UNIT 11 MEASURES OF INEQUALITY

Structure

11.0 OBJECTIVES

After going through this unit, you will be able to:

- explain the various positive measures of inequality;
- discuss the computational device for construction of Gini index; and
- describe the normative measures of inequality propounded by Dalton, Atiknson, Sen, Kakwani and Theil.

11.1 INTRODUCTION

Improvement in well being of the poor has been one of the important goals of economic policy and to a significant extent it is determined by the growth and distribution of its income. Distribution patterns have an important bearing on the relationship between average income and poverty levels. Extreme inequalities are economically wasteful. Further, income inequalities also interact with other life-chance inequalities. Hence reducing inequalities has become priority of public policy.

It therefore, becomes pertinent to measure income inequalities. Various measures have been developed over a period of time to study the level of inequalities in different situations. Broadly these measures can be put under two categories (i) positive measures, and (ii) normative measures. The measures which capture the inequality of income without value judgment about social well-being are known as positive measures. Range, quartile range, standard deviation, Gini ratio, Lorenz curve etc., are positive measures of inequality. On the other hand, the measures that essentially involve value judgement about social welfare are called normative measures. The index propounded by Dalton, Atkinson, Sen, Theil and Kakwani are normative measures. We shall discuss all these measures one by one in this unit. Gini Coefficient of Inequality and Lorenz Curve will receive particular attention, as they are very popular in literature.

11.2 POSITIVE MEASURES

If all values in a distribution are not equal, which means that there is dispersion in the distribution, there exists inequality in the distribution. If a measure is developed to capture this non-equality in values without giving explicit consideration to its consequences with respect to social well-being or economic significance in a particular context, the measure is known as **positive**. It means that the measure is bothering about the fact whether it is measuring inequality of lengths of iron nails or incomes of wage earners in a village. Nevertheless, many of them are standard statistical measures and their social implications and/or social consequences can still be studied. Some of these measures can be arrived from normative approach as well.

Let us consider an income distribution x_i i=1,2,...,N over N persons and with mean income μ. Let the relative share of total income with person i be designated as q_i, which is naturally given by $x_i / N\mu$. The cumulative share of total income with the persons not having more than x_i income can be given as Q_i. However, when we have an income having frequency more than one, the proportion of people with income x_i can be denoted by p_i and the cumulative proportion of people with income no less than x_i as P_i. In this case, obviously the relative share of total income would be given by $f_i x_i / N\mu$ where f_i denotes the frequency of occurrence of income x_i and $N=\sum f_i$. It will not be necessary to use two different subscripts for distinguishing the two cases for the purpose as the context would make it clear whether subscript i stands for a person with income x_i or for the group of persons with income x_i.

We have implicitly assumed that the data are arranged in the increasing (non-decreasing) order by magnitude of income so that symbolic representation is easy. The same could be accomplished by decreasing (non-increasing) order.

We intend to cover important measures along with their variants in this Unit. We shall also, albeit briefly, discuss their properties and weaknesses.

To recapitulate, we recount what we said above in a formal way:

$$q_i = \frac{x_i}{N\mu} \text{ or } \frac{f_i x_i}{N\mu}$$

$$Q_i = \sum_{i-1}^{i} q_i,$$

$$p_i = \frac{1}{N} \text{ or } \frac{f_i}{N}$$

$$P_i = \sum_{i-1}^{i} p_i.$$

11.2.1 Relative Range

A measure of relative dispersion can be taken a measure of inequality. It is defined as the relative range by

$$RR_1 = \frac{Max_i x_i - Min_i x_i}{\mu} \qquad \text{(RR.1)}$$

that is, the relative difference between the highest income and the lowest income. If income is equally distributed, then RR_1=0 and if one person received all the income, then RR_l is maximum. If one wants to make the index lie in the interval between 0 and 1, one can define it as

$$RR_2 = \frac{Max_i x_i - Min_i x_i}{N\mu} \qquad \text{(RR.2)}$$

which means it is the gap between the maximum share and the minimum share. That is,

$$RR_2 = Max_i q_i - Min_i q_i \qquad \text{(RR.3)}$$

Though Cowell has suggested division of range by $Min_i x_i$, which does not serve, in our view, any purpose. Two other normalization or standardization procedures that make it unit-free and contain it in (0,1) interval are suggested below:

$$RR_3 = \frac{Max_i x_i - Min_i \bar{x}_i}{Max_i x_i} \qquad \text{(RR.4)}$$

and

$$RR_4 = \frac{Max_i x_i - Min_i x_i}{Max_i x_i + Min_i x_i} \qquad \text{(RR.5)}$$

The basic weaknesses of these range-based measures are that they are not based on all values and therefore they do not reflect the change in inequality if there is any transfer of income between two non-extreme recipients.

Instead of considering extreme values at either end, which may not be even known, some scholars have toyed with the idea of the ratio between the mean income of the highest fractile (percentile, quintile or decile) and that of the lowest counterpart.

They term it as the extreme disparity ratio (EDR). Naturally, this ratio is not contained in the interval (0.1). This ratio is independent of μ as well. The measure will not reflect the transfer of income that does not involve the extreme fractiles.

11.2.2 Relative Inter-Quartile Range

Sometimes, extremism of the relative range is sought to be moderated by restricting the distribution between the 10th and 90th percentile or sometimes to interquartile range. Bowley (1937) suggested relative quartile deviation as the index of inequality:

$$B = \frac{x^{q^3} - x^{q^1}}{x^{q^3} + x^{q^1}} \tag{B.1}$$

where x^{q3} represents the income level which divides the population in r and (4-r) quartiles. B is zero for degenerate distribution where everybody has the same income and unity if the lowest 75 percent people have no income at all.

Though the extremes are moderated in comparison to the measure of range, it has an obvious weakness that the measure takes into account only 50 percent of the distribution. Further, a transfer of income between two persons without causing either or both of them cross x^{q1} or x^{q3} would not change the measured level of inequality. Thus, the index suffers from all weaknesses of the earlier proposals except that of extremism. Its highest value reaches when the lowest 75 percent people do not possess any income.

A variant of this measure is inter-quartile ratio, which can be defined as the 75th percentile (3rd quartile) income minus 25th percentile (1st quartile) income divided by the median (x^{q2}) income.

11.2.3 Relative Standard Variation

The standard deviation divided by the mean can be used as one measure of dispersion. It is:

$$RSD = \frac{\sigma}{\mu} \tag{RSD.1}$$

where σ and μ are standard deviation and mean of the distribution.

It can be equivalently defined as the standard deviation of relative incomes. Using definition of σ, one can find out that it lies in the interval of 0 and $(N\text{-}1)^{1/2}$, not in (0,1). The highest value depends on the size of distribution.

Since the measure uses all values, any transfer of income would be reflected in the measure. However, it should be noted that the measure is equi-sensitive to transfers at all levels. Whether a given amount d is transferred between x_j=Rs.400 and x_k=Rs.500, or between x_j=Rs.10,000 and x_k=Rs.10,100, the change in RSD is exactly the same.

We may finally note that the square of RSD is also quite often used as another measure of inequality, which is known as the coefficient of variance. Quite a few scholars suggest use of variance as a measure of inequality but we have not considered it here primarily because it is not unit-free. We think that an inequality measure must be unit-free.

11.2.4 Standard Deviation of Logarithms

One way of attaching greater importance to transfers at lower end (as required by Sen) is to consider some transformation of incomes. This transformation can easily be attained by considering the logarithms that stagger the income at lower levels.

This measure is defined in either of the following two ways:

$$SDL_1 = \left(\frac{1}{N} \sum_{i=1}^{N} (\log \mu - \log x_i)^2 \right)^{1/2} \quad \text{(SDL.1)}$$

$$SDL_2 = \left(\frac{1}{N} \sum_{i=1}^{N} (\log \hat{\mu} - \log x_i)^2 \right)^{1/2} \quad \text{(SDL.2)}$$

where μ and $\hat{\mu}$ are the arithmetic and geometric means respectively. While standard statistical literature prefers use of geometric mean the more common practice in literature on income inequality is one of using arithmetic means.

Cowell (1995) prefers to define these in terms of variance and calls the square of SDL_1 as the logarithm variance (V_l) and the square of SDL_2 as the variance of logarithms (V_2). Name of the second is clear from the expression but that of the first is derived from the fact that ($\log x - \log \bar{x}$) could be written as $\log (x/\bar{x})$. One can see that V_1 is equal to V_2 plus log ($\hat{\mu}$ /μ).

As these measures are in terms of ratios of incomes, any proportionate change in incomes would leave the magnitude of inequality unchanged when measured by these indices. But, unfortunately, a transfer from a richer person to a poorer person may raise the magnitude of inequality, particularly if the poorer person has income more than 2.72 times the mean of the distribution.

While the lower limit, irrespective of formula, is zero when everybody has the same income, the upper limit depends on the size of distribution and approaches infinity when N is large and when everybody except the richest, receives income equal to one unit (as zero is inadmissible in logarithmic transformation.) Further, if we face grouped data, it is convenient to use μ in place of $\hat{\mu}$ and μ_i in place of x_i.

The variance of logarithms is however, decomposable. It is a property that is being given emphasis of late. It can be shown that V_2 is the sum of between group component and within group component, latter being population-weighted sum of within-group V_2's.

11.2.5 Champernowne Index

Champernowne (1973) makes use of the idea of geometric mean. It is a well known fact of an unequal distribution that its geometric mean is smaller than the arithmetic mean. The additive inverse of the ratio of geometric mean to arithmetic mean can duly be considered as an index of inequality. Formally, the index could be written as:

$$CII = 1 - \frac{\hat{\mu}}{\mu} \quad \text{(CII.1)}$$

where μ and $\hat{\mu}$ are, as stated earlier, arithmetic and geometric means of the income distribution. It is easy to see that its value is bound between 0 and 1.

One can obviously think of another measure where geometric mean is replaced by harmonic means.

These measures are sensitive to transfer to income and change is greater when the transfer takes place at lower end of the distribution. They are sensitive to transfer of income between two persons. One can try it by replacing x_j and x_k by (x_j – d) and (x_k + d) respectively and finding out the direction of the change. Or, one can use differential calculus.

The trouble with these indices is that they cannot be defined when any of the income is zero.

11.2.6 Hirschman-Herfindahl Indices

These indices were developed in the course of studying the commodity concentration in trade by Hirschman (1945) and in characterizing market monopoly in industry by Herfindahl (1950). Later, they were more used in capturing autonomy and dependence of units in a federation.

If each unit is a class in itself, p_i =1/N, i=1,2...,N. Then concentration could be captured through use of q_i's. As the sum of q_i's is always 1, Hirschman devised a measure which would capture the inequality among them. He proposed square root of the sum of squares of shares q_i i=1,2..., N. That is,

$$H_1 = \left(N \sum_{i=1}^{N} q_i^2 \right)^{1/2} \tag{H.1}$$

which could be generalized as

$$H_1^{*} = \left(N \sum_{i=1}^{N} q_i^a \right)^{1/a}, a > 1$$

Herfindahl devised a very similar measure, which has been more popular than the original (H.1). This is just the sum of share squares:

$$H_2 = \sum_{i=1}^{N} q_i^2 \tag{H.3}$$

$$H_2^{*} = \sum_{i=1}^{N} q_i^a, a \geq 1. \tag{H.4}$$

It is clear that, besides inequality among the shares, the value of these measures depends on N–the fewness or largeness of the number of units. For $N = 2$, it has been suggested that ($1/N$) could be subtracted from (H.3)

$$H_3 = \sum_{i=1}^{N} q_i^2 - \frac{1}{N} \tag{H.5}$$

The minimum value of H_3 is zero. But it serves no great purpose. When N=2, for q_1=0.99 and q_2=0.01, while H_2=0.98, H_3=0.48. H_2 scores definitely better than H_3 in characterizing the scene of monopoly.

11.2.7 Kolm's Index

Let there be N incomes such that $N= nm$ where n is the number of different incomes and each income has m recipients. The number of equal pairs with a given income would be $m(m-1)/2$ and total number of equal pairs would be $n.m(m-1)/2$. Total number of all pairs would obviously be $N(N-1)/2$-$nm(nm-1)/2$. One can think of an 'equality' index in terms of $nm(m-1)/nm(nm-1)=(m-1)/(N-1)$. The inequality index could then be constructed by subtracting it from 1: $1-(m-1)/(N-1)-(N-m)/(N-1)=m(n-1)/(nm-1)=(nm-m)/(nm-1)$. In case, income x_1 has f_i recipients, the measure is:

$$K = 1 - \frac{\Sigma f_i^2 - N}{N(N-1)} = \frac{N^2 - \Sigma f_i^2}{N(N-1)} \tag{K.1}$$

The purpose of developing this curiosum due to Kolm (1996) is just to make one feel that there could be a variety of simple ways to approach the issue of measurement of inequality.

Check Your Progress 1

1) Define relative range measures of inequality. List out relative merits.

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2) Discuss how relative inter-quartile range is better than relative range.

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3) What is the relative mean deviation? If a transfer of income is between two persons both having income lower than the mean, will it change the magnitude of this index?

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4) Compare the two versions of standard logarithmic deviations.

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5) What is import of Champernowne Index?

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6) What is Hirfindahl index? What are its areas of application?

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7) What is the message from the Kolm's index? Calculate the Kolm index for a distribution, which frequency 5 for value Rs.5 lakh and frequency 5 with value Rs.10 lakh and therefore total size 10 and the arithmetic mean 7.5.

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11.3 GINI INDEX

This coefficient of concentration, as it is usually called, owes to an Italian statistician by the name of Corrado Gini (1912). Modern practice is to call it just Gini. This index in its origin is positive. There are a number of ways in which this coefficient can be expressed. There are also a number of ways in which it can be interpreted. People have also derived it as a measure of inequality under plausible axioms in welfare theoretic framework. The index satisfies good many axioms proposed in literature for an index of inequality.

First, we shall discuss those definitions and expressions, which can be derived as a measure of dispersion. Besides giving its expressions for its frequency data for grouped observations, we shall discuss its welfare theoretic interpretations.

11.3.1 Gini as a Measure of Dispersion

Recall that mean deviation and standard deviation, which are measures of dispersion, seek the deviation from arithmetic mean. Also recall that one of the logarithmic measure sought deviation from the geometric mean. However, one may ask why to seek dispersion in terms of deviations from any mean? Why not compare all pairs and seek the differences. In order to consider positive values of differences, we either take modal values of deviations before averaging (mean deviation) or sum the squares of deviations and take root of the mean of the squared differences.

Corrado Gini (1912) proposed to consider all the differences, that is all pairs of values. By contrast, the range measure of dispersion considers only one pair of highest value and lowest value. When x_i and x_j denote ith and jth incomes respectively and $i, j = 1, 2, \ldots, N$, we can see that the aggregate of absolute differences is given by

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\left|x_{i}-x_{j}\right| \tag{G. 1}$$

and because total number of differences is N^2, the mean of absolute differences can obviously be written as

$$\frac{1}{N^2}\sum_{i=1}^{N}\sum_{i=1}^{N}\left|x_{i}-x_{j}\right| \tag{G. 2}$$

where the differences with the self have also been counted and the difference of x_i with x_j is treated as separate from that of x_j and x_i though numerically they are the same. This is also said to be the case with replacement. Expression (G 2) ranges between 0 and 2μ.

In the case of without replacement, the sum is obviously to be divided by $N(N\text{-}1)$ as there are N deviations with the self. It is not difficult to see that the numerical value of the sum remains the same.

In order to make it serve as a measure of inequality, (G 2) can be divided by μ to produce what can be called coefficient of mean difference (CMD):

$$CMD = \frac{1}{N^2\mu}\sum_{i=1}^{N}\sum_{i=1}^{N}\left|x_i - x_j\right| \tag{G. 3}$$

CMD fulfils the idea of scale independence. However, the expression (G. 3) ranges between 0 when everybody has the same income and $2[=2N/(N\text{-}1)]$ when only one person has all the income. In order to make it satisfy the interval (0,1), we can further divide it by 2. The result is Gini coefficient of concentration or Gini index of inequality:

$$G = \frac{1}{2N^2\mu}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|x_i - x_j\right| \tag{G. 4}$$

or

$$G = \frac{1}{N^2\mu}\sum_{i=1}^{N}\sum_{x_j \le x_i}^{N}(x_i - x_j) \tag{G. 5}$$

conceived as an aggregate of only positive differences, though normalized by the number of all differences and the mean income. Kendall and Stuart define this as 'one half of the average value of absolute differences between all pairs of incomes divided by the mean income'.

The index can also be defined in terms of population proportions and income shares. If the income-share of individual i is denoted by q_i, that is,

$$q_i = \frac{x_i}{N\mu}, \tag{G.6}$$

then then the expression (G. 4) can also be written as

$$G = \frac{1}{2N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|q_i - q_j\right| \tag{G. 7}$$

In the case of a discrete distribution, each individual constitutes (1/*N*)th of the population, that is,

$$p_i = \frac{1}{N}. \tag{G. 8}$$

Therefore one can also write (G. 7) as

$$G = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\left|p_j q_i - p_i q_j\right|. \tag{G. 9}$$

An obvious question is: why not $\left|p_i q_i - p_j q_j\right|$ in the expression (G. 9)? For understanding this, let us consider the Gini coefficient for the groups.

Let μ_r and μ_s denote mean incomes of r^{th} and s^{th} groups (say, families) respectively and r,s=1,2…g. Then, Gini for the groups can be defined as

$$G = \frac{1}{2N^2\mu}\sum_{r=1}^{g}\sum_{s=1}^{Ng}\left|\mu_r - \mu_s\right| f_r f_s \tag{G. 10}$$

where f_r and f_s are frequencies of the groups r and s respectively. This can obviously be written as

$$G = \frac{1}{2}\sum_{r=1}^{g}\sum_{s=1}^{g}\left|\frac{\mu_r}{\mu} - \frac{\mu_s}{\mu}\right| p_r p_s \tag{G. 12}$$

where

$$p_r = \frac{f_r}{N} \text{ and } p_s = \frac{f_s}{N} \tag{G. 12}$$

Now, let us note the share of total income with the group r:

$$q_r = \frac{\mu_r f_r}{N\mu} = p_r\frac{\mu_r}{\mu} \tag{G. 13}$$

Then, the expression (G.12) can be written in either of the two ways (G. 14) and (G. 15)

$$G = \frac{1}{2}\sum_{r=1}^{g}\sum_{s=1}^{g}\left|\frac{q_r}{p_r} - \frac{q_s}{p_s}\right| \tag{G. 14}$$

or

$$G = \frac{1}{2}\sum_{r=1}^{g}\sum_{s=1}^{g}\left|p_s q_r - p_r q_s\right| \tag{G. 15}$$

It is easy to see that $g=N$ and $p_s=p_r=(1/N)$ when f_r and f_s are all equal to 1.

In statistics literature we emphasize frequency aspects, in economics literature we find it convenient, expression-wise, to treat each individual with single income though there is no bar for $x_i=x_j$.

11.3.2 Simple Computational Device

Two years after giving his index to terms of relative mean differences, Gini (1914) showed that the index is exactly equal to one minus twice the area under Lorenz curve (to be discussed later). That is,

$$G = 1 - 2\overline{A} \qquad \text{(LG.1)}$$

where $\overline{A}$ is the area under the Lorenz curve, as shown in Fig. 11.1

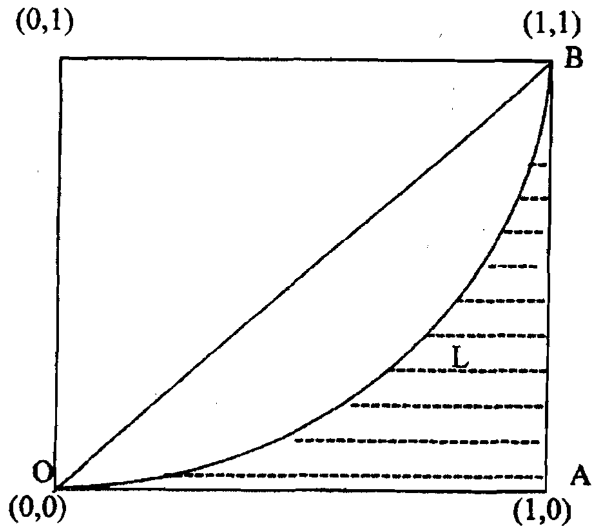

Fig. 11.1

However, normally, we do not specify and estimate a smooth relationship between Q and P. Instead, we obtain the curve by plotting cumulative proportions of people in classes and cumulative shares of their incomes, where classes are arranged according to increasing per capita income values:

$$\mu_1 \leq \mu_2 \leq \mu_3 \leq .... \leq \mu_r \leq ... \leq \mu_g . \qquad \text{(LG4)}$$

strictly speaking, equality sign is useless in this presentation.(We have written it in deference to Theil (1967). Plotting P_r and Q_r, we obtain the Fig. 11.2. We can see that the area below the Lorenz curve can be conceived as consisting of several trapeziums. A trapezium could be seen as consisting of a rectangle and a triangle. Summing the areas of all trapeziums (say g in number), we can get the area $\overline{A}$. Substituting it in (LG.1), we get the following expression for computing Gini coefficient G:

$$G = 1 - \sum_{r=1}^{g} (P_r - P_{r-1})(Q_r + Q_{r-1}) \qquad \text{(LG.5)}$$

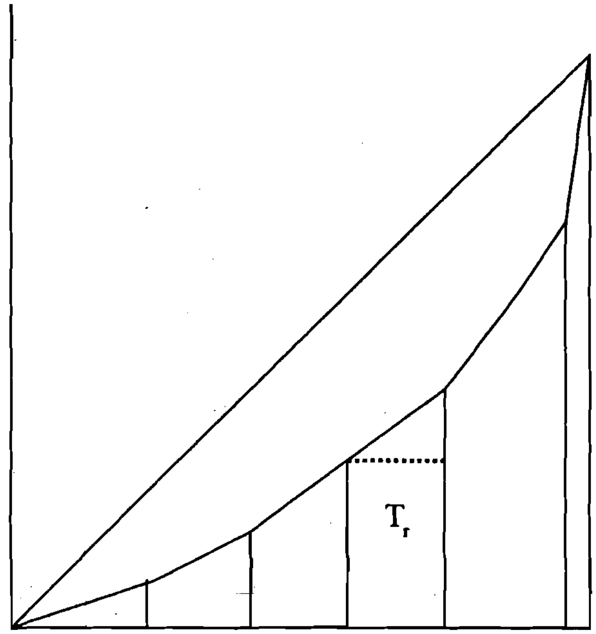

Fig. 11.2

Check Your Progress 2

1) Define Gini ratio.

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2) Show the difference in approach in defining Gini from other measures of dispersion.

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3) Write the expression for computing Gini.

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4) How can you compute Gini ratio?

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11.4 LORENZ CURVE

Lorenz curve is a powerful geometrical device to compare two situations of distribution with regard to their level of inequality. Devised some hundred years ago by Max O. Lorenz (1905) to measure concentration of wealth, it is still very widely used in empirical studies on inequality. The device can be used for comparing inequality of distribution of any measurable entity such as income, wealth (land, capital), consumption, expenditure on an item (say, food or education), etc. The distribution may be over persons or households. But the device can also be used to measure inequality of tax collection or expenditure incurred by states or federal grants received by different states. We can compare pre-tax and post-tax distributions in order to study the efficacy of instrument of tax.

Lorenz (1905) studied a number of methods then in use to gauge the level of, or change in the level of, inequality. Most of these measures used fixed-income classes in data tabulation and made inter-temporal comparison, employed changes in percentage of recipients of class incomes in each of the fixed-income classes or movement of persons from one class to another and so on. Finding them unsatisfactory, he comes to the conclusion that changes in income and changes in population both have to be simultaneously taken into account and in a manner that 'fixed-ness' of income classes gets neutralized.

In fact, this measurement relates to comparison and in most cases, we are in a position to compare but there are situations of non-comparability. However, a few of inequality measures that are capable of numerical representation in terms of a scalar number, and therefore called summary measure, are found to be based on the Lorenz curve.

It may be pointed out that the curve was independently introduced by Gini (1914). It is therefore, quite often referred to as Lorenz-Gini curve as well. We shall, however, stick to more common usage and call it Lorenz Curve.

11.4.1 Geometrical Definition

The Lorenz curve of concentration of incomes is the relationship between the cumulative proportions of recipients, usually plotted on the abscissa, and the corresponding cumulative shares of total income with the recipients, usually plotted on the ordinate. If population proportions and income shares of class j are denoted by p_j and q_j and cumulative proportions and shares upto class i, by P_i and Q_i then

$$P_i = \sum_{j=1}^{i} p_j, \qquad 1 \geq p_j \geq 0 \qquad \text{(GD.1)}$$

and

$$Q_i = \sum_{j=1}^{i} q_j, \qquad 1 \geq q_j \geq 0 \qquad \text{(GD.2)}$$

The relationship between P_i and Q_i is given by the curve

$$Q_i = L(P_i), \qquad 1 \geq P_i \geq 0, \quad 1 \geq Q_i \geq 0 \qquad \text{(GD.3)}$$

and the point on the curve by (P_i, Q_i). Naturally, the first point is (0,0) and the last one on the curve, (1,1). It is also clear that $Q_i \leq P_i$ i=1, 2,..., N-1 if there are N classes of incomes. It means no point will make an angel of more than 45° with the abscissa at the origin. Then, one can be sure that the Lorenz curve lies in the lower triangle of Lorenz Box of the unit square. See Fig. 11.3 in which OLB shows the Lorenz curve (Fig. 11.3).

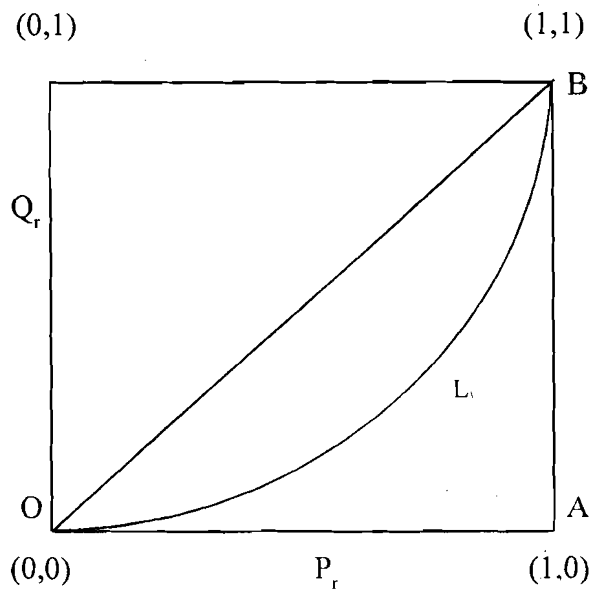

Fig. 11.3

11.4.2 Properties of the Lorenz Curve

Now it is easy to see that the extreme case of perfect equality is given by the diagonal OB which represents $P_i = Q_i$, $i=1, 2, \ldots, N$. The other extreme of perfect inequality will be given by a curve OAB. The diagonal OB is often designate as the egalitarian line or line of equality. The other diagonal CA is known as the alternative diagonal and is useful to study the symmetry of the curve. The line OAB with sharp kink of 90° at A can be said to be the line of perfect inequality. See Fig. 11.4.

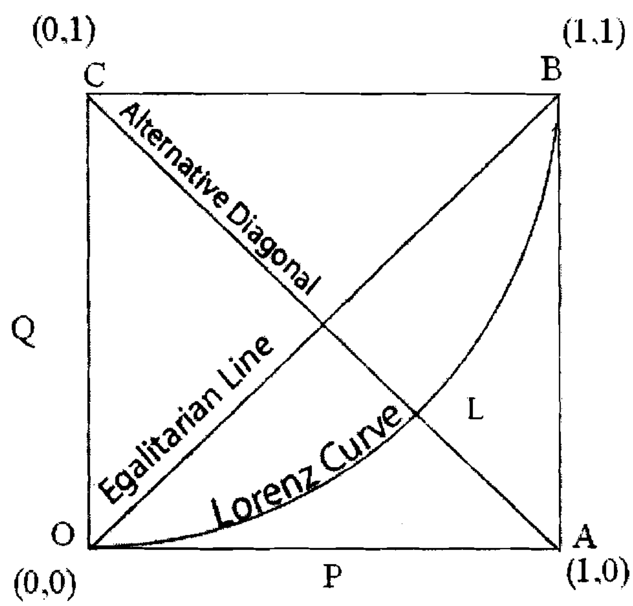

Fig. 11.4

We can finally note the following properties:

i) $1 \geq p_i \geq 0; 1 \geq q_i \geq 0, \; i = 1,2,\ldots,N$

ii) $1 \geq P_i \geq 0; 1 \geq Q_i \geq 0, i = 1,2,\ldots,N-1$

iii) $P_0 = Q_0 = 0; \; P_N = Q_N = 1$

iv) $P_i \geq Q_i, i = 1,2,\ldots, N-1$

By drawing a Lorenz Curve, we can know whether a given distribution is equal or unequal. We do not yet know how much unequal a given distribution is. When we draw two or more Lorenz Curves, we can compare the distributions as regards their levels of inequality. The curve closer to the diagonal of equality has lower level of inequality than the one away from it (Fig. 11.5). But we do not know yet the level of inequality. And even this comparison is possible only when the curves do not intersect (Fig. 11.6).

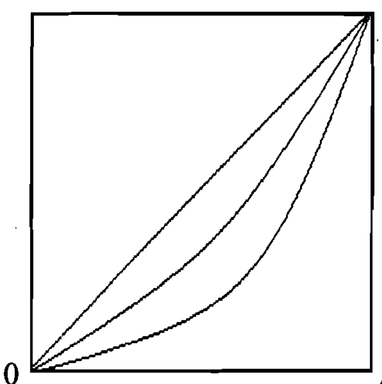

Fig. 11.5

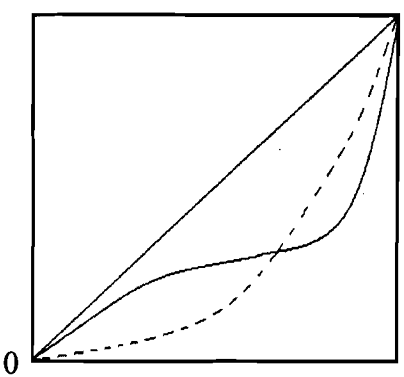

Fig. 11.6

However, we can devise some measures, which are based on the Lorenz curve. In case the Lorenz curves intersect, reducing the distributions into single real number is the only option. So we shall discuss only two such proposals.

11.4.3 A Measure Based on Area

We have noted that if Lorenz curve coincides with the diagonal of equality, the inequality is nil and if Lorenz curve coincides with the two sides of the square, the inequality is full. In the case of non-intersecting Lorenz curves, it is clear that the curve closer to the diagonal of equality will circumscribe smaller area between itself and the diagonal of equality than the one, which is farther. Which is what it should be. We can therefore devise a measure of inequality by dividing the area OLB by the area of triangle OAB, which is the maximum possible area between the diagonal of equality and Lorenz curve. As the area of OAB is (1/2), the measure turns out to be twice the area between the diagonal of equality and the Lorenz curve. In other words, Lorenz coefficient of concentration (LCC) is:

$$LCC = \frac{Area\,OLB}{\Delta OAB} = 2Area\,OLB$$

Since this turns out to be exactly equal to Gini coefficient, we are not elaborating it any further.

11.4.4 A Measure Based on Length

This is a measure proposed by Kakwani (1980). The length of the Lorenz curve, denoted by l, cannot fall below $\sqrt{2}$, which is the length of the egalitarian line and cannot exceed 2, which is the sum of the lengths of the two arms of the lower triangle. In order to produce a measure with the minimum value 0 and the maximum value 1, following exercise can be suggested:

	Minimum	Actual	Maximum
Length of the Curve	$\sqrt{2}$	ℓ	2
Length of the Curve - $\sqrt{2}$	0	$\ell - \sqrt{2}$	$2-\sqrt{2}$
$\dfrac{\text{Length of the Curve} - \sqrt{2}}{\text{Maximum length} - \sqrt{2}}$	0	$\dfrac{\ell - \sqrt{2}}{2 - \sqrt{2}}$	1

So this measure is clearly:

$LK = (\ell - \sqrt{2}) / (2 - \sqrt{2})$

In both the cases, one can draw actual graphs and actually measure the area and the length and calculate the indices for level of inequality. Those who wish to carry out a more sophisticated exercise will have to estimate smooth functions.

Check Your Progress 3

1) Enumerate the properties of Lorenz curve.

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2) When will comparison between two Lorenz curve fail to compare inequality in two distribution?

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3) What is the relationship between Lorenz curve and Gini coefficient.

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4) What is Kakwani's measure of inequality, which is based on the Lorenz curve.

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11.5 NORMATIVE MEASURES

The measures that essentially involve judgement about values through specification of social welfare function are called normative measures. The arguments of this nature were first advanced by Dalton, pretty eight decades ago in 1920 for constructing what are today called normative measures of inequality.

Reacting to an observation by Pearsons (1909) that 'the statistical problem before the economists in determining upon a measure of inequality in the distribution of wealth is identical with that of the biologist in determining upon a measure of the inequality in the distribution of any physical characteristic', Dalton (1920) pointed out that 'economist is interested, not in distribution as such, but in effects of the distribution upon the distribution (and total amount) of economic welfare which may be derived from income'. The objection to great inequality of income, he further points out, is due to the resulting loss of potential economic welfare that could accrue to people in the absence of it.

Yet, it has to be noted that inequality though defined in terms of economic welfare, has to be measured in terms of income. This idea due to Dalton has been conceded by subsequent contributions. Using the notion of social welfare function in construction gives rise to **normative** measures of inequality.

It may be instructive to remember that the discussion would revolve around three issues:

1) the relationship between income of a person and his welfare;

2) the relationship between personal income-welfare functions; and

3) the relationship between personal welfare and social welfare.

It may be noted that utility is the word mostly used for personal welfare whereas for welfare of society the phrase social utility is rarely used.

There are two major indices in this category: Dalton's index and Atkinson's index. In Atkinson's index a new idea is introduced and that is of equally distributed equivalent income. Actually there are two sub-approaches within normative approach. One is Dalton's and the other is Atkinson's. While in Dalton's approach present social welfare is compared with that could be obtained by equally distributing the total income, in Atkinson's approach the present level of income is compared with that of equally distributed level of income, which generates the present level of social welfare. Sen has generalised the Atkinson's index. Theil's index based on information theory could be suggested here only to sort of complete the unit.

11.5.1 Dalton Index

For each individual, Dalton assumes, marginal economic welfare diminishes as income increases. It means income-welfare function

$$U_i = U_i(x_i), i = 1,2,...N \qquad \text{(D.1)}$$

(where U_i is welfare of person i possessing income x_i

is concave, suggesting that $(\partial U_i/x_i) > 0$ but $(\partial^2 U_i/x_i^2) < 0$. Dalton further assumes that economic welfare of different persons is additive. Thus, in his scheme, social welfare is a simple aggregation of personal welfares. In other words, social welfare W is given by

$$W = \sum_{i=1}^{N} U_i(x_i) \qquad \text{(D.2)}$$

He further assumes that the relation of income to economic welfare is the same for all members of the community. That is,

$$U_i = U(x_i), \text{i=1,2,...,N} \qquad \text{(D.3)}$$

In that case, the relation (D.2) can be expressed as

$$W = \sum_{i=1}^{N} U(x_i) \tag{D.4}$$

which makes it clear that whosoever gains in welfare, the addition to the social welfare is the same. For any given level of social welfare, any distribution of welfare among the members of the society is permissible. However, one must remember that the relation of individual income to their welfares is concave. Therefore, transfer of income from A to B will not lead to symmetric change in welfares of those two persons involved in the transaction. The result is some impact on W the measure of social welfare.

From Fig. 11.7, we may compare the situation when two individuals, both possessing the same relation, have two different income levels, with that when they have the same (mean) income. We may note that the sum of the welfare of person 1(BB') and the welfare of person 2(DD') is less than the twice of CC' which is the level of welfare enjoyed by both the persons when they have equal income. It is easy to see that the loss suffered by person 2, that is D'E, is overcompensated by the gained by person 1, which is C'F.

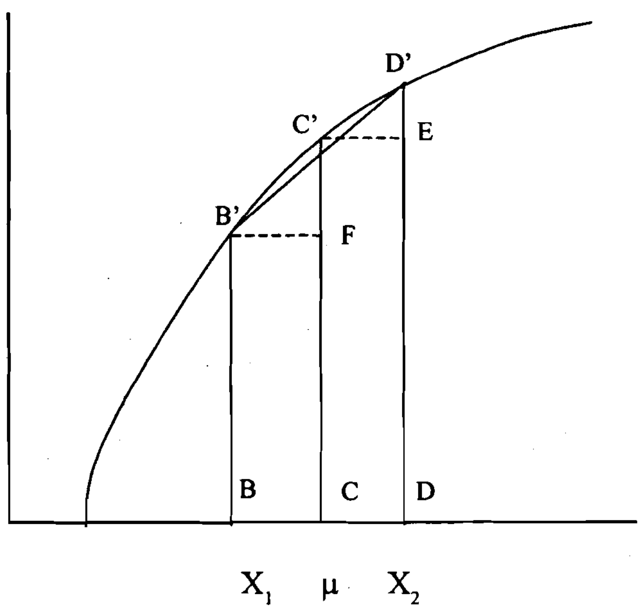

Fig. 11.7

This demonstrates that, under assumptions by Dalton, an equal distribution is preferable to an unequal one for a given amount of total income from the viewpoint of social welfare. In fact, for a given total of income, the economic welfare of the society will be maximum when all incomes are equal. The inequality of anv given distribution may therefore be defined as

$$D_1 = \frac{\sum_{i=1}^{N} U(\mu)}{\sum_{i=1}^{N} U(x_i)} = \frac{NU(\mu)}{\sum_{i=1}^{N} U(x_i)} \tag{D.7}$$

which is equal to unity for an equal distribution and greater than unity for an unequal one. It may therefore, be preferred to define the Dalton's index as

$$D_2 = \frac{NU(\mu)}{\sum_{i=1}^{N} U(x_i)} \tag{D.8}$$

which is obviously zero for an equal distribution. How large can it be? It will depend on the values of $U(0)$, $U(\mu)$ and $U(N\mu)$ when N and μ are given, not necessarily 1. Later writers have therefore, preferred to define Dalton's index in the following form, which inverts the arguments of D_2 subtract it from 1:

$$D = 1 - \frac{\sum_{i=1}^{N} U(x_i)}{NU(\mu)} = 1 - \frac{\overline{U}}{U(\mu)} \tag{D.9}$$

It looks as if the index is contained in the interval (0,1). However, there are many valid concave functions where it may not hold true. For example, if we have $U(x_i) = \log x_i$, then D=1- $\{\log \hat{\mu} / \log \mu \}$. Given the fact that $\hat{\mu} < \mu$, D would turn out to be a negative number for $\mu<1$. And μ could be less than 1 as x can be measured in any unit. It would be the same case $U(x_i) = 1/x_i$.

However, in order to obtain numerical magnitude, it is not sufficient to define the index. Dalton (1920) points out that though defined in terms of economic welfare, inequality has to be measured in terms of income. Then, no unique measure of inequality will emerge. It will verily depend on the particular functional relationship assumed. Dalton himself considered two such functions for the purpose of illustration. The first is related to Bernaulli's hypothesis. It holds that proportionate additions to income (in excess of that required for bare subsistence-poverty line) make equal additions to personal welfare, That is,

$$dU_i = \frac{dx_i}{x_i} \, or \, U_i = \log x_i + c_i \tag{D.10}$$

Under the assumption that every person has the same functional relationship, the Dalton's index can be given as

$$D = 1 - \frac{\log \hat{\mu} + c}{\log \mu + c} \tag{D.11}$$

where $\hat{\mu}$ is the geometric mean of personal incomes. The other formulation he discusses is given as

$$dU_i = \frac{dx_i}{x_1^2} \, or \, U_i = c - \frac{1}{x_i} \tag{D.12}$$

where c is the maximum welfare one can obtain when $x \to \infty$. Dalton's index in this case would turn out to be:

$$D = 1 - \frac{c - (1/\tilde{\mu})}{C - (1/\mu)} \tag{D.13}$$

where $\tilde{\mu}$ is the harmonic mean.

11.5.2 Atkinson Index

Atkinson (1970) objects to Dalton's measure because D is not invariant with respect to positive linear transformations of personal income-welfare functions. This was pointed out by Dalton himself but he could not resolve it.

Atkinson seeks to redefine the index in such a way that measurement would be invariant with respect to permitted transformations of welfare numbers. Atkinson does it through devising what he calls 'equally distributed equivalent income'. Both the distributions, the original and the new one, are supposed to yield the same level of welfare.

In order to make the concept clear, we put a few artifacts along with the actual distribution. Let us first note that for an actually distributed income vector x_i, i=1,2...,N (call it vector a), there is only one equally distributed income vector with each element equal to μ (call it vector b) but there are a number of equivalently distributed, vectors (call them vectors c). See Chart 1. An equivalent income distribution is one, which has the same level of welfare as that of currently given distribution. However, one of these equivalent distributions (vectors c) is 'equal' as well. This is called equally distributed equivalent income vector, shown as vector (d) in the Chart 1. As μ is the mean level of current distribution, μ^* may be used for designating the level of equally distributed equivalent income.

CHART-I

Vector (a) Actually distributed income vector	x_1,	$x_2, \ldots,$	$x_i, \ldots,$	x_N.
Vector (b) Equally distributed income vector	μ,	μ, ...,	μ, ...,	μ.
Vector (c) Equivalently distributed income vector	x_1^*,	$x_2^*, \ldots,$	$x_i^*, \ldots,$	x_N^*
Vector (d) Equally distributed equivalent income vector	μ*,	μ*, ...,	μ*, ...,	μ*

It should be obvious that $W_b \geq W_a = W_c$ *and* $W_c = W_d$. *Then*, $W_a = W_d$. W represents social welfare with respective distributions of income vectors. It is clear that $\mu \geq \mu^*$. μ^* is defined by the additive social welfare function having symmetric individual utility functions such as:

$$U(\mu) = \frac{1}{N}\sum_{i=1}^{N} U(x_i) \tag{A.1}$$

or equivalently

$$\mu^* = U^{-1}\left[\frac{1}{N}\sum_{i=1}^{N} U(x_i)\right] \tag{A.2}$$

The index due to Atkinson is then defined as the additive inverse of the ratio of equivalent mean income to actual mean income:

$$A = 1 - \frac{\mu^*}{\mu} \tag{A.3}$$

which is said to lie between zero (complete equality) and 1 (complete inequality).

We can see that A cannot be 1 unless μ^* is zero, which is an impossibility for any distribution with $\mu>0$. If complete inequality is defined as the situation when only one person grabs all the income, we can see that

$$A = 1 - \frac{\mu^*_m}{\mu} \tag{A.4}$$

where,

$$NU(\mu^*) = \sum_{i=1}^{N} U(x_i)$$

and

$$NU(\mu^*_m) = (N-1)U(0) + U(N\mu).$$

This index is not scale independent unless some restriction is imposed on the relationship U. If this requirement has to be met, Atkinson points out, we may have to have the following form

$$U(x_i) = \begin{cases} \alpha + \dfrac{\beta}{1-\varepsilon} x_i^{1-\varepsilon}, & \varepsilon \neq 1 \\ \log_e x_i, & \varepsilon = 1 \end{cases} \tag{A.5}$$

Note that we need $\varepsilon = 0$ for ensuring concavity and $\varepsilon > 0$ for ensuring strict concavity. This is a homothetic function and is linear when $\varepsilon = 0$. We may note that ε cannot exceed 1 as in that case the varying component assumes inverse relationship. α is usually negative so that $U(x_i)$ is not positive for $x_i = 0$. Otherwise, when $x_i = 0$, Ui = α which means that welfare is positive even when income is zero. This is generally not acceptable. On the contrary, a negative α would be more acceptable. When $\varepsilon = 1$, α is infinitely large and negative.

Since ε can be zero, Atkinson's requirement is not strict concavity. Sen (1973) has a question. He asks to consider two distributions (0,10) and (5,5) along with

$$U(x_i) = \alpha + \beta(x_i) \tag{A.6}$$

Then, he points out the level of social welfare would be ($2\alpha + 10\beta$) whatever the distribution. μ^* would be 5 in both the cases. μ is of course 5. The measure of inequality A is therefore zero. So, both the distributions are ethically equal. This is obviously absurd. Therefore, the relation (A.5) should be defined with the restriction $\varepsilon > 0$. We should also note that (A.4) is an iso-elastic marginal utility function.

ε is the inequality-aversion parameter and has close resemblance with risk-aversion parameter. Atkinson proposed to draw on the parallel formally with the problem of measuring risk. He finds his concept of equally distributed equivalent income very closely resembles with risk-premium or certainty equivalent income as used in the theory of decision-making under uncertainty.

In case we seek to introduce this restrictive personal income-welfare function along with simple aggregation of individual welfares to constitute the social welfare into the inequality measure A, we will have

$$A = 1 - \left[\frac{1}{N} \sum_{i=1}^{N} \left(\frac{x_i}{\mu} \right)^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \quad , \varepsilon \neq 1 \tag{A.7}$$

The question is now narrowed down to choosing ε. As ε rises, more weight is attached to transfers at the lower end of the distribution and less to that at the top. When ε rises, (A.7) assumes the function $\min_i (x_i)$, which only takes account of transfers to the very lowest income group (and is therefore not strictly concave). When $\varepsilon = 0, U_i$ is linear. As a consequence, A is always zero. This means A has no descriptive content at all. When $\varepsilon \to 1, A$ turns out to be

$$A = 1 - \prod_{i=1}^{N} \left(\frac{x_i}{\mu}\right)^{1/N} = 1 - \frac{\hat{\mu}}{\mu} \tag{A.8}$$

which is the same as Champernowne index (CII.1). For values of ε between 0 and 1, the expressions may not be very neat. Parameter ε is often chosen to be ½ or 1/3 or 2/3.

11.5.3 Sen Index

There are people who feel rather strongly that the social valuation of the welfare of individuals should depend crucially on the incomes of their neighbours too. Then, why should society add simply individual welfares? One may also question the assumption of one welfare function for all individuals. If we do so, we should go for broad social welfare function such as

$$W = W(x_i, x_2, \ldots, x_N) \tag{S.1}$$

which is just symmetric, quasi-concave and increasing in individual income levels. Then, a more general normative measure of inequality can be defined by devising the concept of 'generalized equally distributed equivalent income'. This is obviously the level of per capita income x^* which, if shared by all, would produce the same level of W as is generated by the present actual distribution. That is,

$$x^* = x \big| W(x^*, x^*, .., x^*) = W(x_1 x_2, \ldots, x_N) \tag{S.2}$$

Under the assumption that (S.1) is quasi-concave, $x^* \leq \mu$ for every distribution of income. The index S would then be

$$S = 1 - \frac{x^*}{\mu} \tag{S.3}$$

which is but a generalized version of A. If utilitarian framework is employed, then S and A turn out to be indistinguishable.

These measures, it may be noted, clearly suggest that there exists a redistribution equivalent of growth so far as the concern is about raising the welfare.

11.5.4 Theil Entropy Index

Theil (1967) poses a question: Does information theory supply us with a 'natural' measure of income inequality among N individuals, which is based on income shares? He answers: Yes. Here is a short introduction.

Let us start with income share of individual i:

$$q_i = \frac{x_i}{N_\mu} > o \quad such\,that \sum_{i=1}^{N} q_i = 1 \tag{T.1}$$

When $x_i = \mu$, i=1, 2,..., N, that is, when distribution is equal, we have

$$q_i = \frac{1}{N} \quad i = 1,\ 2,...N \tag{T.2}$$

We have complete inequality when some $x_i = N\mu$ and $x_j = 0, j \neq i$. It implies that $q_i = 1$ for some i and $q_j = 0, i \neq j$.

In information theory, one way of defining entropy of probabilities p_i is

$$H = \sum_{i=1}^{N} p_i \log \frac{1}{P_i} \tag{T.2}$$

Replacing probabilities by shares, we have

$$H = \sum_{i=1}^{N} q_i \log \frac{1}{q_i}. \tag{T.3}$$

which can be taken as a measure of equality. For the situation of complete equality, we can see that H is equal to $log N$ and for that of complete inequality H is zero. We can therefore define Theil index T as

$$T = \log N \sum_{i=1}^{N} q_i \log \frac{1}{q_i}.$$

$$= \sum_{i=1}^{N} q_i \log N - \sum_{i=1}^{N} q_i \log \frac{1}{q_i}$$

$$= \sum_{i=1}^{N} q_i \log N.q_i \tag{T.4}$$

This measure is motivated by the notion of entropy in information theory. But one can see that it can be interpreted in the traditional normative framework with

$$U_i = q_i \log \frac{1}{q_i} \tag{T.5}$$

and

$$W = \sum_{i=1}^{N} U_i(q_i). \tag{T.6}$$

We may note that (T.5) depends on x_i as well as on μ along with N and U that it is concave with respect to x_i.

While the lower limit of T is zero, its upper limit $log\ N$ increases as the number of individuals increases. To many people, it is objectionable. However, Theil (1967) chooses to defend it. When society consists of two crore persons and one grabs all and when society consists of two persons and one grabs all, cannot have the same level of inequality. The former case is equivalent to the situation in which one crore out of two crore people have nothing and the other one crore have equal income. Maximum value for two-person society is log 2, and that for two crore-person society is 7 log 2. Some researchers still insist that the measure should be normalized by dividing it by log N.

11.5.5 Kakwani Index

From (A.2), we can see that

$$M^* = \mu \ (1\text{-}A) \tag{K.1}$$

The underlying social welfare function i therefore

$$W - NU(\mu^*) + NU\{\mu \ (1\text{-}A)\}, \tag{K.2}$$

which is an increasing function of u and decreasing function of A. Another function with these properties which can be though of is:

$$W - NU\left(\frac{\mu}{1+K}\right)$$

Where K is inequality measure due to Kakwani. Under additive social welfare function,

$$\mu^* = \frac{\mu}{1+K} \tag{K.4}$$

and thereore

$$K = \frac{\mu}{\mu^*} - 1, \tag{K.5}$$

which is identical to A in all respects except in its sensitivity to mean income vis-a-vis inequality. It may be noted (1+K) is reciprocal to (1-A).

Check Your Progress 4

1) What is social welfare function, according to Dalton?

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2) Discuss Dalton index of inequality.

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3) Give the logic behind Atkinson index.

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4) How is Sen's index distinct from Atkinson's index?

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5) Discuss Theil's entropy index of inequality.

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11.6 LET US SUM UP

Owing to adverse impacts of economic inequality both on poverty and on growth, reducing inequality has been a priority of public policy. Various measures of income inequality can be put under two categories: positive measures and normative measures. The positive measures capture the inequality of income without value judgements. These include range quartile range, standard deviation, Gini ratio, etc. Lorenz curve belongs to this category. It measures inequality to the extent of comparing two distributions. The measures, which essentially involve value judgements about social welfare, are normative measures. These include indices propounded by Dalton, Atkinson, Sen, and Theil.

Easy comprehension and easy computation, range of variation and amount of information needed the desirable properties of the measures of economic inequality. In order to judge the efficacy of an inequality index, several axiom have been set up. However, these axioms have been relegated to the Appendix.

11.7 KEY WORDS

Coefficient of Mean Difference : Mean of all pair-wise differences divided by the mean of differences has been termed as coefficient of mean difference in this text.

Dispersion : The fact that values of a variable are not all the same is known as dispersion. The spread or scattering of the distribution is measured by a measure of dispersion.

Extreme Disparity Ratio : The ratio of the highest value to the lowest value is known as extreme disparity ratio.

Normative Measures of Inequality : Measures of inequality, which are articulated through the explicit incorporation of social welfare function or social welfare considerations, are known as the normative measures of inequality.

Positive Measures of Inequality : Measures of inequality, which are based in statistical properties of distribution, are known as the positive measures of inequality.

Relative Standard Deviation : Standard deviation of a distribution divided by its mean is known as Relative Standard Deviation.

Standard Logarithmic Deviation : Standard deviation of logarithms of values in a distribution is known as Standard Logarithmic Deviation. Though logically the deviations of logarithmic values should be taken from the logarithm of geometric mean but at times they are taken from logarithm of arithmetic mean. Therefore, there are two versions.

Social Welfare Function : An index of social well-being, often articulated as a function of individual utilities or individual incomes or individual consumption baskets with or without labour disposition, or individual rankings of potential state of affairs.

11.8 EXCERCISES

1) Following is the adapted distribution of monthly per capita expenditure (in Rs.) in rural India in the 60th round of the NSS over January to June 2004:

Class	50-225	225-255	255-300	300-340	340-380	380-420	420-470	470-525	525-615	615-755	755-950	950-1200
Avg. Exp.	100	240	280	325	365	405	450	500	580	700	850	1100
Percentage of Persons	2.4	2.7	6.4	8.3	9.6	9.6	10.8	10.0	12.2	12.6	6.7	8.8

Calculate as many positive measures as you can.

2) Following is the distribution data of operational holdings from agriculture census 1976-77:

	Holding	Marginal	Small	Semi-medium	Medium	Large	All
Definition	Unit	0.0-1.0 Ha	1.0-2.0 Ha	2.0-4.0 Ha	4.0-10.0 Ha	10.0 Ha	Above
Number	in '000	44523	14728	11666	8212	2440	81569
Area	in '000 Ha	17509	20905	32428	49628	42673	163343

Draw Lorenz curve and compute Gini ratio.

11.9 SOME USEFUL BOOKS

Chaubey, P. K. (2004), *Inequality: Issues and Indices*, Kanishka Publishers, Distributors, Delhi.

Cowell, Frank A. (1995), *Measuring Inequality*, Prentice Hall/Harvester Wheatsheaf, London.

Sen, A.K. (1997) *On Economic Inequality*, Oxford University Press, Oxford. with Annexe by James E. Foster.

11.10 ANSWERS OR HINTS TO CHECK YOUR PROGRESS

Check Your Progress 1

1) Range is the difference between the maximum and minimum value. With a view to ensure that the index of inequality based on this measure of dispersion is unit-free and/or is confined in the interval of (0,1), various ways of normalizations could be considered. See Section 11.2.1.

2) In the relative range only extreme values are considered, which may not be representatives of the distribution. It is like comparing the poorest (who may be a few) with the richest (who may be one). Inter-quartile measures take into account the middlemost distribution with 50 percent recipients.

3) In the mean deviation, all values in a way are considered. The mean of absolute deviations is divided by the mean of the distribution to yield relative mean deviation. See Section 11.2.3. Since the sum of deviations on one side of the mean will not change with the transfer of income contemplated, the magnitude of the index would not change.

4) See Section 11.2.4. In one version the deviations are taken with respect to logarithm of geometric mean and in the other with respect to that of arithmetic mean. With some mathematical manipulation, one can find out that former is smaller than the latter by square of the difference between the logarithms of geometric mean and arithmetic mean.

5) It is a straight application of the fact that geometric mean of a distribution is smaller than its arithmetic mean. Of course, when the values are greater than 1.

6) Hirfindahl index is sum of squares of the shares with each recipient, which of course varies with the number of recipients. Equal distribution of shares between two recipients will yield a value of 0.5 and between three recipients, 0.333. It is therefore, more used as a measure of concentration.

7) The message is that one can try on one's own to devise new methods. For second part, the answer is 5/9.

Check Your Progress 2

1) Gini ratio is one half of the average value of absolute differences between all pairs, including with self, of values divided by the arithmetic mean.

2) The basic difference lies in the fact that in articulating gini index all differences are considered while in others either few differences are considered or deviations from arithmetic/geometric means are considered.

3) See section 11.3.2.

4) By writing out in a table, columns for class intervals or values, frequencies, class total values, cumulative frequencies, cumulative total values, cumulative proportions and cumulative shares in respect of each class. For using expression (LG.5), consecutive moving differences (or sums) of proportions and consecutive moving sums (or differences) need to be computed in two additional columns. MS-Excel will do well.

Check Your Progress 3

1) Look at the Fig. 11.4 and Section 11.4.2. and write the properties in language.

2) When the two Lorenz curve will intersect, it will not be possible to say on balance which distribution is more unequal. In fact, one section in that case will be more unequal and the other section less unequal in distribution 1 in comparison to their counterparts in the distribution 2.

3) The value of Gini coefficient is equal to twice the areas inscribed between line of equality and the Lorenz curve.

4) Kakwani's measure of inequality is normalized length of the Lorenz curve.

Check Your Progress 4

1) Dalton's social welfare function is a simple aggregation of welfare (utility) functions of the individuals constituting the society. In addition, all individuals are supposed to have the same income utility function.

2) Section 11.5.1. Write Dalton's proposal and its modern version. Also point out that though conceived in terms of utility, Dalton held that the index has to be measured in terms of income only.

3) An inequality measure should not change with linear transformation of personal utility function. Since Dalton's index does not respect this property, Atkinson is not happy. He therefore, devises a new artifact called 'equally distributed equivalent income' and suggests a new utility function called iso-elastic marginal utility function.

4) Sen's index is different from the Atkinson's index in one respect that he opts for a social welfare function, which has individual incomes as its arguments and is symmetric and quasi-concave. See Section 11.5.3.

5) See section 11.5.4.

Appendix 11.1

AXIOMS OF INEQUALITY MEASURES

For any statistical measure, some of the desirable properties that are described in standard textbooks are (i) simplicity of comprehension, (ii) ease of computation, (iii) range of variation, and (iv) amount of information needed. However, we discuss below only those properties, which are peculiar to the measures of economic inequality.

We are often faced with situations where we have to compare two distributions with the help of an index with regard to their level of inequality. The two distributions may belong to two different countries at a point of time, to a country at two points of time, or to two situations – say, one before tax and the other after tax or before and after interpersonal transfers etc.

People have set up some intuitively appealing properties in order to judge the efficacy of an inequality index. The first set of properties was given as 'principles' by Dalton (1920). Today, in literature, they are known as axioms. We propose to discuss some common axioms. It may be pointed out at the outset that these axioms almost ignore the question whether inequality is an issue, which matters more (or less) in an affluent society or in a poor one. The whole discussion will assume that all incomes are positive though we know for sure in case of business failure or crop failure, incomes can well be negative or zero.

1) **Axiom of Scale Independence**

If there are two distributions of equal size such (N) that each element of one distribution is a multiple θ of the corresponding element of the other distribution, i.e.,

$$x_1^2 = \theta x_1^1 \qquad\qquad i=1,2,\ldots,N,$$

then the numerical magnitudes of inequalities of both the distributions should be the same, i.e.,

$$I(x_1^1, x_2^1, \ldots, x_N^1) = I(x_1^2, x_2^2, \ldots, x_N^2)$$

where the inequality measure I is shown as a function of the distribution

$$(x_1, x_2 \; \ldots, \; x_N).$$

Obviously, it also satisfies the idea that the level of inequality should not change when the scale of measurement changes, say, from rupees to paise or bushels to quintals.

It does also mean that equal proportionate additions to all incomes would not change the level of inequality for

$$x_i(1 + \lambda) = \theta x_i \qquad\qquad i=1,2,\ldots,N,$$

The proportionate addition could even be negative. Thus, it is a question of shares in the cake, not the size of the cake. It is very obvious that an inequality measure is defined in terms of shares s_i because a proportionate change in all incomes leaves the shares unchanged.

However, this axiom goes against Dalton's principle of proportionate additions to income, which stated that equal proportionate additions (subtractions) should diminish (increase) the level of inequality. Perhaps, Dalton could not see that, equal proportionate addition is theoretically, equivalent to change in the scale of measurement. It should so happen in the case of a measure of relative dispersion is obvious enough.

The axiom covers the cases of proportionate taxation/subsidies. It may be noted that such additions do not change individual (class) shares of total income and, therefore, the Lorenz curve remains unchanged. All measures based on the Lorenz curve shall therefore satisfy this axiom.

It should not mean that change in the size of cake is immaterial. In the social welfare, size of cake and distribution of cake both matter. It is only in a limited context of measurement of inequality that this property is considered desirable.

Lorenz (1997) mentions an objection raised against this axiom in terms of non-proportionate increase in well-being of different income holders, which means diffusion of well-being when incomes increase but concentration when incomes decrease. Thus, this idea existed much before Dalton mentioned it. One could easily see that this objection incorporates the idea of diminishing marginal utility. The true province of the axiom then is the unit of measurement.

2) **Axiom of Population Size Independence**

The level of inequality remains unaffected if a proportionate number of persons is added to each income level.

This suggests that the magnitude of inequality in the distribution of the cake should depend on the relative number of receivers with different levels of income. If we merge two economies of identical distributions of the same size *N*, then, in the consequent economy of size *2N*, there shall be the same proportion of the merged population for any given income. Such replications will leave the inequality level unchanged. The axiom is also known as the Principle of Population Replication.

This exactly corresponds to Dalton's principle of proportionate additions of persons. Since the Lorenz curve remains unchanged so long as proportions of people in each class remains the same, the measures based on the Lorenz curve would satisfy this axiom.

Let us have however, a counter-intuitive example. Let us a have two-person world in which one person is having no income and the other is having all. Let us replicate the economy. Now there is a four-person world in which two are sharing destitution with zero income but the other two are sharing positive income equally. Earlier there was no equality; now each 50 percent of population is sharing the income equally. So, some scholars do not accept it.

3) **Axiom of Equal Income Addition**

If the distribution 2 x_i^2,i=1,2,…,N is obtained by addition of equal amount d (say, through pension) to each element of distribution 1 x_i^1, i= 1,2,...N, i.e.,

$$x_i^2 = x_i^1 + d$$

then inequality level of distribution 2 should be lower than that of distribution 1 i.e.,

$$I(x_1^2, x_2^2, ..., x_N^2) < I(x_1^1, x_2^1, ... x_N^1)$$

naturally, subtractions (say, taxation) of equal amount from each income would reverse the inequality sign. It can be noted that in the former situation, the shares of the poorer persons increase and in the latter, they decrease. This axiom exactly corresponds to Dalton's principle of equal additions to incomes.

Now, we propose to discuss two very important axioms relating to transfer of an income from a person to another when other things remain the same. The former may be called Pigou-Dalton condition and the latter, Sen condition.

4) **First Axiom of Income Transfer (Pigou-Dalton Condition)**

If an equalizing transfer from a richer person to a poorer person takes place, then the level of inequality is strictly diminished, provided that the equalizing transfer amount is not more than the difference between two incomes involved. Any number of such transfers taking place between any two consecutive income units will not cause any change in the ranking of income units and therefore, such a process of transfers may be called the rank-preserving equalization.

This axiom requires an inequality measure to be sensitive to transfers at all levels of income and, thus, at least a function of all incomes.

This axiom corresponds to Dalton's principle of income transfer. Dalton (1920) argued that an inequality measure must have this minimal property. Since in this context Pigou's contribution (1912) is found significant, Sen (1973) designated this axiom as Pigou-Dalton condition. Following him, a number of contributors in the field have given it the name of 'P-D condition'.

Most of the indices, barring relative range and relative mean deviation, pass this test. This axiom is also known as weak transfer axiom because it suggests the direction but not the magnitude of change in the level of inequality.

5) **Second Axiom of Income Transfer (Sen Condition)**

If we consider two transfers, one at a time, at different points of scale, then the transfer at lower end of scale should have greater impact than its counterpart at higher end of the scale. According to Sen, (1973), the impact on the index should be greater if the transfer takes place from a person with an income level of, say Rs.1000 to someone with Rs.900 than a similar transfer from a man with Rs.1000100 to someone with Rs.1000000.

We may see many measures do not satisfy any of the two conditions of transfer and some satisfy only the first one. Those that satisfy the second transfer axiom automatically satisfy the first transfer axiom.

6) **Axiom of Symmetry**

If distribution $(x_1^p, x_2^p, ..., x_N^p)$ were a permutation of distribution $(x_1, x_2, ..., x_N)$, then the inequality level of both the distributions would be the same.

This implies that if two persons interchange their income positions, inequality measure does not change. Thus the axiom ensures impartiality between individuals for non-income characteristics. The evaluator does not distinguish between Amar, Akabar and Anthony; nor between Shiela and Peter; or between Mr. Pygmy and Ms. Dwarfy. Further, it means that the inequality depends only on the frequency distribution of incomes.

7) **Axiom of Interval**

The inequality measure should lie in the closed interval of (0,1).

The measure is supposed to assume the value of zero when all incomes are equal, which means when all persons have equal income and the value of unity when only one individual gets all the income (and other have zero incomes, not negative incomes).

Most people tend to agree with the axiom. A few, notably Theil (1967) and Cowell (1995), disagree. They hold that the situation of one person grabbing all the income in a society of 2 persons cannot be described by the same level of inequality as that of one person doing so in a society of 2 crore persons. It would not be easy to assert that in the case of 2-person society the level of inequality is unity when one person has all the income and the other has none. In one case, 50 percent population is having positive income, in the other only 0.00000005 percent. Some people therefore, qualify the axiom by saying that when one person gets all the income the measure approaches unity in the limit as the number increases.

When a measure has a finite maximum, it is easy to transform such an index into the one, which has maximum value 1. Most measures, though not all, have zero as their minimum value. But question that Cowell (1995) raises is that there are many ways in which the measure could be transformed so that it lies in the zero-to-one range but each transformation has different cardinal properties.

8) **Axiom of Decomposability**

Suppose population can be sub-divided into several groups and an over-all index of inequality was a function of group-wise indexes and if the population mean can be expressed as weighted average of group means, the population index of inequality can be regarded as decomposable. The groups might be defined as comprising of people in different occupations, residents of different areas, with different religious or educational backgrounds etc.

However, we find a lot of overlapping in these groups. This leads sum of the weights to differ from unity.

Not all indices are found to be decomposable. Gini coefficient, a very popular measure is decomposable only if the constituent groups are non-overlapping. Cowell (1995) has conducted a beautiful experiment. First, he computes four inequality measures for two distributions of same size and same mean-each divided into two groups of equal size in a manner that there is no overlapping:

Population A: (60,70,80), (30,30,130)

Population B: (60,60,90), (10,60,120)

Now, it is found that the group means and population means in two distributions are the same and group inequalities in B are higher than their counterparts in A. But when we compute overall inequalities, one of the measures suggests that the magnitude of inequality in B is lower than that in A. And the measure used is Gini, which is very popular among economists. As he says, 'strange but true'. If the component inequality magnitudes are higher and the weights are the same, how could overall measure be lower? It is therefore, impossible to express overall inequality (change) as some consistent function of inequality change in the consisted groups.

These are all intuitively appealing axioms. There does remain scope for formulating other axioms. In the literature on poverty measurement one finds a plethora of axioms developed by a number of contributors working in that areas. But we shall be content with these only.