "procedure of celestial element" as the "counting board algebra" in the same way as Seki Takakazu did in *the Methods for Solving the Hidden Problems.* Through these monographs, Takebe showed the "method of side writing" is a method to handle polynomials with several unknowns and applied them to various kinds of problems.

He collaborated with his master in many mathematical researches and in editing *the Great Accomplished Classic of Calculation* (*Taisei Sankei*). In his thirties and forties, Takebe was busy as a government officer but he resumed his mathematics in his fifties and wrote, in 1722, *the Mathematical Treatise on the Technique of Linkage* (*Tetsujutsu Sankei*) and *the Fukyû's Technique of Linkage* (*Fukyû Tetsujutsu*) on the "Technique of Linkage" (*tetsujutsu/zhuishu*). In these books, Takebe described, among others, his calculation of the circular constant π with more than forty digits accuracy and three formulas to represent the length of the arc when the sagitta (the length of the arrow) is given. One of the formulas coincides with the Taylor expansion of the square of the inverse trigonometric function $(\arcsin x)^2$. We can say that with these results he could compete with his European contemporaries (For the details, see Morimoto [6]). For lives of Seki and Takebe and their mathematics, see Horiuchi [3].

5 *The Great Commentary on the Introduction to Mathematics*

While *the Introduction to Mathematics* consists of four volumes, Summary, Upper volume, Middle volume, and Lower volume with total 137 sheets, Takebe's *Great Commentary* consists of seven volumes; Summary, Upper first, Upper second, Middle first, Middle second, Lower first, and Lower second volumes with total 219 sheets. In *the Great Commentary*, Takebe's annotation is printed with half-sized characters, Takebe's *Great Commentary* is more than two times of the original book. (Note that one sheet consists of two pages.)

	Introduction to Math.	*Great Commentary*
Preface	2 sheets	2 sheets
Table of Contents	1 sheets	1 sheets
Summary	7 sheets	13 sheets
Upper Volume	35 sheets	54 sheets
Middle Volume	44 sheets	62 sheets
Lower Volume	48 sheets	87 sheets
Total	137 sheets	219 sheets

The Japanese name of *the Great Commentary* is *the Genkai Taisei*, where *Genkai* means the colloquial explanation and *Taisei* the great accomplished book. In Japan, the Chinese classics used to be read literally word by word, but Takebe tried to annotate the original text in colloquial Japanese. Note that the Japanese in Takebe's commentary was written using *kata-kana* and Chinese characters. This means Takebe's readers were supposed to belong to the "warrior" (*samurai*) class, while *the Book on Things Small and Large* was written using *hira-kana* and Chinese characters and widely used in private primary schools (*terakoya*) for the merchant class.

The last chapter of *the Introduction to Mathematics* is named *Chapter for Extracting the Root* (*Kaihô Sekisa Mon/Kaifang Shisuo Men*) is composed of 34 problems. The first seven problems deal with the "procedure for extracting the root" (*kaihô jutsu/kaifangshu*) and the other 27 problems concern with the "procedure of celestial element".

In order to explain the Takebe's understanding of the "procedure of celestial element", we are going to explain the counting tools of the *wasan*, i.e., the counting-rods and the counting board.

6 Counting-rods

The *wasan* is in the stream of Chinese traditional mathematics, where the numbers are, basically, natural numbers represented decimally using counting-rods. There are two ways to represent numbers; in the orders of 1, 100, 10^4, etc. the counting-rods are placed vertically on the counting board, while in the order of 10, 10^3, 10^5, etc., they are placed horizontally.

	1	2	3	4	5	6	7	8	9
Order of 1, 100, 10^4, $\cdots$	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
Order of 10, 10^3, 10^5, $\cdots$	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

There are two kinds of counting-rods, red and black. The red rods represent positive numbers and the black rods negative numbers. If one has to write numbers on paper with black ink, negative numbers are written with oblique line.

	1	2	3	4	5	6	7	8	9
Negative numbers	𝍠̸	𝍡̸	𝍢̸	𝍣̸	𝍤̸	𝍥̸	𝍦̸	𝍧̸	𝍨̸

If there is no counting-rods on the counting board, it means the digit is 0. On the paper, the empty digit is represented by the sign 〇DFor exampleC 𝍫𝍠〇𝍡𝍮𝍧 represents 310268D

The notion of positive and negative numbers had been clearly established. But their operation was complicated and required some techniques because the numbers were closely related with counting tools like counting board, counting-rods, or abacus.

For example, the addition of integers was called "if same, add; if different, subtract" (*dôka igen/tongjia yijian*). This means that if two numbers are represented by the rods of the same color, we add and that if two numbers are of different color, we subtract. As in the case of abacus, the addition and the subtraction of natural numbers were fundamental operation; the notion of addition of integers were secondary operation and thus required some explanation.

Similarly, the subtraction of integers were called "if same, subtract; if different, add" (*dôgen ika/tongjian yijia*).

This rule had been known, since *the Nine Chapters on the Mathematical Art*, as the sign (*seifu/zhengfu*) rule (see p.404, [9]) and was repeated by Zhu Shijie in *the Introduction to Mathematics*. Takebe recognized its importance and, in Summary of *the Great Commentary*, he stated the rule of addition and subtraction of positive and negative numbers and zero with great care.

7 Counting Board

The counting board looks like the following: The rows of the counting board are named, from top to bottom, Quotient (*shô/sheng*), Reality (*jitsu/shi*), Square (*hô/feng*), Side (*ren/lien*), and Corner (*gû/yu*).

10^3	10^2	10	1	10^{-1}	10^{-2}	10^{-3}	
							Quotient
							Reality
							Square
							Side
							Corner

The counting board was used in many kinds of calculation, the most important of which was the extraction of root.

For example, an algebraic equation of order 3 with numerical coefficients

$$a_0 + a_1x + a_2x^2 + a_3x^3 = 0 \tag{1}$$

was represented on the counting board in the *wasan*. The constant term a_0 was placed in the Reality row, a_1 in the Square row, a_2 in the Side row, and a_3 in the Corner row; that is, the algebraic equation (1) was represented by the configuration of counting-rods on the counting board:

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{2}$$

Problem No. 1 in *Chapter for Extracting the Root* of *the Introduction to Mathematics* reads as follows: (A part of this chapter was translated into English by A. Yamaguchi [10].)

> There is a square shaped area of 4096 *bu* [squared]. (*bu* is a unit for length.) Question: how much is one side? Answer: 64 *bu*.

The "equation to be extracted" (*kaihô no shiki/kaifangshi*)

$$4096 - x^2 = 0 \tag{3}$$

was represented on the counting board with counting-rods as follows:

	10^3	10^2	10	1	10^{-1}	10^{-2}	10^{-3}	
								Quotient
Red rods	𝍬		𝍱	𝍥				Reality
								Square
Black rods				𝍠				Side
								Corner

An equation of any order could be solved numerically by the "procedure for extracting the root".

We regard the Reality row A_0, the Square row A_1, the Side row A_2, and the Corner row A_3 as memories in a computer and the coefficients of (1) a_0, a_1, a_2, and a_3 as values of A_0, A_1, A_2, and A_3. If we place a value q in the Quotient row Q, then the calculation is done on the counting board as follows: (we use here the BASIC-like language)

$$
\begin{aligned}
A_2 &= A_2 + A_3 \times Q \\
A_1 &= A_1 + A_2 \times Q \\
A_0 &= A_0 + A_1 \times Q \\
\\
A_2 &= A_2 + A_3 \times Q \\
A_1 &= A_1 + A_2 \times Q \\
\\
A_2 &= A_2 + A_3 \times Q
\end{aligned}
$$

Let us denote by a'_0, a'_1, a'_2, and a'_3 the values of A_0, A_1, A_2 and A_3 after these operations. Then we have

$$a_0 + a_1x + a_2x^2 + a_3x^3 = a'_0 + a'_1(x-q) + a'_2(x-q)^2 + a'_3(x-q)^3.$$

Further, if we add q' to q in Q, then the values a''_0, a''_1, a''_2, and a''_3 in

$$a_0 + a_1x + a_2x^2 + a_3x^3 = a''_0 + a''_1(x-q-q') + a''_2(x-q-q')^2 + a''_3(x-q-q')^3.$$

can be calculated from a'_0, a'_1, a'_2, and a'_3 by the same program. If we can make the value in the Reality row empty, i.e., zero, after several operations, the value $q + q' + \cdots$ in the Quotient row becomes a root of the equation. Usually, the root is sought in this way, digit by digit from the top digit. This is the principle of the "procedure for extracting the root". This principle was well known in Chinese traditional mathematics since the age of *the Nine Chapters on the Mathematical Art*. In Summary of *the Introduction to Mathematics* Zhu Shijie stated succinctly this "procedure for extracting the root" saying

> "Place the product in the Reality row and operate in Square, Side, Corner rows adding if same and subtracting if different."

Zhu Shijie explained also this procedure in *Chapter for Extracting the Root* and Takebe, in *the Great Commentary*, commented further how to manipulate counting-rods in the "procedure for extracting the root".

8 Counting Board Algebra

The "procedure of celestial element" can be said, in today's terminology, a method to represent a polynomial

$$a_0 + a_1x + a_2x^2 + a_3x^3 \tag{4}$$

by a configuration of the counting board $\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$. Because, in the "procedure for extracting the root", the same configuration represents the algebraic equation (1), it was difficult, psychologically, to admit this ambiguity of the meaning of an configuration. For example, making the Reality row empty and placing one rod in the Square row, we form the following configuration:

$$\begin{bmatrix} \bigcirc \\ | \end{bmatrix}. \tag{5}$$

In the "procedure for extracting the root", the configuration (5) represents the equation $x = 0$ but in the "procedure of celestial element" the same configuration (5) represent a virtual number x. To make the configuration (5) on the counting board was called "to place the celestial element unit" in the *wasan*.

Let us examine how the argument goes in the procedure of celestial element. Problem No. 8 in *Chapter for Extracting the Root* of *the Introduction to Mathematics* reads as follows:

> There is a rectangular rice field of area 8 *mu* 5 *fen* 5 *li* [squared]. Given: the sum of the length and width is 92 *bu*. Question: how much is the length and width respectively? Answer: width 38 *bu*, length 54 *bu*.

Because 1 *mu* is equal to 240 *bu* [squared], 8 *mu* 5 *fen* 5 *li* $= 8.55 \times 240 = 2052$ *bu* [squared].

Zhu wrote as follows:

> Method of Solving: Place the celestial element unit $\begin{bmatrix} \bigcirc \\ | \end{bmatrix}$ as the width. Take this and subtract from the given sum, and let this be the length. Multiply this with the width, and we get the area: $\begin{bmatrix} \bigcirc \\ \text{𝍨}\,\| \\ \text{⊁} \end{bmatrix}$. Move this aside to the left. Take the area and convert the unit from *mu* to *bu*. This we subtract from the area and we obtain the algebraic equation $\begin{bmatrix} \text{═○𝍨╫} \\ \text{𝍨}\,\| \\ \text{⊁} \end{bmatrix}$. Extracting the root from this, we obtain the width. Taking the given sum and subtracting the width, we obtain the length. End of Problem.

Zhu's method can be translated into today's terminology almost literally as follows: Let x be the width. Then $92 - x$ is the length and

$$x(92 - x) = 92x - x^2$$

is the area. As the area is equal to the given 2052, we obtain the equation

$$x(92 - x) - 2052 = 0.$$

Solving this equation, we find the width.

Takebe interpreted Zhu's method as follows: Place the celestial element unit and consider the configuration $\begin{bmatrix}\text{〇}\\ \text{𝍠}\end{bmatrix}$ as the virtual width. Subtracting this configuration from the sum 92, we obtain the configuration $\begin{bmatrix}\text{𝍱𝍡}\\ \text{𝍠̸}\end{bmatrix}$, which is considered as the virtual length.

Here Takebe inserted a long explanation on addition of configurations on the counting board. In today's terminology this amounts to the addition of column vectors.

Now multiplying the two configurations, the virtual width and the virtual length, we obtain the configuration $\begin{bmatrix}\text{〇}\\ \text{𝍱𝍡}\\ \text{𝍠̸}\end{bmatrix}$, which is considered as the virtual area. Canceling the virtual area with the true area 2052, we find the "equation to be extracted" $\begin{bmatrix}\text{𝍪〇𝍭𝍡̸}\\ \text{𝍱𝍡}\\ \text{𝍠̸}\end{bmatrix}$. Extracting the root from this equation by the "procedure for extracting the root", we find the width.

Takebe recognized the configuration of counting-rods on the counting board as the virtual number (*kari no sû*) and formulated the three rules of arithmetic, i.e., addition, self-multiplication, and mutual multiplication. As we mentioned earlier, the addition was defined as vector addition. The rule of powers was formulated as follows:

Method of self-multiplication and mutual multiplication

If the configuration with 2 rows $\begin{bmatrix}\text{𝍠} & \text{Reality}\\ \text{𝍠} & \text{Square}\end{bmatrix}$ is to be multiplied by itself, the Reality multiplied by itself is placed in the Reality, the doubled product of the Reality and the Square is placed in the Square, and the Square multiplied by itself is placed in the row below, thus we obtain the configuration with 3 rows. For example, if we multiply $\begin{bmatrix}\text{𝍦}\\ \text{𝍡}\end{bmatrix}$ by itself, we obtain $\begin{bmatrix}\text{𝍬𝍨}\\ \text{𝍪𝍧}\\ \text{𝍣}\end{bmatrix}$.

If the configuration with 3 rows $\begin{bmatrix}\text{𝍠} & \text{Reality}\\ \text{𝍠} & \text{Square}\\ \text{𝍠} & \text{Side}\end{bmatrix}$ is to be multiplied by itself, the Reality multiplied by itself is placed in the Reality, the doubled product of the Reality and the Square is placed in the next row, the doubled product of the Reality and the Side, added by the squared Square, is placed in the third row, the doubled product of the Square and the Side is placed in the fourth row, and the squared Side is placed in the fifth row. For example, if we multiply $\begin{bmatrix}\text{𝍡̸}\\ \text{𝍢}\\ \text{𝍠}\end{bmatrix}$

by itself, we obtain $\begin{bmatrix} \text{𝍣} \\ \text{𝍩}\not{\text{𝍡}} \\ \text{𝍤} \\ \text{𝍥} \\ \text{𝍠} \end{bmatrix}$

.

In today's terminology, the method of self-multiplication described above can be stated as follows:

$$(7 + 2x)^2 = 49 + 28x + 4x^2,$$
$$(-2 + 3x + x^2)^2 = 4 - 12x + 5x^2 + 6x^3 + x^4,$$

or more generally

$$(a + bx + cx^2)^2 = a^2 + 2abx + (2ac + b^2)x^2 + 2bcx^3c^2x^4.$$

Takebe also stated the rule of mutual multiplication of configurations and gave the following examples (in today's terminology)

$$(-7 + 2x)(3 + x) = -21 - x + 2x^2,$$
$$(1 - 6x + 2x^2)(2 - 3x + x^2) = 2 - 15x + 23x^2 - 12x^3 + 2x^4.$$

Thus, Takebe knew that the configurations on the counting board could be regarded as "virtual numbers" and could be operated addition, self-multiplication and mutual multiplication in the same way as "true numbers". In today's terminology, Takebe recognized that the "procedure of celestial element" was a way of manipulating polynomials. In this sense, I would like to say that the configurations on the counting board form the "counting board algebra", which is canonically isomorphic to the ring of polynomials of one variable with numerical coefficients.

Seki wrote *the Methods for Solving Hidden Problems* (*Kai Indai no Hô*) around 1683 and developed the "counting board algebra" in a systematic way. A traditional Chinese book on mathematics followed the style of *the Nine Chapters on the Mathematical Art* and looked like a problem book. But in *the Methods for Solving Hidden Problems* Seki stated the rule of operations on configurations without introducing any problem. Horiuchi [2] argues that this book was a separating point of the *wasan* from the tradition of Chinese mathematics.

9 Method of side writing

Seki introduced the "method of side writing" in *the Methods for Solving Visible Problems* (*Kai Kendai no Hô*) and then combined it with the "procedure of celestial element" in *the Methods for Solving Concealed Problems* (see [1]). The "method of side writing" can be considered as a generalization of the "procedure of celestial element" and allowed Seki and Takebe to obtain the "equation to be extracted" even if the data were not given numerically. Note that the Seki's *Trilogy*, i.e., *the Methods for Solving Visible Problems*, *the Methods for*

Solving Hidden Problems, and *the Methods for Solving Concealed Problems*, was completed around 1685 as manuscripts but was being kept secretly in Seki's school. It was Takebe who first published results relying on the "method of side writing".

If we state Problem No. 8 above in a manner of Takebe's *Clarifying Slight Signs*, it reads as follows:

> There is a rectangular rice field of given area A. The sum of the length and width is given to be B. Question: how much is the length and width respectively?

In the "method of side writing", we place the celestial element unit $\begin{bmatrix} \bigcirc \\ | \end{bmatrix}$ and consider it as the virtual width, and let the configuration $\begin{bmatrix} |\,A \\ \not| \end{bmatrix}$ be the virtual length, and the configuration $\begin{bmatrix} \bigcirc \\ |\,A \\ \not| \end{bmatrix}$ the virtual area. Then the "equation to be extracted" can be represented as $\begin{bmatrix} \not|\,B \\ |\,A \\ \not| \end{bmatrix}$.

In *Clarifying Slight Signs* many problems were given in this form and the final equations were described in Chinese. But in *the Colloquial Commentary on the Exploring Subtle Points*, Takebe showed how the equations were derived with the "method of side writing". In this way, in the *wasan* polynomials with polynomial coefficients could be manipulated easily although the notation was cumbersome. Because of this, in the Meiji period Japanese abandoned the *wasan* and could switch to the western mathematics with almost no difficulties.

K. Sato concluded his article [8] stating "While *Tengenjutsu* experienced a rigorous change in Japan, we have another question whether this technique became the counter part of "algebra" or not. Straightforwardly, the anser is negative". But as we explained above, Seki and Takebe recognized the configurations on the counting board can be calculated as true numbers, and thus form an algebra. As the "counting board algebra" is canonically isomorphic to the algebra of polynomials of one unknown with numerical coefficients, they can be identified naturally.

References

[1] Hirayama Akira and Shimodaira Kazuo and Hirose Hideo, editor. *Seki Takakazu Zenshû, (Collected Works of Seki Takakazu).* Osaka Kyôiku Tosho, Osaka, 1974.

[2] Annick Horiuchi. Sur un point de rupture entre les traditions chinoise et japonaise des mathématiques. *Rev. Hist. Sci.*, XLII/4:375–390, 1989.

[3] Annick Horiuchi. *Les mathématiques japonaises à l'époque d'Edo.* J.VRIN, 1994.

[4] Li Yan and Du Shiran. *Chinese Mathematics, A concise history.* Clarendon Press Oxford, 1987. translated by John N. Crossley and Anthony W.-C. Lun.

[5] H.-C. Martzloff. *A History of Chinese Mathematics*. Springer, 1997. The French original edition: *Histoire des mathématiques chinoises*, Masson, Paris 1987.

[6] Morimoto Mitsuo. Japanese mathematics in the 18th century, 2001, Manila. (reproduced in Science Research Reports, International Christian University,@2003, http://science.icu.ac.jp/srr)

[7] Ogawa Tsukane. A review of the history of Japanese mathematics. *Revue d'Histoire des Mathematiques*, 7:137–155, 2001.

[8] Sato Ken'ichi. Reevaluation of *tengenjutsu* or *tianyuanshu*; in the context of comparison between China and Japan. *Historia Scientiarum*, 5:57–67, 1995.

[9] Kanshen Shen, John N. Crossley, and Athoney W.-C. Lun. *The Nine Chapters on the Mathemtatical Art, Companion and Commentary.* Oxford University Press and Science Press, Beijing, 1999.

[10] Yamaguchi Ai. Solving polynomial equations based on the *Suanxue Qimeng*, 2003. Senior thesis, International Christian University, June, 2003.