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Institutions, Madhyamaka, and Universal Model Theory

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Abstract

The theory of "institutions" [40] is a categorical universal model theory which formalises the intuitive notion of logical system, including syntax, semantics, and the satisfaction between them. It provides the most complete form of abstract model theory, free of commitement to any particular logic, the only one including signature morphisms, model reducts, and even mappings (morphisms) between logics as primary concepts. This essay discusses the relationship between institution theory, considered together with its thinking and methodologies, and Madhyamaka philosophical school within Mahayana Buddhism. We also discuss applications of this school of thought to model theory in the form of the so-called 'institution-independent model theory'.

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1 Introduction

The concept of institution arose within computing science (algebraic specification) in response to the population explosion among logics in use there, with the ambition of doing as much as possible at a level of abstraction independent of commitment to any particular logic [40, 72, 31]. Besides its extensive use in specification theory (it has become the most fundamental mathematical structure in algebraic specification theory), there have been several substantial developments towards an "institution-independent" (abstract) model theory, [77, 78, 23, 25, 24, 45, 44, 68, 19, 67]. A monograph [20] dedicated to this topic is under preparation and [26] is a relatively recent survey. Recently institutions have also been extended towards proof theory [65, 27] in the spirit of categorical logic [50].

Institution-independent model theory is emerging as an important form of universal model theory part of the grand project of universal logic promoted by Béziau and others [6]. Like universal logic in general, universal model theory in particular is not seeking for one single model theory in which all other model theories can be expressed. This would be an unrealistic approach based upon an essentialist view on logic and model theory. The universal model theory ideal rather means a mathematical framework for developing model theoretic concepts and results in a non-essentialist and groundless manner, and which may be reflected at the level of actual logics in the form of concrete model theories. Here by 'non-essentialist' we mean the absence of a lasting, individual essence of any-thing in general, and of logics and logical theories in particular, and by 'groundless' we mean a true absence of commitement to actual logical systems.

In this paper we discuss the philosophical aspects of the institutional approach to universal model theory, which are rooted within the doctrine of Shunyata as presented by the Madhyamaka school of thinking within Mahayana Buddhism. We also show how various aspects of the Madhyamaka thinking are reflected at the level of the institution theoretic approach to model theory. The practical benefits of applying a non-essentialist approach to logic and model theory are hard to argue against especially in the context of the recent developments in informatics and computing science asking for a relativistic approach to logic, opposed to the rather Platonic approach promoted by most of the current trends in mathematical logic and model theory. Moreover, institution-independent model theory illuminates various model theoretic concepts and phenomena by liberating them of the unnecessary ornament of concrete structures. Often this has led to a better understanding and important new results even in well studied classical areas [25, 68, 45].

There have been other relationships between Madhyamaka and contemporary 'western' sciences and philosophical trends. As shown in [81], Madhyamaka thinking is coherent to important ideas from neuroscience, quantuum physics, astrophysics (for the physical sciences see also [85]). There have also been a series of comparative studies [10, 54, 53, 55] between Madhyamaka and the postmodern philosophy of Jacques Derrida. In the case of the institution theory, there seems to be a more direct relationship to Madhyamaka given by the fact that a number of prominent authors in institution theory have had systematic access to the Madhyamaka philosophy.

The structure of the paper is as follows.

1. We recall the concept of institution.
2. We give a brief introduction to the Madhyamaka system.
3. We discuss some reflections of the Madhyamaka thinking to institution theory.
4. We give a brief overview of the current status of institution-independent model theory including some issues in model theory whose understanding have been illuminated by the institution-independent approach.

2 Institutions

Institutions have been defined by Goguen and Burstall in [13], the seminal paper [40] being printed after a delay of many years. (As I have learnt from one of the authors, this delay had been caused mainly by the reluctance of the editors of the respective journal to let this extremely important paper being printed in spite of its final acceptance!) The concept of institution relies heavily on concepts from category theory [52]. For the definition below we assume the reader is familiar with basic category theory concepts and notations.

Definition 2.1 (Institutions) *An* institution $\mathcal{I} = (\mathbb{S}ig^{\mathcal{I}}, \mathsf{Sen}^{\mathcal{I}}, \mathsf{Mod}^{\mathcal{I}}, \models^{\mathcal{I}})$ *consists of*

1. *a category* $\mathbb{S}ig^{\mathcal{I}}$*, whose objects are called* signatures*,*
2. *a functor* $\mathsf{Sen}^{\mathcal{I}} : \mathbb{S}ig^{\mathcal{I}} \to \mathbb{S}et$*, giving for each signature a set whose elements are called* sentences *over that signature,*
3. *a functor* $\mathsf{Mod}^{\mathcal{I}} : (\mathbb{S}ig^{\mathcal{I}})^{\mathrm{op}} \to \mathbb{CAT}$ *giving for each signature* Σ *a category whose objects are called* Σ-models*, and whose arrows are called* Σ-*(*model*)* morphisms[1] *and*
4. *for each* $\Sigma \in |\mathbb{S}ig^{\mathcal{I}}|$*, a relation* $\models^{\mathcal{I}}_{\Sigma} \subseteq |\mathsf{Mod}^{\mathcal{I}}(\Sigma)| \times \mathsf{Sen}^{\mathcal{I}}(\Sigma)$ *between* Σ*-models and* Σ*-sentences, called* Σ-satisfaction*,*

such that for each morphism $\varphi\colon \Sigma \to \Sigma'$ *in* $\mathbb{S}ig^{\mathcal{I}}$*, the* satisfaction condition

$$M' \models^{\mathcal{I}}_{\Sigma'} \mathsf{Sen}^{\mathcal{I}}(\varphi)(\rho) \quad \textit{iff} \quad \mathsf{Mod}^{\mathcal{I}}(\varphi)(M') \models^{\mathcal{I}}_{\Sigma} \rho$$

holds for each $M' \in |\mathsf{Mod}^{\mathcal{I}}(\Sigma')|$ *and* $\rho \in \mathsf{Sen}^{\mathcal{I}}(\Sigma)$*.*

The following quote (from a referee report on a submission of a paper in institution-independent model theory to a 'top' logic journal) gives a very informal but accurate description of the concept of institution.

> ...the institutional framework the authors adopt is very weakly informative (it basically reduces logic to a simple satisfaction relation between the abstract set of sentences and the abstract set of models, with no strongly characterizing property).

From an essentialist viewpoint these attributes of the institutional framework appear as rather strong negative points. (In fact this perception of institution theory in general was the main cause for rejecting that submission, only to have the very same paper re-submitted to a 'topmost' logic journal and accepted there rather easily!). It is also interesting to note that an essentialist mind perceives

[1] $\mathbb{CAT}$ is the category of all categories; strictly speaking, it is only a quasi-category living in a higher set-theoretic universe. See [52] for a discussion of foundations.

groundless, which in this case means generality and mathematical abstraction, as a form of reduction. This is similar to thinking that the concept of group *reduces* various groups from the practice of algebra to an abstract set together with an abstract binary operation satisfying certain concrete axioms.

And in fact this parallel between institution theory and group theory can be drawn further. While the definition of group is also very weakly informative, yet the theory of groups and its applications constitute a very rich mathematical world, without which modern algebra together with all its consequences could not exist. In the same way, the theory of institutions constitutes an ideal mathematical framework for the development of deep model theory (see Sect. 5) and also for much of the formal specification theory with important applications to software engineering.

Coming back to the definition of institutions, the so-called satisfaction condition may be considered as its main axiom. Its meaning is that the (semantic) truth is invariant with respect to change of notation and extension of context. This captures formally a very important insight in logic.

The thesis of institution theory is that each "logic" can be formalized as an institution. A myriad of logical systems from logic and computing science have been captured formally as institutions. Let us review few of the most conventional ones.

Example 2.1 Let **FOL** be the institution of *many sorted first order logic with equality*. Its signatures (S, F, P) consist of a set of sort symbols S, a set F of function symbols, and a set P of relation symbols. Each function or relation symbol comes with a string of argument sorts, called *arity*, and for functions symbols, a result sort. $F_{w\to s}$ denotes the set of function symbols with arity w and sort s, and P_w the set of relation symbols with arity w.

Signature morphisms map the three components in a compatible way. Models M are first order structures interpreting each sort symbol s as a set M_s, each function symbol σ as a function M_σ from the product of the interpretations of the argument sorts to the interpretation of the result sort, and each relation symbol π as a subset M_π of the product of the interpretations of the argument sorts. Sentences are the usual first order sentences built from equational and relational atoms by iterative application of Boolean connectives and quantifiers. Sentence translations rename the sorts, function, and relation symbols. For each signature morphism φ, the reduct $M'\restriction_\varphi$ of a model M' is defined by $(M'\restriction_\varphi)_x = M'_{\varphi(x)}$ for each x sort, function, or relation symbol from the domain signature of φ. The satisfaction of sentences by models is the usual Tarskian satisfaction defined inductively on the structure of the sentences.

Example 2.2 The institution **PL** of *(classical) propositional logic* can be defined as the 'sub-institution' of **FOL** obtained by restricting the signatures to those with empty set of sort symbols. This means that **PL** signatures consist only of sets (of zero arity relation symbols), therefore $\mathbb{S}ig^{\mathbf{PL}}$ is just $\mathbb{S}et$, the category of sets, for each set P the set of P-sentences consists of the Boolean expressions formed with variables from P, and the model functor is the contravariant power set functor $\mathcal{P}\colon \mathbb{S}et \to \mathbb{CAT}^{\mathrm{op}}$ (the category of P-models is the partial order $(\mathcal{P}(P), \subseteq)$ regarded as category). Note that a P-model $M \subseteq P$ satisfies $\pi \in P$ when $\pi \in M$.

Example 2.3 The institution **EQL** of *equational logic*, which is the institution underlying general algebra, can also be regarded as a sub-institution of **FOL** which restricts the signatures only to the *algebraic signature*, which are just **FOL** signatures without relation symbols, and the sentences to the universally quantified equations.

Example 2.4 The institution **MPL** of *modal propositional logic* extends propositional logic **PL** with modal connectives for the sentences and considers Kripke structures as models. The satisfaction relation is defined using possible-world semantics in the usual way. The institution **IPL** of *intuitionistic propositional logic* differs from **PL** in having Kripke structures as models, and possible-world satisfaction. These may be extended to institutions for modal or intuitionistic first order logic.

A very brief list of other examples of institutions from computing science include institutions for rewriting [58], higher-order [8], polymorphic [73], various modal logics such as temporal [36], process [36], behavioural [7], coalgebra [18], object-oriented [41], quantuum [14] and multi-algebraic (non-determinism) [51] logics.

But why 'institution' as name for a mathematical concept which captures the informal notion of logical system? This name appeared as a half joke reaction of Goguen and Burstall to the dogmatic, almost fundamentalist, thinking which dominated formal specification theory and practice at the time. Many people used to define their own specification logic which they soon believed to be the best, and that a lot if not everything can be solved in that logic. They used to write scientific papers about it or using it, to build research groups, to build supporting tools, to promote it by organizing dedicated conferences or by defending its superiority in general conferences. All these looked very much like social institutions built around logical systems.

A typical way to promote a newly defined logical system used to consist of showing how as many as possible logics can be encoded into that logic. Often such encodings are rather shallow, involving only the proof theoretic side of logics and discarding the model theoretic one. A special case is some type theoretic trend in logic which even considers semantics unnecessary; this constitutes an extreme viewpoint which may lead to gross failures such as inconsistency. All these essentialist tendencies are still very much present in the logic and computing science academic communities. For example there is still a general strong inclination among adepts of classical logic to attempt to understand any logical system as a fragment of (eventually some extension of) first order logic. A case is that of modal logic, which many logicians prefer to study at the propositional level rather than at the significantly more expressive first order level. This particular tendency might be motivated by the fact that propositional modal logic can be regarded as a fragment of (classical) first order logic.

3 A brief introduction to Madhyamaka

Madhyamaka is the gate to understanding the non-essentialist philosophical aspect of institution theory and of its approach to universal model theory. This section is organised as follows:

1. We give a brief presentation of Buddhist thinking and investigation methodologies in relationship with contemporary 'western' science.
2. We present the place of the Madhyamaka system within the Buddhist thinking, and
3. We present the main philosophical aspects of Madhyamaka.

3.1 Buddhism and contemporary science

Buddhism is a 2500 years old non-theistic spiritual path which shares several important aspects with most of the religious traditions but also with the contemporary scientific traditions. It shares with the most major religions universal ethical values and their importance for achieving happiness. The ethical component is very regretably largely absent in the contemporary scientific approach, with potential dramatic consequences for our life. Buddhism differs from most religions by being non-theistic and by strongly emphasizing non-dogmatic thinking and analysis in the search for a correct understanding of reality. These differences might be exactly what in principle Buddhism shares with modern science. ('In principle' here means that for various reasons parts of modern science are becoming increasingly dogmatic and some of them are even based on strong metaphysical beliefs; for instance some parts of biology are obvious examples of this.) Therefore validating the truth of a claim appears as a point of convergence between Buddhism and science as explained clearly by the following paragraph from [81]

> Although Buddhism has come to evolve as a religion with a characteristic body of scriptures and rituals, strictly speaking, in Buddhism scriptural authority cannot outweigh an understanding based on reason and experience. In fact, the Buddha himself, in a famous statement, undermines the scriptural authority of his own words when he exhorts his followers not to accept the validity of his teachings simply on the basis of reverence to him. Just a seasoned goldsmith would test the purity of his gold through a meticulous process of examination, the Buddha advises that people should test the truth of what

> he has said through reasoned examination and personal experiment. Therefore, when it comes to validating the truth of a claim, Buddhism accords greatest authority to experience, with reason second and scripture last.

This corresponds exactly to the usual practice of the correct approaches to science [69]. For example physics constitutes a very transparent example for this order of priorities. Although scriptural authority may appear to play no role in science, actually it does through the body of scientific literature. Unless contradicted by reasoning or experience, a scientist usually accepts claims from the literature.

These aspects of the Buddhist approach to understanding reality and its focus on mind made many people consider Buddhism as the 'science of the mind'. The correct understanding of reality plays a central role in Buddhist thinking which considers ignorance (both in gross or subtle forms) as the fundamental cause for our unsatisfactory existence. Here 'ignorance' means exactly an incorrect understanding of physical and mind realities, and of the relationship between them. A correct understanding of the human existential condition gives a logical basis to ethics [79], thus one can say that Buddhism has a 'scientific' approach to ethics based upon logical and experiential analysis.

Since from the Buddhist viewpoint sound reasoning plays a crucial role in correct understanding of reality, logic as a topic of study plays an important role in the Buddhist tradition. This goes back to the Indian tradition of monastic universities and has been continued by the Tibetan tradition which made logic an important part of the monastic curriculum of higher philosophical studies. Buddhist scholars either from the Indian or from the Tibetan traditions have developed over the centuries a sophisticated system of logic. A rather impressive survey of Buddhist logic is [76].

Differences between Buddhism and science consist in the facts that Buddhism contains a larger spectrum of investigation methodologies (such as contemplative techniques) and in that the scope of the Buddhist investigation is significantly wider, especially in the mind realm. The absence of the focus of contemporary sciences on the mind realm and on the existential condition of human beings might provide an explanation why ethics does not occupy a central position in science.

Although ancient, Buddhism has maintained unbroken lineages of transmisions. By being independent of the cultural context, it adapts easily across various cultures. One of the consequences of the arrival of Buddhism to the western culture (arrival linked to the Chinese invasion and occupation of Tibet in the fifties) is a substantial dialogue between Buddhist thinking and the contemporary sciences. An expression of the success of this dialogue is the Mind and Life Institute (see `www.mindandlife.org`) started by The Fourteenth Dalai Lama together with a group of prominent scientists from various fields ranging from physics to biology and neurosciences. This institute has already organized a series of interesting workshops and has undertaken a series of systematic experiments especially in the areas directly related to the mind realm, such as in the field of neuroscience. The survey of the relationship between Buddhist thinking and science given in [81] owes to the activity of the Mind and Life Institute.

The connection between the Buddhist thinking and the non-essentialist approach to logic and computer science promoted by institution theory is even more direct since there is a list of prominent authors in institution theory (and applications) which have been life long students of Tibetan Buddhism. This list includes both the authors of the concept of institution, Joseph Goguen and Rod Burstall (see [12] for details). In our essay we will not elaborate on this historical dimension of the relationship between Madhyamaka and institution theory.

3.2 The place of Madhyamaka within Buddhist thinking

Madhyamaka represents the highest from the four main philosophical schools of ancient India explaining and interpreting the doctrine of *Shunyata*, usually translated as *emptiness* or *vacuity* or *selflessness* [of phenomena]. Shunyata is one of the most fundamental teachings of Buddhism and perhaps represents the most characteristic teaching of Mahayana Buddhism (i.e. the 'Greater Vehicle' in Buddhism). It is also one of the most important topics of the large body of the Perfection of Wisdom literature, with the *Heart Sutra* as its most famous text. A definitive presentation of the latter can be found in [80].

The presentation of Shunyata becomes progressively subtler with the arising of each of the above mentioned philosophical schools. These schools, in the order of their arising, are the *Vaibashika* School, the *Sautrantika* School, the *Chittamatra* (Mind-only) School, and the *Madhyamaka* (Middle Way) School. Although Madhyamaka is considered by all Tibetan Buddhist traditions to represent the highest philosophical presentation on Shunyata, the Tibetan monastic curriculum contains detailed studies of the tenets of all four schools. As explained in [80], this helps avoiding being stuck in what is only a partial understanding of Shunyata, and gives a greater appreciation for the profundity of the most subtle standpoint.

The Madhyamaka philosophy was founded by the great Indian teacher Arya Nagarjuna around the second century C.E., his most famous writing being *Fundamentals of the Middle Way* (an English translation being [66]). Among his Indian successors writing commentaries on his works, Buddhapalita, Bhavaviveka, and Chandrakirti were particularly influential. Some differences of interpretation arose between Buddhapalita and Chandrakirti on the one hand, and Bhavaviveka on the other hand. These had lead to two principal strains of Madhyamaka philosophy, called *Prasangika* and *Svatantrika*, respectively. The differences between these two strains are reflected also at the level of their corresponding reasoning methodologies. While Prasangika methodology (used by Nagarjuna himself and later by Buddhapalita and Chandrakirti) is based upon a consequentialist (*reductio ad absurdum*) form of reasoning, the followers of Svatantrika (such as Bhavaviveka) reason from the basis of established syllogisms. For a deeper presentation of the Madhyamaka and its two strains, Prasangika and Svatantrika, we recommend [80].

3.3 The Madhyamaka philosophical view

So, what is the explanation of Shunyata given by Madhyamaka? Since a full answer to this question is not possible here, we will try to give only a brief explanation based on [80].

Shunyata is not a view nor a method. It is rather a correct onto/logical understanding of the nature of things and phenomena. This understanding applies both to the physical ('external') or the mind ('internal') realm. According to shunyata all phenomema are devoid, or 'empty', of any intrisic mode of existence. In other words they are empty of any lasting, individual essence (sanskrit: *svabhava*).

Shunyata has been often mistakenly understood as a nihilistic view, that nothing exists. Firstly, shunyata is not a view or a theory about reality. On the contrary, shunyata is the extinction of all views as illustrated by the following paragraphs from [66].

> So, because all entities are empty,
> Which views of permanence, etc., would occur,
> And to whom, when, why, and about what
> Would they occur at all?

Then misunderstanding Shunyata as nihilist comes from the confusion between non-existence of things and the non-essentialist mode of existence of things. In fact the significance of the name 'Madhyamaka', meaning Middle Way, is exactly that it avoids both the extremes of the nihilism and of the essentialism. Therefore phenomena, things and events, do exist but not in the essentialist way we usually tend to perceive them.

Before examining more closely the mode of existence advocated by Madhyamaka, we mention that from the four above mentioned philosophical schools, only the last two, Chittamatra and Madhyamaka accept the emptiness of phenomena, the other two accepting only the emptiness of the person (meaning our strong sense of self). The Chittamatra school discriminates between the 'internal' (i.e. mind) and the 'external' phenomenal world and completely acknowledges the emptiness of phenomena in the external world but not of the emptiness of the internal world. Understanding the emptiness of *all* phenomena by not discriminating between an internal and an external world is unique to Madhyamaka.

So, in what way do phenomena exist according to Madhyamaka? In his seminal writing *Fundamentals of the Middle Way*, Nagarjuna explains the existential status of phenomena by *dependent origination* (sanskrit: *pratiya-samutpada*). Another English word for this is interdependence: no-thing exists apart of its relationship with other 'things'. While for the other Mahayana philosophical schools this means dependence upon causes and conditions according to the *law of cause* *and effect*, for the Madhyamaka Prasangika school this primarily means dependence upon the conceptual designation of a subject.

An important aspect of Madhyamaka is that emptiness applies to all levels of existence of a phenomenon, from the gross to the subtler ones, including the causes and the conditions of the phenomenon. While it is easier to understand the lack of intrisic existence at gross levels, it is less easy as we go to the subtler levels of existence. *Fundamentals of the Middle Way* consists mainly of strings of logical arguments showing the inconsistency of the belief in any trace of intrisic existence of things and events. Nagarjuna applies this to motion, cause and effect, becoming and destruction, compounded phenomena, and even some of the basic tenets of Buddhism itself, including Shunyata itself. He thus undermines all views by showing them to be, by their very nature, untenable. Holding any view, including the 'view of Shunyata' is a grave error of understanding as written by Nagarjuna [66]:

> The Victorious ones [Buddhas] have said
> That emptiness is the relinquishing of all views
> For whomever emptiness is a view,
> That one will accomplish nothing.

Non-essentialist philosophical approaches to are not unique to Buddhism. While the so-called 'postmodern' contemporary philosophical trend started by Jacques Derrida seem to re-discover Shunyata under the name of *Différance*, several authors, while analyzing the similarities between Madhyamaka and postmodernism, argue about the superiority of Madhyamaka [54] or even question the originality of postmodern philosophy [10].

4 Emptiness in logic and model theory

In this section we discuss several aspects of the institutional approach which may be considered as a reflection of the Madhyamaka thinking. These are the following:

1. The absence of commitement of institution theory to any particular logical system.
2. The top down development methodology promoted by institution theory.
3. The intensive use of category theory.
4. The significance of mappings between institutions.

4.1 No logical view

Institution theory is the only approach to logic which is completely free of commitement to any actual logical system, and even to any type of logical systems.

This groundless aspect of the institution theoretic approach can be considered as a reflection of the Madhyamaka relinquishing of all views. In the particular case of the realm of logic, this just means the absence of any 'logical view'. In institution theory all entities of a logic, signatures, models, and sentences, are fully abstract, and the satisfaction relation between the models and the sentences is axiomatized rather than being defined. It is interesting that this absence of a logical view results in a conceptual framework with a strong flavour of voidness, which has been the main reason for frequent complaints from mainstream logicians.

The institution theoretic study of logic emerged in the context of an unprecedented high proliferation of increasingly unconventional logical systems given by computing science and from the understanding of the absence of an inherent essence of particular logics. But what do we mean by this? This is just the mistaken view, shared by most logicians, of logical systems as a kind of Platonic systems rather than as conceptual fabrications arising in dependence of several patterns of thinking cultivated through common mathematical practice, education, etc. At this point of discussion it is important to remember that the institutional view does not negate the existence of particular logical systems or their adequacy for certain applications but the Platonic way to regard their existence.

An example of how logical views are determined by practice and education is the the situation of many sortedness in classical logic. Classical logic arose mainly from the foundations of mathematics and remains linked to the conventional mathematical practice, hence it is almost always considered in its single sorted version. The strong belief in the single sorted aspect of first order logic has led to the mistaken view that many sorted logics are 'inessential variations' of the single sorted ones [60].[2] By contrast, in computing science, due to the high importance of typing, first order logic is mostly considered in its many sorted version. And this is often how computing science students learn logic.

Another example is given by logic with partial functions, i.e. the logic of the so-called 'partial algebras' [11]. Although partial functions arise everywhere in mathematics, the conventional developments of logic largely ignored this phenomenon and modelled it rather indirectly by the use of relations. That partial functions can be a good substitute for relations, in fact a mathematically more refined option, has been often and succesfully argued by the researchers of partial algebra. However partial functions remain a marginal concept in mathematical logic which due to cultural and social determinations and conceptual inertia still prefers to work with total entities even in situations when this appears to be an awkward option. This is not the case though in computing science which recognized the importance of partial functions and which is also less committed to the traditional thinking patterns of mainstream mathematics.

The essentialist Platonic view on logic has also lead to several misuses of classical logic. For example classical logic has been pushed rather hard in com-

[2]That this view is completely wrong is shown by the case of interpolation (see [45] for details).

puting science in spite of its rather poor computational power and in spite of being rather inappropriate as a formalism for certain computing paradigms.

There have been attempts other than institution theory to liberate logic and model theory from particular logical views. However all of them fell short of this goal. First there was the partial recognition of the fact that traditional first order logic was not adequate for certain things. This has to extension of classical logic to new logical systems such as higher order logic [17, 47] or modal logics [49]. Another important step was constituted by attempts to study logic at an abstract level. One of such development in the area of model theory was the so-called 'abstract model theory' of Barwise [5]. But this still keeps a strong commitment to first order classical logic by explicitly extending it and retaining many of its features, such as its models, but also some syntactic features too. In this context even the remarkable Lindström characterization of first order logic by some of its properties should be rather considered as a first order logic result rather than as a true abstract model theoretic one. Another abstract model theoretic framework is given by the so-called 'categorical model theory' best represented by the works on sketches [34, 46, 84] or on satisfaction as cone injectivity [1, 2, 3, 57, 59, 56]. The former just develops another language for expressing (possibly infinitary) first order classical realities and in the latter the sentences and their satisfaction are just categorical reflections of classical concepts.

4.2 Top down development

Perhaps the most important contribution of institution theory lies not in the theory itself but in the proposed thinking and development methodology. This is new to logic and model theory and one can say it goes opposite to the established ways of doing things in these areas. It can be seen as a practical consequence of the absence of particular logical views.

Before discussing more deeply the institutional development methodology let us look briefly into the familiar methodology based upon the essentialist view of logic. This can be considered as a 'bottom up' methodology since it is based upon a fixed solid concrete framework, in fact an actual logical system, and it uses the conceptual infrastructure available there for developing concepts and results. There are several problems related to the bottom up approach. One of the most grave is that having a solid base hinders a clear understanding of the concepts and of the causes of the results. This happens because such understanding is suffocated by the often irrelevant details of the concrete framework. In other words, each concept or result seem to 'live' best at a particular level of abstraction and in the bottom up approach this is frozen at the lowest level given by the concrete framework. In this way not only that the scope of the results is not clear, but even important concepts are mistakenly formulated (some examples from classical logic will be discussed in the next section). Consequently, even important results in classical intensly worked areas of logic have been missed.

By contrast to the bottom up approach, institution theory proposes a 'top down' methodology which starts from an 'empty' framework and introduces the concepts as presumptive features that "a logic" might exhibit or not, by defining them at the *most appropriate* level of abstraction. Hypotheses are kept as general as possible and introduced on a by-need basis, and thus results and proofs are modular and easy to track down regardless of their depth. This top down development methodology is thus guided by structurally clean causality, which means a clear and deep understanding of the complex network of conceptual interdependencies in logic and model theory. This is very related to the Madhyamaka explanation since Shunyata can also be understood in terms of a complete understanding at the most subtle levels of the causality of phenomena.

The top down institution theoretic methodology has important pragmatic consequences. For example access to highly non-trivial results is also considerably facilitated. It also brings a large array of new results for particular non-classical logics, unifies several known results, produces new results in well studied classical areas, reveals previously unknown causality relations, and demounts some which are usually assumed as natural. The discussion in the next section gives several examples supporting these claims.

4.3 Category theory

Institution theory makes intensive use of category theory, in fact institutions can be regarded as a happy marriage between category theory and model theory. This role played by category theory can be seen at a first glance by inspecting the definition of the concept of institution: three out of the four entities that form the concept of institution are abstract categorical entities. However this is only an expression of a deeper relationship between the thinking promoted by these two areas. It is not wrong to think of institutions as a reflection of categorical thinking in model theory.

Category theory started in mainstream mathematics (algebraic topology) [35], raised quickly to the status of a big promise only to become a rather controversial branch of modern mathematics. It regained however a prominent position in the mathematical sciences with the sharp rise of computing sciences. The elegant essay [39] surveys some of the role played by category theory in computing science. Although category theory has been promoted by well respected names in mathematics, such as Mac Lane, Eilenberg, or Grothendieck, it also has many foes who accuse it of being 'abstract nonsense', a mere form without (conceptual) substance. But from a non-essentialist perspective this can be considered as a rather positive aspect. Since anyway, according to Madhaymaka thinking, form is the existential status of all things of the conventional reality, including (especially!) mathematical theories, category theory just keeps honest to this truth, an aspect which may be regarded as a sign of theoretical health. It is also ironic to see the success of the 'abstract nonsense' in applied areas such as software engineering. For example the design of a number of prominent formal specification languages, such as CASL [4], Specware [75, 48], or CafeOBJ [29, 30] are based on institution theory and/or category theory.

The category theoretic approach to mathematical structures seems to reflect Shunyata, understood as dependent origination (pratitya-samutpada). Category based mathematical theories (such as institution theory) do not give any importance to the internal structure of particular objects, considering them as a kind of 'black boxes', instead they think of the objects in terms of their relationships to all other objects given by the homomorphisms. This reminds a lot of the Buddhist illustration of Shunyata/pratitya-samutpada by the so-called 'Indra net' of interdependency [71].

This precedence of homomorphisms over the objects means that in institution theory the signature morphisms, and consequently the model reducts corresponding to signature morphisms, play a prime role. This multi-signature aspect is actually a distinctive feature of institution-independent model theory with respect to other categorical model theories (such as those mentioned at the end of Section 4.1) which do not seem to use category theory in its full spirit. Although classical model theory do use model reducts to some extent it does this only for signature extensions, thus not considering a proper concept of signature morphism. The wider perspective on signature morphisms promoted by institution theory has also a practical significance related to the use of formal specifications. The use of model homomorphisms is another aspect which distinguishes the institution-independent from the traditional model theory. While in the former these are used rather intensly, the latter makes rather limited use of proper model homomorphisms.

The conceptual circularity in the arising of categorical concepts also reminds strongly of Shunyata/pratitya-samutpada. For example the important concepts of initiality, colimits, and adjunctions are inter-definable concepts. Initial objects are a particular case of colimit, colimits are universal arrows, and universal arrows are initial objects (in comma categories).

4.4 Mappings between institutions

In institution theory concepts of mappings between institutions play a prime role. This is very much in the spirit of category theory since institutions as a mathematical structure can be understood properly only through concepts of institution mappings. We use the plural 'concepts of institution mappings' here because there are two main concepts of structure preserving mappings between institutions, the so-called institution 'morphisms' and 'comorphisms', respectively. A survey on the relevance of morphisms and comorphisms in institution theory can be found in [42]. Both morphisms and comorphisms provide exactly the same level of preservation of mathematical structure, thus from this point of view one cannot say that one is more adequate than the other one. In fact this case is a clear example which illustrates the idea that a category is defined by its arrows rather than by its objects. This means that one should speak of categories of institution morphisms and of institution comorphisms rather than of the 'category of institutions' which is unprecise terminology.

The structural symmmetry between morphisms and comorphisms however breaks down at the level of methodologies, these two concepts of institution mappings having rather different meanings and uses in actual situations. Institution morphisms and comorphisms provide the means for relating between logical systems, supporting thus the non-essentialist view that looks at a particular logical system through its relations with other logical systems. Moreover this has lead to a novel rather efficient method of systematic development of results and investigation of logical properties by translation between logical systems [16, 62, 28].

One of the emblematic applications of institution co/morphisms is given by the recent heterogenuous multi-logic specification paradigm in computing [29, 30, 21, 63, 64]. In this paradigm one needs to work with a system of institutions, related by institution co/morphisms, rather than with a single institution. There is an obvious non-essentialist aspect to this paradigm, which comes from practice, and which acknowledges openly the impossibility to find a logic which may serve all necessary specification purposes. The main technical problem raised when working with a system of institutions was the lack of theoretical uniformity, hence there was the need for the construction of a single institution representing the whole given system of institutions. The essentialist approach to this problem would try to construct a big institution by a logical combination of the individual institutions in the system. However this showed to be unrealistic for at least two reasons. One was the difficulty to define such construction at a general level, and the other was that in the special cases when there was a natural combination of the institutions of the system, this proved to be technically inadequate due to the collapse of important semantic information [30]. The solution to this problem came from a construction having a non-essentialist flavour, the so-called 'Grothendieck institutions' [22, 61]. The basic idea comes from algebraic geometry [43], which in the case of systems of institutions represents a flattening of the system to a single 'Grothendieck institution' still retaining informationally the individuality of the component institutions and of the relationships between them. These Grothendieck institutions do not carry any logical meaning, they are just purely technical constructions representing the whole system of institutions as a single institution.

5 Towards universal model theory

In this section we give a brief overview of the current status of institution-independent model theory. This way of doing model theory, without a logic, a hence without concrete models, may be regarded as an application of the 'no logical view' approach to model theory. Here we can draw an analogy with the practical role played by Madhyamaka in Buddhism. Instead of leading to spiritual paralisy as the mistaken nihilistic reading of Madhyamaka would suggest, the Madhyamaka philosophy constitute the basis for a rich body of practices leading to spiritual growth and development, and ultimately to liberation from suffering (sanskrit: *dukha*). In the same way, but of course in the much more modest context of model theory, the institution-independent approach has led to a rich body of significant model theoretic results. As its name suggests institution-independent model theory is a model theory liberated from any particular logical view, a model theory without (concrete) models, fullfilling the ideal of universal logic. The material of this section is organized as follows:

1. We first present very briefly the list of model theoretic methods developed at the institution-independent level and of the results obtained.
2. We discuss some examples of model theoretic concepts and phenomena which have been illuminated and put into a correct shape by the institution-independent approach.

5.1 Model theoretic methods and results

That model theory is mostly a collection of methods is quite widely recognized (see for example the classic textbook [15]). In model theory it is not uncommon to have certain important results obtained in different ways by different methods. Although the rather rich spectrum of model theoretic methods have been developed within the context of classical model theory, which means mainly the model theory of first order logic, recent work using institutions show that they are in fact independent of any actual context. Institution-independent model theory has already developed general versions of the method of diagrams [24], ultraproducts [23], and saturated models [32]. These have been used for developing general compactness [23], axiomatizability [77, 78, 20], interpolation [25, 45], definability [68], completeness [19, 67] results generating a big array of novel concrete results in actual unconventional, or even classical well studied, logics. Moreover, the institution-independent approach has lead to the redesign of important fundamental logic concepts such as interpolation and definability, to the clarification of some causality relationships between model theoretic phenomena including the demounting of some deep theoretical preconceptions. We will briefly discuss these issues in the section below. The institution-independent approach to model theory makes the access to highly difficult model theoretic results considerably easier, an example being the Keisler-Shelah isomorphism theorem [32].

Although algebraic logic is not a model theoretic approach, the π-institutions of [37], which replace the model theoretic structure of institutions with abstract consequence relations, have been used as the framework for the development of a general institution-independent approach to algebraic logic [83].

5.2 Illuminating model theoretic phenomena

Interpolation is one of the central topics in logic and model theory [15, 38]. It has an important theoretical significance and many applications. In spite of the constant attention given to this topic over many decades, even the concept of interpolation has not been correctly understood and defined. As we will see below this situation can be directly linked to the traditional bottom up development approach in which concrete infrastructures irrelevant to interpolation clouds the correct understanding of the concept.

A first important fault of the traditional definition of interpolation refers to the fact that this is defined in terms of single sentences rather than sets of sentences. Since there are no applications requiring the single sentence formulation, and this also cannot be motivated theoretically, this choice seems arbitrary. Most probably this has been caused by the confusion arising from the particular situation of classical first order logic being compact as having conjunctions. In any such logic the single and the sets of sentences formulations are equivalent. However such misunderstanding about interpolation has led to the mistaken view that institutions such as equational or Horn logics lack the interpolation properties badly needed by their computing applications for instance. It is the merit of [70] to correct this misunderstanding for the case of equational logic, while the paper [31] gives the general institution-independent definition of interpolation with sets of sentences and [25] develops the method of [70] for abstract institutions generating a myriad of new interpolation results in many logics including fragments of classical first order logic. More importantly is that this work has also revealed an unknown causality relationship between the Birkhoff-style axiomatizability properties of the logic and its interpolation properties.

Another misunderstanding about interpolation is that this is usually considered in its Craig rather than Craig-Robinson formulation [74]. For the readers without enough knowledge of Craig-Robinson interpolation we mention here that this stregthen the Craig formulation by adding to the set of the premises a set of 'secondary' premises from the second signature. In classical first order logic these two are not distinguishable since they can be shown equivalent. But this is caused by particularities of first order logic, namely that it has implications and it is compact [20]. Institution-independent studies revealed the fact that the main applications of interpolation requires it in its Craig-Robinson rather than its Craig form, the latter being the form under which it is almost always used traditionally. These applications include definability [68, 20], borrowing techniques [28, 20], modularisation of formal specifications [31, 82, 33], completeness (of proof systems) in structured specifications [9, 20].

A third contribution of institution-independent model theory to the correct understanding of interpolation is the generalisation from intersection-union to pushout squares of signature morphisms. The traditional intersection-union square formulation constitutes an unnecessary limitation due to the fact traditional studies of logic never consider proper signature morphisms, prefering instead to work only with signature extensions. The need to consider a proper concept, potentially non-injective, of signature morphism (i.e. the natural concept of mapping which preserve the mathematical structure of the signatures) comes mainly from computing science, which is a place of many applications of interpolation. Once interpolation is considered in this way, a significant difference between single sorted logics and many sorted logics shows up. While single sorted first order logic has interpolation for all pushout squares of signature morphisms, its many sorted version has it only for those pushouts for which one of the signature morphisms is injective on the sorts. This many sorted interpolation result, which stayed as a conjecture in the algebraic specfication community for several years, has been obtained elegantly as an instance of the general institution-independent interpolation result of [45]. The institution-independent studies of interpolation also showed that the correct way to think of the interpolation properties of a logic is not globally, i.e. that a logic has it or not, but locally, i.e. that certain classes of pushout squares have it.

Leaving now the territory of interpolation, a somehow similar situation can be seen for the (Beth) definability. Again it has been traditionally wrongly thought in a single sentence form, thus blocking definability results for logics without implication, such as equational or Horn logic. This has been corrected by the institution-independent approach to definability of [68] which also revealed an unknown causality relationship between axiomatizability and Beth definability.

The last example of how the non-essentialist approach represented by institution-independent model theory projects a correct light on the model theoretic phenomena that we discuss here (more examples can be found in the forthcoming monograph [20]) is given by the completeness phenomenon. Both Birkhoff [19] and Gödel-Henkin [67] completeness have received generic institution-independent treatments leading to completeness results for a multitude of Horn or first order style logics. Both above mentioned completeness results have been obtained by separating the completeness phenomenon on several layers. The base layer is given by the 'atomic' sub-institution of the given institution, and represents the institution-dependent layer of completeness. This means that one needs to start with a sound and complete proof system for the base institution, however this is usually easy to do since it concerns only the atomic sentences. The upmost layer of completeness, of the full institution, is institution-independent in the sense that the proof rules can be formulated at the level of abstract institutions. In between there can be other layers corresponding to simpler shapes of sentences. In other words, we should think of the proof rules of logical systems, such as equational logic or first order logic, as being of two kinds: 'atomic' and 'general'. Moreover, in this way it is very easy to construct sound and complete proof systems for various styles of logics by taking care only of the base level.

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