Plasma Jet-Driven Magneto-Inertial Fusion: Ignition Considerations

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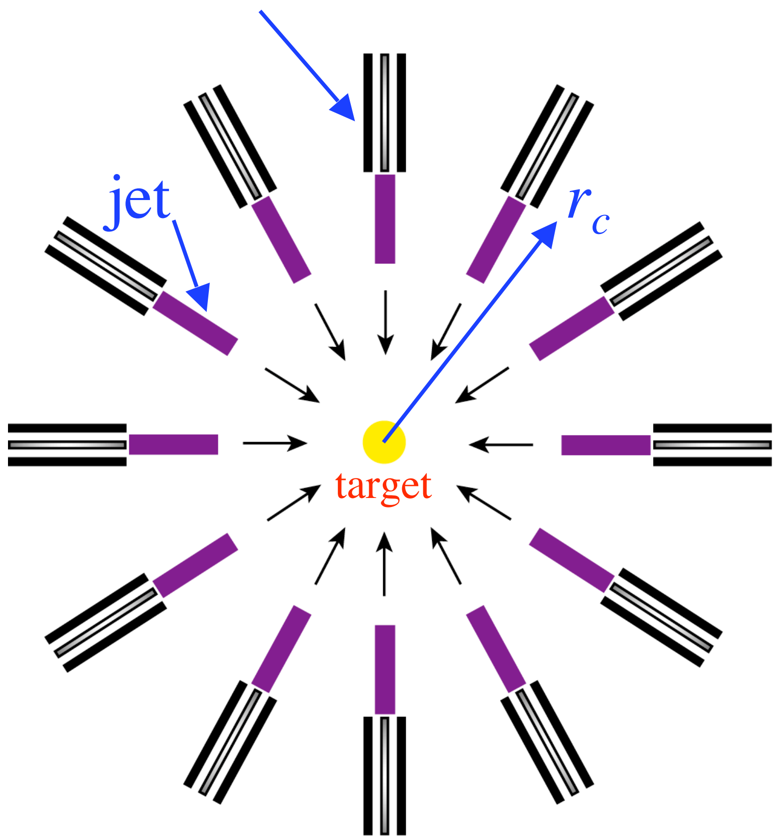
Contents of Talk

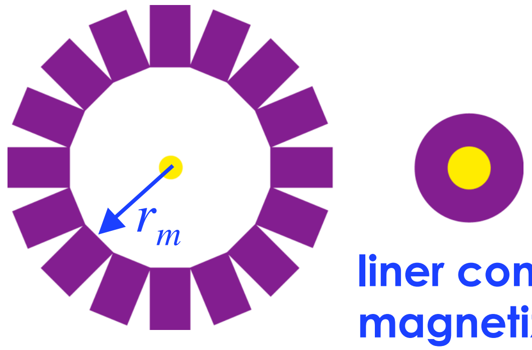
- Description of the supersonic PJMIF concept
- Review ignition criteria for a hot, compressed magnetized target
- Liner implosion dynamics determine plasma jet parameters
- A theory for the fuel disassembly time
- Heating the surrounding liner fuel by an alpha-driven thermal wave
- Conclusion and future work needs

Supersonic plasma jets create imploding plasma liner

Plasma guns at chamber wall r_c ~ 6 m allow good clearance distance



Thio et al (1999)



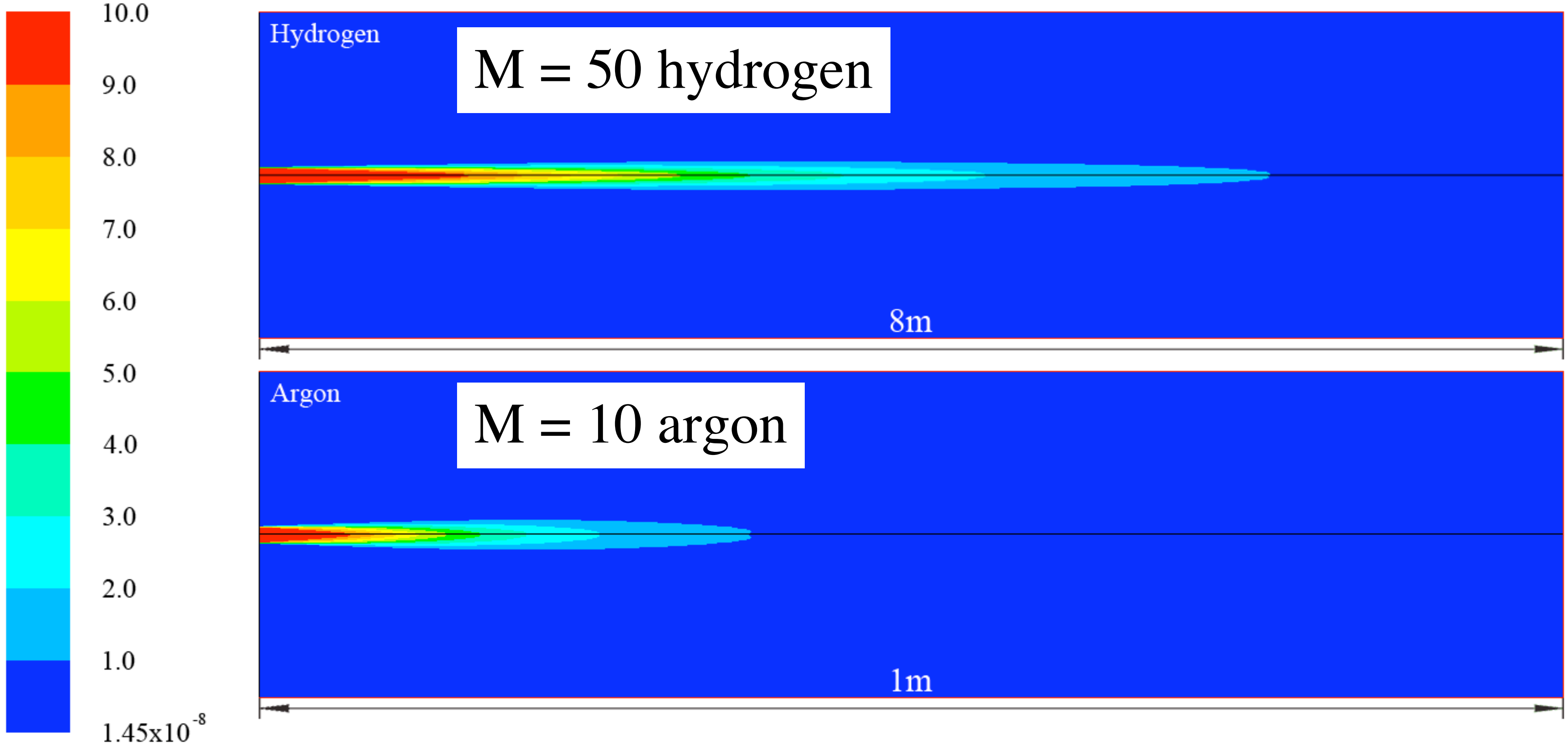
plasma liner formed by jet merging

liner compresses magnetized target to fusion condition

$N_{jet} \sim 70$ (number of jets)

$M_{jet} \sim 10 - 60$ (Mach Number)

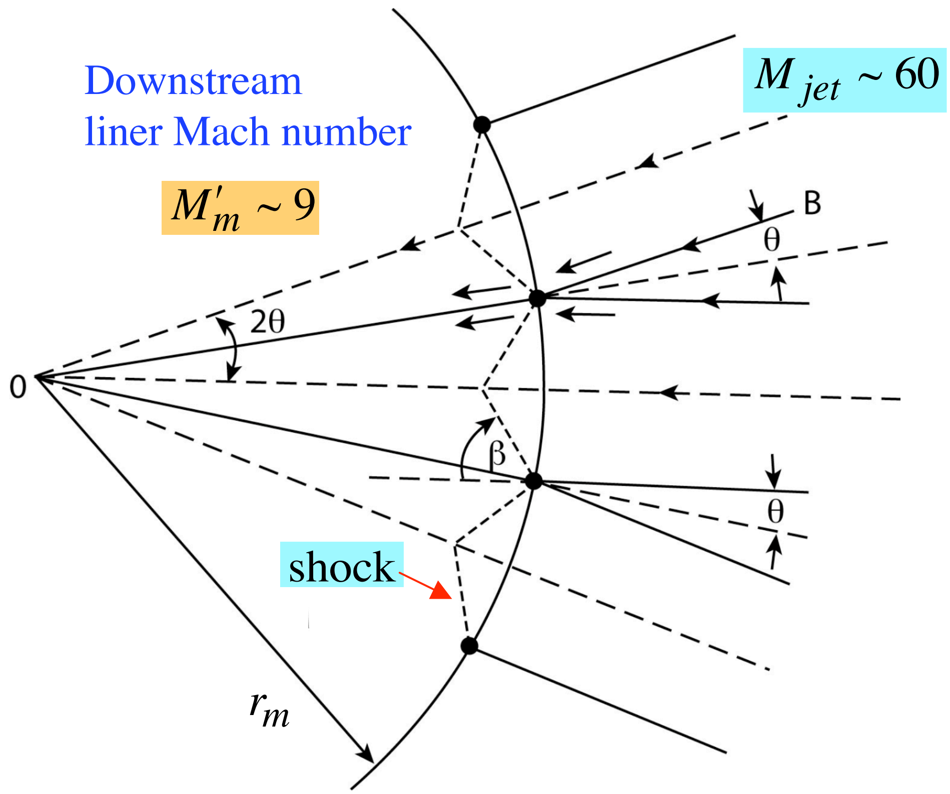
2D axisymmetric Simulation of high-M jet propagation using FLUENT CFD code



Countours of Density (x10^{24}(/m^3))

u_{jet} = 100 km/s, n_{jet} = 10^{25} m^{-3}
Gun diameter = 10 cm

Oblique shocks formed at merging radius downshift the liner Mach number



– Problem is related to planar supersonic flow past a wedge with turning angle

– $\theta \sim (\pi/N_{jet})^{1/2} \sim$ 12 deg

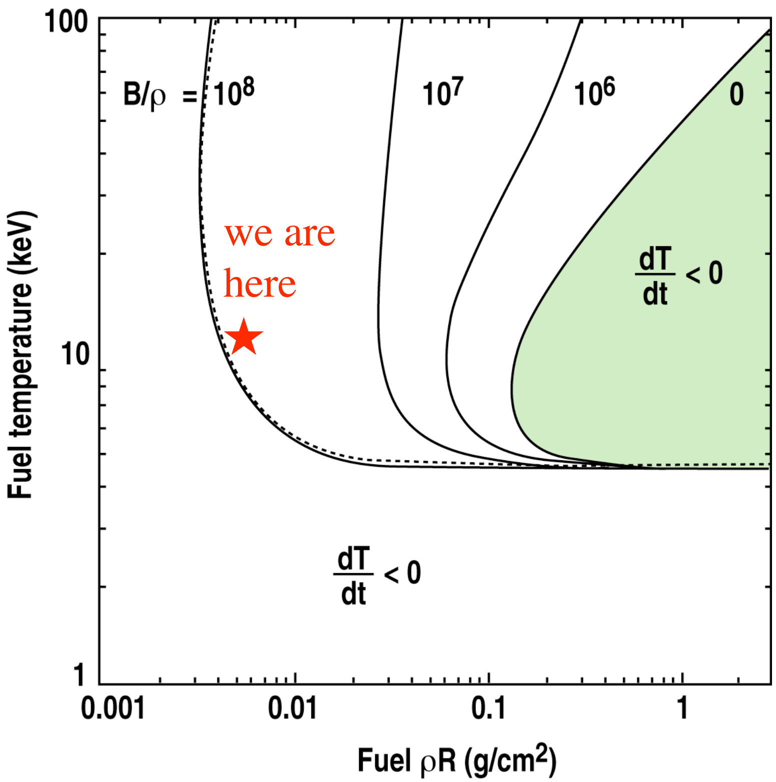
$$M_m \approx \sqrt{\frac{2N_{jet}}{\pi\gamma(\gamma-1)}}$$

– D. Ryutov's improved round-jet free-energy model: $M'_m \approx \sqrt{2}M_m$

• Mitigate the oblique shocks by radiation cooling using trace impurities (F. Thio, D. Ryutov)

Ignition criterion for a magnetized target

- Magnetized target fuel has $\rho R <<$ 0.3 g/cm^2 (nominal ICF value)



Basco, Kemp, Meyer-ter-Vehn (2000)

Select: $R = 0.5 \;\text{cm}, \;\; B = 1 \;\text{MG}$

$$BR = (\rho R)\left(\frac{B}{\rho}\right) = 5 \times 10^5 \;\text{G} \cdot \text{cm}$$

★

$$B/\rho = 7.8 \times 10^7 \;\; \text{G} \cdot \text{cm}^3/\text{g}$$

$$\rho R = 0.0064 \;\; \text{g}/\text{cm}^2$$

$$n_{hot} = \frac{\rho}{m_{DT}} = \;\; 3.2 \times 10^{21} \text{cm}^{-3}$$

– Fraction of α energy escaping target (from B-K-M)

$$f_{esc} = 0.965$$

Ignition conditions set target parameters

- Plasma Liner needs to implode target to these pressures and energies:

target pressure*

$$p_{hot} = \frac{2(\rho R)T_{hot}}{mR} = 100 \ \ \text{Mbar}$$

magnetic field pressure

$$p_{mag} = \frac{B^2}{2\mu_0} = 0.04 \ \ \text{Mbar}$$

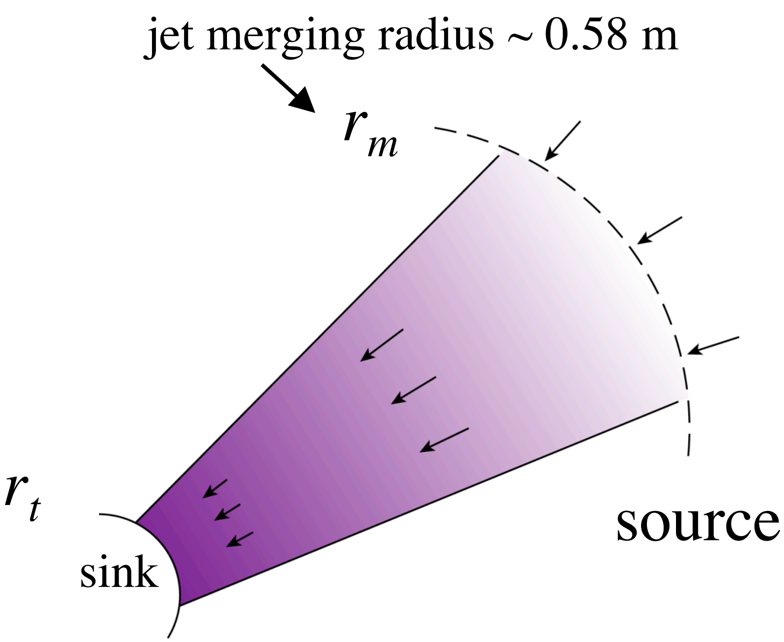
target thermal energy

$$E_t = 4\pi(\rho R)R^2 T_{hot}/m = \ 8 \ \ \text{MJ}$$

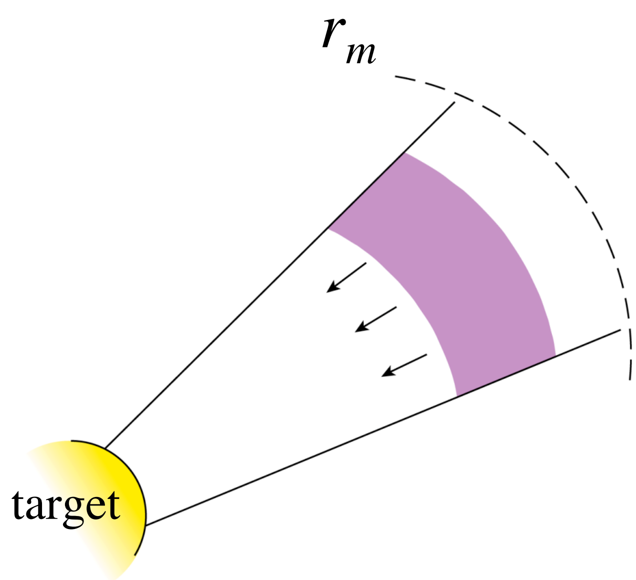
* The advantage of enlisting magnetic fields is a factor 300 lower hot spot pressure compared with conventional ICF

Supersonic flow field within imploding liner

$$\frac{\rho}{\rho_m} = \left(\frac{2 + (\gamma - 1) M_m^2}{2 + (\gamma - 1) M^2} \right)^{\frac{1}{\gamma - 1}} \qquad \frac{p}{p_m} = \left(\frac{\rho}{\rho_m} \right)^{\gamma} \qquad \left(\frac{r}{r_m} \right)^2 = \frac{M_m}{M} \left(\frac{2 + (\gamma - 1) M^2}{2 + (\gamma - 1) M_m^2} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}}$$



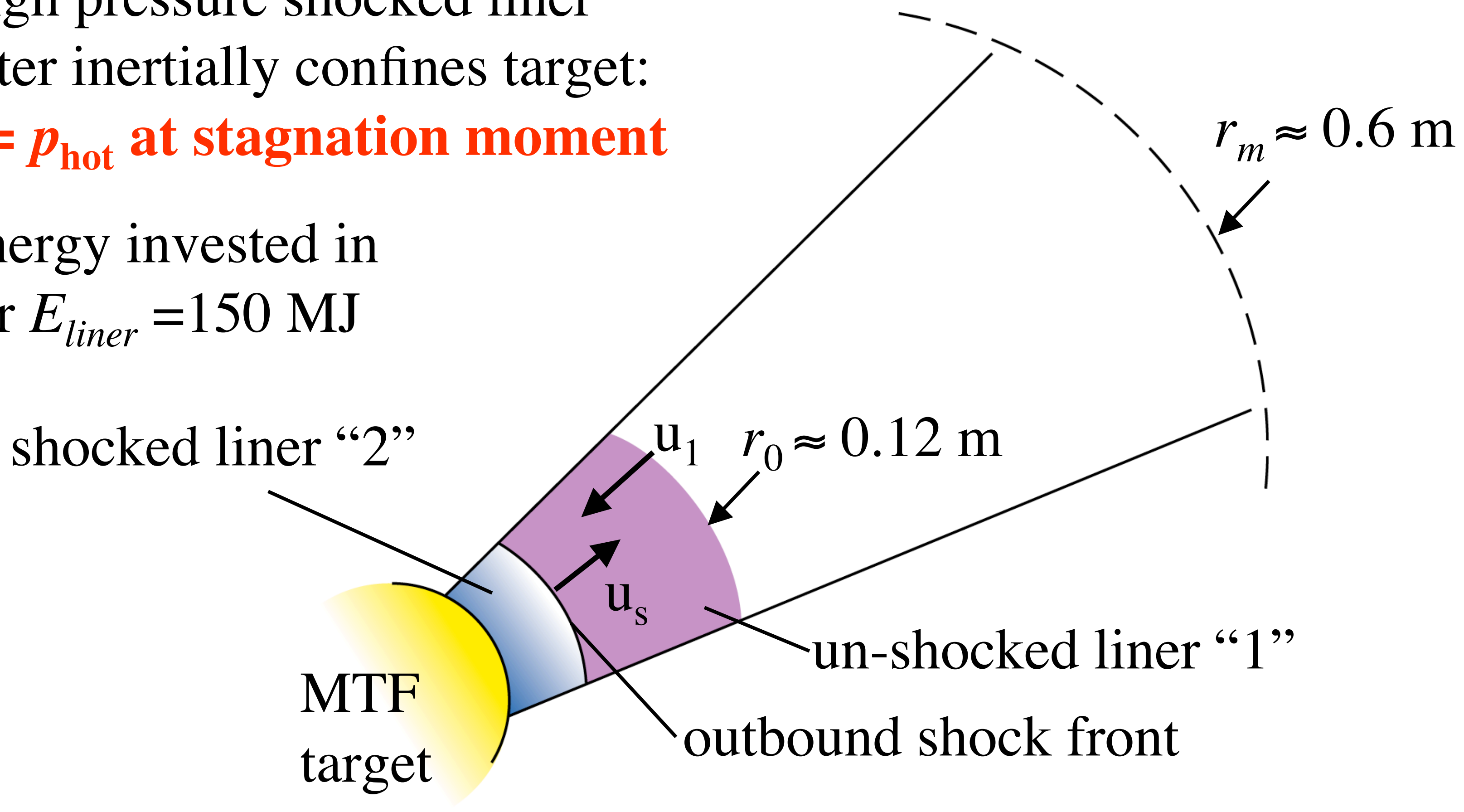
Steady-state fictitious flow



Liner flow is represented by a spherical annulus of the fictitious flow

Plasma liner just after stagnation time

- Supersonic liner flow disappears into outbound shock.
- High pressure shocked liner matter inertially confines target: **$p_2 = p_{hot}$ at stagnation moment**
- Energy invested in liner E_{liner} =150 MJ



Analysis of post-shocked stagnation region determines required jet Mach number

- Shock jump conditions $\dfrac{\rho_2}{\rho_1} = \dfrac{(\gamma+1)M^2}{(\gamma-1)M^2+2}$ $\dfrac{p_2}{p_1} = 1 + \dfrac{2\gamma}{\gamma+1}(M^2-1)$
- Shock frame Mach number: $M = M_1(1 + u_s/u_1)$
- Shock speed $\dfrac{dr_s}{dt} = u_s = \dfrac{\gamma+1}{4}(u_1+u_2) - u_1 + \left\{\dfrac{(\gamma+1)^2}{16}(u_1+u_2)^2 + c_1^2\right\}^{1/2}$
- At stagnation moment ($u_2 = 0$) and taking $\gamma = 5/3$

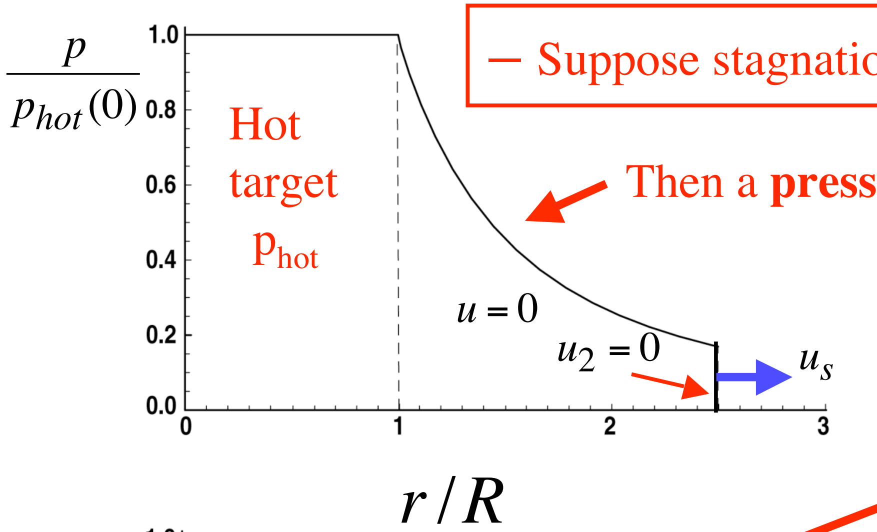
$$p_2 \cong \frac{3}{5}\Phi(M_1) m_j n_j u_j^2 M_j^3 \qquad \Phi(M_1) = (3+M_1^2)^{-5/2}\left\{1 + \frac{10}{9}M_1^2 + \frac{5M_1}{3}\left(1+\frac{4}{9}M_1^2\right)^{1/2}\right\}$$

– Cubic Mach number dependence reflects strong density amplification by convergence

- M_1=1 maximizes $\Phi(M_1)$. Now set $p_2 = p_{hot}$, getting for DT jets

$$M_j = 60, \quad \text{for} \quad n_j = 1.5 \times 10^{19} \ \text{cm}^{-3}, \ u_j = 100 \ \text{km/s}$$

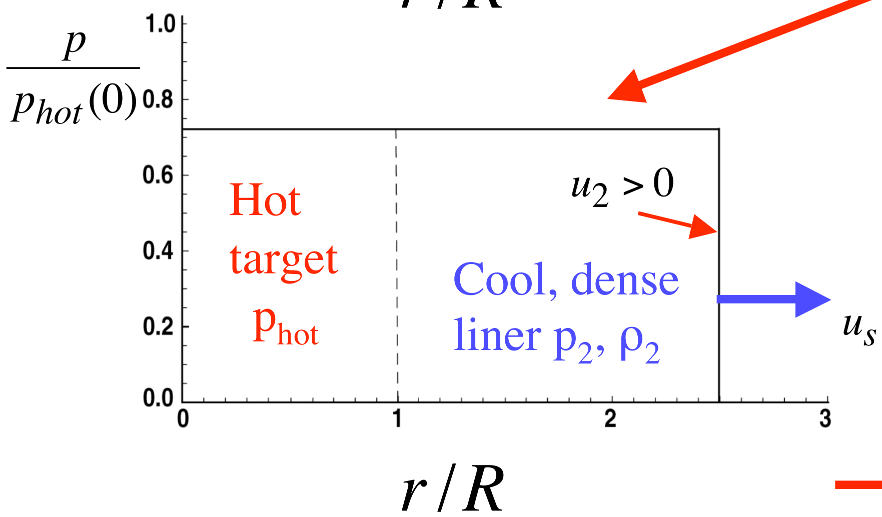
Hot spot (target) decompression time t_{HS}



– Suppose stagnation region were **immoveable**. Then $u_2 = 0$

Then a **pressure gradient** develops behind shock front since

$$\frac{p_2(r=r_s)}{p_{hot}(0)} = \frac{\Phi[M_1(r=r_s)]}{\Phi[M_1=1]} \xrightarrow[M_1>>1]{} \frac{20}{9M_1^3}$$



– This means that flow cannot be immovable. Instead, the pressure becomes nearly isobaric such that $p(t) \sim p_2(t)$ and $u_2(t) > 0$

– Expansion is detonation-like

$$p_{hot} = p_2 \sim \rho_2 u_2^2 \quad \therefore \; u_2 \sim c_{s,hot}\sqrt{\frac{\rho_{hot}}{\rho_2}} \sim 52 \text{ km/s}$$

$$t_{HS} \sim R/u_2 = 98 \text{ ns}$$

Caution: this time is an underestimate

Re-evaluate decompression problem using a new isobaric model (work in progress)

- Integrate energy equation over volume behind outbound shock ($0 < r < r_s$)

$$\frac{d}{dt}\int_0^{r_s} r^2\left(\frac{p}{\gamma-1}\right)dr + r_s^2 u_2 p_2 - r_s^2(u_s - u_2)\left(\frac{p_2}{\gamma-1}\right) = \int_0^{r_s} r^2 Q_s dr$$

fusion & radiation power densities

- Isobaric approximation $p(r,t) \approx p_2(t) = \text{post} - \text{shocked pressure}$

$$\frac{dp_2}{dt} + \frac{3\gamma}{r_s}u_2 p_2 = (\gamma-1)\left(\frac{R}{r_s}\right)^3 Q_s \qquad (1)$$

- Integrate energy equation over volume of hot target ($0 < r < R$)

$$\frac{dp_2}{dt} + \frac{3\gamma}{R}\frac{dR}{dt}p_2 = (\gamma-1)Q_s \qquad (2)$$

- Two equations involving four unknowns: $p_2,\ u_2,\ R,\ r_s$. The system is closed by consideration of the shock jump relations (slide 10).

- Finally, mass conservation in the hot region gives the density and temperature:

$$n_{hot}(t) = n_{hot}(0)[R(0)/R(t)]^3 \qquad T_{hot}(t) = p_2/2n_{hot} \quad \longrightarrow \quad Q_s(n_{hot}, T_{hot})$$

Target expansion speed rule

- If the hot target is still producing sufficient fusion power such that: $Q_s > 0$

- then the expansion speed of target radius is bounded

$$\rightarrow \quad \frac{R}{r_s} < \frac{\dot{R}}{u_2} < 1$$

$R =$ target radius

$r_s =$ shock radius

$u_2 =$ fluid velocity just behind shock

Can α-driven thermal wave "fireup" liner?

- Alphas escaping target at stagnation heat inner layer of liner

Alpha continuity equation in liner region x > 0

$$\frac{\partial n_\alpha}{\partial t} + n_\alpha \frac{\partial \mathrm{v}_\alpha}{\partial x} + \mathrm{v}_\alpha \frac{\partial n_\alpha}{\partial x} = 0$$

x = 0 is plane of separation

Alpha momentum equation with electron drag

$$\dot{\mathrm{v}}_\alpha(t) = \frac{\partial \mathrm{v}_\alpha}{\partial t} + \mathrm{v}_\alpha \frac{\partial \mathrm{v}_\alpha}{\partial x} = -\mathrm{v}_\alpha \nu_s \qquad \nu_s(x,t) = \text{slowing down rate}$$

- Assumed all alphas escaping compressed target have birth velocity $\mathrm{v}_{\alpha 0}$ normal to target/liner interface & no energy spread.

Slowing down rate of fusion alpha particles

- We have a weakly coupled, non-degenerate liner, so the stopping formula is based on liner response theory, valid when $\Gamma = Z_\alpha / N_D < 1$

$$\nu_s = \omega_{pe} \frac{m_e}{m_\alpha} \left(\frac{Z_\alpha^2}{2^{1/2} 8\pi N_D} \right) \left(\frac{\mathrm{v}_{te}}{\mathrm{v}_\alpha} \right)^3 \left[G(\mathrm{v}_\alpha / \mathrm{v}_{te}) \ln(\lambda_D / b_{\min}) + H(\mathrm{v}_\alpha / \mathrm{v}_{te}) \ln(2^{1/2} \mathrm{v}_\alpha / \mathrm{v}_{te}) \right]$$

$$G(\eta) = \mathrm{erf}(\eta) - (2\eta / \pi^{1/2}) \exp(-\eta^2) \ , \quad H(\eta) = -\frac{2\eta^3 \exp(-\eta^2)}{3\pi^{1/2} \ln(2^{1/2}\eta)} + \frac{\eta^4}{3 + \eta^4}$$

Peter & Meyer-ter-Vehn (1991)

- On **low-velocity** side of Bragg peak $\mathrm{v}_\alpha / \mathrm{v}_{te} < 1, \quad (T_e > 400 \ \ \mathrm{eV})$

$$\nu_s \to \frac{C_0 n_e}{T_e^{3/2}} \ln \Lambda \quad C_0 = 1.597 \times 10^{-9} \ \ \mathrm{eV}^{3/2} - \mathrm{cm}^3/\mathrm{s}$$

– **Plasma heating causes alpha particle range increase**

Model for α-particle heat deposition

– Slowing down time $0.002\ ns < \nu_s^{-1} < 0.66\ ns$ much shorter than times characterizing changes in liner temperature $t_{HS} \sim 74$ ns

$O \sim 1/t_{HS}$

– Alpha momentum equation simplifies $$\frac{\partial \mathrm{v}_\alpha}{\partial x} \cong -\nu_s - \cancel{\frac{\partial \mathrm{v}_\alpha}{\mathrm{v}_\alpha \partial t}}$$

– Alpha continuity equation simplifies $n_\alpha \mathrm{v}_\alpha = \Gamma_{\alpha 0}$

$$\Gamma_{\alpha 0} t_{HS} = \frac{f_{esc} f_b}{6 m_{DT}} (\rho R), \qquad f_b = \frac{\langle \sigma \mathrm{v} \rangle_{\mathrm{DT}} (\rho R)}{2 m_{DT} u_2} = \text{fuel burnup fraction}$$

Alpha heat flux $q_\alpha(x,t) = \Gamma_{\alpha 0}(t) \cdot (1/2) m_\alpha \mathrm{v}_\alpha^2(x,t)$

Alpha heat source $\nabla \cdot q_\alpha(x,t) = -\Gamma_{\alpha 0}(t) \cdot m_\alpha \mathrm{v}_\alpha(x,t) \nu_s(x,t)$

Dynamics of α-driven thermal wave

– Two coupled PDE's for v_α and T

$$\left.\frac{\partial v_\alpha}{\partial u}\right|_t \cong -\frac{C_0 \ln\Lambda}{T^{3/2}} \qquad 3k\left.\frac{\partial T}{\partial t}\right|_u \cong \Gamma_{\alpha 0} m_\alpha C_0 \ln\Lambda \frac{v_\alpha}{T^{3/2}} \qquad u = \int_0^x n\,dx$$

– Recast in non-dimensional variables (eV-cgs)

$$\boxed{V = v_\alpha / v_{\alpha 0}, \quad Z = T^{5/2}/H_0} \qquad \boxed{\tau = t/t_{HS}, \quad U = u/u_0,}$$

$$u_0 = v_{\alpha 0} H_0^{3/5} / C_0 \ln\Lambda \qquad H_0 = \frac{5C_0 \ln\Lambda}{18 m_{DT}} \frac{E_{\alpha 0}}{v_{\alpha 0}} f_{esc} f_b(\rho R)$$

3.5 MeV

$$\left.\frac{\partial V}{\partial U}\right|_\tau = -Z^{-3/5}$$

$$\left.\frac{\partial Z}{\partial \tau}\right|_U = V$$

$$\boxed{\begin{array}{l} V(U=0) \to 1, \quad \text{for} \quad \tau > 0 \\ Z(\tau = 0) \to 0, \quad \text{for} \quad U > 0 \end{array}}$$

Single ODE describes thermal wave

– Convert PDE's to ODE by means of transformed variables:

$$W = Z/U^{5/3} \qquad \xi = \tau/U^{5/3}$$

– Non-dimensional ODE

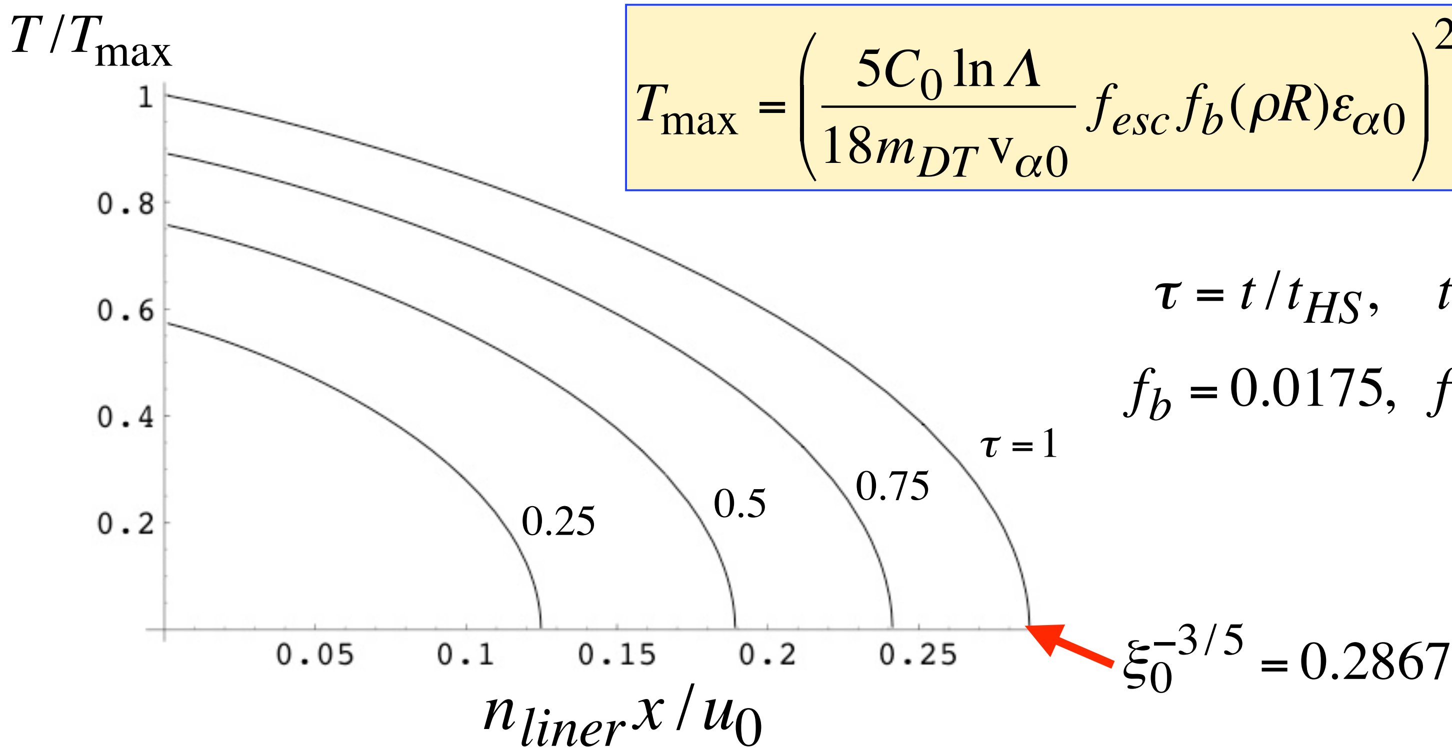
$$\frac{dV}{d\xi} = \frac{3}{5\xi W^{3/5}} \qquad \frac{dW}{d\xi} = V \quad \rightarrow \quad \frac{d^2W}{d\xi^2} - \frac{3}{5\xi W^{3/5}} = 0$$

$$\boxed{\xi \in (\xi_0, \infty) \quad \text{BCs}: W(\xi_0) = 0, \quad W'(\infty) = 1}$$

– Motion of the heat front $(W = T = 0)$ is $U_F = (\tau/\xi_0)^{3/5}$

– Numerical shooting scheme finds that $\xi_0 = 8.0209$

α-driven thermal wave profiles



$$T_{max} = \left(\frac{5C_0 \ln \Lambda}{18 m_{DT} v_{\alpha 0}} f_{esc} f_b (\rho R) \varepsilon_{\alpha 0} \right)^{2/5} \sim (\rho R)^{4/5}$$

$$\tau = t / t_{HS}, \quad t_{HS} \sim 100 \text{ ns}$$

$$f_b = 0.0175, \quad f_{esc} = 0.965$$

– Maximum Liner temperature $T_{max} = 1806 \text{ eV}$

– Maximum penetration depth $x_F(\tau = 1) = \dfrac{v_{\alpha 0} T_{max}^{3/2}}{n_{liner} C_0 \xi_0^{3/5}} = 45.4 \ \mu\text{m}$

Conclusion and future work needs

- The B-K-M ignition criteria tells us that a nominally-sized compressed magnetized target will have a pressure of ~ 100 Mbar.
- High M ~ 60 DT jets are needed to implode plasma liner to these high pressures. Possibly "macroparticle" jets instead of gas/plasma jets can achieve the needed high Mach numbers needed for PJMIF.
- Hot spot disassembly time is key issue. Need to complete the target decompression problem using the isobaric expansion model presented here. We plan to use the Hyades 1-D hydro code for future verification.
- We formulated a new analytical model for alpha-driven thermal wave, and discovered that the DT plasma liner does not easily "fireup" or burn, because of the low ρR: this presents itself as a very challenging problem.
- Plasma jets may have special advantages when used to compress B-fields to > 50 MG. Jets can potentially manipulate or collimate low-beta "magnetized winds".