

**The ECB's Directorate General Statistics releases euro area yield curves every TARGET working day at 12 noon Central European Summer Time (or Central European Time).**

### **General description of ECB yield curve methodology**

A yield curve (which is known as the term structure of interest rates) represents the relationship between market remuneration (interest) rates and the remaining time to maturity of debt securities. The information content of a yield curve reflects the asset pricing process on financial markets. When buying and selling bonds, investor include their expectations of future inflation, real interest rates and their assessment of risks. An investor calculates the price of a bond by discounting the expected future cash flows.<sup>1</sup>

The ECB estimates zero-coupon yield curves for the euro area and derives forward and par yield curves. A zero coupon bond is a bond that pays no coupon and is sold at a discount from its face value. The zero coupon curve represents the yield to maturity of hypothetical zero coupon bonds, since they are not directly observable in the market for a wide range of maturities. They must therefore be estimated from existing zero coupon bonds and fixed coupon bond prices or yields. The forward curve shows the short-term (instantaneous) interest rate for future periods implied in the yield curve. The par yield reflects hypothetical yields, namely the interest rates the bonds would have yielded had they been priced at par (i.e. at 100).

### **Data availability**

Daily yield curves are now available as from 6 September 2004, and are calculated and released on a daily basis according to the TARGET calendar.

### **Data source**

- Bond and price information is provided by **EuroMTS Ltd.**  
[www.euromts-ltd.com](http://www.euromts-ltd.com)
- The ratings are provided by **Fitch Ratings.**  
[www.fitchratings.com](http://www.fitchratings.com)

### **Selection of bonds**

The following criteria are applied when selecting bonds:

- Only bonds issued in euro by euro area central government (European System of Accounts 1995: sector code 'S.1311') are selected.

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<sup>1</sup> Please see the section "Price and yield calculations" for more information.

- Only bonds with an outstanding amount of at least € billion are included.
- Bonds with special features, including ones with specific institutional arrangements, are excluded.<sup>2</sup>
- Only fixed coupon bonds with a finite maturity and zero coupon bonds are selected, including STRIPS<sup>3</sup>. Variable coupon bonds, including inflation-linked bonds, and perpetual bonds, are not included.
- Only actively traded central government bonds with a maximum bid-ask spread per quote of three basis points are selected. The prices/yields are those at close of market on the reference day.
- In order to reflect a sufficient market depth, the residual maturity brackets have been fixed as ranging from three months up to and including 30 years of residual maturity.

An outlier removal mechanism is applied to bonds that have passed the above selection criteria. Bonds are removed if their yields deviate by more than twice the standard deviation from the average yield in the same maturity bracket. Afterwards, the same procedure is repeated.

### **Two credit risk yield curves**

The spot, forward and par yield curves, and their corresponding time series, are calculated using two different datasets reflecting different credit default risks.

- One sample contains “AAA-rated” euro area central government bonds, i.e. debt securities with the most favourable credit risk assessment.
- The second dataset contains (AAA-rated and other) euro area central government bonds.

Please refer to the yield curve technical notes file for further technical details, attached as a PDF file on this webpage.

### **Price and yield calculations**

- The estimation of the curve is done by means of a modelling algorithm that minimises the sum of the quadratic difference between the yields that can be computed from the curve and the yields actually measured. Yields are calculated,

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<sup>2</sup> Such as Brady bonds, convertible bonds, bonds with non-regular structures (bonds with embedded options – calls, puts and step-up/step-down bonds).

<sup>3</sup> Separate Trading of Registered Interest and Principal Securities.

where applicable, according to the International Securities Market Association formula for a fully paid fixed coupon bond with an assumed single redemption date.<sup>4</sup>

For a fully paid fixed coupon bond paying h times interest per year with an assumed single redemption date, the redemption yield formula can be written as follows:

$$P = v^{f_1} \cdot \left( k + \sum_{i=1}^{n-1} \frac{g}{h} \cdot v^i \right) + \left( C + \frac{g}{h} \cdot f_2 \right) \cdot v^{n+f_1+f_2-1}$$

where

P = gross price of the bond at the close of the previous business day

g = annual coupon interest rate as a percentage

k = first/next coupon payment as a percentage

h = number of coupon payments a year

n = number of coupon payments to redemption

f<sub>1</sub> = fraction of the number of calendar days from value date to the first/next interest payment

f<sub>2</sub> = fraction of the number of calendar days from the last normal coupon date to redemption

C = redemption value

v = discount factor.  $v = 1/(1+y)$ , where y = required redemption yield compounded h times per annum.

- The continuous method is used to compound interest rates.<sup>5</sup>
- Calculations used to derive cash flows are based on settlement dates according to individual bond market practices.
- The day count calculations are performed using market practices<sup>6</sup> such as Actual/360, Actual/Actual or are based on 30E/360 depending on the type of bonds and the residency of the issuer.
- No adjustments for tax or coupon effects are made.
- The previous TARGET day parameter values are used as the starting values for the next TARGET day calculations.

## Model description

The methodology used, the Svensson model<sup>7</sup>, is a parametric model which specifies a functional form for the **spot rate**,  $z(TTM)$  as follows:

<sup>4</sup> See International Securities Market Association (1992), "Formulae for Yield and Other Calculations".

<sup>5</sup> Continuous compounding implies assuming that the compounding period is infinitely small.

<sup>6</sup> "An Introduction to Bond Markets" by Moorad Choudhry, Wiley, 2006, p. 36.

$$z(TTM) = \beta_0 + \beta_1 \left[ \frac{1 - e^{\left(\frac{-TTM}{\tau_1}\right)}}{TTM / \tau_1} \right] + \beta_2 \left[ \frac{1 - e^{\left(\frac{-TTM}{\tau_1}\right)}}{TTM / \tau_1} - e^{\left(\frac{-TTM}{\tau_1}\right)} \right] + \beta_3 \left[ \frac{1 - e^{\left(\frac{-TTM}{\tau_2}\right)}}{TTM / \tau_2} - e^{\left(\frac{-TTM}{\tau_2}\right)} \right]$$

where  $TTM$  = term to maturity, and  $\beta_i$  and  $\tau_j$  are the parameters to be estimated.

The relationship between the spot rate and the forward rate can be written as follows:

$$z(TTM) = \frac{1}{TTM} \int_t^{TTM} f(s) ds .$$

The first derivative of this expression with respect to  $TTM$  yields the relation

$$f(TTM) = z(TTM) + TTM \frac{dz(TTM)}{dTMM} .$$

Therefore the instantaneous **forward curve** is equal to:

$$f(TTM) = \beta_0 + \beta_1 e^{\left(\frac{-TTM}{\tau_1}\right)} + \beta_2 \frac{TTM}{\tau_1} e^{\left(\frac{-TTM}{\tau_1}\right)} + \beta_3 \frac{TTM}{\tau_2} e^{\left(\frac{-TTM}{\tau_2}\right)}$$

An instantaneous forward rate contracted at time  $t$  for duration  $d$  measures the short-term interest rate that investors can lock-in at time  $t$  in order to be received at  $t + d$ .

For the **par yield curve**, the continuously compounded par yield can be estimated according to Anderson et. al.<sup>8</sup> as follows:

$$y_{par}(TTM) = \frac{C \cdot (1 - disc(TTM))}{\int_0^{TTM} disc(t) dt}$$

The discrete relationship can be derived by approximating the integral of the discount function as follows:

$$y_{par}(TTM_i) = \frac{C \cdot (1 - disc(TTM_i))}{\sum_{j=1}^i disc(TTM_j) \cdot \Delta TTM}$$

where

- $TTM_i$  = term to maturity
- $\Delta TTM$  = constant interval between terms to maturity
- $C$  = redemption value (principal value)
- $disc(TTM_i)$  = discount function evaluated at  $TTM_i$
- $TTM_i$  =  $i \cdot \Delta TTM$ , and  $i$  denotes a positive integer

<sup>7</sup> Svensson, L. E., 1994, "Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994", Centre for Economic Policy Research, Discussion Paper No 1051.

<sup>8</sup> "Estimating and Interpreting the Yield Curve", Nicola Anderson, Francis Breedon, Mark Deacon, Andrew Derry, Gareth Murphy (1996).