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BURMESTER AND ALLIEVI: A THEORY AND ITS APPLICATION FOR MECHANISM DESIGN AT THE END OF 19-TH CENTURY

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ABSTRACT

The second half of 19-th century can be considered the Golden Age of TMM for the achieved theoretical and practical results that stimulated and obtained enhancements of machinery for Industrial Revolution. Burmester and Allievi can be considered as significant examples of that time for their personality and professional experiences as well as for their works on Kinematics of Mechanisms. In this paper a survey is presented on their curriculum and scientific main works on Mechanism Design with the aim also to stress similarities and differences in the life of kinematicians and in developments of Mechanism Design at the end of 19-th century.

1. INTRODUCTION

In this paper we briefly discuss aspects of the work of two kinematicians: Ludwig Burmester and Lorenzo Allievi. After some biographical remarks we argue that Burmester brought about an influential synthesis of theoretical kinematics and kinematics of mechanisms with his 'Theoretische Kinematik' of 1888. Burmester's book is very general. Among Burmester's original contributions the nowadays so-called Burmester Theory is particularly important. Allievi's 'Cinematica della biella piana' of 1895 deals with a more specific subject: results on curvature theory – related to the Burmester Theory – in planar instantaneous kinematics are applied to the four-bar linkage, with specific applications in mind. Although the scope of Allievi's book seems to be limited, his treatment of the curvature theory of planar motion through four-bar linkages is elegant, quite exhaustive and it includes several original features even with hints for generalization to any class of planar mechanisms.

Allievi has approached the problems as being an engineer and Burmester treated the subject as a theoretician being a geometrician. The paper presents similarities and complementary views in the main works of Burmester and Allievi as significant examples of the historical developments of kinematics of mechanisms in the second half of 19-th century during the enhancements for mechanism design at the time of Industrial Revolution.

2. BIOGRAPHICAL NOTES

2.1. Ludwig Burmester

Ludwig Ernst Hans Burmester was born on May 5, 1840 as son of a gardener in the village of Othmarschen near Hamburg in Germany. He died on April 20, 1927 as a respected professor emeritus of descriptive geometry and kinematics at the University of Munich and a member of the Bavarian Academy of Sciences. Between these dates lies a rich and varied professional life dominated by two great loves: mechanics and geometry. At the age of fourteen he became apprentice in the workshop of a Hamburg precision mechanic. He was allowed to go to the Polytechnical Preparatory School in Hamburg run by a man called Otto Jensen. Jensen was an excellent man who was not only a great teacher of mathematics but good at spotting talented children, like Ludwig Burmester. He, moreover, repeatedly succeeded in convincing rich citizens of the city of Hamburg to support such children. Because Burmester thought that there was a good future in telegraph machines he left Hamburg and went to Berlin where Siemens & Halschke were building such machines. Yet, in the end his desire to study more prevailed. In 1862 he enrolled at the Polytechnical School in Dresden in the department for future teachers of mathematics, science and technology. There he attended the classes of Otto Schlömilch on several areas of mathematics. It is remarkable that he seems to have acquired the considerable knowledge of geometry, that later dominated his activities, all by himself. In 1864 Burmester received his diploma in Dresden with honours.

Fig.1 - Ludwig Burmester (1840-1927)

In the next year Burmester studied in Göttingen where he got his doctor's degree in 1865. His thesis was called Ueber die Elemente einer Theorie der Isophoten (About the elements of a theory of isophotes). It is about the representation of surfaces in 3-dimensional space. Isophotes are lines on a surface defined by a light direction: they are the sets of points for which the cosine of the angle between the light direction and the surface normal has a constant value. So isophotes are lines of constant light intensity. It is remarkable that Burmesters treatment of the subject is entirely analytical. The contrast with Burmester's later works, in which his method is more and more synthetic, is huge. In 1868 Burmester got married to Gabriele Schallowetz. Out of the marriage a daughter that died very early and three sons were born. After 1865 Burmester worked for four years as a secondary school teacher in Lodz in Russian Poland. In that period he wrote several papers on geometrical subjects. In 1871 in Leipzig his first book appeared, Theorie und Darstellung gesetzmäßig gestalteter Flächen (Theory and representation of well-defined surfaces). It was reprinted in 1875. The book is about the subject of his dissertation, the theory of isophotes. Interesting is Burmester's distinction between lines of "true light intensity", the isophotes and lines of "observed light intensity", which he called isophengs. They are the sets of points for which the product of, on the one hand, the cosine of the angle between the light direction and the surface normal, and, on the other hand, the cosine of the angle between the direction of the eye and the normal, has a constant value.

In 1870 Burmester had to leave Lodz after the Russians had closed down the secondary school he was teaching in. Two difficult years followed. However, in 1872 Burmester's luck changed, Burmester became professor of descriptive and synthetic geometry in Dresden. Here, under the influence of Trajan Rittershaus, professor of pure and applied kinematics, Burmester turned to kinematics. Most of his work in his period is related to kinematics. Yet in 1883, he published in Leipzig his Grundzüge der Reliefperspective (Foundations of relief perspective) dealing with the principles of relief perspective. It is remarkable that both the subject of isophotes and the subject of relief perspective have recently attracted interest in the area of computer aided geometric design. In 1888 Burmester's magnum opus, the Lehrbuch der Kinematik, Erster Band, Die ebene Bewegung (Textbook of kinematics, First Volume, Planar Motion), appeared in Dresden. On 941 pages accompanied by an atlas with 863 figures Burmester gave a survey of everything he knew about planar kinematics, including obviously many of his own results. A year earlier, in 1887, he had been appointed to the chair of descriptive geometry and kinematics in Munich.

The turn to kinematics was crucial in Burmester's life. It needs some explanation. Before the sixties of the 19th century kinematics of mechanisms was hardly a coherent discipline and results from theoretical kinematics were not applied systematically to kinematics of mechanisms. In the second half of the 19th century things changed. Kinematics enjoyed great popularity among both mechanical engineers and mathematicians. The work of the German engineer Franz Reuleaux (1829-1905) was in particular very influential. Reuleaux' Theoretische Kinematik (a theoretical treatise on kinematics of mechanisms, in spite of its title), published in book form in 1875, paved the way for a further mathematization of kinematics of mechanisms. On the other hand mathematicians also played an essential role. Ludwig Burmester was a prominent example. With the completely new classification of mechanisms by means of kinematical chains already in the 1860s Reuleaux had introduced a very abstract point of view in kinematics of mechanisms: a planar mechanism came to be seen as a collection of coinciding Euclidean planes moving all (with one degree of freedom) with respect to each other. This was a major and not trivial step forward which helped to turn kinematics of mechanisms into a much more coherent discipline. It involves two related elements: considering the frame of a mechanism as a link and, moreover, abstraction from the particular shape of the links in a mechanism and concentration on the way in which the links are connected. Without mentioning Reuleaux, Sylvester (1875) wrote: "The true view of the theory of linkages is to consider every link as carrying with it an indefinitely extended plane and to look upon the question as one of relative motion [...] Fix any of these planes and the linkage becomes a link-work [...]". In the investigation of bar-mechanisms Sylvester attributed the extension of the consideration to the planes connected with bars to Samuel Roberts.

In many ways Burmester's turn to kinematics can only be understood against the background of Reuleaux' influence. The appointment of Trajan Ritterhaus in Dresden had taken place because of the impact of Reuleaux' kinematics. Moreover, the

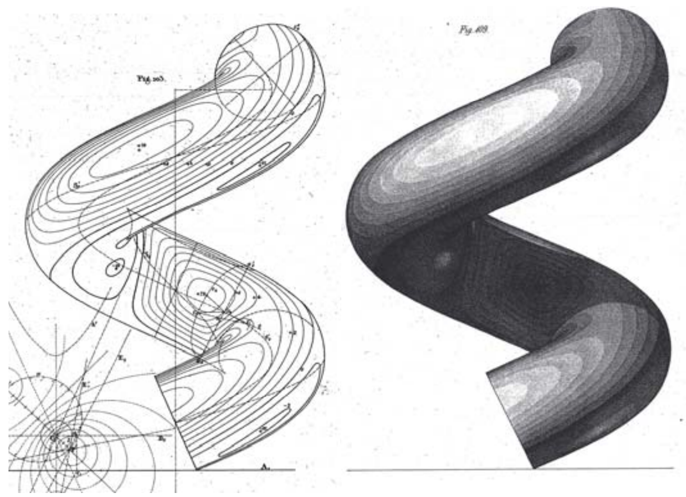


Fig.2 - Figures from Burmester's book on lines of equal light intensity on surfaces.

abstract point of view introduced by Reuleaux in the theory of mechanisms combined with Burmester's great interest in the recently developed geometrical theories enabled Burmester to do his kinematical work. (As for geometry K. T. Reye's Geometrie der Lage (Projective geometry) revised and reprinted several times, is representative of the geometrical background of Burmester's work in kinematics).

In Munich Burmester was highly respected. In 1905 he became a member of the Bavarian Academy of Sciences. In the last years of his life he developed a great interest in the mechanical engineering side of cinematography. He lectured on this subject and he studied different methods to guide the film in the projector. He was buried on the cemetery of Kreuth near the lake Tegernsee. The necrologies that appeared in 1927 and 1930, written by Finsterwalder and Müller, clearly show that one of the giants of 19th century German descriptive geometry and kinematics had died.

2.2 Lorenzo Allievi

Several biographical notes are written on Lorenzo Allievi from several viewpoints and dates, (Angelini 1992; Anonimus 1952, Ceccarelli 1999; Evangeleisti 1956, Marzolo 1942; Enciclopedia Italiana 1960, Marchetti 1941;Roger 1995; Roger Allievi 1980), even in the family (grandchildren Mirta Lancellotti and Luigi Allievi are gratefully acknowledged for the material used for these notes; Luigi Allievi granted a collection of technical documents to the Historical Archive of ENEL that unfortunately is not yet available for the public).

Lorenzo Allievi, Fig.3, was born in Milan on 18 November 1856 and died in Rome on 30 October 1941.

He was son of Francesca Bonacina Spini and Antonio Allievi, who was Senator in the Italian Parliament of recently established Italian Kingdom. Lorenzo started the school in Como but when the father was appointed Senator, in 1871 the family moved to Rome where he completed the college and got the Engineer degree on 24 October 1879. His thesis on 'Internal equilibrium of metallic pylons according to elastic behavior', Fig.4, was also published in 1882 in Rome and was circulated successfully in Italy, as stated by the fact that it was stored in the libraries of main Italian Royal Schools of Engineering. He received a grant as visiting scholar in Germany and successively he got a position at the Royal School of Engineering in Rome where he was devoted mainly to TMM. During this period he spent efforts also in other design problems, such as the Metro in Rome and the railway branch to Castelgandolfo.

Fig. 3 – Lorenzo Allievi (1856-1941)

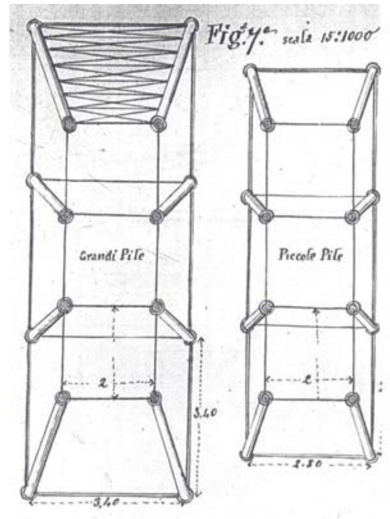


Fig. 4 – Schemes of pylons in the thesis for Engineer degree by Lorenzo Allievi. (Courtesy of Mirta Lancellotti)

On 31 August 1855 Lorenzo Allievi married Anna Brenna who gave him three children: Francesca, Raimondo, and Antonio.

In 1893 he left Rome to get the position of Director of the industrial enterprise 'Risanamento di Napoli' in Naples where he promoted industrial developments until 1901 when he came back to Rome. There he got several positions in many industrial enterprises (Carburo Calcio, Risanamento della Romana Gas, Anglo-Romana, Terni, Romana Elettricità, Banca Commerciale, Meridionale di Elettricità, Elettrochimica, Saline Eritreee) and he became President of the Association of User Electrical Companies. This successful activity brought him to the position of President of the Industrial Union of Region Lazio and later he became vice-President of the Italian Industrial Union.

Particularly interesting is his activity in the company Elettrochimica for which he designed plant enlargements in Popoli but mainly he studied problems in the plant at Papigno in Terni where in 1902 a hydraulic pipe exploded with great damages for the structures. Indeed, since then Lorenzo Allievi continuously addressed attention to the study on perturbed motion of water in pipelines by working mainly during the night after day-work duties for industrial companies. He often remained home in the smoke of his cigarettes as absorbed in the study of hydraulic phenomena for attempting rigorous formulation for design and operation purposes.

Nevertheless, he never disregarded his family to whom he dedicated attention and time, mainly in the holyday periods in Anzio, Fig.5

The study of Hydraulics always attracted his attention, even after he solved the problems in Papigno plants by solving the regulation of Water Hammer as in his first publication in 1902 that was reprinted in 1903. He never considered again problems on Kinematics of mechanisms that were the subject of his first scientific publication, (Allievi 1895).

In his activity as engineer and industrial manager he always paid attention also to the satisfaction of the employers since he considered the work of all as fundamental for achieving scheduled goals for the company and undergoing job. Since he was involved in the Economy, Lorenzo Allievi approached also subjects of Finance in articles that were published later in a volume 'Spunti polemici di attualità' in Rome in 1918.

Fig. 5 – Lorenzo Allievi with his grandchild AnneMarie in Anzio during her celebration for the first communion on 9.3.1929. (Courtesy of Mirta Lancellotti)

Lorenzo Allievi carried out successfully activity as professional engineer and industrial director. But the activity that gave him international fame is the scientific study on the water hammer that he treated in several publications since 1902 until 1936 (Allievi 1902, 1913, 1932, 1933, 1934, 1936) and he was still investigating on the subject when he died in 1941.

In the hydraulic plant in Papigno a big marble plague reminds his contributions, Fig.6, (ENEL 1996).

His approach that is still today known as Allievi's Theory gave him several prizes in Italy, like for example the Jona for Industrial Engineering achievements, and abroad, such as the immediate translations of his publications into French, German, and English. Significant is the premium that ASME, American Society of Mechanical Engineers, gave him as recipient for Honorary Membership in 1937, (ASME 1937), in a period of great international tensions before the second World War.

Recently he has still received honors in the form of entitling to his name technical schools and even streets in several Italian cities, and particularly in Rome and Terni.

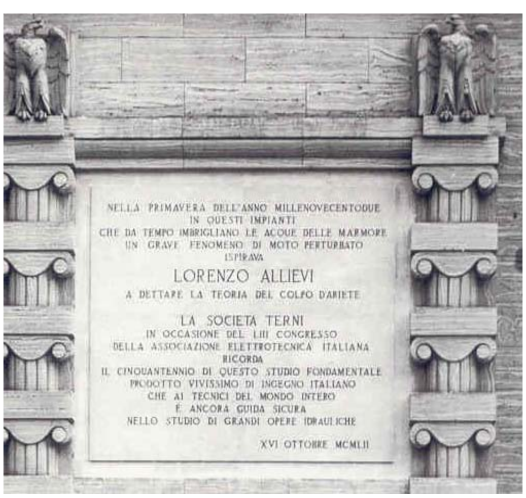


Fig. 6 – The marble plague acknowledging Allievi's contributions on water hammer at Papigno plant in Terni

3. BURMESTER'S BOOK

The 'Lehrbuch der Kinematik' book consists of 12 chapters. In a sense it is a compilation of everything that was known at the time. On the one hand Burmester gave a survey of the results from theoretical planar kinematics. On the other hand he systematically studied their application to practically all planar mechanisms that had been identified at the time.

The first chapter is devoted to the general properties of planar motion: velocity, the instantaneous center of rotation (the pole), the polhodes, the motion of three coinciding planes with respect to each other, construction of tangents and centers of curvature, with many examples of special motions.

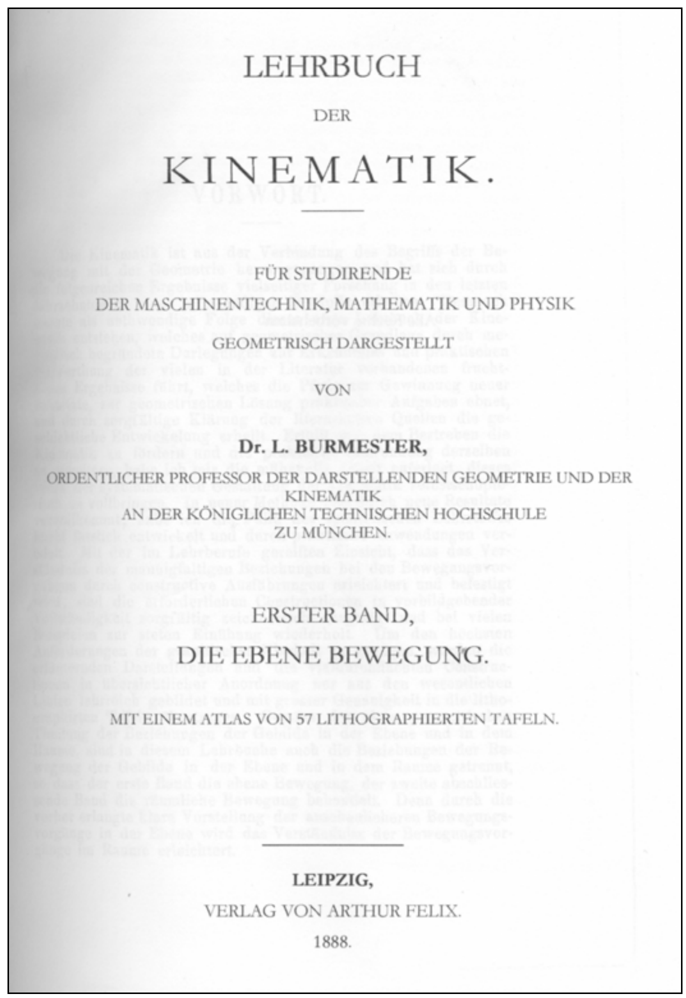
Chapter two concerns cyclic curves and their properties.

Chapter three deals with cylindrical gears, cycloid gearing, involute gearing.

Chapter four deals with the gear in internal gear pumps (Kapselraeder). In this chapter Burmester discusses all existing types of such pumps.

Burmester always goes from the very general to the specific. The theories given in the first chapter are applied in the chapters two through four. In the fifth chapter Burmester again starts from very general considerations, which are applied in the chapters six and seven.

In the fifth chapter he deals with the theory of constraints (Zwangsbedingungen): restrictions on the motion of a system.



LEHRBUCH

DER

KINEMATIK.

FÜR STUDIRENDE

DER MASCHINENTECHNIK, MATHEMATIK UND PHYSIK

GEOMETRISCH DARGESTELLT

VON

Dr. L. BURMESTER,

ORDENTLICHER PROFESSOR DER DARSTELLENDEN GEOMETRIE UND DER KINEMATIK
AN DER KÖNIGLICHEN TECHNISCHEN HOCHSCHULE
ZU MÜNCHEN.

ERSTER BAND,

DIE EBENE BEWEGUNG.

MIT EINEM ATLAS VON 57 LITHOGRAPHIERTEN TAFELN.

LEIPZIG,

VERLAG VON ARTHUR FELIX.

1888.

Fig.7 - Title page of Burmester's Lehrbuch der Kinematik.

The goal of this theory, which was defined by Reuleaux, is to determine the most favorable ways to realize a particular one degree of freedom constrained motion. When the motion of a pair of links with respect to each other is constrained in such a way that we get the same motion whether we fix the one or the other of the two, such a pair is a lower closure (Niedere Paarung). A revolute closure (Drehpaarung) is an example of a lower closure. And so is the closure that only allows two bodies to translate with respect to each other in a particular direction: prismatic closure (Richtpaarung). (Nota bene. In space there is one other example of a lower closure: screw closure). Other closures are higher closures.

The sixth chapter is very long, 122 pages. It deals with the simple planar mechanisms. When the links (or elements including the one that is fixed, the frame) of a mechanism form a closed sequence and all elements execute a one degree of freedom constrained motion with respect to each other, the mechanism is by definition simple mechanism. By the way, Burmester distinguishes elementary (elementar) mechanisms consisting of one pair of elements from simple (einfach) mechanisms. The main simple planar mechanisms are the ones with four revolute pairs (or hinges, that allow only rotary motion between the links). Such mechanisms consist at most of four hinges. The four bar mechanism with only revolute closure occurs in three forms: the crank rocker, the double crank and the double rocker. When we replace in the four bar mechanism revolute closure by prismatic closure we get four special elementary mechanisms: the slider-crank mechanism (Schubkurbelmechanismus), the Scotch-Yoke mechanism (Kreuzkurbelmechanismus) which is a four bar mechanism in which two adjacent revolute pairs have been replaced by prismatic pairs, the mechanism (Schleifschiebermechanismus) which is a four bar mechanism in which two opposite pairs have been replaced by prismatic pairs and the mechanism (Dreirichtmechanismus) with three prismatic closures. Burmester then turns to simple mechanism consisting of three links in combination with two lower closures (revolute or prismatic) and one higher closure: cam mechanisms (Mechanismen mit Kurvenführung). Burmester has in this chapter a section on non-circular gears. There is a section on simple mechanisms with belt transmission and a section on locking and switching devices as well.

Burmester then proceeds in the seventh very long chapter of 143 pages to compound planar mechanisms. In such mechanisms there will be at least one element that is connected through closure to more than two others. At the beginning of the chapter Burmester discusses Grübler's results concerning constraints and results concerning the configuration of poles at a particular instant in the case of several coinciding moving planes. Then he turns to the discussion of all compound mechanisms that he was familiar with. Wherever possible Burmester applied the pertinent theoretical results to the many special mechanisms that he discusses: Watt's mechanism, Stephenson's mechanism and many others.

In the eighth chapter Burmester deals with what he calls "guided mechanisms", like the pantograph. A one-degree of freedom kinematic chain is attached to the fixed plane by means of only one hinge. Then if we guide a point of one of the other links along a curve we have a guided mechanism. In the same chapter he discusses overconstrained mechanisms.

In the ninth chapter he deals with straight line mechanisms.

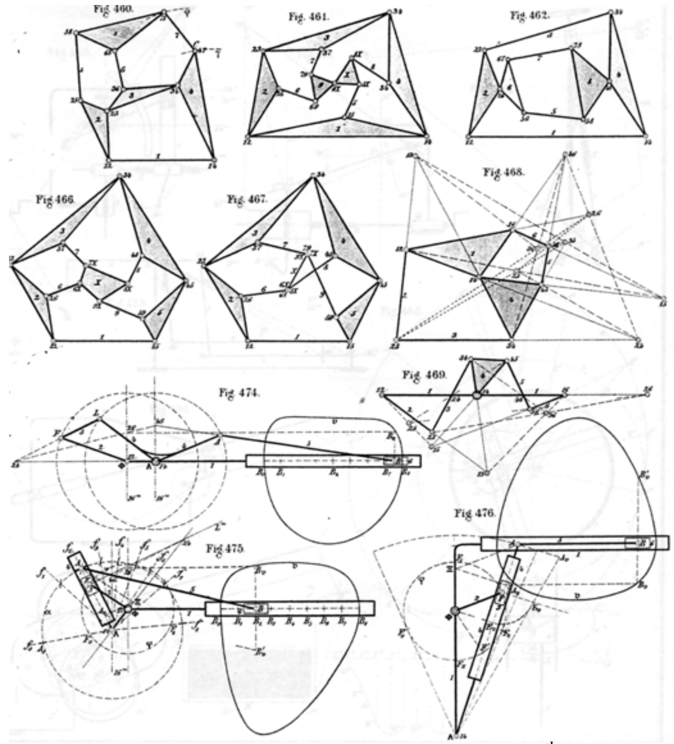


Fig. 8 - Figures accompanying Burmester's 7th chapter on compound planar mechanisms.

Also here he develops a very general theory, before it is applied. This time it is quite new: it is the theory nowadays called Burmester theory. Burmester's Lehrbuch der Kinematik is the culmination of a long development and frequently based on papers that Burmester had written earlier.

The treatment of the Burmester theory is based on the content of two papers from 1876 and 1877. We will look at these papers in some detail. The 1876 paper which clearly represents Burmester's at the time growing interest in applications of kinematics in mechanical engineering consists of three parts. The paper is on the design of four-bar straight-line mechanisms by considering discrete positions of a moving plane.

In 1876 Burmester first considered three discrete positions S_1, S_2 and S_3 of a moving plane and determines the points in the moving plane that are in those positions on a straight line. Right from the start Burmester attached the problem by means of projective geometry. By considering projective pencils of points and lines he proves that the set of all points in S_2 that are in the three positions on a straight line is a conic section. Because the circle points I and J are on it, it is a circle c_2 in S_2. In 1876 Burmester restricted four position theory to a proof of the theorem that in general precisely one point is in the four positions on a straight line.

In 1877 Burmester attached the more general problem of the loci of points that are in a number of discrete positions on a circle. First he considered three positions and proved that the locus of the centres of the circles that are determined by triples of homologous points on three homologous lines is a conic section through the three poles. This theorem immediately yields: There exists either one or three circles that go through four homologous points on four homologous lines. The circles correspond to real points of intersection of conic sections. By applying this theorem to the homologous lines of a pencil, the locus of centres of quadruples of four homologous points turns out to consist of the points of intersection of corresponding elements of two pencils of conic sections. Burmester proved analytically (the methods in the rest of the paper are synthetic) that this Mittelpunktskurve (centre point curve) is a circular curve of third degree. He also shows that it is a so-called focal curve, which is the locus of the foci of all conic sections that touch four given straight lines. Burmester then showed that the locus of points that are in four positions on a circle is also a focal curve, which he calls the Angelpunktkurve.

In his book he called this curve the “circle point curve”. From the text of the 1877 paper it is clear that Burmester's ideas were still in development. For example, Burmester identifies the fixed plane with position 1, which means that he did not seem to realize that by considering the situation from the point of view of the moving plane, we are also dealing with four positions of the fixed plane with respect to the moving one and it is because of that obvious that the "Angelpunktkurve" must be of the same nature as the "centre point curve".

Finally Burmester considered five positions and he found the points that are all five positions on a circle by intersecting two focal curves. Because from the nine points of intersection the circle points and three poles must be subtracted, he easily proves the existence of the four so-called Burmester-points. One of the admirable characteristics of Burmester’s work is the fact that he combined a great interest in theoretical results with an interest in applications.

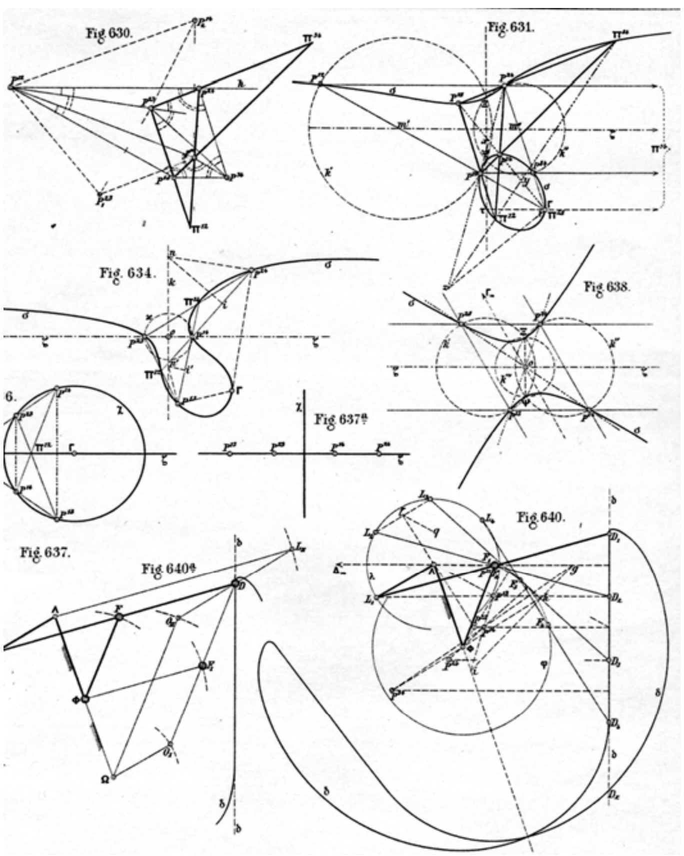


Fig.9 - Figures accompanying the treatment of the “Burmester theory” in the Lehrbuch der Kinematik. The curve σ in Fig. 634 is the centre point curve. In Fig. 638 σ consists of the line at infinity and a hyperbola.

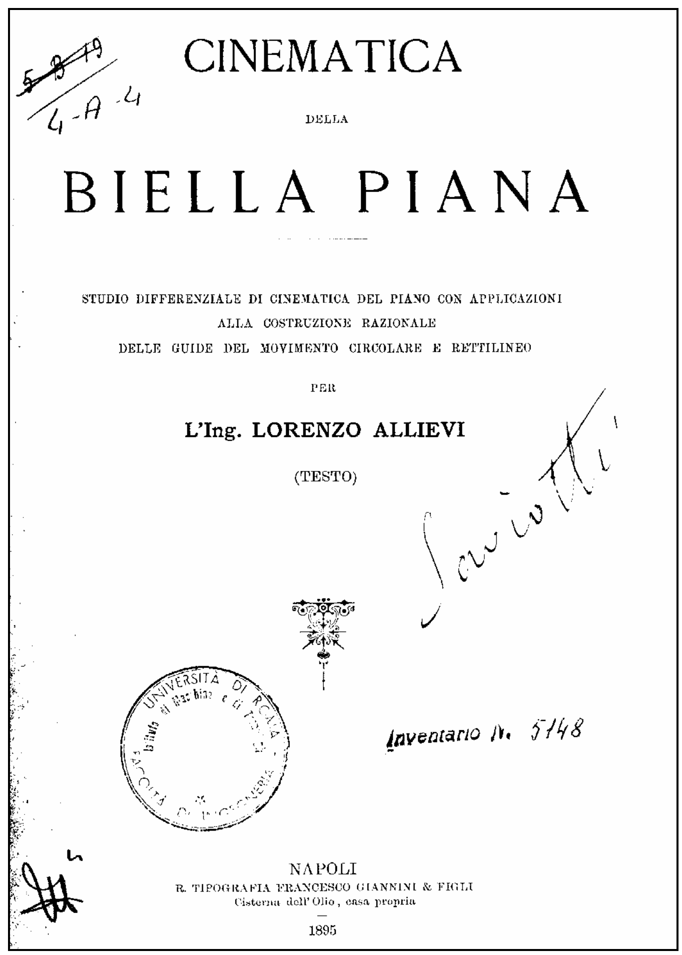
His 1877 paper on discrete position theory is no exception. The Burmester theory is immediately applied to Stephensons link mechanism for controlling the steam valve of a locomotive.

The treatment of the discrete position theory given in the Lehrbuch der Kinematik is a somewhat more elegant summary of the results of the 1876 and 1877 papers.

In the tenth chapter of the Lehrbuch der Kinematik he discusses slider controls for locomotives like Stephenson’s control or Heusinger von Waldegg’s control. In the long eleventh chapter of 122 pages Burmester gives an extensive theory of acceleration with applications to many simple and compound mechanisms. Also here Burmester starts from the general theory and only then considers its applications. Finally in the twelfth chapter Burmester deals with equiform and affine kinematics.

4. ALLIEVI’S TREATISE

Allievi wrote the treatise ‘Cinematica della Biella Piana’ (it can be translated as Kinematics of Planar Couplers), Fig.10, in Rome in 1892 very probably as a consequence of his experience in Germany, but he published it in Naples only in 1895 when he already left the academic position. In several Allievi’s biographies this treatise is considered as a minor work and even is often not cited.


CINEMATICA

DELLA

BIELLA PIANA

STUDIO DIFFERENZIALE DI CINEMATICA DEL PIANO CON APPLICAZIONI
ALLA COSTRUZIONE RAZIONALE
DELLE GUIDE DEL MOVIMENTO CIRCOLARE E RETTILINEO

PER

L’Ing. LORENZO ALLIEVI

(TESTO)

NAPOLI
R. TIPOGRAFIA FRANCESCO GIANNINI & FIGLI
Cisterna dell'Olio, casa propria

1895

Fig. 10 – Title page of treatise "Cinematica della biella piana" by Lorenzo Allievi in 1895

The treatise (Allievi 1895) is presented as a survey of Kinematics of planar motion as applied to mechanism design with a specific reference to the work by Burmester and Schoenflies, but also to original contributions by Allievi himself. Allievi refers to Burmester's book with date 1886 (instead of 1888) since probably he got an early edition of the book during his stay in Germany.

The treatise is organized in seven chapters: the first five chapters deal with a general theory and last two chapters show the application of the theory in design solutions of mechanisms. In particular, in the preface Allievi stresses the novelty of his work both in theoretical arguments and design applications for a rational classification of mechanisms for planar motion and particularly for approximate straight-line and circular guides.

In the Introduction a survey is presented on the correlation between points and lines as generators of geometric loci in planar motion. Characteristics of coupler curves are analyzed in terms of singularities through a mathematical characterization from Differential Geometry and a graphical characterization from Descriptive Geometry. This systematic analysis gives a complete classification of stationary singularities in coupler point trajectory that are named as cusps (cuspidi in Italian), inflections (flessi in Italian), cuspidates (cuspidazioni in Italian, which are arcs due to p cusps with infinite curvature or very short cusps), undulations (ondulazioni in Italian, which are due to q inflections with long inflected trajectory or with zero curvature), falcates (falcate in Italian for the sickle shape, which are due to p = q as cusps with finite curvature and concavity of trajectory branches that are oriented in same direction), and their iper-shapes as a function of the order p of cusp and degree q of inflection as well as a function of their generation and shape. This classification is clearly summarized in Table that is reported in Fig.11. This classification is still a novel way to classify mechanisms in a very elegant and general way for all planar mechanisms.

In the first chapter there is a survey of theories on trajectory curvature; the circles of inflections and cusps are introduced; and an expression for curvature analysis is derived from a quadratic transformation that can be useful for a new synthetic classification of mechanisms for trajectory generation. Formulation are presented in simple expressions all throughout the treatise by means of synthetic methods mixing nicely approaches from Analytic Geometry and Descriptive Geometry.

Although the treatise is directed to four-bar linkages, Allievi approaches the generality of planar motion by considering also mechanisms that can be derived from four-bar linkages when its fixed and mobile joints are constrained on suitable trajectories modeling different kinematic chains. Besides the common revolute and prismatic joints, he defined head-cross (testa-croce in Italian) a joint with straight-line mobility as connected to a fixed joint that is located at infinity and he named as link-block (glifo in Italian) a joint whose center of motion is at infinity. Therefore, he identifies six families of elementary mechanisms that are the basis of the study and are represented in Fig.12 reproducing Figs.10-13 of the treatise: four-bar linkages, slider-crank mechanisms, crank-slider mechanisms, slide-crosshead mechanisms, cross-sliders mechanisms, and so-called Oldham Joint.

For each mechanism type a simple graphical procedure is outlined to determine the circles of inflections and cusps, which

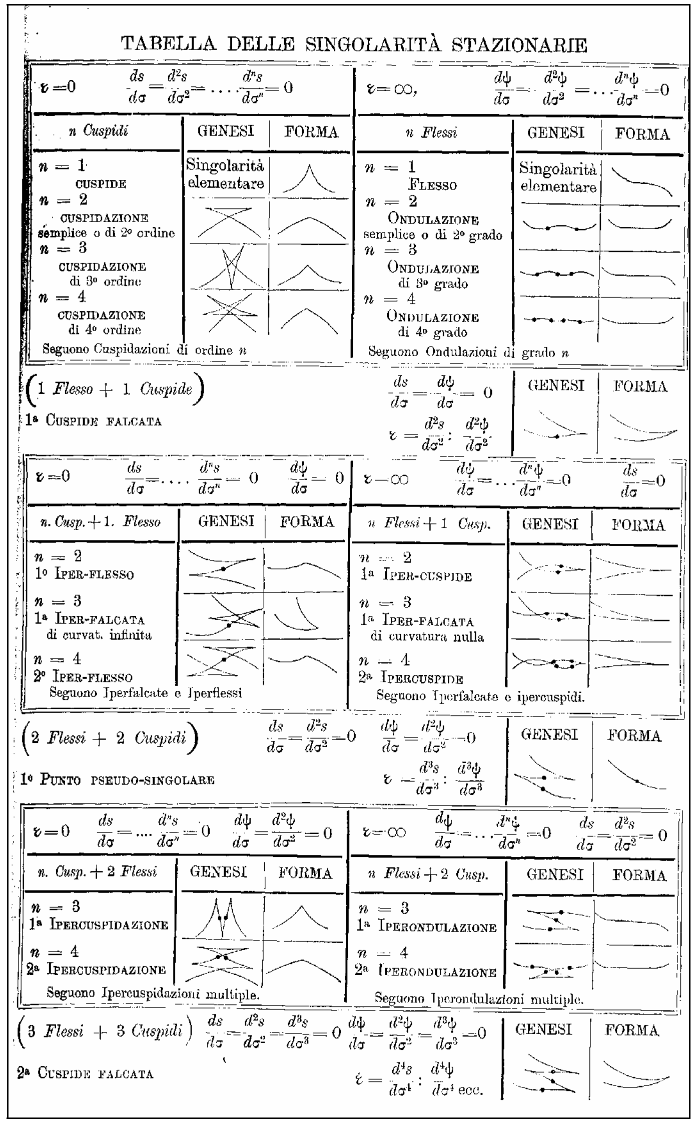


Fig. 11 – Table summarizing stationary singularities in planar coupler curves from Allievi's treatise.

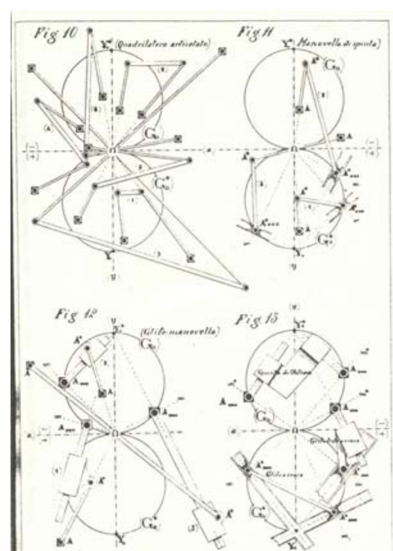


Fig. 12 Six elementary mechanisms for generation of planar coupler curves as from Figs.10 to 13 in Allievi's treatise.

are useful to compute the curvature of any point of the mobile plane through the Euler-Savary equation.

Second chapter deals with Kinematics of two infinitesimal movements. A calculus of the curvature variation gives a mathematical characterization of the circles of inflections and cusps as loci of trajectory points with stationary curvature and of curvature centers of trajectory points with stationary curvature, respectively. The example in Fig.13 reproducing Fig.27 of the treatise illustrates graphically such a characterization.

The loci of the points with stationary curvature in fixed plane and in mobile plane can be expressed as the cubics in Eqs.(12) of the treatise

$$\left(x^2 + y^2\right)\left(\frac{1}{Ry} + \frac{1}{Sx}\right) = 1 \qquad (1)$$

$$\left(x^{*2} + y^{*2}\right)\left(\frac{1}{Ry^*} + \frac{1}{Sx^*}\right) = 1$$

in which R and S, R* and S* are coefficients representing diameters of osculating circles in the acnode of the cubic as shown in Fig.14, reporting Fig.22 of the treatise.

In addition, manipulating the cubic expressions of these loci gives a proof and lemmas for kinematic characterizations of undulations and cuspidates as correlations that for the continuous motion can be expressed as: ‘the loci of successive points of undulations and cuspidates are the loci of the successive intersections among the inflection circles and cusp circles respectively, for successive motions’ (pag. 43). Those mathematical arguments led finally to graphical procedures of generation of the loci ‘ by means of the use of squares only’ (pag.48) as outlined in the construction of Fig.17 of the treatise.

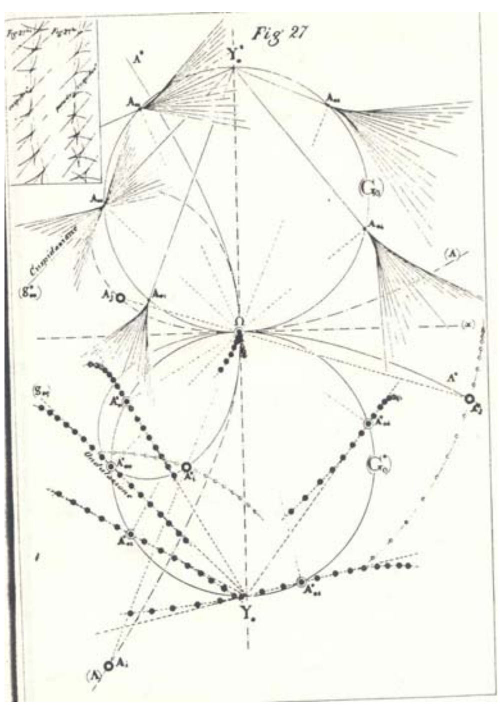


Fig.13 Graphical representation of characteristics of points of inflection and cusp circles as from Fig. 27 in Allievi’s treatise.

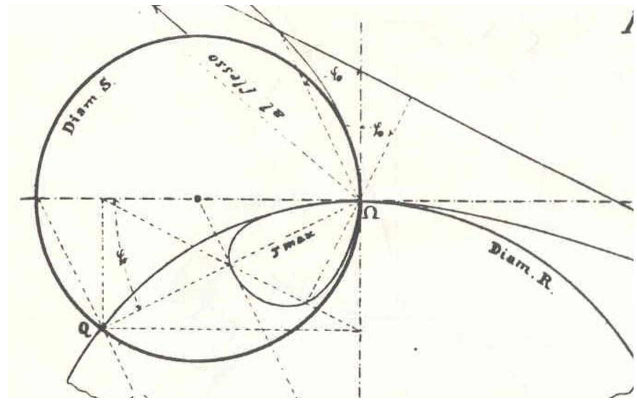


Fig. 14 Graphical interpretations of coefficients in the cubic of stationary curvature from Fig.22 in Allievis’ treatise.

In the third chapter the study is extended to the case for three infinitesimal motions in order to characterize so-called pseudo-undulations and pseudo-cuspidates that are points with stationary curvature with multiple contacts with osculating circles. This characterization is made by discussing Eq. (20) up to its form (23) in the treatise, which are additional manipulations of Eqs.(1) reproducing Eqs.(12) of the treatise. In particular, in this short chapter Allievi has shortened the original heavy treatment of instantaneous Kinematics by Burmester that was extended by Schoenflies to the case of continuous motion for determining the four cyclic points. In fact Allievi outlines a handy procedure for graphical constructions by using analytic differentiation of (r-r*) with r and r* radii of polhodes leading to Eq. (20) of the treatise.

In chapter four degeneration of the loci of points with stationary curvature is discussed by using the cubic expression in Eq.(1) (Eq.(12) in the treatise) from chapter two. Degenerations into circles and straight-lines are analyzed through conditions on the cubic coefficients and corresponding kinematic relations for the motions they represent. Five classes of degenerated mechanisms are identified.

In the first class for 1/S = 0 each cubic becomes a circle and a straight-line giving three series of mechanisms only depending of the location of the joints on them. In the first series the relative location line of fixed joints gives only four-bar linkages, being the cranks convergent, crossed, or diverging. These mechanisms can show pseudo-undulations and pseudo-cuspidates; double undulations and double cuspidates. In the second series, with joints on circle and line, mechanism types are related to crack position giving four-bar linkages with two followers and slider-crank mechanisms as shown in Fig.32 of the treatise with the possibility of having pseudo-undulations or pseudo-cuspidates that are expressed by a simplified expression of the cubic in the form of Eq. (31). The third series, with joints on a line only, is composed of mechanisms of the previous series at dead-lock configurations.

In the second class with 1/S = 1/R = 0 a duality of series is identified as corresponding to the case in which a cubic degenerates into either the inflection circle or the cusp circle with a line joining their centers. In this class there is a great variety of mechanisms with symmetric and asymmetric motion capability. Those mechanisms with symmetric motions are related to the possibility to have symmetrical motions of cyclic and paracyclic types. All the mechanisms can show several types of stationary singularities that are discussed with mathematical and graphical characterizations by using algebraic manipulations of Eq. (20) and illustrations from Fig. 34 to 45, and then they are summarized in a synoptic view from pag 92 to pag.98 of the treatise.

A third class is identified by the condition $1/R = 0$ or $1/R^* = 0$, which corresponds to the case in which one of the loci does not degenerate and the corresponding mechanism types are characterized as having pseudo-cuspidates and pseudo-undulations in the coupler curves. Those mechanisms are illustrated with their typical structures in Figs 46 to 49 of the treatise.

In the fifth chapter a fourth class of mechanisms is introduced as deduced from degeneracy of the cubics that is due to the location of the instantaneous center of rotation at infinity. Therefore possible mechanisms like four-bar linkages and slider-crank mechanisms are configured with parallel cranks as shown in Fig.52 to 54 in the treatise. The corresponding coupler curves are characterized by having iper-falcates and iper-undulations.

In the last two chapters detailed analyses are reported for mechanisms with coupler curves for approximate circular and straight-line guides, respectively. The discussion is also focused on practical design constraints using the proposed classification in classes and series. Design practical solutions with very detailed graphical representations are reported in Fig. 56 to 107, which is the last one of the treatise.

An example of the rich graphical details is reported in Fig.15 reproducing Fig.89 of the treatise for a case of straight-line guide mechanism in the second class as an example in which the coupler curve of point Ω shows a falcate

In particular at the beginning of chapter six the mathematical structure of synthesis problems is formulated and discussed in terms of available equations and conditions in order to have the possibility to determine the eight design parameters that corresponds to the coordinates of the four joints

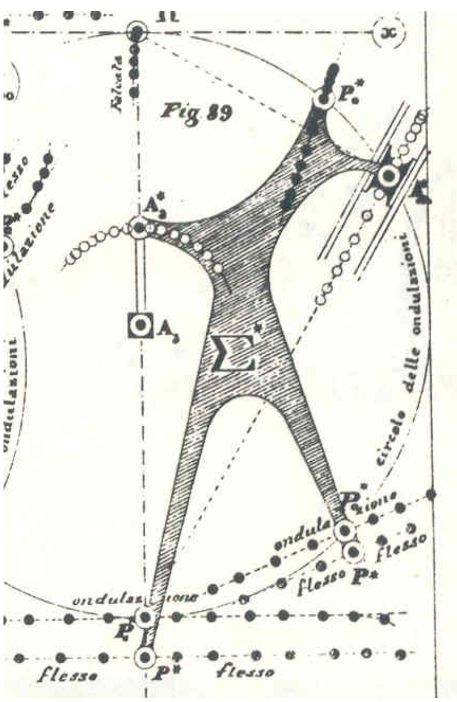


Fig.15 An example of detailed design solutions for guide mechanisms from Fig.89 in Allievi's treatise

of a mechanism for planar motion. Allievi outlines that in general it is possible to design guide mechanisms for point trajectory up to the fourth order and in the case of guiding two points up to third order so that the variety of stationary singularities in the proposed Table in Fig.6 can be used to obtain suitable coupler curves.

As an example of the practical design-oriented approach of the Allievi's treatise, the case of Watt mechanism is illustrated as referring to Fig.16, reproducing Figs 105 to 107 of the treatise. The Watt mechanism of Fig.106 is a mechanism whose coupler curve is used with three inflections coinciding in a point to give a pseudo-undulations. The three inflections can be separated by means of enlarging or shortening the cranks, as stated by a proposed mathematical characterization of the corresponding mechanism series, and therefore it is possible to obtain an excursion of the approximate straight-line trajectory as long as required for a design application. In particular, referring to Fig.105 in Fig.11 Allievi deduces the following proposition: 'once the inflections are separated, if the cranks are fixed with length $(m^2+n^2)^{1/2}$, the two extremity inflections are located at a distance ± n from the central inflection, by being m the crank length and 4n the distance between them' (pag 150). Similar considerations can be applied to a slider-crank mechanism of the fourth class to obtain the solution in Fig.107 of the treatise as an efficient alternative to Watt mechanism in Fig.106.

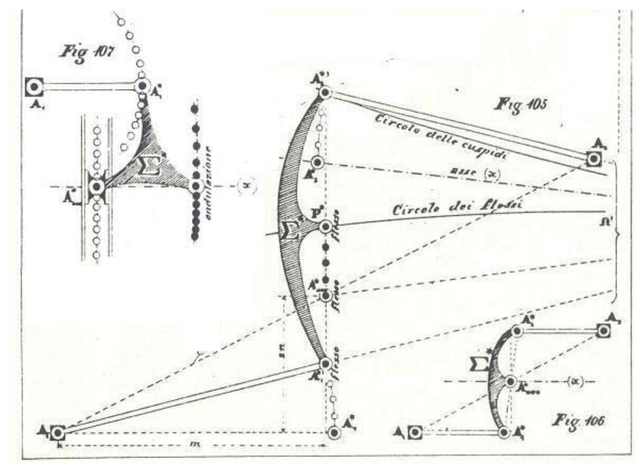


Fig. 16 – Schemes and solutions for designing approximate straight-line mechanism with Watt coupler curve from Allievi's treatise

CONCLUSIONS

Both Burmester's 'Lehrbuch der Kinematik' and Allievi's 'Cinematica dell biella piana' are still of modern interest, since they are cited in teaching and research publications (like for example in the book 'Geometric Design of Linkages' by J.M: McCjharty in 2000), and they are even reprinted for practical professional purposes (like for example in the anastatic reprint by CFR-FIAT in 1999).

Although both worked in kinematics, Burmester and Allievi were very different in many respects. Ludwig Burmester had a humble background. He was the son of a gardener. However, because he was very talented, he was given the opportunity to study. He developed into a typical German university professor, who spent his life teaching and writing papers and books. Lorenzo Allievi was the son of an Italian senator. He did some brilliant theoretical work but he spent his life as an engineer and technical manager. Burmester was a theoretician with a great interest in applications. In his work he excelled in the application of synthetic methods in geometry. Allievi was an engineer interested in theory. In contrast to Burmester in his theoretical work we find the elegant application of analytical methods.

It is remarkable to see how the so-called Burmester theory concerning four or five subsequent positions of a moving plane in a fixed plane in fact led to Allievi's book. If the four or five positions of Burmester's theory are infinitesimally close, the Burmester theory gives us results on points of stationary curvature. The chapters two through five of Allievi's seven chapter book could not have been written without Burmester's results. That is why we have given some special attention to the Burmester theory.

Both Burmester's 'Lehrbuch der Kinematik' and Allievi's 'Cinematica dell biella piana' show that kinematics had become a mature discipline in the second half of the 19th century. In both books specific mechanisms are studied only after first having developed a body of general theoretical knowledge (i.e. results in principle applicable to many other mechanisms).

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