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Thoughts from the front line:
Current issues in real-time graphics and areas where Geometric Algebra can help

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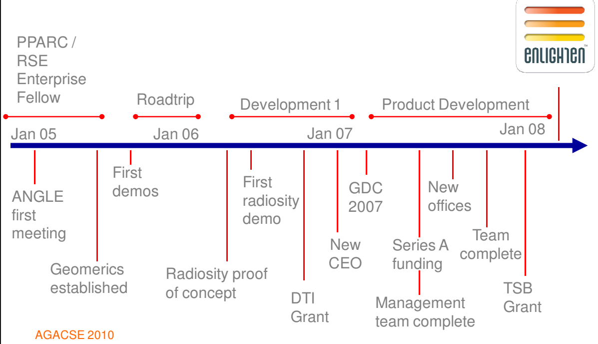
Introduction

- In 2005 I decided to take a break from academic research and formed Geomerics
- Looking for a new challenge
- Looking for other ways to win people over to GA

Geomerics

Geomerics Timeline



What we do

- Enlighten is a revolutionary lighting technology available for PC, Xbox 360 and PS3
- Enlighten is being used in games by EA Dice (Battlefield), CCP, Funcom, all pushing for the highest quality lighting and graphics

What we do

All images here are re-lit in real time at 30 fps, using Enlighten

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This talk

- Three areas which look interesting for future work on GA
  - Graphics
  - Discrete exterior calculus
  - Functional programming
- These are chosen not for their academic interest
- Areas where there is a real opportunity for GA to make an impact on a wider stage
- Also throwing in 3 puzzles / hobbyist topics

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Graphics

- An enormous topic, covered in depth in this conference
  - Radiosity
  - Global Illumination
  - Photon mapping
  - Ray tracing
  - Shadowing
  - Visibility
  - Ambient Occlusion
  - BRDF
  - Pre-computed radiance transfer

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The biggest problems?

- For games, the key problems are:
- Shadowing dynamic objects from direct lights
  - Shadow maps …
- Soft shadows from area lights
  - Convolution shadow maps …
- Dynamic object radiosity
  - Dynamic ambient occlusion, screen-space techniques …

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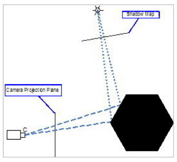
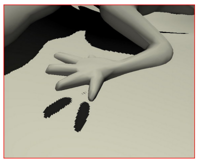
Shadows from Direct Lights

- A solved problem to some extent:
- Create a shadow map for each light source by rendering depth information
- Use this to look up whether or not a point is in shadow
- Gives rise to jaggies, aliasing artifacts …





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Refinements

- Many ways to improve basic shadow maps
  - Deform the geometry so that the shadow map better reflects the camera orientation (paraboidal SMs)
  - Introduce a 'cascade' of shadow maps to prevent horrible blocky shadows from distant sources
- But basic problems remain:
  - High quality results using shadow maps requires high resolution maps
  - These are slow and limit the number of direct light sources that can be used in real time
  - Not obvious how to filter

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What we would like!

- A solution based on rendering from light sources that:
  - Massaged the geometry in a useful way before rendering
  - Stored more than just depth (a plane, point + line …)
  - Ideally in a form that could be low resolution and amenable to filtering
  - Implemented as a simple screen-space step (potentially where filtering came in)
- Remember:
  - Will always trade off accuracy for speed
  - A nicely blurred approximate answer often works well

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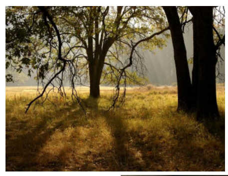
Soft Shadows

- Soft shadows are generated by area lights and are everywhere



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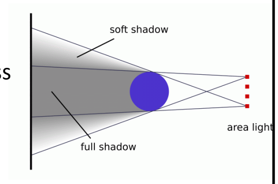
Soft Shadows

- This is a really hard problem!
- Can break into two aspects
  - Area lights
  - Full blown radiosity
- We have made good progress with radiosity
- But accurate area lights are unsolved for real-time graphics



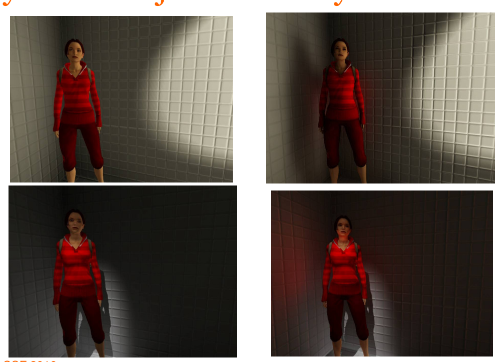
Area Lights

- The ability for an artist to dynamically place area lights with correct soft shadows would revolutionise work flow
- Any GA tricks?
  - Light sources as circles
  - Fractional / approximate visibility
  - Ability to blur simple shadows in an appropriate texture (see eg convolutions shadow maps)

Dynamic object radiosity

- In Enlighten we make a number of compromises:
- Radiosity is computed for static geometry
  - Involves an off-line pre-compute
- Light sources can move and change in real time
- Dynamic objects are lit by the radiosity
  - Appear to be rooted in their world
- But dynamic objects do not shadow the radiosity or bleed colour

Dynamic object radiosity



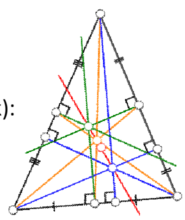
Dynamic Radiosity

- The big unsolved problem
- Need fast, approximate visibility updates
- Re-creation of form factors is less important
- Need to replace hierarchical data structures with something more malleable
- Incorporation of surface reflection properties
- Possibly screen-space type approach (caution!)
- Volume based or surface based?

Interlude 1

- Occasional frustrations with conformal GA
- Often want to drop back to affine or projective framework
- Somehow this is never easy
- Elementary pieces of geometry turn into lengthy un-inspired algebra
- Consider same basic triangle results:

Simple Triangle



- Circumcenter (green) is easy
- Centroid (orange) is (after some work):

$$G \wedge n = I(L_1 \times L_2 + L_2 \times L_3 + L_3 \times L_1)$$

- Tricky, but at least it is transparently symmetric
- Orthocentre (blue) is yet more difficult

$$H \wedge n = I \langle nL_1L_2L_3 \rangle_2$$

- Anyone got a simple proof of the Euler line?

Discrete exterior calculus

$$\int_M d\omega = \int_{\partial M} \omega.$$

- Work of Desbrun, Marsden, Hirani and others
- An attempt to develop a formal discrete theory of differential forms
  - Every continuous concept has a discrete analog
- We MUST develop a GA version of this theory
  - Otherwise the graphics community will be lost to exterior geometry for good!

Objects in DEC

- Discrete versions of each of
  - Differential forms
  - wedge product
  - Vector fields (and higher dimensions)
  - exterior derivative
  - Codifferential
  - Hodge star
  - Flat and sharp operators
  - Contraction
  - Lie derivative, Laplace – deRham operator, etc…

Foundations of DEC

- All defined in such a way that the main theorems are automatically true
- All very reminiscent of Hestenes and Sobczky's approach to the foundations of geometric calculus
- Chose your definitions carefully so that the key result is transparent

$$\oint L(dS) = \int \dot{L}(\dot{\nabla} \cdot dX)$$

Concepts in DEC

- 1-forms are numbers attached to edges
- 2-forms are numbers attached to planes
- And so on. All seems utterly obvious.
- But no useful notion of direction – the 1-form has to have the direction of the edge
- We need a notion of a vector field to discretise Maxwell equations (or anything else useful)
- At this point a dual manifold is introduced, based on either barycentric duals or centroids

Dual Manifold

- The dual manifold is the first point where things go awry
  - Vector fields look un-natural
- The wedge product is quite horrific

$$\langle \alpha^k \wedge \beta^l, \sigma^{k+l} \rangle = \frac{1}{(k+l)!} \sum_{\tau \in S_{k+l+1}} \mathrm{sign}(\tau) \frac{|\sigma^{k+l} \cap \star v_{\tau(k)}|}{|\sigma^{k+l}|} \alpha \smile \beta(\tau(\sigma^{k+l})) \,,$$

- It takes pages to prove the main results of the product
  - They should be obvious by definition
- From then on it all feels like a struggle

Hasn't this all been done?

- NO!
- Discrete exterior calculus is a recent development and actively ongoing
- Despite its difficulties it is comfortably the most complete and impressive theory we have
- With work, discrete analogs of most continuum results can be found
- We have no equivalent discrete theory within GA
  - This was not what Hestenes and Sobczyk were after

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Simple Example

- 2D vector derivative (aka the Cauchy-Riemann equations)

$$\nabla\psi = 0$$

- This is surprisingly hard to discretise
- Partly because the operator only propagates the part of the boundary data consistent with analyticity
- Can start from the Cauchy integral formula
- But then lose the ability to extend to curved surfaces
- And this is a problem of real practical significance!

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The right approach

- Some wild speculation:
  - The idea of defining scalars at points, 1-forms on lines, 2-forms on surfaces etc may not be the way to go
  - Instead, should we be defining a complete GA at discrete points?
  - Then need an operator for connecting adjacent algebras
  - This approach is more in the spirit of jet theory (see Olver: Equivalence, Invariants and Symmetry)
  - In jet theory differential equations are reduced to algebraic equations at a point, plus contact relations

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What is required

- A discrete vector manifold theory
- Based on the geometric product in the obvious way
- With a discrete vector derivative, and a discrete version of the fundamental theorem

$$\oint L(dS) = \int \dot{L}(\dot{\nabla}\cdot dX)$$

- The applications for such a theory would be vast
  - EMM, elasticity, re-meshing, numerical pdes …
- This is the problem I would be focussing all efforts on!

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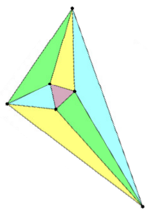
Interlude 2

- The Morley triangle, formed from angle tri-sectors
- Alain Connes has an algebraic proof of the result at www.alain**connes**.org/docs/**morley**.pdf
- This proof involves
  - Complex projective geometry
  - Rotations from reflections
  - Fixed points of twists
- A conformal GA version please!



Functional Programming

- Recently become interested in the functional programming language Haskell
- Will talk through its main features, and why it looks perfect for GA
- Functional languages are currently generating considerable interest:
  - Haskell, ML, ocaml …
  - Microsoft developing F#, and supporting Haskell

Haskell is a functional language

- Key objects are functions that take in arguments and return values (or functions)
- Mathematically this is simple, but far removed from modern object-oriented programming
- Means we give up on mutable objects
  - Never change a variable
  - Always create a new variable, then let garbage collector free up memory
- Focussing on functions gives compiler much better chance of parallelising code

Haskell is a ‘pure’ language

- Pure functions have no I/O side effects
- Un-used results can be discarded
- Compiler can use tricks like memoization
- Evaluations are thread-safe
  - Good for parallelisation again
- Pure functional code can have various compiler optimisations applied
- In practice, Haskell code is mostly pure with a small amount of I/O

Haskell is strongly typed

- Haskell contains a powerful type system
- Everything has a type
  - Functions map types to types, eg Int -> Int
- All code is checked for type integrity before compilation
- A lot of bugs are caught this way!
- Ties in with the concept that GA multivectors can remove ambiguity
  - Are 4 numbers are quaterion, a projective vector …
  - Tracking blades removes all ambiguity

Haskell has recursive functions

- In functional programming traditional for .. from ..to loops are replaced by other constructs
- Recursive functions are particularly useful

```
qsort [] = []
qsort (x:xs) = qsort (filter (< x) xs) ++ [x] ++ qsort (filter (>= x) xs)
```

- Use of recursion can shrink code dramatically
- Driving recursive definitions of functions is a powerful pattern matching framework
- Again, for mathematicians this is all natural!

Haskell is a higher-order language

- Functions can take functions as arguments
- Functions can return functions as results
- Under the hood, functions are curried
  - Concept due to Haskell Curry
- All functions take in one parameter, and return a function / parameter
- Great for mapping functions to lists, etc

Haskell is 'lazy'

- A defining property of Haskell is that function evaluation is lazy
- Functions are only evaluated when the result is needed elsewhere
  - Avoids unnecessary computation
  - Ensures programmes terminate where possible
  - Encourages good programming style
  - Allows for infinite lists
- Eg can define the 'infinite' list of all integers, and at a later date ask for the 10^{th} element

Haskell and GA

- This combination of properties makes Haskell uniquely suitable for GA
- Define blade and multivector data types

```
type GaBlade = (Float, GaBasis)
type GaMulti = [GaBlade]
```

- Says that a multivector is a list of blades
- Define a geometric product of blades, trivial to build up everything else
- Write code that mirrors hand-written algebra

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Laziness and GA

- Laziness is the key to Haskell's suitability

$$\langle AB \rangle_0$$

- Lazy evaluation ensures that only terms of grade zero are actually computed
- Can avoid vast amounts of hand optimisation this way
- Haskell will never be as fast as hand optimised C++ or intrinsics
- But it is far easier to write and debug, and promised much on multicore devices

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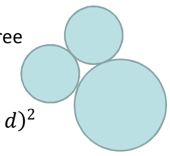
One final problem

- A fun problem from Martin Gardner's mathematical recreations
- Given three kissing circles:
  - Can always find two circles to kiss all three
- Inverse radii satisfy



$$a^2 + b^2 + c^2 + d^2 = \frac{1}{2}(a + b + c + d)^2$$

- A neat problem in conformal GA!

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Conclusions

- Many interesting open problems to explore with GA
- Opportunities to make a real difference in areas that will get GA widely noticed
  - Graphics, discrete theory, functional programming
- Plenty of drive from industry in setting the problem space, if people are interested
- And please come and talk to me if you make serious progress in any of these areas!

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